A Coded DHA FH OFDMA System with a Noncoherent ML Detector under Multitone Jamming

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Abstract. In what follows an upper bound for the probability of erroneous decoding in a coded DHA FH OFDMA system with a noncoherent ML detector under multitone jamming is introduced.

Keywords: multiple access, coded DHA FH OFDMA, maximum likelihood, upper bound.

1 Introduction

Dynamic Hopset Allocation Frequency Hopping OFDMA (DHA FH OFDMA) has been initially proposed in [1]. Since then a number of modifications has been considered. Noncoherent DHA FH OFDMA with threshold reception seems the most promising one since it is much less vulnerable to multitone jamming than the conventional FH OFDMA. However, it was shown in [2] that in classical DHA FH OFDMA using q-ary FSK modulation the probability of erasure grows drastically as q grows. Thus in a classical DHA FH OFDMA system the value of q is to be relatively small and therefore the data transmission rate in such a system is bound to be relatively low too. In [3] a modification of a classical DHA FH OFDMA model has been proposed: a coded DHA FH OFDMA model. The basic idea underlying the model under consideration is a combination of q-ary FSK modulation and noncoherent reception utilized in a classical DHA FH OFDMA system with a q-ary error correcting code with good relative distance (e.g. Reed-Solomon code) and correspondingly a replacement of symbol-wise decision by a codeword decoding. Due to additional redundancy in the time domain introduced by an error-correcting code the value of q can be much larger in use in a coded DHA FH OFDMA; and even though additional redundancy is introduced it turns out that the overall transmission rate in a coded DHA FH OFDMA is much higher than that ensured by a classical DHA FH OFDMA system (number of active users, signal-to-noise ratio and signal-to-inference ratio being fixed). Therefore the problem of giving a probabilistic description of a coded DHA FH OFDMA (especially under jamming) is of great importance. In what follows an upper bound for the probability of erroneous decoding

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in a coded DHA FH OFDMA system with noncoherent ML reception under multitone jamming will be introduced.

This paper is organized as follows. In section 2 a short description of a coded DHA FH OFDMA with a noncoherent ML detector will be given. In section 3 a probabilistic model of the system in question under multitone jamming is introduced. In section 4 the model under consideration will be used to upper bound the probability of erroneous decoding.

2 A Coded DHA FH OFDMA System with a Noncoherent ML Detector

Let us consider a multiple access system in which m active users transmit information via an AWGN channel split into Q identical nonoverlapping subchannels by means of OFDM. Information that is to be transmitted is encoded into a codeword of a (n,k,d) q-ary code ($q << Q$).Whenever a user is to transmit a q-ary symbol it places 1 in the position of the vector $\overline{a}_g$ corresponding to the symbol in question within the scope of the mapping in use (in what follows it will be assumed that all positions of the vector are enumerated from 1 to Q, moreover without loss of generality we shall assume that the 1st subchannel corresponds to 0, the 2nd subchannel corresponds to 1 and so on). Than a random permutation of the aforesaid vector is performed and the resulting vector $\pi_g\left(\overline{a}_g\right)$ is used to form an OFDM symbol (permutations are selected equiprobably from the set of all possible permutations and the choice is performed whenever a symbol is to be transmitted). Therefore in order to transmit a codeword a user is to transmit n OFDM symbols. A sequence of OFDM symbols, corresponding to a certain codeword that has been sent by a certain user, will be referred to as a frame. Note that frames transmitted by different users need not be block synchronized, i.e. if within the time interval a certain user transmits a frame that corresponds to a codeword, symbols transmitted by another user within the same time period do not necessarily all comprise one codeword. Moreover, it will be assumed that transmissions from different users are uncoordinated, i.e. none of the users has information about the others. In what follows we shall assume that all users transmit information in OFDM frames and the transmission is quasisynchronous. In terms of the model under consideration this assumption means that transmissions from different users are symbol synchronized.

Within the scope of reception of a certain codeword (let us designate it with v_z where z is the number of the codeword) the receiver is to receive n OFDM symbols corresponding to the codeword in question. Note that the receiver is assumed to be synchronized with transmitters of all users. Therefore all the permutations done within the scope of transmission of the codeword in question are known to the user. The receiver applies inverse permutation to each vector b_g corresponding to the respective OFDM symbol thus reconstructing initial order of elements and obtaining vector $\tilde{b}_g = \pi_g^{-1}(b_g)$. Let us designate a matrix that consists of vectors $\bar{a}_g$ corresponding to the codeword v_z with V_z. Furthermore we shall consider a matrix X that consists of vectors $\tilde{b}_g = \pi_g^{-1}(b_g)$ that correspond to the very same codeword v_z. Note that matrix V_z corresponds to the transmitted codeword whereas matrix X corresponds to the received codeword. The detector is to decide on the transmitted codeword matrix. Let us designate each element of matrix X with $X(i,j)$, where i is the column number, whereas j is the row number. Let M be the mapping that associates number t of a certain column of matrix V_z with the number of the nonzero element of the vector in question j_t (i.e. $V_z(t, j_t) = 1$)

$$\begin{aligned} &M(V_z) = [j_1, j_2 \dots j_n] \ \forall t = 1:\tilde{n} \ \exists! j_t : V_z(t, j_t) = 1, \\ &V_z(t, j) = 0 \ \forall j \neq j_t. \end{aligned} \tag{1}$$

To decide on the codeword transmitted by the active user the detector is to compute the value

$$y_z = \sum_{i=1:n} X^2(i, j_z(i)) \ \ \bar{j}_z = M(V_z), \tag{2}$$

where $\bar{j}_z$ - is a vector of numbers of rows, corresponding to the nonzero elements of V_z for each codeword v_z. The value y_z is the sum of powers of the elements corresponding to the codeword v_z. Detection boils down to finding $z^* = \arg\max_z (y_z)$.

Since transmissions from different users are uncoordinated it is possible that at some instant more than one user will use a certain subchannel. Thus, the values of the summands in (2) are affected both by the background noise and other users' signals Therefore erroneous decision can occur. It is the probability of erroneous decision that predetermines the capacity of the system under consideration. Therefore the problem of obtaining upper bound on error probability is of great importance. This problem will be considered in what follows.

3 A Coded DHA FH OFDMA System with a Noncoherent ML Detector under Multitone Jamming: A Probabilistic Description

Let us assume that the codeword t is the codeword that was transmitted by the user under consideration. Let us now consider the reception procedure described above. Erroneous decision is possible if

$$\exists z \neq t \ \ y_t - y_z < 0. \tag{3}$$

Probability of (3) can be upper bounded with

$$p_e < \sum_{z \neq t} P\left(\sum_{i=1:n} \left(X^2\left(i, j_t(i)\right) - X^2\left(i, j_z(i)\right)\right) < 0 \right). \tag{4}$$

Note that since the minimum distance of the code in use is equal to d any two codewords coincide at most in $n-d$ symbols. Let us designate the set of positions in which codewords v_t and v_z coincide with Θ, while the rest (i.e. those, in which v_t and v_z differ) will be designated by Φ. Let us designate the decision statistic corresponding to the codeword v_z with S_z. This value is given by

$$\begin{aligned} S_z &= \sum_{i=1:n} \left(X^2\left(i, j_t(i)\right) - X^2\left(i, j_z(i)\right)\right) = \\ &= \sum_{i \in \Theta} \left(X^2\left(i, j_t(i)\right) - X^2\left(i, j_z(i)\right)\right) + \sum_{i \in \Phi} \left(X^2\left(i, j_t(i)\right) - X^2\left(i, j_z(i)\right)\right). \end{aligned} \tag{5}$$

Note that $\forall i \in \Theta \;\; X^2\left(i, j_t(i)\right) = X^2\left(i, j_z(i)\right)$. Therefore (5) can be rewritten:

$$S_z = \left(\sum_{i \in \Phi} X^2\left(i, j_t(i)\right) - \sum_{i \in \Phi} X^2\left(i, j_z(i)\right) \right). \tag{6}$$

And (4) can be rewritten:

$$p_e < \sum_{z \neq t} P\left(S_z < 0\right) = \sum_{z \neq t} P\left(\left(\sum_{i \in \Phi} X^2\left(i, j_t(i)\right) - \sum_{i \in \Phi} X^2\left(i, j_z(i)\right) \right) < 0 \right). \tag{7}$$

Note that summands in (7) are statistically independent (though, generally speaking, not identically distributed), and the number of summands in each sum is at least d. Therefore if d is sufficiently great (which is exactly the case that is of interest to us, since to guarantee high data rates and jamming-proofness the minimum distance of the outer code in use is to be great) (7) is well approximated by normal distribution. Note that mean and variance of the distribution of each value S_z depend on the values of means and variances of the summands. In what follows we shall obtain this values in order to estimate the expression at the right side of (7). Due to random permutations the elements of the matrix X (and thus the summands in (7)) correspond to randomly chosen subchannels. Distributions of the value $X\left(i, j_z(i)\right)$ (and thus the moments of this value) depend on the situation at the subchannel corresponding to the $j_z(i)$ th row of marix at the time interval corresponding to the i th time interval. The element on the matrix can correspond to the subchannel via which an authorized user has transmitted a signal (in what follows we shall assume that the optimal power control is maintained in the system under consideration therefore the amplitude of the signals from all authorized users at the receiver side is A). On the other hand, we assume that there is an intruder in the system that transmits a multitone jamming signal. The jamming signal occupies $\Omega = \lceil \alpha Q \rceil$ subchannels ($\alpha < 1$) and the amplitude of each jamming signal is equal to λ (λ can be any positive number depending on the power available to the intruder). Thus the signal transmitted by a certain user (not necessarily the user under consideration) can be jammed. However since due to the use of random permutations the subchannels that are used by the authorized users are chosen in a random fashion the jamming signal might as well affect the subchannel that has not been used for transmission. Moreover, it is possible that the subchannel corresponding to a certain element of matrix $X\left(i, j_z\left(i\right)\right)$ was not used for information transmission, nor was it jammed. In this case the value $X\left(i, j_z\left(i\right)\right)$ is predetermined by the influence of background noise only.

First of all let us consider the case of jamming. For the case under consideration the received signal is given by

$$\overline{r} = \overline{s} + \overline{s}_j + \overline{\eta} = \overline{s} + \overline{z}, \tag{8}$$

where $\overline{s}$ is a random vector with a constant amplitude $|s| = A$ (i.e. the signal transmitted by the authorized user), $\overline{s}_j$ is the jamming signal, i.e. a random vector with a constant amplitude $\left|\overline{s}_j\right| = \lambda A$, $\overline{\eta}$ is the vector corresponding to a two-sided additive white Gaussian noise with a standard deviation σ , $\overline{z} = \overline{s}_j + \overline{\eta}$.

The power of the signal is given by

$$|\overline{r}|^2 = |s|^2 + \left(2 \cdot |s| \cdot |z| \cdot \cos\alpha\right) + |z|^2, \tag{9}$$

where α is the angle between $\overline{s}$ and $\overline{z} = \overline{s}_j + \overline{\eta}$ Note that since phases of the vectors $\overline{s}$, $\overline{s}_j$ and $\overline{\eta}$ are uniformly distributed on $[0, 2\pi]$ α is also uniformly distributed on $[0, 2\pi]$.

Let us find the mean and the variance of the value $|\overline{r}|^2$. The former is given by

$$E\left(|\overline{r}|^2\right) = E\left(|s|^2 + \left(2 \cdot |s| \cdot |z| \cdot \cos\alpha\right) + |z|^2\right) = E\left(|s|^2\right) + E\left(2 \cdot |s| \cdot |z| \cdot \cos\alpha\right) + E\left(|z|^2\right). \tag{10}$$

Note that $|s|, |z|, \alpha$ are uncorrelated random values. Therefore

$$E\left(2 \cdot |s| \cdot |z| \cdot \cos\alpha\right) = 2E\left(|s|\right) E\left(|z|\right) E\left(\cos\alpha\right). \tag{11}$$

Since α is uniformly distributed on $[0, 2\pi]$

$$E(\cos\alpha) = \int_0^{2\pi} \frac{\cos\alpha}{2\pi} d\alpha = 0. \tag{12}$$

Note that $|z|^2$ has a noncentral χ^2 distribution with 2 degrees of freedom and therefore its mean is known [4] and is given by

$$E\left(|z|^2\right) = \lambda^2 A^2 + 2\sigma^2. \tag{13}$$

Thus (10) can be rewritten in the following form:

$$E\left(|\overline{r}|^2\right) = E\left(|s|^2\right) + E\left(|z|^2\right) = A^2 + \lambda^2 A^2 + 2\sigma^2 = \left(1+\lambda^2\right)A^2 + 2\sigma^2. \tag{14}$$

Let us now find the variance of the value $|\overline{r}|^2$. The latter can be derived as in [5]:

$$D\left(|\overline{r}|^2\right) = E\left(\left(|\overline{r}|^2\right)^2\right) - E\left(|\overline{r}|^2\right)^2. \tag{15}$$

The first summand is given by

$$\begin{aligned} E\left(\left(|\overline{r}|^2\right)^2\right) &= E\left(\left(\left(|s|^2+|z|^2\right)+\left(2\cdot|s|\cdot|z|\cdot\cos\alpha\right)\right)^2\right) = \\ &= E\left(|s|^4 + 2|s|^2|z|^2 + |z|^4 + 4\cdot|s|^2\cdot|z|^2\cdot\cos^2\alpha + \left(2\left(|s|^2+|z|^2\right)\left(2\cdot|s|\cdot|z|\cdot\cos\alpha\right)\right)\right). \end{aligned} \tag{16}$$

Note that

$$E\left(2\left(|s|^2+|z|^2\right)\left(2\cdot|s|\cdot|z|\cdot\cos\alpha\right)\right) = 4E\left(\left(|s|^2+|z|^2\right)\left(|s|\cdot|z|\right)\right)E(\cos\alpha) = 0. \tag{17}$$

Therefore

$$\begin{aligned} &E\left(\left(|\overline{r}|^2\right)^2\right) = \\ &= E\left(|s|^4\right) + \left(2E\left(|s|^2\right)E\left(|z|^2\right)\right) + E\left(|z|^4\right) + 4E\left(|s|^2\right)E\left(|z|^2\right)E\left(\cos^2\alpha\right). \end{aligned} \tag{18}$$

The last multiplier is given by

$$E\left(\cos^2\alpha\right) = \frac{1}{2\pi}\cdot\int_0^{2\pi}\cos^2\alpha d\alpha = \frac{1}{\pi}\cdot\int_0^{\pi}\cos^2\alpha d\alpha. \tag{19}$$

Note that [6]:

$$\int_0^{\pi} \cos^2 \alpha d\alpha = \frac{\pi}{2}.$$

$E\left(|z|^4\right)$ is given by [4]:

$$\begin{aligned} E\left(|z|^4\right) &= 4\sigma^4 + 4\sigma^2\lambda^2 A^2 + \left(2\sigma^2 + \lambda^2 A^2\right)^2 = \\ &= 4\sigma^4 + 4\sigma^2\lambda^2 A^2 + 4\sigma^4 + 4\sigma^2\lambda^2 A^2 + \lambda^4 A^4 = 8\sigma^4 + 8\sigma^2\lambda^2 A^2 + \lambda^4 A^4 . \end{aligned} \tag{20}$$

and $E\left(|s|^4\right) = A^4$. Substituting respective summands in (18) we obtain

$$E\left(\left(|\overline{r}|^2\right)^2\right) = 8\sigma^4 + 8\sigma^2\lambda^2 A^2 + \lambda^4 A^4 + A^4 + 2\lambda^2 A^4 + 4\sigma^2 A^2. \tag{21}$$

Now let us consider the second term:

$$E\left(|\overline{r}|^2\right)^2 = \left(\left(1+\lambda^2\right)A^2 + 2\sigma^2\right)^2 = \left(1 + 2\lambda^2 + \lambda^4\right)A^4 + 4\sigma^2\left(1+\lambda^2\right)A^2 + 4\sigma^4. \tag{22}$$

Therefore $D\left(|\overline{r}|^2\right)$ is given by:

$$D\left(|\overline{r}|^2\right) = 4\sigma^4 + 4\sigma^2\lambda^2 A^2. \tag{23}$$

For the sake of convenience let as designate the presence of the signal transmitted by the active user as $S=1$ and the absence of the signal in question as $S=0$ whereas the presence and the absence of the jamming signal as $J=1$ and $J=0$ respectively; the presence and the absence of the signal transmitted by another authorized user will be designated as $I=1$ and $I=0$ respectively . Thus the tuple (S,J,I) describes the respective subchannel completely. Hereinabove it has been shown

$$E(1,1,0) = \left(1+\lambda^2\right)A^2 + 2\sigma^2 \tag{24}$$

and

$$D(1,1,0) = 4\sigma^4 + 4\sigma^2\lambda^2 A^2. \tag{25}$$

However since all active users transmit information in uncoordinated fashion the signal is affected by other users' interference, i.e. collision can occur.

For the sake of simplicity we shall further consider only the most probable case, i.e. . collision of multiplicity two (see [1]). However the approach that is to be introduced can be generalized for the case of collision of any multiplicity.

We can use the technique that has been presented hereinabove to obtain the mean and variance of the output of the subchannel described by $(1,1,1)$, i.e. the subchannel where the signal transmitted by the user under consideration has collided with the signal transmitted by another authorized user and was jammed by the signal transmitted by the intruder. The output of the subchannel in the situation under consideration is given by

$$\overline{r} = \overline{s} + \overline{s}_i + \overline{s}_j + \overline{\eta} = \overline{w} + \overline{z}, \tag{26}$$

where $\overline{s}$ is a random vector with a constant amplitude $|s| = A$ (i.e. the signal transmitted by the authorized user), $\overline{s}_j$ is the jamming signal, i.e. a random vector with constant amplitude $|\overline{s}_j| = \lambda A$, $\overline{s}_i$ is the signal transmitted via the same subchannel by another authorized user (i.e. is a random vector with a constant amplitude $|s| = A$), $\overline{\eta}$ is the vector corresponding to the two-sided additive white Gaussian noise, $\overline{z} = \overline{s}_j + \overline{\eta}$, $\overline{w} = \overline{s} + \overline{s}_i$.

Therefore

$$E\left(|\overline{w}|^2\right) = 2A^2 \tag{27}$$

$$E\left(|\overline{w}|^4\right) = 3A^4 \tag{28}$$

$$E(1,1,1) = E\left(|\overline{r}|^2\right) = 2A^2 + \lambda^2 A^2 + 2\sigma^2 \tag{29}$$

$$\begin{aligned} &E\left(\left(|\overline{r}|^2\right)^2\right) = E\left(|w|^4\right) + E\left(|z|^4\right) + 2E\left(|w|^2\right)E\left(|z|^2\right)E\left(\cos^2\alpha\right) = \\ &3A^4 + 8\sigma^4 + 8\sigma^2\lambda^2A^2 + \lambda^4A^4 + 2\lambda^2A^4 + 4\sigma^2A^2 = \\ &= \left(3 + \lambda^4 + 2\lambda^2\right)A^4 + (4 + 8\lambda)\sigma^2A^2 + 8\sigma^4 \end{aligned} \tag{30}$$

$$E\left(|\overline{r}|^2\right)^2 = \left(\left(2+\lambda^2\right)A^2 + 2\sigma^2\right)^2 = 4A^4 + 4\lambda^2A^2 + \lambda^4A^4 + 8\sigma^2A^2 + 4\lambda^2A^2\sigma^2 + 4\sigma^4 \tag{31}$$

$$D(1,1,1) = E\left(\left(|\overline{r}|^2\right)^2\right) - E\left(|\overline{r}|^2\right)^2. \tag{32}$$

In other cases we are to consider obtaining moments is not that cumbersome. The output of the subchannel described by $(1,0,0)$ is given by

$$\overline{r} = \overline{s} + \overline{\eta}, \tag{33}$$

where $\bar{s}$ is a random vector with a constant amplitude $|s| = A$ (i.e. the signal transmitted by the authorized user), $\bar{\eta}$ is the vector corresponding to the two-sided additive white Gaussian noise. Therefore $|\bar{r}|^2$ has a noncentral χ^2 distribution and its mean and variance are given by [4]:

$$E(1,0,0) = A^2 + 2\sigma^2 \tag{34}$$

$$D(1,0,0) = 4\sigma^2 A^2 + 4\sigma^4 \tag{35}$$

respectively.

The output of the subchannel described by $(0,1,0)$ is given by

$$\bar{r} = \bar{s}_j + \bar{\eta}. \tag{36}$$

Since $\bar{s}_j$ is a random vector this value also has a noncentral χ^2 distribution. Since $|\bar{s}_j| = \lambda A$ the mean and the variance of the value $|\bar{r}|^2$ are given by [4]:

$$E(0,1,0) = \lambda^2 A^2 + 2\sigma^2 \tag{37}$$

$$D(0,1,0) = 4\sigma^2 \lambda^2 A^2 + 4\sigma^4. \tag{38}$$

Finally the output of the subchannel described by $(0,0,0)$ is predetermined by the influence of the additive white Gaussian noise and therefore the value $|\bar{r}|^2$ has a χ^2 distribution. Thus, we can claim that

$$E(0,0,0) = 2\sigma^2 \tag{39}$$

$$D(0,0,0) = 4\sigma^4. \tag{40}$$

4 A Coded DHA FH OFDMA System with a Noncoherent ML Detector under Multitone Jamming: An Upper Bound

Let us once again consider the reception of a codeword by a certain user. Within the scope of the process under consideration the user transmits n signals (n OFDM symbols). As has been stated above it is assumed that the intruder transmits signals in $\Omega = \lceil \alpha Q \rceil$ subchannels within the scope of transmission of every OFDM symbol. Therefore each signal transmitted by the user under consideration is jammed with probability $\frac{\Omega}{Q}$. Let us consider two codewords: codeword $\bar{v}_t$ (the one that has been transmitted by the user under consideration) and codeword $\bar{v}_z$. Let us assume that these codewords differ in n positions ($\tilde{n} \geq n \geq d$). The probability of the fact that α of n signals will be jammed is given by

$$p_{SJ}(\alpha) = C_n^{\alpha} \left(\frac{\Omega}{Q} \right)^{\alpha} \left(1 - \frac{\Omega}{Q} \right)^{n-\alpha} . \tag{41}$$

Moreover signals transmitted by the user under consideration can collide with the signals transmitted by other active users (let us further on refer to them as "interfering" users). Since there are $m-1$ interfering users the probability of the fact that a certain signal will collide is given by

$$p_c(m) = 1 - \left(1 - \frac{1}{q} \right)^{m-1} . \tag{42}$$

The probability of the fact that β signals of α signals that were jammed will undergo collision (i.e. the respective signals will interfere both with the intruder and with other authorized users) is given by

$$p_{SJI}(\alpha,\beta) = C_{\alpha}^{\beta} \left(p_c(m) \right)^{\beta} \left(1 - p_c(m) \right)^{\alpha-\beta} . \tag{43}$$

and the probability that γ signals of $n-\alpha$ signals that were not jammed will undergo collision (i.e. the respective signals will be affected by the signals transmitted by other authorized users but not by a jamming signal) is given by

$$p_I(n-\alpha,\gamma) = C_{n-\alpha}^{\gamma} \left(p_c(m) \right)^{\gamma} \left(1 - p_c(m) \right)^{n-\alpha-\gamma} . \tag{44}$$

Note that in this case the mean of the first sum in (7) will be given by

$$E_t(\alpha,\beta,\gamma) = \left(\beta \cdot E(1,1,1)\right) + \left((\alpha-\beta) \cdot E(1,1,0)\right) + \left(\gamma \cdot E(1,0,1)\right) + \left((n-\alpha-\gamma) E(1,0,0)\right) \tag{45}$$

and the variance is given by

$$D_t(\alpha,\beta,\gamma) = \left(\beta \cdot D(1,1,1)\right) + \left((\alpha-\beta) \cdot D(1,1,0)\right) + \left(\gamma \cdot D(1,0,1)\right) + \left((n-\alpha-\gamma) D(1,0,0)\right) \tag{46}$$

Let us now consider the vector of elements corresponding to the second codeword.

Let p_s be the probability of the fact that a elements of the vector correspond to the subchannels, via which only one authorized user has transmitted a signal whereas b elements of the vector correspond to the subchannels were collisions occurred. The probability of this is given by

$$p_s(a,b) = \frac{n!}{a!b!(n-a-b!)} p_1^a \left(1 - p_1 - p_0\right)^b p_0^{n-a-b} , \tag{47}$$

where

$$p_1 = C_m^1 \left(\frac{1}{Q-1} \right) \left(1 - \frac{1}{Q-1} \right)^{m-1} \tag{48}$$

$$p_0 = \left(1 - \frac{1}{Q-1} \right)^{m-1}. \tag{49}$$

Now let us assume that f subchannels, via which only one user has transmitted, h subchannels, in which collisions occurred, and u subchannels, via which none of the users transmitted, were jammed. Respective probabilities are given by

$$\tilde{p}_{SJ}(f,a) = C_a^f \left(\frac{\Omega}{Q} \right)^f \left(1 - \frac{\Omega}{Q} \right)^{a-f} \tag{50}$$

$$\tilde{p}_{SJI}(h,b) = C_b^h \left(\frac{\Omega}{Q} \right)^h \left(1 - \frac{\Omega}{Q} \right)^{b-h} \tag{51}$$

$$\tilde{p}_J(u,n-a-b) = C_{n-a-b}^u \left(\frac{\Omega}{Q} \right)^u \left(1 - \frac{\Omega}{Q} \right)^{n-a-b-u}. \tag{52}$$

In this case the mean of the second sum in (7) is given by

$$\begin{aligned} E_z(a,b,f,g,u) &= \big((a-f)E(1,0,0)\big) + \big(f \cdot E(1,1,0)\big) + \big((b-h)E(1,0,1)\big) + \\ &+ \big(h \cdot E(1,1,1)\big) + \big(u \cdot E(0,1,0)\big) + \big((n-a-b-u) \cdot E(0,0,0)\big) \end{aligned} \tag{53}$$

and the variance is given by

$$\begin{aligned} D_z(a,b,f,g,u) &= \big((a-f)D(1,0,0)\big) + \big(f \cdot D(1,1,0)\big) + \big((b-h)D(1,0,1)\big) + \\ &+ \big(h \cdot D(1,1,1)\big) + \big(u \cdot D(0,1,0)\big) + \big((n-a-b-u) \cdot D(0,0,0)\big) \end{aligned} \tag{54}$$

Therefore the decision statistic

$S_z = \left(\sum_{i\in\Phi} X^2(i, j_t(i)) - \sum_{i\in\Phi} X^2(i, j_z(i)) \right)$ has a normal distribution with mean $E = E_t(\alpha,\beta,\gamma) - E_z(a,b,f,g,u)$ and variance $D = D_t(\alpha,\beta,\gamma) + D_z(a,b,f,g,u)$.

Let $\bar{\varpi} = [\varpi_1,\ldots,\varpi_M]$ where $M = q^k$ be the spectrum of the code

Then the probability of error is upper bounded by

$$p \le \sum_{z \ne t} \bar{P}\left(S_z < 0\right) =$$

$$\le \sum_{i \ne t} \sum_{\alpha=0}^{\varpi_i} \sum_{\beta=0}^{\alpha} \sum_{\gamma=0}^{\varpi_i - \alpha} \sum_{a=0}^{\varpi_i} \sum_{b=0}^{\varpi_i - a} \sum_{f=0}^{a} \sum_{h=0}^{b} \sum_{u=0}^{\varpi_i - a - b} \Big[p_{SJ}(\alpha) \cdot p_{SJI}(\alpha,\beta) \cdot p_I(\varpi_i - \alpha, \gamma) \cdot p_s(a,b) \cdot \tilde{p}_{SJ}(f,a) \cdot \tilde{p}_{SJI}(h,b) \times$$

$$\times \tilde{p}_J(u, \varpi_i - a - b) \cdot \left(\int_{-\infty}^{0} f_N\left(x, E_t(\alpha,\beta,\gamma) - E_z(a,b,f,g,u), D_t(\alpha,\beta,\gamma) + D_z(a,b,f,g,u)\right) \right) \Big] dx.$$

where $f_N(x,E,D)$ is probability density function of the normal distribution with mean E and variance D, $\bar{P}$ designates an upper bound on probability P.

5 Conclusion

Hereinabove an upper bound on the probability of erroneous decoding in a DHA FH OFDMA system with a noncoherent ML detector for the case of multitone jamming has been introduced. The approach that has been used to obtain the bound in question can be easily applied to obtain bounds on erroneous decoding probability for other cases (e.g. partial band noise jamming, follower jamming etc.).

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