Optimal Information Disclosure in Contests with Stochastic Prize Valuations *

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Abstract

We study optimal information design in static contests where contestants do not know their values of winning. The designer aims at maximizing the total expected effort. Before the contest begins, she commits to the information technology that includes (1) a signal distribution conditional on each values profile (state) and (2) the type of signal disclosure to contestants – public, private or none at all. Upon observing the signal, contestants simultaneously choose effort that maximizes their expected payoff in an all-pay auction game. We find that the optimal information technology involves private signals, which are slightly positively correlated and never reveal the true state precisely if the contestants' values of winning are different. In settings where public disclosure must be used, the optimal signal distribution generates symmetric beliefs about the values profile, so that, for example, a complete information concealment is optimal, while public and precise disclosure of each state is not.

KEY WORDS: contest, all-pay auction, information disclosure, signal distribution, signal precision

JEL CLASSIFICATION: C72, D82, D83

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1 Introduction

Many real life competitions can be viewed as contests in which the contestants' values of winning/prize valuations are not known neither to the contest organizer (designer), nor to the contestants themselves. For example, competition for promotion at a firm often has the feature that neither the employer, nor the candidates know how much the "winner" will benefit from getting the position. This is because on the one hand, neither of the two parties know from the start how good the match between each candidate and the new position is. On the other hand, the bonuses from promotion often depend on financial health and projected revenue of the firm, which can be highly uncertain, especially for the candidates who have little means to obtain this information. A particular case of such situation is a promotion to become a *partner* in a law, accounting or consulting firm. Most law firms are organized as partnerships: a lawyer becomes a partner of the firm when he or she transitions from being an employee, who is paid a salary, to a part-owner of the firm, who is sharing in the firm's profits (and liabilities). Clearly, these prospective profits and liabilities often bear a great deal of uncertainty.

The information about the contestants' values of winning, however, is an important determinant of how much effort the contestants will be willing to exert in the competition, and that, in turn, is frequently what the designer cares about the most. For example, if the contestants know that they all have a high value of winning, they are likely to compete aggressively and exert much effort, while the opposite might be the case if the competition is known to be "uneven". Then, it is reasonable to presume that the designer, who often has an advantage in acquiring the relevant information (in the form of a report, expert evaluation, etc.),[1] may wonder whether she can benefit from (a) soliciting an informative signal about the contestants' values of winning and (b) passing this signal on to the contestants. Even if this signal is of no intrinsic interest to the designer and is meant to be used exclusively for stimulating the competitive efforts, the designer may be concerned with how precise she wants the signal to be and in what way, if any, she wants to convey it to the contestants. This is the essence of the research question that we address in this paper:

[1] For example, she has access to application materials of all competitors, and she can invite an expert/committee or solicit a report to evaluate the risks and unknown characteristics of both, the job and the contestants themselves.

can the organizer of a contest with unknown prize valuations increase the total competitive effort by choosing optimally the precision of the signal and the way of communicating it to the contestants?

In general, players in a contest can be informed publicly or privately or not informed at all. We consider these three information regimes and assume that for each regime the designer chooses the precision of the signal and whether to make it dependent on the true (unknown) values profile. For example, in case of public information disclosure, the designer can choose to have a perfectly precise signal about any values profile, or a signal that is perfectly precise only when both contestants have high values of winning, or any other signal with predetermined precision. This provides the designer with a very broad set of possible disclosure rules.

We formalize these ideas in a stylized all-pay auction model with two ex ante identical players who compete for a single prize, and a designer who aims at maximizing the expected aggregate effort.[2] The value of a prize can be either high or low for each contestant and is determined randomly according to a symmetric prior distribution. The designer has access to a certain information technology that allows her to (costlessly) draw signals about the contestants' prize values. She can choose (1) the precision of this information technology, conditional on each actual values profile (state of the world) and (2) whether to pass the signals on to both contestants privately, or publicly, or keep them uninformed. In the spirit of the Bayesian persuasion literature, we assume that the designer commits to the disclosure regime before she receives the signal.[3] Moreover, irrespective of the chosen regime, she always reveals the obtained signal truthfully. After that the contestants update their beliefs about own and the opponent's type and choose an action from continuous effort space.

To infer the optimal choice of information technology by the designer, we first study equilibria of the contest game and find the best signal distribution (i.e., the signal precision parameters) for each type of information disclosure separately. Then we compare the resulting values of the expected aggregate effort across all the cases and characterize the information technology that constitutes the global optimum. Our main result is that the

[2]For example, in promotion competitions, the goal of the designer is often to achieve the highest aggregate work effort among employees.

[3]For example, in the promotion story, the rules of information disclosure are often set by HR or the labor law, so they cannot be adjusted ex post.

optimal disclosure policy employs private signals. Moreover, these signals are (slightly) positively correlated and have the property that they are precise if and only if both contestants have the same values of winning, that is, the state is symmetric. This means that upon receiving a private message, each contestant is led to believe that conditional on the prior, the state is most likely to be symmetric[4], and if it is asymmetric, then either of the two contestants can have a higher valuation. Such structure of private signals makes contestants perceive their expected prize valuations as similar and thus, allows the designer to "level the playing field". This, in turn, is what the designer strives for in our model because, in line with the common finding in the contest literature (Baik 1994; Baye et al. 1996; Dechenaux et al. 2015; Fu and Wu 2019), the aggregate effort is higher when contestants are homogeneous.

Under public disclosure the situation is different. Here more information is passed to the contestants, and this leaves the designer less room for belief manipulation. Indeed, since a full value profile is communicated to both contestants, there is no way the designer can make them assign a relatively high probability to symmetric states under *any* (two-dimensional) signal. For example, if she chooses the signal to be precise or close to precise when the true values profile is symmetric (as under optimal private disclosure policy), then upon receiving an asymmetric signal (with one high and one low valuation), the contestants are certain or almost certain that the state is actually *not* symmetric.[5] In this sense, the commitment of the designer to reveal a full signal profile publicly reduces her ability to induce effort.

The finding that some form of private disclosure can be optimal has been also reported in earlier studies (e.g., Chen 2019, 2021b; Kuang et al. 2019; Melo-Ponce 2020), but not in the setting where contestants are a priori identical and have no private information about their own type. The latter produces a different information structure in our model, as it means that it is the designer who decides whether to provide the contestants with private information and what kind of information it should be. As a result (and in contrast to other studies), we find that disclosure rules inducing asymmetric beliefs are never optimal, and the

[4]High (low) private signal has probability one in the symmetric state where both valuations are high (low). By contrast, in asymmetric states, the probability of any private signal is less than one because the information disclosure in such states is noisy.

[5]Similarly, if the signal precision is low when the true values profile is symmetric, then depending on the exact specification of the signal distribution, contestants will not assign much belief to the symmetric states either when their received signal profile is symmetric or when it is asymmetric. See Section 5.2 for more detail.

designer extracts the full surplus.

We further find that although public information disclosure is not globally optimal, in cases when it *has* to be used, an optimal signal distribution must generate symmetric beliefs. That is, upon receiving a public signal, the contestants should believe that each of them is equally likely to have high valuation, even if they do not believe that the state is actually symmetric. For example, complete nondisclosure of information, which can be regarded as public disclosure with a completely uninformative signal, satisfies this property. On the other hand, a precise public revelation of *every* state is never optimal. The key intuition here is the same as before: symmetric beliefs make contestants perceive their chances of winning as equal, leading to the highest competitive efforts.

Despite the simple intuition behind the results, our paper is the first to provide a precise theoretical characterization of optimal disclosure policies in a setting with (i) a broad range of available disclosure rules, including private and partial (private or public) disclosure, (ii) initially symmetric contestants who have no private information about their values of winning and (iii) effort choices being continuous. In the next section we discuss our contribution to the existing literature in more detail. The rest of the paper is organized as follows. In Section 3 we introduce the model. Section 4 describes equilibria of the contest game under public and private information disclosure. Section 5 characterizes the optimal information disclosure policy. Section 6 provides some insights for the case with ex ante informed contestants. Finally, Section 7 concludes.

2 Related literature

This paper lies at the intersection of two research fields. First, it is related to the Bayesian persuasion literature initiated by Kamenica and Gentzkow (2011) and extended to a setting with multiple signal receivers by a stream of subsequent studies (Alonso and Camara 2016 a,b; Bardhi and Guo 2018; Arieli and Babichenko 2019; Chan et al. 2019). In this literature, the closest to us is Taneva (2019). She uses the concept of Bayes correlated equilibrium introduced by Bergemann and Morris (2016) to solve for the optimal signal structure in an environment with multiple interacting receivers. We borrow Taneva's signal parametrization but allow the precision parameters to be state-dependent. This is important in our setting since in a contest where state captures the players' values of winning, it might be optimal to make the signals more or less precise depending on the state. Our key results confirm this intuition, and perfect precision appears optimal only in those states where the contestants' values of winning are the same. Another important difference from Taneva (2019) is that our model assumes continuous efforts, while her results are derived for a discrete and finite action space, which makes them inapplicable in our setting.

Second, our study contributes to the literature on the optimal information revelation/feedback in contests. Most papers in this field focus on very simple disclosure rules. In a dynamic setting, Aoyagi (2010) investigates the optimal feedback in two-stage tournaments and looks only at no feedback and full feedback cases. He finds that the no-feedback policy maximizes the players' expected effort when the second stage effort cost is convex; otherwise, full feedback is best.[6]

In static contests, the information that can be disclosed, is generally about a number of participants (Feng and Lu 2016) or rivals' types (Zhang and Zhou 2016; Serena 2021 for Tullock-type contests and Chen 2019; Chi et al. 2019; Chen 2021a for all-pay auctions). Unlike us, most studies in this field assume that at least one of the contestants is privately informed about own type (Warneryd 2003; Lu et al. 2018; Lu and Wang 2019; Ewerhart and Lareida 2021). If the designer is benevolent (Azacis and Vida 2015; Chen 2019; Chen 2021b) and aims at maximizing the contestants' expected payoff, then the optimal information disclosure should generally rely on positively but mildly correlated signals, like in our model. Depending on the degree of ex ante heterogeneity, such signals may induce both symmetric and asymmetric beliefs (Chen 2021b). This last result is in stark contrast to our findings where the optimal disclosure policy extracts all the surplus from the contestants and actually always works through the level-playing-field channel if we use the terminology of Chen (2021b).

Similarly to us, Celik and Michelucci (2020) assume no private information on the contestants' side and allow for a rich set of disclosure rules, where a principal chooses the optimal coarseness of information that she provides to the contestants. They report that the solution varies with the size of the effort cost and with the relative likelihood of different

[6]Lai and Matros (2007), Gershkov and Perry (2009), Ederer (2010), Goltsman and Mukherjee (2011) and Mihm and Schlapp (2019) work on similar issues and also look at the restricted set of information disclosure policies.

states. In contrast to us, Celik and Michelucci (2020) consider only public information disclosure and assume binary efforts. We find that private disclosure delivers a higher total effort than public disclosure, at least when there is a committment on the designer's side.

Finally, the closest to our paper are Melo-Ponce (2020) and Kuang et al. (2019). Both of them consider a large set of disclosure rules but, unlike us, assume private information on the contestants' side, which significantly limits the designer's control over the information they have. Melo-Ponce (2020) looks at a setting with binary effort and private but state-dependent signals. Kuang et al. (2019) use the continuous effort space and do not allow the signal distribution to be state-dependent. In the setting of Melo-Ponce (2020), the optimal policy often requires asymmetric (and sometimes uncorrelated) signals that make a weak player more informed and incentivize him to exert higher effort. This is different from what we find in our paper, although the underlying mechanism works through the same level-playing-field channel. With privately informed contestants, the results of Kuang et al. (2019) are similar to those of Melo-Ponce (2020). In an extension, Kuang et al. (2019) relax the assumption of privately informed contestants but assume that the designer's signal about prize valuations must be perfectly precise in this setting. As we show, this does not come without loss of generality. Moreover, differently from us, Kuang et al. (2019) compare just three cases, where both, one or none of the contestants have private information about their prize valuations. They find that the setting where only one player knows his type induces less effort than the setting where no players are privately informed.

3 The model

Two players striving to win a single prize engage in a contest game. The values of winning the prize can be different for the players and are determined by realizations of two i.i.d. discrete random variables v_i, $i = 1, 2$, where:

$$v_i = \begin{cases} H\,(\text{high}), & \text{with probability } \alpha \in (0,\, 1) \\ L \equiv 1\,(\text{low}) < H, & \text{with probability } 1-\alpha \end{cases}$$

This results in four possible *states of the world*: $(H,\ H)$, $(L,\ L)$, $(H,\ L)$, and $(L,\ H)$.[7] Let us denote the set of these states by S. An important feature of our model is that neither the contest designer nor the contestants have any ex ante information about the realized state of the world and share a common symmetric prior, denoted by ξ.[8]

In the contest, players simultaneously choose non-negative effort e_i and pay the effort cost $\gamma_i(e_i) = e_i$ for any $i = 1, 2$. The player exerting higher effort wins and receives the prize, and ties are broken randomly. Thus, in the absence of any signal about the state of the world, player i's payoff is equal to

$$(\alpha H + (1-\alpha) L)\, P(e_i > e_{-i}) - e_i,$$

where $(\alpha H + (1-\alpha) L)$ is the expected prize valuation of player i given the prior, and $P(e_i > e_{-i})$ denotes the probability that player i wins. By making his effort choice e_i, player i maximizes this utility or – if some information about valuations arrives – a utility analogous to this, but calculated with an updated prior.

The designer aims at achieving the highest aggregate effort in the contest. As the contestants, she does not know the actual state of the world. But in contrast to them, she has access to a certain information technology that allows her to (costlessly) draw signals (or messages) $m_1, m_2 \in \{H, L\}$ about contestants' prize valuations v_1 and v_2. The designer can choose (1) the precision of this information technology, conditional on each state of the world, and (2) whether to pass the signals on to both contestants privately, or publicly, or keep them uninformed, with the goal of inducing maximal aggregate effort.

To be more precise, let us define an information technology $\mathcal{I} = (M, \pi, T)$ as a triple that consists of (i) all possible signal profiles $M = \{(m_1, m_2) \text{ s.t. } m_i \in \{H, L\}, i = 1, 2\}$, (ii) conditional signal distributions $\pi : S \times T \to \Delta(M)$, one for each possible state and type of information disclosure, and (iii) the type of information disclosure to contestants $\tau \in T = \{public, private, none\}$.[9] We assume that any type of disclosure $\tau \in T$ is applied

[7] Note that $\alpha = 0$ and $\alpha = 1$ are assumed away as otherwise, the question of information disclosure is irrelevant: the state is *known* to be $(H,\ H)$ or $(L,\ L)$, respectively.

[8] As should become clear from the model description below, the assumption about uninformed designer who draws signals about contestants' prize valuations with a predetermined precision and simply passes them on to the contestants, is equivalent to assuming that the designer holds the best possible knowledge about the state but prior to its realization commits to the precision of signals that she will send to the contestants.

[9] This concept is similar to the concept of "information structure" in Taneva (2019), where it is applied to a discrete-action game in which signals are action recommendations, and they are conveyed to players privately.

to both players *symmetrically*. That is, either both players are informed about the signal publicly (each of them learns the same pair of messages (m_1, m_2)), or both are approached privately (player i observes only message m_i), or both remain uninformed and have to rely only on their prior. Furthermore, suppose that irrespective of whether the disclosure is public or private, the designer always conveys the information *truthfully*.

We restrict attention to signal distributions that are analogous to those introduced in Taneva (2019).[10] Such distributions can be fully characterized by two parameters per state $q_j, r_j \in [0,1]$, $j \in 1:4$, as shown on Figure 1.

$\mathbf{s_1} = (H,\ H)$

	$m_2 = H$	$m_2 = L$
$m_1 = H$	r_1	$q_1 - r_1$
$m_1 = L$	$q_1 - r_1$	$1 - 2q_1 + r_1$

$\mathbf{s_2} = (H,\ L)$

	$m_2 = H$	$m_2 = L$
$m_1 = H$	$q_2 - r_2$	r_2
$m_1 = L$	$1 - 2q_2 + r_2$	$q_2 - r_2$

$\mathbf{s_3} = (L,\ H)$

	$m_2 = H$	$m_2 = L$
$m_1 = H$	$q_3 - r_3$	$1 - 2q_3 + r_3$
$m_1 = L$	r_3	$q_3 - r_3$

$\mathbf{s_4} = (L,\ L)$

	$m_2 = H$	$m_2 = L$
$m_1 = H$	$1 - 2q_4 + r_4$	$q_4 - r_4$
$m_1 = L$	$q_4 - r_4$	r_4

Figure 1: Conditional signal distribution π

These parameters reflect signal *precision* in the following sense. The parameter r_j indicates a probability with which signals about *both* players' prize valuations match the true state $\mathbf{s_j}$:

$$P(m_1 = H, m_2 = H|\mathbf{s_1} = (H,H)) = r_1,\ P(m_1 = H, m_2 = L|\mathbf{s_2} = (H,L)) = r_2,$$

$$P(m_1 = L, m_2 = H|\mathbf{s_3} = (L,H)) = r_3, P(m_1 = L, m_2 = L|\mathbf{s_4} = (L,L)) = r_4.$$

In other words, r_j measures the probability that conditional on the state, the information technology reveals the true prize valuations of both contestants. The parameter q_j represents a probability that a signal regarding *each* contestant matches his true valuation in state $\mathbf{s_j}$, irrespective of whether the same holds for the opponent. This probability is the same for both contestants. For example, in state $\mathbf{s_2} = (H, L)$ this probability for players 1 and 2, respectively, is given by:

$$P(m_1 = H, m_2 = H|\ \mathbf{s_2} = (H,L)) + P(m_1 = H, m_2 = L|\ \mathbf{s_2} = (H,L)) = q_2,$$

$$P(m_1 = H, m_2 = L|\ \mathbf{s_2} = (H,L)) + P(m_1 = L, m_2 = L|\ \mathbf{s_2} = (H,L)) = q_2.$$

[10] Differently from us, Taneva (2019) considers such distributions over action profiles and two states of the world.

Clearly, it must be that $q_j \geq r_j$ for any state $\mathbf{s_j}$.

The choice of q_j and r_j in each state also allows for arbitrary correlation between signals m_1 and m_2. In general, this correlation can be defined as

$$\rho = P(m_{-i} = H|m_i = H) - P(m_{-i} = H|m_i = L) =$$

$$P(m_{-i} = L|m_i = L) - P(m_{-i} = L|m_i = H).$$

It will be the focus of our attention when we study the case of private information disclosure. By calibrating the correlation between private messages the designer can further her agenda of maximizing their aggregate effort.

Note that depending on the parameter values, the distribution over signal profiles π may, in fact, be independent of the state. This is the case when all four probability matrices on Figure 1 are identical. For example, this can be achieved when all entries in each matrix are equal to 1/4, or when the same and equal weight is given only to symmetric signals (H, H), (L, L) and no weight is assigned to asymmetric signals in every state. The former represents a situation where signals are completely uninformative and thus do not affect the prior beliefs. The latter corresponds to an environment with perfectly positively correlated private signals.[11]

The timing of the events is as follows. Before the contest and any signal realization, the designer publicly announces an information technology $\mathcal{I}$ and commits to it. Once the state has been randomly drawn from the prior distribution ξ, the signals are generated according to the announced distribution π and subsequently revealed to each player in line with the chosen type of disclosure τ. Upon observing the signal, each contestant i updates beliefs about the state and about the signal of his opponent. Then, both contestants simultaneously choose a (possibly mixed) action – a probability distribution over effort levels – which maximizes their expected payoffs. The resulting choices conditional on players' signals define a Nash equilibrium or a Bayesian Nash equilibrium of the contest game. The appropriate concept is Nash if the designer sends public messages or transmits no information ($\tau = public$ or $\tau = none$), since in this case the information set of both contestants is the same. Under private disclosure ($\tau = private$), the contest game features incomplete information, and each contestant can be of two types, H or L, as determined by his private message. Then, the

[11] Moreover, the precision of these signals is zero if the realized state is asymmetric (namely, $r_2 = r_3 = 0$).

appropriate concept is Bayesian Nash equilibrium.

In general, there could exist multiple equilibria of the contest game. The designer has to choose an information technology that induces contestants to play the equilibrium that maximizes her ex ante expected payoff, i.e., the ex ante expected aggregate effort of the contestants. This problem can be solved in two steps:

1. First, we characterize the set of equilibria that could emerge under any possible information technology, that is, under any type of information disclosure $\tau \in T$ and any conditional signal distribution $\pi\left(\{q_j, r_j\}_{j\in 1:4}, \tau\right)$.
2. Second, we maximize the objective function of the designer over the set of all possible equilibria. This delivers the optimal information technology $\mathcal{I}^*$, that is, the optimal type of information disclosure τ^* and the optimal signal distribution $\pi(\{q_j^*, r_j^*\}_{j\in 1:4}, \tau^*)$.

Following this approach, in the next section we consider the equilibria of the contest game under both public and private information disclosure. Then in Section 5 we solve the designer's problem in each case to derive an optimal signal distribution for $\tau = \textit{public}$ and $\tau = \textit{private}$,[12] and by comparing the results, we characterize the global optimum, that is, the optimal information technology $\mathcal{I}^*$.

4 Equilibrium of the contest game

We first study the Nash equilibrium of the contest game following public disclosure and then, the Bayesian Nash equilibrium of the contest game following private disclosure.

4.1 Equilibrium under public information disclosure

Let us denote the precision parameters in the case of public disclosure by $\{q_j^{pub}, r_j^{pub}\}_{j\in 1:4}$ so as to distinguish this case from the case of private disclosure (where we impose additional restrictions on distribution π).

Since under public disclosure, the designer truthfully reveals the same signal profile $\mathbf{m} = (m_1, m_2)$ to both players, they update beliefs about own and the opponent's prize valuations

[12]Nondisclosure, or information concealment ($\tau = \textit{none}$), is addressed as a special case of public disclosure.

in the same way:[13]

$$P\left(v_i = H \middle| \mathbf{m} = (H,\, H)\,,\, \pi\right) =$$
$$= \frac{\alpha^2 r_1^{pub} + \alpha\left(1-\alpha\right)\left(\left(q_2^{pub} - r_2^{pub}\right) I_{\{i=1\}} + \left(q_3^{pub} - r_3^{pub}\right) I_{\{i=2\}}\right)}{\alpha^2 r_1^{pub} + \alpha\left(1-\alpha\right)\left(q_2^{pub} - r_2^{pub} + q_3^{pub} - r_3^{pub}\right) + \left(1-\alpha\right)^2\left(1 - 2q_4^{pub} + r_4^{pub}\right)},$$

$$P\left(v_i = H \middle| \mathbf{m} = (H,\, L)\,,\, \pi\right) =$$
$$= \frac{\alpha^2\left(q_1^{pub} - r_1^{pub}\right) + \alpha\left(1-\alpha\right)\left(r_2^{pub} I_{\{i=1\}} + \left(1 - 2q_3^{pub} + r_3^{pub}\right) I_{\{i=2\}}\right)}{\alpha^2\left(q_1^{pub} - r_1^{pub}\right) + \alpha\left(1-\alpha\right)\left(r_2^{pub} + 1 - 2q_3^{pub} + r_3^{pub}\right) + \left(1-\alpha\right)^2\left(q_4^{pub} - r_4^{pub}\right)},$$

$$P\left(v_i = H \middle| \mathbf{m} = (L,\, H)\,,\, \pi\right) =$$
$$= \frac{\alpha^2\left(q_1^{pub} - r_1^{pub}\right) + \alpha\left(1-\alpha\right)\left(\left(1 - 2q_2^{pub} + r_2^{pub}\right) I_{\{i=1\}} + r_3^{pub} I_{\{i=2\}}\right)}{\alpha^2\left(q_1^{pub} - r_1^{pub}\right) + \alpha\left(1-\alpha\right)\left(1 - 2q_2^{pub} + r_2^{pub} + r_3^{pub}\right) + \left(1-\alpha\right)^2\left(q_4^{pub} - r_4^{pub}\right)},$$

$$P\left(v_i = H \middle| \mathbf{m} = (L,\, L)\,,\, \pi\right) =$$
$$= \frac{\alpha^2\left(1 - 2q_1^{pub} + r_1^{pub}\right) + \alpha\left(1-\alpha\right)\left(\left(q_2^{pub} - r_2^{pub}\right) I_{\{i=1\}} + \left(q_3^{pub} - r_3^{pub}\right) I_{\{i=2\}}\right)}{\alpha^2\left(1 - 2q_1^{pub} + r_1^{pub}\right) + \alpha\left(1-\alpha\right)\left(q_2^{pub} - r_2^{pub} + q_3^{pub} - r_3^{pub}\right) + \left(1-\alpha\right)^2 r_4^{pub}}.$$

where $I_{\{i=1\}}$ and $I_{\{i=2\}}$ are index functions equal to one if the condition in curly brackets holds, and zero otherwise. The expected prize valuation of player i, conditional on $\mathbf{m}$, is then given by:[14]

$$E\left(v_i \middle| \mathbf{m},\, \pi\right) \equiv v_i^{\mathbf{m},pub} = P\left(v_i = H \middle| \mathbf{m},\, \pi\right) H + P\left(v_i = L \middle| \mathbf{m},\, \pi\right).$$

Since the signal profile $\mathbf{m}$ is publicly observable, the contest mimics a complete information all-pay auction. This class of games was extensively studied in Baye et al. (1996). With two contestants, the Nash equilibrium is unique and features mixed strategies. For any message $\mathbf{m} \in M$ and any $v_i^{\mathbf{m},pub} \geq v_{-i}^{\mathbf{m},pub}$, the Nash equilibrium is the following:[15]

- Contestant i randomizes uniformly on $\left[0,\, v_{-i}^{\mathbf{m},pub}\right]$ with probability 1.

[13] All probabilities are calculated by Bayes rule using the parameters of the conditional distribution $\pi\left(\{q_j^{pub}, r_j^{pub}\}_{j\in 1:4}, \tau = public\right)$ (see Figure 1) and the prior distribution ξ defined in the previous section. It is easy to verify that these beliefs are Bayes-plausible according to the definition of Kamenica and Gentzkow (2011), that is, for every player i, the expected posterior probability equals the prior.

[14] Recall that $L \equiv 1$.

[15] See Baye et al. (1996) for technical details.

- Contestant $-i$ randomizes uniformly on $\left(0,\, v_{-i}^{\mathbf{m},pub}\right]$ with probability $\frac{v_{-i}^{\mathbf{m},pub}}{v_{i}^{\mathbf{m},pub}}$ and stays inactive with probability $\left(1-\frac{v_{-i}^{\mathbf{m},pub}}{v_{i}^{\mathbf{m},pub}}\right)$.

Note that given these strategies, the contestants' expected equilibrium payoffs are:

$$\begin{aligned} u_i &= \frac{v_{-i}^{\mathbf{m},pub}}{v_{i}^{\mathbf{m},pub}} \cdot \frac{1}{2} \cdot v_{i}^{\mathbf{m},pub} + \left(1-\frac{v_{-i}^{\mathbf{m},pub}}{v_{i}^{m,pub}}\right) \cdot v_{i}^{\mathbf{m},pub} - \frac{v_{-i}^{\mathbf{m},pub}}{2} = v_{i}^{\mathbf{m},pub} - v_{-i}^{\mathbf{m},pub} \geq 0, \\ u_{-i} &= \frac{v_{-i}^{\mathbf{m},pub}}{v_{i}^{\mathbf{m},pub}} \cdot \frac{1}{2} \cdot v_{-i}^{\mathbf{m},pub} - \frac{v_{-i}^{\mathbf{m},pub}}{v_{i}^{\mathbf{m},pub}} \cdot \frac{v_{-i}^{\mathbf{m},pub}}{2} = 0. \end{aligned}$$

Furthermore, we can define J^{pub}, the ex ante expected aggregate effort of the contestants, where "ex ante" refers to the fact that the designer commits to the information technology *before* the realization of a signal:

$$J^{pub} \equiv \sum_{\mathbf{m}\in\tilde{M}_\pi} P(\mathbf{m}) \frac{v_{2}^{\mathbf{m},pub}}{2}\left(1+\frac{v_{2}^{\mathbf{m},pub}}{v_{1}^{\mathbf{m},pub}}\right) + \sum_{\mathbf{m}\notin\tilde{M}_\pi} P(\mathbf{m}) \frac{v_{1}^{\mathbf{m},pub}}{2}\left(1+\frac{v_{1}^{\mathbf{m},pub}}{v_{2}^{\mathbf{m},pub}}\right). \tag{1}$$

Here $\tilde{M}_\pi \subseteq M$ denotes the set of all signal profiles $\mathbf{m}$ for which $v_{1}^{\mathbf{m},pub} \geq v_{2}^{\mathbf{m},pub}$ holds.

4.2 Equilibrium under private information disclosure

Consider now the case of private information disclosure. Upon observing signal profile $\mathbf{m} = (m_1, m_2)$, the designer truthfully reveals m_1 to contestant 1 and m_2 to contestant 2. Thus, private signals create an environment with asymmetric information, where each contestant's message m_i is his privately observed type. To simplify things, let us assume in the analysis of private information disclosure that conditional on realization of either of the asymmetric states (H, L) or (L, H), the signal distribution π is characterized by the same precision parameters (see Figure 2). Although this assumption might not be without loss of generality, it reduces the number of choice variables for the designer and, hence, the dimensionality of her optimization problem. Moreover, as we demonstrate later, even with this restricted choice of the precision parameters, the designer prefers private information disclosure to public. Then, if no such restriction on private policies was imposed, this key result would definitely still hold.

As another simplification, we require the optimal private disclosure rule to be *consistent.* This means that the signal distribution π must be such that for every player i, the

$\mathbf{s_1} = (H,\, H)$

	$m_2 = H$	$m_2 = L$
$m_1 = H$	r_1	$q_1 - r_1$
$m_1 = L$	$q_1 - r_1$	$1 - 2q_1 + r_1$

$\mathbf{s_2} = (H,\, L)$

	$m_2 = H$	$m_2 = L$
$m_1 = H$	$q_2 - r_2$	r_2
$m_1 = L$	$1 - 2q_2 + r_2$	$q_2 - r_2$

$\mathbf{s_3} = (L,\, H)$

	$m_2 = H$	$m_2 = L$
$m_1 = H$	$q_2 - r_2$	$1 - 2q_2 + r_2$
$m_1 = L$	r_2	$q_2 - r_2$

$\mathbf{s_4} = (L,\, L)$

	$m_2 = H$	$m_2 = L$
$m_1 = H$	$1 - 2q_3 + r_3$	$q_3 - r_3$
$m_1 = L$	$q_3 - r_3$	r_3

Figure 2: Conditional signal distribution π in case of private information disclosure

probability of receiving signal $m_i = H$ is equal to the prior probability of high valuation α, i.e., $P(m_i = H) = \alpha$.

Definition. A disclosure rule is consistent if and only if $P(m_i = H) = \alpha$ for $i = 1, 2$, that is, $\alpha^2 q_1 + \alpha(1-\alpha) + (1-\alpha)^2(1-q_3) = \alpha$.

Note that the consistency condition holds for any $\alpha \in (0,1)$, when the disclosure of symmetric states is perfectly precise: $q_1 = q_3 = 1$, so that also $r_1 = r_3 = 1$. However, the condition also holds for other q_1 and q_3, provided that α is defined appropriately.[16]

Having received signal m_i from the designer, each player i updates his beliefs about own valuation and the opponent's type m_{-i} as follows:[17]

$$P(v_i = H|\, m_i = H,\, \pi) = \frac{\alpha^2 q_1 + \alpha(1-\alpha) q_2}{\alpha}, \tag{2}$$

$$P(v_i = H|\, m_i = L,\, \pi) = \frac{\alpha^2(1-q_1) + \alpha(1-\alpha)(1-q_2)}{1-\alpha}, \tag{3}$$

and

$$P(m_{-i} = H|\, m_i = H,\, \pi) = \frac{\alpha^2 r_1 + 2\alpha(1-\alpha)(q_2 - r_2) + (1-\alpha)^2(1 - 2q_3 + r_3)}{\alpha}, \tag{4}$$

$$P(m_{-i} = H|\, m_i = L,\, \pi) = \frac{\alpha^2(q_1 - r_1) + \alpha(1-\alpha)(1 - 2q_2 + 2r_2) + (1-\alpha)^2(q_3 - r_3)}{1-\alpha} \tag{5}$$

[16] We note, again, that despite these restrictions on the signal distribution, our results in the next section imply that the optimal information disclosure with private signals ($\tau = private$) delivers higher expected aggregate effort than the best public disclosure rules ($\tau = public$ and $\tau = none$).

[17] These probabilities are computed by Bayes rule using the conditional distribution π in Figure 2 and the prior distribution ξ. As in case of public disclosure, these beliefs are Bayes-plausible, that is, for every player i, the expected posterior probability of $v_i = H$ equals the prior probability α.

Let us denote the last two conditional probabilities by $P_{H|H}$ and $P_{H|L}$, respectively. Also, define $P_{L|H} = 1 - P_{H|H}$ and $P_{L|L} = 1 - P_{H|L}$. Beliefs in (2)–(3) determine the expected prize valuation of player i, conditional on message m_i:

$$E\left(v_i|\, m_i,\, \pi\right) \equiv v_i^m = P\left(v_i = H|\, m_i,\, \pi\right) H + P\left(v_i = L|\, m_i,\, \pi\right).$$

From this equation it is clear that the type of player i determined by message m_i, can be equivalently defined by his expected valuation v_i^m:

$$m_i = H \text{ if and only if } v_i^m = v_i^H.$$

Moreover, since $v_1^H = v_2^H$ and $v_1^L = v_2^L$, we can drop the player subscript i. Therefore, we will refer to v_i^H and v_i^L as *types* v^H and v^L of player i.

In the analysis, we focus on the case where $v^H \geq v^L$ holds. This describes a situation where receiving signal H means good news and results in higher expected valuation. In other words, we require the designer to stick to signal distributions that preserve the relationship between valuations H and L. Technically, this means

$$P\left(v_i = H|\, m_i = H,\, \pi\right) \geq P\left(v_i = H|\, m_i = L,\, \pi\right) \Leftrightarrow q_1 \geq 1 - \frac{1-\alpha}{\alpha} q_2.$$

To characterize the contestants' equilibrium behavior, we employ the results of Liu and Chen (2016). They provide a closed-form solution for both monotonic and non-monotonic symmetric Bayesian Nash equilibria of an all-pay auction with correlated types. In our setting where types are players' expected valuations, or messages that they observe privately, the relevant correlation is between the contestants' private signals. It is inherent in the signal distribution π and equal to

$$\rho = P_{H|H} - P_{H|L} = P_{L|L} - P_{L|H}.$$

Similarly to Liu and Chen (2016), we define two types of equilibrium. If messages m_1 and m_2 are independent or mildly correlated (namely, $\rho = 0$ or slightly positive/negative), the unique equilibrium is monotonic. With sufficiently correlated messages ($\rho \gg 0$ or $\rho \ll 0$), the equilibrium becomes non-monotonic.[18] The uniqueness of the monotonic equilibrium is guaranteed by condition $\frac{v^H}{v^L} \geq \max\{\frac{P_{L|L}}{P_{L|H}}, \frac{P_{H|L}}{P_{H|H}}\}$ that holds in this case and is equivalent to Condition M (implying uniqueness) in Siegel (2014).[19] The uniqueness of the non-monotonic

[18]Each equilibrium features mixed strategies. It is defined as monotonic if different types of a contestant randomize on non-overlapping intervals. Otherwise, the equilibrium is called non-monotonic.

[19]Chi et al. (2019) also prove that the monotonic equilibrium is unique.

equilibrium is established by Chi et al. (2019) and Chen (2021b) for the cases of sufficiently positive and sufficiently negative correlation, respectively. Each of the cases – with mild, sufficiently positive and sufficiently negative correlation – corresponds to respective intervals for $\frac{P_{L|L}}{P_{L|H}}$ and $\frac{P_{H|L}}{P_{H|H}}$.[20] Using Proposition 1 of Liu and Chen (2016), we now describe the value of the ex ante expected aggregate effort that is generated in each equilibrium. The derivations are provided in Appendix A-3.

Proposition 1. *If correlation between the two messages m_1 and m_2 is zero or mild, i.e., $\frac{v^H}{v^L} \geq \frac{P_{L|L}}{P_{L|H}}$ and $\frac{v^H}{v^L} \geq \frac{P_{H|L}}{P_{H|H}}$, the equilibrium is monotonic. The ex ante expected aggregate effort amounts to:*

$$J^M = (1+\alpha)\, P_{L|L} v^L + \alpha P_{H|H} v^H$$

If correlation between m_1 and m_2 is sufficiently positive, i.e., $\frac{v^H}{v^L} < \frac{P_{L|L}}{P_{L|H}}$, or sufficiently negative, i.e., $\frac{v^H}{v^L} < \frac{P_{H|L}}{P_{H|H}}$, the equilibrium is non-monotonic. The ex ante expected aggregate effort in case of positive correlation is equal to:

$$J^{NM,+} = \frac{v^H \left(v^L \left(P_{L|L} - P_{L|H}\right) + \alpha \left(v^H - v^L\right)\right)}{P_{H|H} v^H - P_{H|L} v^L},$$

and in case of negative correlation it is equal to:

$$J^{NM,-} = \frac{v^L \left(v^H - v^L\right)\left(P_{L|L} - (1-\alpha)\right)}{P_{L|H} v^H - P_{L|L} v^L} + v^L.$$

In all equilibria, a contestant of type L gets zero expected payoff:

$$u_L^M = u_L^{NM,+} = u_L^{NM,-} = 0,$$

while a contestant of type H receives an expected payoff given by:

$$u_H^M = P_{L|H} v^H - P_{L|L} v^L,$$

$$u_H^{NM,+} = 0, \;\; u_H^{NM,-} = v^H - v^L.$$

Note that Proposition 1 does not immediately imply the ranking of the three equilibria in terms of the expected aggregate effort they generate. This depends on the designer's choice of the conditional signal distribution π, that is, on the exact values of $\{q_j, r_j\}_{j=1}^3$ (see Figure 2). In our search for the optimal private disclosure rule we proceed in two steps. First,

[20] Indeed, $\frac{P_{L|L}}{P_{L|H}} = \frac{P_{H|L}}{P_{H|H}} = 1$ holds if and only if the contestants' types are independent, and any deviation from this corresponds to the presence of non-zero correlation.

we study an optimal choice of the signal distribution π for each of the described equilibria separately. Second, we compare the expected aggregate effort associated with each choice and define the best signal distribution under private information disclosure.

5 The optimal signal distribution

In this section, we solve for the optimal signal distribution. First, we state a general result that does not depend on the type of information disclosure τ. Since the contestants are ex ante identical and have no private information, the designer chooses her optimal signal distribution to extract the full surplus.[21]

Lemma 1. *For any type of information disclosure τ, an optimal information technology leaves no surplus to either of the contestants in any of the induced equilibria (see Sections 4.1 and 4.2), i.e., $u_i = 0$ for any $i = 1, 2$.*

The formal proof of this result is rather tedious and, therefore, presented in Supplementary Appendix 2. However, the main idea is simple and is a proof by contradiction. In case of private information disclosure, we consider the equilibria described by Proposition 1 and show that under the optimal signal distribution, not only a contestant of type L but also a contestant of type H always receives a zero expected payoff. This means that $u_H^M \equiv P_{L|H}v^H - P_{L|L}v^L = 0$ in the monotonic equilibrium, and $u_H^{NM,-} \equiv v^H - v^L = 0$ in the non-monotonic equilibrium with sufficiently negative signal correlation. To show this, we first write down the designer's constrained optimization problem and the corresponding Kuhn-Tucker optimality conditions for parameters $\{q_j, r_j\}_j$. Then we suppose, by contradiction, that there exists a solution of the designer's problem such that the above equalities do not hold. By checking all possibilities, we demonstrate that such assumption always leads to an inconsistency either in one of the constraints or in the first-order conditions. Thus, any optimal private disclosure policy that generates monotonic equilibrium or non-monotonic equilibrium with sufficiently negative correlation, must deliver zero expected payoffs to both

[21] In Chen (2021b), the signal that maximizes players' *expected payoff* works either through "unleveling the playing field" for the contestants or through the "information rent" channel. Here, contestants do not have any private information and hence, receive no information rent. This is the result of Lemma 1. Later we will show that the channel through which the optimal signals work in our setting, where the designer maximizes *expected aggregate effort*, is "leveling the playing field".

types of a contestant.[22]

Under public disclosure the argument is also straightforward. In this case, the expected equilibrium payoffs of both contestants are zero if and only if $v_i^{\mathbf{m},pub} = v_{-i}^{\mathbf{m},pub}$ for any message $\mathbf{m} \in M \equiv \{HH, LL, HL, LH\}$ (see Section 4.1). Then the key observation is that in the designer's objective function (1), both the term $\left(1 + \frac{v_2^{\mathbf{m},pub}}{v_1^{\mathbf{m},pub}}\right)$ for signal profiles $\mathbf{m}$ such that $v_1^{\mathbf{m},pub} > v_2^{\mathbf{m},pub}$, and the term $\left(1 + \frac{v_1^{\mathbf{m},pub}}{v_2^{\mathbf{m},pub}}\right)$ for signal profiles $\mathbf{m}$ such that $v_1^{\mathbf{m},pub} \leq v_2^{\mathbf{m},pub}$ increase as $v_1^{\mathbf{m},pub}$ and $v_2^{\mathbf{m},pub}$ get closer to each other – the upper bound of 2 is achieved when $v_1^{\mathbf{m},pub} = v_2^{\mathbf{m},pub}$. This means that whenever $v_1^{\mathbf{m},pub} \neq v_2^{\mathbf{m},pub}$, there is potentially a scope for improving the designer's payoff. We develop this argument carefully and show that, indeed, as soon as parameters $\{q_j, r_j\}_j$ are such that the equality $v_1^{\mathbf{m},pub} = v_2^{\mathbf{m},pub}$ does not hold for all messages $\mathbf{m}$, there exists a deviation to other parameters within the feasible set that strictly improves the designer's ex ante expected payoff.

Equipped with Lemma 1, we can now characterize an optimal signal distribution under public and under private disclosure and deduce a globally optimal disclosure policy.

5.1 Public information disclosure

By Lemma 1, an optimal signal distribution under public disclosure must induce $v_1^{\mathbf{m},pub} = v_2^{\mathbf{m},pub}$ for any message $\mathbf{m} \in M$ or, equivalently, $P(v_1 = H|\,\mathbf{m}, \pi) = P(v_2 = H|\,\mathbf{m}, \pi)$. This equality is true for any signal $\mathbf{m} \in M$ if and only if the following three conditions hold:[23]

$$q_2^{pub} - r_2^{pub} = q_3^{pub} - r_3^{pub}, \tag{6}$$

$$r_2^{pub} = 1 - 2q_3^{pub} + r_3^{pub}, \tag{7}$$

$$r_3^{pub} = 1 - 2q_2^{pub} + r_2^{pub}. \tag{8}$$

In fact, any two of these conditions imply the third. For example, from equations (6) and (7) it follows that $q_2^{pub} = 1 - q_3^{pub}$, or $q_3^{pub} = 1 - q_2^{pub}$. But this turns equation (6) into $r_3^{pub} = 1 - 2q_2^{pub} + r_2^{pub}$, which is exactly (8). Thus, under public disclosure any two of the conditions (6) – (8) are necessary for an optimal signal distribution π. Moreover, it is easy to see that they are also sufficient for an optimum because the designer's objective function in

[22] Note that when the correlation between private messages is sufficiently positive, both types' (and hence both players') equilibrium payoffs are identically zero under *any* signal distribution, even if it is not optimal.

[23] This follows immediately from the definition of probabilities $P(v_i = H|\,\mathbf{m}, \pi)$ in the beginning of Section 4.1 and the assumption that $\alpha \in (0, 1)$.

(1) achieves its upper bound on the set of feasible parameters when these conditions hold.[24]

Proposition 2. *Under public disclosure (τ = public), an optimal signal distribution $\pi\left(\{q_j^{pub}, r_j^{pub}\}_{j\in 1:4}, \tau\right)$ is characterized by any two of the conditions* (6) – (8)*, that is, it generates symmetric beliefs, where $P\left(v_1 = H|\, \mathbf{m},\, \pi\right) = P\left(v_2 = H|\, \mathbf{m},\, \pi\right)\ \forall\, \mathbf{m} \in M$. The ex ante expected aggregate effort associated with the optimal signal distribution is equal to $J^{pub} = \alpha H + (1-\alpha)$.*

The interpretation of this result is the following. By definition of distribution π in Figure 1, conditions (6) – (8) make the probability matrices corresponding to states $\mathbf{s_2} = (H, L)$ and $\mathbf{s_3} = (L, H)$ identical, without imposing any further restrictions on the probability matrices for states $\mathbf{s_1} = (H, H)$ and $\mathbf{s_4} = (L, L)$. This means that for any signal realization (m_1, m_2), the asymmetric states (H, L) and (L, H) receive equal weight in contestants' beliefs. Such policy smooths out the asymmetry in contestants' perceptions of their own and competitor's valuations. This is optimal from the designer's perspective because if the contestants believe that their prize valuations are different, they exert the lowest possible effort.[25] In particular, an information technology with precise revelation of *every* state is never optimal, because at least in asymmetric states, a noisy signal is better.

Note that despite the symmetry in the contestants' expected valuations under this optimal public disclosure rule, they may not assign high belief to the fact that the state is actually symmetric. This depends on the signal distribution parameters for the states (H, H), (L, L) and the exact signal realization. For example, if $r_1^{pub} = r_4^{pub} = 1$ and the received signal profile is (H, L) or (L, H), then both contestants are *certain* that the state is *not* symmetric. But in that asymmetric state they assign equal probability to the valuation being high for either of them.

Another immediate observation is that the optimal signal distribution under public disclosure is not unique. In fact, any distribution that results in symmetrically updated beliefs, where for any signal $\mathbf{m}$ each contestant is equally likely to have a high prize valuation, is optimal. Here we provide two examples of optimal public disclosure rules. The first one is equivalent to complete nondisclosure, or concealment. The second features

[24]Simple algebra implies that this upper bound is $\alpha H + (1-\alpha)$. The details are available from the authors.

[25]The latter is easy to verify given the contestants' strategies under public disclosure: the aggregate effort is highest when contestants prize valuations are the same (particularly if they are high), and it is lowest when the valuations are different.

precise revelation of the symmetric states (H, H) and (L, L) and uninformative signal when prize valuations are asymmetric.

Example 1. ***Concealment***

Suppose the designer chooses to send signals that are completely uninformative about the state: $r_j^{pub} = \frac{1}{4}$ and $q_j^{pub} = \frac{1}{2}$ for any $j \in 1 : 4$. Then, upon receiving any signal $\mathbf{m}$, contestants stay with their (symmetric) prior. It is easy to see that the expected aggregate effort in this case is, indeed, equal to $J^{pub} = \frac{v_1^m}{2} + \frac{v_1^m}{2} = v_i^m = \alpha H + (1 - \alpha)$.

Example 2. ***Partially precise information revelation***

Another policy that respects symmetric belief updating is to disclose precisely both symmetric states and convey no information otherwise: $r_1^{pub} = r_4^{pub} = q_1^{pub} = q_4^{pub} = 1$ and $r_2^{pub} = r_3^{pub} = q_2^{pub} = q_3^{pub} = \frac{1}{2}$. The expected aggregate effort generated by this policy is $\alpha^2 H + 2\alpha(1-\alpha)\left(\frac{H}{2} + \frac{1}{2}\right) + (1-\alpha)^2$, which is exactly equal to $\alpha H + (1 - \alpha)$.

However, it turns out that even the optimal public disclosure regime is never globally optimal, as there exists a private disclosure rule that delivers a higher payoff to the designer.

Proposition 3. *Public information disclosure is never optimal. The optimal information technology features private disclosure, $\tau^* = private$.*

The intuition for this result is that private messages communicate less information to the contestants, which leaves the designer more room for belief manipulation. In particular, in the next section we will show that by means of optimal private disclosure, the designer is able to make contestants assign a sufficiently high belief to a symmetric state for *any* private message, which is not possible under public disclosure. This leads to the highest expected aggregate effort.

5.2 Private information disclosure

To find an optimal signal distribution π under private disclosure, we analyze the designer's optimal choice of a signal distribution for each of the equilibria in the corresponding contest game separately (see Proposition 1). First, we derive an optimal signal distribution that supports the monotonic equilibrium. Then, we consider optimal distributions that induce non-monotonic equilibria (with positive and negative signal correlation) and show that the designer's payoff in this case is strictly lower than under optimal distribution supporting the monotonic equilibrium. Thus, private disclosure with strongly negatively or positively correlated signals is never the designer's best choice. We provide a detailed discussion of this case in Appendix A-2 and we focus here on what turns out to be the best signal distribution under private information disclosure – the optimal policy generating the monotonic equilibrium. In view of Proposition 3, this distribution will also determine the global optimum in our model.

An optimal signal distribution that supports the monotonic equilibrium is a solution to the following maximization problem:

$$\max_{\pi=\{q_j, r_j\}_{j=1}^{3}} \left\{J^M \equiv (1+\alpha) P_{L|L} v^L + \alpha P_{H|H} v^H\right\} \tag{9}$$

$$\text{s. t. } \alpha^2 q_1 + \alpha(1-\alpha) + (1-\alpha)^2 (1-q_3) = \alpha \tag{10}$$

$$q_1 \geq 1 - \frac{1-\alpha}{\alpha} q_2 \tag{11}$$

$$v^H P_{L|H} - v^L P_{L|L} \geq 0 \tag{12}$$

$$v^H P_{H|H} - v^L P_{H|L} \geq 0 \tag{13}$$

$$q_j \geq r_j \tag{14}$$

$$r_j \geq \max\{0,\, 2q_j - 1\} \tag{15}$$

$$q_j,\, r_j \in [0,\, 1],\, j = 1, .., 3 \tag{16}$$

The set of constraints includes the consistency requirement in (10), the inequality stating that $v^H \geq v^L$ in (11),[26] two "monotonicity" conditions to sustain a monotonic equilibrium in (12)–(13), and the feasibility constraints ensuring that the signal distribution π is well defined in (14)–(16).

Lemma 2 formulates a general property of any solution to this problem.

Lemma 2. *Under private disclosure, an optimal signal distribution must satisfy* $P_{L|H} v^H = P_{L|L} v^L$*, i.e., condition* (12) *binds.*

The proof follows immediately if we recall from Lemma 1 that the expected payoff of both contestant types must be zero, and $u_H^M = P_{L|H} v^H - P_{L|L} v^L$ by Proposition 1. The fact that condition (12) binds implies that private signals of the contestants must be (mildly)

[26] Recall that this is the assumption we made in Section 4.2.

positively correlated. This benefits the designer as it makes the contestants give less weight to asymmetric states, leading to higher expected aggregate effort.

Note that Lemma 2 gives a necessary but not a sufficient condition for an optimum. Thus, to find an optimal signal distribution, it is not enough, in general, to characterize all the distributions that induce the monotonic equilibrium and allow the designer to extract the full surplus. Actually, finding an optimum requires solving the constrained optimization problem in (9) - (16), where Lemma 2 turns out to be very useful.

Proposition 4 describes an optimum for the case when the prior probability α of having a high valuation is above a certain threshold: the optimal signal distribution is such that the signals are perfectly precise in the symmetric states $(H,\ H)$ and $(L,\ L)$, but noisy when the states are asymmetric.[27]

Proposition 4. *For* $\alpha \geq \max\left\{0,\ \frac{H-3}{3(H-1)}\right\}$*, the optimal signal distribution* $\pi\left(\{q_j, r_j\}_{j\in 1:3}, \tau = private\right)$ *is characterized by such precision parameters* $\{q_j, r_j\}_{j\in 1:3}$ *that generate a small but strictly positive correlation between private messages (condition* (12) *binds), perfectly precise signals in case of the symmetric states* $(H,\ H)$ *and* $(L,\ L)$ *and imprecise signals in case of the asymmetric states* $(H,\ L)$ *and* $(L,\ H)$*. Specifically,* $q_1 = r_1 = q_3 = r_3 = 1$*,* $q_2 = \min\{\hat{q}_2,\ \bar{q}_2\}$ *and* $r_2 = q_2 - \frac{(1-\alpha)\left(v^H - v^L\right)}{2((1-\alpha)v^H + \alpha v^L)} \equiv \hat{r}_2(q_2) > 0$*, where* $\bar{q}_2$ *and* $\hat{q}_2$ *are functions of* H *and* α *defined in Appendix A-3.*

Intuitively, the policy of Proposition 4 allows the designer to keep the competition between contestants even, which, in line with our results on public disclosure, induces the highest ex ante expected aggregate effort. Upon receiving any private message, $m_i = H$ or $m_i = L$, contestant i is made to believe that conditional on the prior, the state is most likely to be symmetric: message H has probability 1 in state (H, H), and message L has probability 1 in state (L, L). There is also a probability that the state is asymmetric but (a) the probability of any private signal, H or L, in such state is less than 1, so the weight of such states in contestants' beliefs is lower, and (b) each contestant is uncertain whether the state is actually (H, L) or (L, H). Thus, whatever the message, the expected valuations of contestants are

[27] Due to non-linearity and multi-dimensionality of the constrained optimization problem in (9) - (16), finding all solutions analytically does not appear feasible. However, the signal distribution of Proposition 4 delivers not just a local but the unique global maximum to J^M in this range of α's (across all distributions inducing the monotonic equilibrium). This is confirmed by numerical simulations in Supplementary Appendix 1.

similar, *and* they assign high probability to the fact that the state is actually symmetric. Positive correlation between contestants' messages is also a step towards "leveling the playing field" for the contestants, particularly given that the prior is strong (α is sufficiently high), that is, high prize valuations are likely.[28]

This signal structure, together with the fact that only one "dimension" of the signal profile is revealed to each contestant, is also the reason why private disclosure dominates public. By revealing a two-dimensional message under public disclosure, the designer has less power over contestants' beliefs, and there is no way she can make them assign a relatively high probability to symmetric states under *any* signal. For example, if she chooses precision parameters r_1^{pub} and r_4^{pub} (in the probability matrices for the symmetric states) close to 1, as under optimal private disclosure policy,[29] this will make states (H, H) and (L, L) quite likely (or even certain) to the contestants *only if* the signal profile is actually (H, H) or (L, L). If the signal profile is (H, L) or (L, H), the contestants will know that there is only a very small (or no) chance that the state is symmetric.[30]

To gain some insights into how an optimal private disclosure rule may look like for a range of α that are below the threshold in Proposition 4, in Appendix A-1 we compare the optimal signal distribution of Proposition 4 with its counterpart under which the asymmetric states (H, L) and (L, H) are *never* revealed precisely, that is, where $r_2 = 0$. We show that such signal distribution is a good candidate for an optimum because the resulting ex ante expected aggregate effort is higher than under the optimal policy of Proposition 4 (see Proposition 5 in Appendix A-1). Moreover, through numerical simulations in Supplementary Appendix 1 we show that the signal distributions of Propositions 4 and 5 are, in fact, the *only optimal distributions* for the specified ranges of α and beyond. A different disclosure policy may lead to a higher expected aggregate effort only when α is small enough and valuations H and $L \equiv 1$ are sufficiently different.

[28]Recall that the highest effort is exerted when contestants know that both of them have high valuation.

[29]She can do this because optimal public disclosure imposes restrictions only on the probability matrices corresponding to the asymmetric states.

[30]Vice versa, if she chooses precision parameters r_1^{pub} and r_4^{pub} close to 0, then depending on q_1^{pub} and q_4^{pub}, this may make states (H, H) and (L, L) quite likely to the contestants *only if* the signal profile is actually asymmetric (when q_1^{pub} and q_4^{pub} are high enough), or *only if* the signal profile is symmetric (when q_1^{pub} and q_4^{pub} are low).

5.3 Globally optimal private disclosure rule

Summing up the results of our analysis, we conclude that the globally optimal information technology, that achieves the highest ex ante expected aggregate effort, has $\tau^* = private$, and its signal distribution π^* induces the monotonic equilibrium of the contest game. Under this policy, private signals are slightly positively correlated, and they are perfectly precise if and only if the state is symmetric. The results of numerical simulations in Supplementary Appendix 1 confirm that no other private disclosure rule outperforms such policy at *any* α if heterogeneity in prize valuations is mild or moderate. A deviation from this policy can be profitable only when heterogeneity in prize valuations is sufficiently strong, while the prior probability α of a high valuation is low.

6 Insights for the case with ex ante informed contestants

Let us now relax the central assumption of our model that the contestants do not have any private information about their prize valuations. To do this in the simplest possible way, suppose that they receive imprecise information about their own and the opponent's prize valuations in the form of a public signal, while the designer only knows the distribution from which this signal is drawn. In this setting we check the robustness of our key results. Formally, let $\mathbf{m^0} = (m_1^0, m_2^0) \in M$ denote a signal profile about the prize valuations that is observed by both contestants but not by the designer. Let π^0 be the distribution from which this signal profile is drawn. We define π^0 in the same way as π on Figure 1, where instead of parameters $\{q_j, r_j\}_{j\in 1:4}$, π^0 is fully characterized by four parameters $\left\{q_j^0, r_j^0\right\}_{j\in 1:4}$ (a pair for each state). Having received a signal from the designer (public, private or none) and an "own" public signal $\mathbf{m^0}$, the contestants update their beliefs and choose the effort levels simultaneously.

The equilibrium analysis of the contest game under both public and private information disclosure by the designer is, in fact, not very different from the analysis in Section 4. But the characterization of the optimal disclosure rules in case when the contestants have some information of their own is much more complicated. First, the result of Lemma 1 is no longer valid. Intuitively, in the new setting the designer does not fully control the information that is available to the contestants, and thus, extracting the full surplus is, in general, no longer possible. Second, the designer's ability to persuade the contestants is limited by the informativeness of $\mathbf{m^0}$. For example, in the extreme case when the contestants receive perfectly precise own signals, i.e., $q_j^0 = r_j^0 = 1$ for any $j \in 1:4$, the designer's message is ignored.However, what we can show is that as long as the contestants' own signal is sufficiently uninformative, the key take-away message from our baseline model continues to hold. Namely, the designer still has the incentive to choose a signal distribution that keeps the competition as symmetric as possible for any pair of messages $\mathbf{m}$, $\mathbf{m^0}$. Moreover, there exists a private disclosure rule that allows the designer to achieve a higher expected aggregate effort than any public disclosure rule.

In case of public information disclosure by the designer, the equilibrium analysis mimics the one in Section 4.1. The main difference is that instead of just four probabilities $P(v_i = H|\, \mathbf{m}, \pi)$, we now have to consider sixteen such probabilities (one for each possible combination of $\mathbf{m}$ and $\mathbf{m^0}$). The expected prize valuation of player i, conditional on $\mathbf{m}$ and $\mathbf{m^0}$, is then given by:

$$E\left(v_i|\, \mathbf{m}, \mathbf{m^0}, \pi, \pi^0\right) \equiv v_i^{\mathbf{m},\mathbf{m^0},pub} = P\left(v_i = H|\, \mathbf{m}, \mathbf{m^0}, \pi, \pi^0\right) H + P\left(v_i = L|\, \mathbf{m}, \mathbf{m^0}, \pi, \pi^0\right).$$

Since both signal profiles $\mathbf{m}$ and $\mathbf{m^0}$ are observed by both contestants, the game again represents a complete information all-pay auction. Thus, the Nash equilibrium strategies are analogous to those described before, and the ex ante expected aggregate effort of the contestants J^{pub} is equal to

$$J^{pub} \equiv \sum_{(\mathbf{m},\mathbf{m^0}) \in \tilde{M}_{\pi,\pi^0}} P\left(\mathbf{m},\mathbf{m^0}\right) \frac{v_2^{\mathbf{m},\mathbf{m^0},pub}}{2} \left(1 + \frac{v_2^{\mathbf{m},\mathbf{m^0},pub}}{v_1^{\mathbf{m},\mathbf{m^0},pub}}\right) + \\ + \sum_{(\mathbf{m},\mathbf{m^0}) \notin \tilde{M}_{\pi,\pi^0}} P\left(\mathbf{m},\mathbf{m^0}\right) \frac{v_1^{\mathbf{m},\mathbf{m^0},pub}}{2} \left(1 + \frac{v_1^{\mathbf{m},\mathbf{m^0},pub}}{v_2^{\mathbf{m},\mathbf{m^0},pub}}\right),$$

where $\tilde{M}_{\pi,\pi^0} \subseteq M \times M$ denotes the set of all *combinations* of signal profiles $\mathbf{m}$, $\mathbf{m^0}$ for which $v_1^{\mathbf{m},\mathbf{m^0},pub} \geq v_2^{\mathbf{m},\mathbf{m^0},pub}$.

Now, despite the fact that Lemma 1 is not applicable in this setting, we can show that the expected aggregate effort J^{pub} does not exceed $\alpha H + (1-\alpha)$, which is its optimal level in our baseline case. Moreover, this upper bound is achieved whenever the contestants believe that their values of winning are the same, i.e.,

$P(v_1 = H|\,\mathbf{m},\, \mathbf{m^0},\, \pi,\, \pi^0) = P(v_2 = H|\,\mathbf{m},\, \mathbf{m^0},\, \pi,\, \pi^0)$ holds. In our baseline model, the symmetry of the contestants' beliefs was generated by three conditions on the parameters of the distribution π. Now, it involves not only the parameters of π, that are chosen by the designer, but also the parameters of π^0, that are beyond the designer's control. As a result, the upper bound on the total expected effort is not achievable, in general.

In case of private information disclosure by the designer, the equilibrium analysis is, again, conceptually similar to the one in Section 4.2, and at least *conditional* on every $\mathbf{m^0} \in M$, a result analogous to that of Proposition 1 can be stated. The characterization of the optimal policy, though, is challenging. First, without Lemma 1 the analysis is more complex. Moreover, the introduction of the contestants' private information makes the problem analytically intractable: the designer's program has no closed-form solution for arbitrary $\left\{q_j^0, r_j^0\right\}_{j\in 1:4}$. However, what is straightforward to show is that as long as the contestants' own signal is sufficiently uninformative, there exists a private disclosure rule by the designer that dominates any public disclosure rule. Also, this private disclosure rule is such that the signal is precise if and only if the contestants' prize valuations are the same.

To see this, consider the extreme case, when the contestants' own message is *completely* uninformative, which requires $q_j^0 = \frac{1}{2}$ and $r_j^0 = \frac{1}{4}$ for any $j \in 1:4$. Intuitively, this makes the designer the only source of information for the contestants and essentially brings us back to the original model with no private information on the contestants' side. Then all our baseline results apply, and in particular, private disclosure dominates public. Let us now perturb parameters $\left\{q_j^0, r_j^0\right\}_{j\in 1:4}$ a little bit, so that the contestants' signal is still sufficiently uninformative. Then, as soon as the monotonicity conditions analogous to (12) – (13) still hold, the value of the ex ante expected aggregate effort induced by the optimal private disclosure rule for the case of $q_j^0 = \frac{1}{2}$ and $r_j^0 = \frac{1}{4}$ must still exceed the upper bound on J^{pub}.[31] This means that if the contestants' own information is sufficiently imprecise, then the designer still prefers private information disclosure to public.

[31] This is due to the continuity of the designer's objective function in (9) with respect to $\left\{q_j^0, r_j^0\right\}_{j\in 1:4}$, which, in turn, follows from the continuity of the conditional probabilities $P_{L|L,m^0}$, $P_{H|H,m^0}$ and the expected prize valuations v^{L,m^0}, v^{H,m^0} with respect to these parameters.

7 Conclusion

In this paper we study, whether the designer of a contest with unknown prize valuations whose objective is to stimulate as much competitive efforts as possible, can gain by choosing the information technology, that is, the precision of the signal about valuations in each state and the way of communicating the signal to the contestants. A combination of three important features distinguishes our model from other studies on the optimal information disclosure/feedback in contests: (i) a broad range of disclosure rules available to the designer, including private and partial disclosure, (ii) an assumption that both contestants initially have no private knowledge about their values of winning, and (iii) the continuous space for the contestants' possible effort choices.

We find that the highest expected aggregate effort is obtained when the designer employs private signals, while public information disclosure is never optimal. Intuitively, the reason for this is that private signals leave the designer more room for belief manipulation and allow her to level the competition among the contestants more effectively. This is achieved through the policy where private signals are (slightly) positively correlated (the monotonicity condition binds) and reveal the true contestants' values of winning precisely if and only if these values are the same. Finally, although public information disclosure is not optimal, when it has to be used (as may be the case in many real-life contests) the optimal signal distribution must induce symmetric beliefs. This is, again, in line with a common observation in the contest literature that the highest aggregate effort is achieved when the competitors perceive their values of winning, and hence the incentives to compete, as equal.

Appendix

A-1 Candidate for an optimal signal distribution inducing the monotonic equilibrium at lower α

Proposition 5 describes a signal distribution that for a range of α lower than in Proposition 4 and a range of sufficiently different H and $L \equiv 1$ generates the monotonic equilibrium with higher expected aggregate effort than under the optimal policy of Proposition 4. This is, therefore, a candidate for an optimum for such lower values of α, which it, indeed, turns out to be according to the numerical simulations in Supplementary Appendix 1.

This signal distribution is a close counterpart of the optimal distribution in Proposition

4. The only difference is that $r_2 = 0$ holds, that is, the asymmetric states (H, L) and (L, H) are *never* revealed precisely. In what follows we denote by $J^M(q_1, r_1, q_2, r_2, q_3, r_3)$ the value of the ex ante expected aggregate effort (in the monotonic equilibrium) when the precision parameters are fixed at $\{q_i,\, r_i\}_{i=1}^3$.

Proposition 5. *Assume $q_1 = r_1 = q_3 = r_3 = 1$ and monotonicity condition (12) binds. Then, for $H \geq \left(9 + 4\sqrt{5}\right)$, there exist $\tilde{\alpha}_1 > 0$ and $\tilde{\alpha}_2 \in \left(\tilde{\alpha}_1,\, \frac{H-3}{3(H-1)}\right)$ such that for any $\alpha \in (\tilde{\alpha}_1,\, \tilde{\alpha}_2)$, a counterpart of the signal distribution in Proposition 4 where the asymmetric states $(H,\, L)$ and $(L,\, H)$ are never revealed precisely (or $r_2 = 0$) induces a higher ex ante expected aggregate effort:*

$$J^M\left(1,\, 1,\, \underline{q}_2,\, 0,\, 1,\, 1\right) > J^M\left(1,\, 1,\, \min\left\{\hat{q}_2,\, \bar{q}_2\right\},\, \hat{r}_2\left(\min\left\{\hat{q}_2,\, \bar{q}_2\right\}\right),\, 1,\, 1\right),$$

where $\underline{q}_2 = \frac{H-3-3\alpha(H-1)}{2(1-2\alpha)(H-1)}$ and $\hat{q}_2,\, \bar{q}_2,\, \hat{r}_2(q_2)$ are defined in Proposition 4.

A-2 Optimal signal distributions supporting the non-monotonic equilibria under private disclosure

To understand the properties of the optimal signal distributions that induce non-monotonic equilibria under private disclosure, let us refer to Lemma 1. First, consider the case of a sufficiently negative correlation between contestants' messages, that is, $\frac{v^H}{v^L} < \frac{P_{H|L}}{P_{H|H}}$. By Proposition 1, the expected equilibrium payoff of a contestant of type H is $u_H^{NM,-} = v^H - v^L$. Combining this with Lemma 1, we obtain the following result.

Lemma 3. *Under private disclosure, an optimal signal distribution that supports a non-monotonic equilibrium with negative correlation must satisfy $v^H = v^L$, i.e., condition (11) binds.*

Lemma 3 implies that with negatively correlated messages, the meaning of private signals gets lost, and the case becomes equivalent to public disclosure. In particular, one can show that the ex ante expected aggregate effort of the contestants in this case is exactly the same as under the optimal signal distribution in case of public disclosure, i.e., $J^{NM,-} = \alpha H + (1-\alpha)$. Indeed, since the optimal signal distribution must satisfy $v^H = v^L$, the ex ante expected aggregate effort $J^{NM,-}$ reduces to (see Proposition 1):

$$J^{NM,-} = \frac{v^L\left(v^H - v^L\right)\left(P_{L|L} - (1-\alpha)\right)}{P_{L|H}v^L - P_{L|L}v^L} + v^L = v^L.$$

By substituting $q_1 = 1 - \frac{1-\alpha}{\alpha}q_2$ (which is condition (11) when it binds) into $J^{NM,-} = v^L$, we obtain:

$$J^{NM,-} = \alpha H + (1-\alpha) = J^{pub}.$$

The intuition behind this result is the following. If a signal distribution features sufficiently negative correlation, each contestant believes that most likely, the opponent is of a different type. As a result, both competitors assign more weight to asymmetric signals, and their efforts decline. To mitigate this effect, the best the designer can do is to choose a signal distribution that induces $v^H = v^L$, independently of the exact signal realizations. This equivalence with the case of public disclosure, together with Proposition 3, implies that private disclosure involving strongly negatively correlated signals is never optimal.

Now, if the signals are sufficiently positively correlated, that is, $\frac{v^H}{v^L} < \frac{P_{L|L}}{P_{L|H}}$, Lemma 1 does not provide any new insights because the expected equilibrium payoffs of both contestant types are already zero for *any* signal distribution π (see Proposition 1). However, by solving the designer's optimization problem directly, we find that also in this case v^H cannot be strictly greater than v^L under the optimal signal distribution, and $v^H = v^L$ makes the ex ante expected aggregate effort of the contestants the same as under optimal public disclosure, i.e., $J^{NM,+} = \alpha H + (1-\alpha)$. Thus, strongly positively correlated private signals are also not optimal. We summarize these findings in Proposition 6.

Proposition 6. *Private disclosure with strongly negatively or positively correlated signals is never optimal.*

To get a better idea why even with perfectly aligned messages of the contestants, their ex ante expected aggregate effort is lower than under the optimal disclosure rule that induces the monotonic equilibrium, let us consider the contestants' strategies in the non-monotonic equilibrium with positive correlation:[32]

- Type L randomizes uniformly on $[0, \underline{e}]$ with probability 1, where $\underline{e} \in \left(0, v^H\right)$.
- Type H randomizes on the same interval with probability $p_B^H = \frac{P_{L|L}v^L - P_{L|H}v^H}{P_{H|H}v^H - P_{H|L}v^L}$ and exhausts the remaining probability in $\left[\underline{e}, v^H\right]$.

Using this, we can rewrite the ex ante expected aggregate effort $J^{NM,+}$ (see Proposition 1) as follows:

$$J^{NM,+} = \underline{e} + \alpha v^H (1 - p_B^H).$$

Notice that $J^{NM,+}$ strictly decreases in p_B^H. Then it is intuitive that the designer should reduce this probability and, hence, prompt type H to play in the top interval more often. This would bring the equilibrium play closer to the one in the monotonic equilibrium, where type H chooses his effort in a strictly higher interval than type L. Also, $p_B^H = 0$ holds in the monotonic equilibrium under the optimal signal distribution (where condition (12) binds). This indicates that the designer should, indeed, benefit from moving towards the monotonic equilibrium.[33]

A-3 Proofs

Proof. [Proof of **Proposition 1**] To characterize the equilibrium of a symmetric contest game with private signals, we use Proposition 1 of Liu and Chen (2016). In our setting

$$v^H \equiv H_{LC},\ v^L \equiv L_{LC},\ \text{and}\ V_{LC} = 1$$

[32] See Proposition 1 of Liu and Chen (2016).

[33] This is an informal argument. In general, the fact that $J^{NM,+}$ strictly decreases in p_B^H is not sufficient to prove that the designer will necessarily improve $J^{NM,+}$ by choosing parameters of the signal distribution π so as to set $p_B^H = 0$. This is because the same parameters enter other components of $J^{NM,+}$ – $\underline{e}$ and v^H, – and the effect on them can be suboptimal.

where the LC subscript corresponds to the original model of Liu and Chen (2016). Then, our equilibrium characterization follows immediately if one substitutes v^H and v^L into Proposition 1 of Liu and Chen (2016).

First, consider a monotonic equilibrium that requires $\frac{v^H}{v^L} \geq \frac{P_{L|L}}{P_{L|H}}$ and $\frac{v^H}{v^L} \geq \frac{P_{H|L}}{P_{H|H}}$. In this equilibrium

- A contestant who gets message L, or type L, randomizes on the $[0, \underline{e}]$ interval with probability 1, where $\underline{e} = P_{L|L}v^L$, and
- A contestant who receives message H, or type H, randomizes on the $[\underline{e}, \bar{e}]$ interval with probability 1, where $\bar{e} = \underline{e} + P_{H|H}v^H$

and the ex ante expected aggregate effort reaches

$$J^M = P_{HH}\left(\underline{e} + \bar{e}\right) + 2P_{HL}\left(\frac{\underline{e}}{2} + \frac{\underline{e} + \bar{e}}{2}\right) + P_{LL}\underline{e} = \underline{e} + \left(P_{HH} + P_{HL}\right)\bar{e} = \left(1 + \alpha\right)P_{L|L}v^L + \alpha P_{H|H}v^H$$

The expected equilibrium payoffs the two contestants' types get are equal to

$$u_H^M = P_{H|H}\frac{1}{2}v^H + P_{L|H}v^H - \frac{\underline{e} + \bar{e}}{2} = P_{L|H}v^H - P_{L|L}v^L$$

$$u_L^M = P_{L|L}\frac{1}{2}v^L - \frac{\underline{e}}{2} = 0$$

Second, take a non-monotonic equilibrium with strong positive correlation that arises when $\frac{v^H}{v^L} < \frac{P_{L|L}}{P_{L|H}}$ holds. In this equilibrium

- Type L randomizes on the $[0, \underline{e}]$ interval with probability 1, where $\underline{e} = \frac{v^H v^L\left(P_{L|L} - P_{L|H}\right)}{P_{H|H}v^H - P_{H|L}v^L}$, and
- Type H randomizes on the $[0, \underline{e}]$ interval with probability $p_B^H = \frac{P_{L|L}v^L - P_{L|H}v^H}{P_{H|H}v^H - P_{H|L}v^L}$ and exhausts the remaining bidding probability on the $[\underline{e}, \bar{e}]$ interval, where $\bar{e} = v^H$.

The ex ante expected aggregate effort is

$$J^{NM,+} = P_{HH}\left(\underline{e} + \left(1 - p_B^H\right)\bar{e}\right) + 2P_{HL}\left(\frac{\underline{e}}{2}\left(1 + p_B^H\right) + \left(1 - p_B^H\right)\frac{\underline{e} + \bar{e}}{2}\right) + P_{LL}\underline{e} =$$

$$\underline{e} + \bar{e}\left(P_{HH} + P_{HL}\right)\left(1 - p_B^H\right) = \frac{v^H v^L\left(P_{L|L} - P_{L|H}\right)}{P_{H|H}v^H - P_{H|L}v^L} + \frac{v^H\alpha\left(v^H - v^L\right)}{P_{H|H}v^H - P_{H|L}v^L} =$$

$$\frac{v^H\left(v^L\left(P_{L|L} - P_{L|H}\right) + \alpha\left(v^H - v^L\right)\right)}{P_{H|H}v^H - P_{H|L}v^L}$$

and the expected equilibrium payoffs amount to

$$u_H^{NM,+} = P_{H|H}\frac{1}{2}v^H + P_{L|H}v^H\left(p_B^H\frac{1}{2} + \left(1 - p_B^H\right)\right) - p_B^H\frac{\underline{e}}{2} - \left(1 - p_B^H\right)\frac{\underline{e} + \bar{e}}{2} =$$

$$P_{H|H}\frac{1}{2}v^H + P_{L|H}v^H\frac{1}{2} + P_{L|H}v^H\frac{1}{2}\left(1 - p_B^H\right) - \frac{\underline{e}}{2} - \left(1 - p_B^H\right)\frac{v^H}{2} = \frac{v^H}{2} - P_{H|H}\left(1 - p_B^H\right)\frac{v^H}{2} - \frac{\underline{e}}{2} =$$

$$\frac{v^H}{2}\left[1-\frac{P_{H|H}\left(v^H-v^L\right)+v^L\left(P_{L|L}-P_{L|H}\right)}{P_{H|H}v^H-P_{H|L}v^L}\right]=\frac{v^H}{2}\left[1-\frac{P_{H|H}v^H-P_{H|L}v^L}{P_{H|H}v^H-P_{H|L}v^L}\right]=0$$

$$u_L^{NM,+}=P_{L|L}\frac{1}{2}v^L+P_{H|L}p_B^H\frac{1}{2}v^L-\frac{\underline{e}}{2}=\frac{v^L}{2}\left[P_{L|L}+P_{H|L}\frac{P_{L|L}v^L-P_{L|H}v^H}{P_{H|H}v^H-P_{H|L}v^L}-\frac{v^H\left(P_{L|L}-P_{L|H}\right)}{P_{H|H}v^H-P_{H|L}v^L}\right]=$$

$$\frac{v^Lv^H}{2\left(P_{H|H}v^H-P_{H|L}v^L\right)}\left[P_{L|L}P_{H|H}-P_{H|L}P_{L|H}-\left(P_{L|L}-P_{L|H}\right)\right]=$$

$$\frac{v^Lv^H}{2\left(P_{H|H}v^H-P_{H|L}v^L\right)}\left[P_{L|L}\left(1-P_{L|H}\right)-P_{L|H}\left(1-P_{L|L}\right)-\left(P_{L|L}-P_{L|H}\right)\right]=0$$

Finally, take a non-monotonic equilibrium with strong negative correlation, that is, $\frac{v^H}{v^L}<\frac{P_{H|L}}{P_{H|H}}$. In this equilibrium

- Type L randomizes on the $[0,\,\underline{e}]$ interval with probability $p_B^L=\frac{v^H-v^L}{P_{L|H}v^H-P_{L|L}v^L}$ and exhausts the remaining bidding probability on the $[\underline{e},\,\bar{e}]$ interval, where $\underline{e}=\frac{P_{L|L}\left(v^H-v^L\right)v^L}{P_{L|H}v^H-P_{L|L}v^L}$ and $\bar{e}=v^L$,
- Type H randomizes on the $[\underline{e},\,\bar{e}]$ interval with probability 1.

The ex ante expected aggregate effort achieves

$$J^{NM,-}=P_{HH}\left(\underline{e}+\bar{e}\right)+2P_{HL}\left(\frac{\underline{e}}{2}p_B^H+\left(1+1-p_B^L\right)\frac{\underline{e}+\bar{e}}{2}\right)+P_{LL}\left(\underline{e}+\left(1-p_B^L\right)\bar{e}\right)=$$

$$\underline{e}+\bar{e}\left(\alpha+\left(1-p_B^L\right)(1-\alpha)\right)=\underline{e}+\bar{e}-\bar{e}p_B^L(1-\alpha)=v^L+\frac{\left(v^H-v^L\right)v^L\left(P_{L|L}-(1-\alpha)\right)}{P_{L|H}v^H-P_{L|L}v^L}$$

and the expected equilibrium payoffs are

$$u_H^{NM,-}=P_{H|H}\frac{1}{2}v^H+P_{L|H}v^H\left(p_B^H+\left(1-p_B^H\right)\frac{1}{2}\right)-\frac{\underline{e}+\bar{e}}{2}=$$

$$P_{H|H}\frac{1}{2}v^H+P_{L|H}v^H\frac{1}{2}+P_{L|H}v^H\frac{p_B^H}{2}-\frac{\underline{e}+\bar{e}}{2}=\frac{v^H}{2}\frac{v^L}{2}+P_{L|H}v^H\frac{p_B^H}{2}-\frac{\underline{e}}{2}=$$

$$\frac{v^H}{2}-\frac{v^L}{2}+\frac{1}{2}\left[\frac{P_{L|H}v^H\left(v^H-v^L\right)-P_{L|L}v^L\left(v^H-v^L\right)}{P_{L|H}v^H-P_{L|L}v^L}\right]=v^H-v^L$$

$$u_L^{NM,-}=P_{L|L}\frac{1}{2}v^L+P_{H|L}\left(1-p_B^L\right)\frac{1}{2}v^L-p_B^L\frac{\underline{e}}{2}-\left(1-p_B^L\right)\frac{\underline{e}+\bar{e}}{2}=$$

$$\frac{v^L}{2}-p_B^LP_{H|L}\frac{1}{2}v^L-\frac{\underline{e}}{2}-\left(1-p_B^L\right)\frac{v^L}{2}=p_B^LP_{L|L}\frac{1}{2}v^L-\frac{\underline{e}}{2}=0$$

□

Proof. [Proof of **Proposition 3**] We prove the statement by constructing an example of a private disclosure policy that generates a higher ex ante expected aggregate effort than the best public disclosure policy.

Consider a private disclosure policy that induces a monotonic equilibrium in the contest game and has the property that it is perfectly precise when the state is symmetric and not precise otherwise:

- Take $q_1 = r_1 = q_3 = r_3 = 1$, $q_2 = \frac{1}{2}$ and let r_2 be such that one of the monotonicity conditions supporting the monotonic equilibrium binds: $v^H P_{L|H} = v^L P_{L|L}$.
- With $q_1 = q_2 = 1$, the contestants' types look as follows (see Section 4.2):
$$v^H = \frac{(1+\alpha)\,H + (1-\alpha)}{2} \equiv v^H\,(1,\,1)\,,\; v^L = \frac{\alpha H + (2-\alpha)}{2} \equiv v^L\,(1,\,1)\,.$$
- Then r_2 can be written as:
$$r_2 = \frac{v^L\,(1,\,1)}{2\left(\alpha v^L\,(1,\,1) + (1-\alpha)\,v^H\,(1,\,1)\right)} \equiv \hat{r}_2\left(\frac{1}{2}\right) \in [2q_2 - 1,\, q_2]\,.$$
- With this policy, the ex ante expected aggregate effort J^M (see Proposition 1) as a function of $(q_1, r_1, q_2, r_2, q_3, r_3)$ is equal to:
$$J^M\left(1,\,1,\,\tfrac{1}{2},\,\hat{r}_2\left(\tfrac{1}{2}\right),\,1,\,1\right) = (1+\alpha)\left(1 - 2\alpha\hat{r}_2\left(\tfrac{1}{2}\right)\right) v^L\,(1,\,1) +$$
$$\alpha\left(1 - 2\,(1-\alpha)\,\hat{r}_2\left(\tfrac{1}{2}\right)\right) v^H\,(1,\,1)\,.$$

Now we show that this policy outperforms the best public disclosure rule. Notice that the ex ante expected aggregate effort under optimal public disclosure, $J^{pub} \equiv \alpha H + (1-\alpha)$, can be expressed as follows:
$$J^{pub} \equiv \alpha H + (1-\alpha)\,L = v^L\,(1,\,1)\,(1-\alpha) + \alpha v^H\,(1,\,1)\,.$$
Then the constructed private disclosure policy dominates the optimal public disclosure policy if and only if $J^M\left(1,\,1,\,\frac{1}{2},\,\hat{r}_2\left(\frac{1}{2}\right),\,1,\,1\right) > v^L\,(1,\,1)\,(1-\alpha) + \alpha v^H\,(1,\,1)$, that is:
$$(1+\alpha)\left(1 - 2\alpha\hat{r}_2\left(\frac{1}{2}\right)\right) v^L\,(1,\,1) + \alpha\left(1 - 2\,(1-\alpha)\,\hat{r}_2\left(\frac{1}{2}\right)\right) v^H\,(1,\,1) >$$
$$v^L\,(1,\,1)\,(1-\alpha) + \alpha v^H\,(1,\,1) \Leftrightarrow$$
$$v^L\,(1,\,1)\left(1 - \hat{r}_2\left(\frac{1}{2}\right)\right) - \hat{r}_2\left(\frac{1}{2}\right)\left(\alpha v^L\,(1,\,1) + (1-\alpha)\,v^H\,(1,\,1)\right) > 0 \Leftrightarrow$$
$$v^L\,(1,\,1)\left(\frac{1}{2} - \hat{r}_2\left(\frac{1}{2}\right)\right) > 0.$$
Here, the last inequality holds for any $v^H\,(1,\,1) > v^L\,(1,\,1)$, i.e., any $H > L \equiv 1$. $\square$

Proof. [Proof of **Proposition 4**] Let us write down the Lagrangian for the designer's constrained optimization program (9) – (16) when the monotonic equilibrium is played:
$$L\left(\{q_i,\,r_i\}_{i=1}^3\,,\,\mathscr{M}\right) = J^M\left(\{q_i,\,r_i\}_{i=1}^3\right) + \eta_1\left(v^H P_{L|H} - v^L P_{L|L}\right) +$$
$$\eta_2\left(v^H P_{H|H} - v^L P_{H|L}\right) + \Sigma_{i=1}^3 \lambda_i\,(q_i - r_i) + \Sigma_{i=1}^3 \gamma_i\,(r_i - \max\{0,\,2q_i - 1\}) +$$
$$\sum_{j=1}^3 \phi_j\,(1 - q_j) + \chi\left(q_1 - \left(1 - \frac{1-\alpha}{\alpha} q_2\right)\right) + \omega\left(\alpha^2\,(q_1 - 1) + (1-\alpha)^2\,(1 - q_3)\right)$$

where

- $\mathscr{M} = \left\{\{\lambda_j,\, \gamma_j,\, \phi_j\}_{j=1}^3,\, \chi,\, \omega\right\}$ denotes a set of non-negative Kuhn-Tucker multipliers and
- $J^M\left(\{q_i,\, r_i\}_{i=1}^3\right) \equiv (1+\alpha)\, P_{L|L} v^L + \alpha P_{H|H} v^H$.

If we express q_3 as a function of q_1 from the consistency condition (10) and substitute this into the objective function $J^M(\cdot)$, the system of the first-order conditions with respect to q_1, r_1, q_2, r_2 and r_3 looks as follows:

$$\begin{cases} \frac{\partial L(\cdot)}{\partial r_i} = \frac{\partial J^M(\cdot)}{\partial r_i} + \eta_1 \frac{\partial\left(v^H P_{L|H} - v^L P_{L|L}\right)}{\partial r_i} - \lambda_i + \gamma_i = 0,\; i \in \{1,\, 2,\, 3\} \\ \frac{\partial L(\cdot)}{\partial q_1} = \frac{\partial J^M(\cdot)}{\partial q_1} + \eta_1 \frac{\partial\left(v^H P_{L|H} - v^L P_{L|L}\right)}{\partial q_1} + \lambda_1 + \frac{\alpha^2}{(1-\alpha)^2}\lambda_3 - 2\gamma_1 - 2\frac{\alpha^2}{(1-\alpha)^2}\gamma_3 = 0 \\ \frac{\partial L(\cdot)}{\partial q_2} = \frac{\partial J^M(\cdot)}{\partial q_2} + \eta_1 \frac{\partial\left(v^H P_{L|H} - v^L P_{L|L}\right)}{\partial q_2} - 2\gamma_2 I\left\{q_2 \geq \frac{1}{2}\right\} = 0 \end{cases}$$

We will now construct a (local) optimum by (a) choosing a set of precision parameters and values of Kuhn-Tucker multipliers, and (b) checking that they satisfy all Kuhn-Tucker conditions.[34]

Assume $q_1 = r_1 = q_3 = r_3 = 1$ holds in the optimum. This makes the constraint $q_1 \geq \left(1 - \frac{1-\alpha}{\alpha} q_2\right)$ strict for any $q_2 > 0$, and from the slackness condition it follows that $\chi = 0$. Moreover, the consistency condition is always satisfied for $q_1 = q_3 = 1$, and we impose $\omega = 0$. We also take $\phi_j = 0$ for $j \in \{1,\, 3\}$.

By Lemma 2, the first monotonicity condition binds in the optimum, i.e., $v^H P_{L|H} = v^L P_{L|L}$, which implies that the second monotonicity condition is strict, and so, $\eta_1 \geq 0$, $\eta_2 = 0$. Solving the first monotonicity condition $v^H P_{L|H} = v^L P_{L|L}$ for r_2 delivers:

$$r_2 = q_2 - T(q_2) \equiv \hat{r}_2(q_2),$$

where[35]

$$T(q_2) = \frac{(1-\alpha)\left(v^H(1,\, q_2) - v^L(1,\, q_2)\right)}{2\left((1-\alpha)\, v^H(1,\, q_2) + \alpha v^L(1,\, q_2)\right)} > 0 \text{ for any } v^H(1,\, q_2) > v^L(1,\, q_2).$$

In our case $v^H(1,\, q_2) > v^L(1,\, q_2)$ since condition $q_1 > \left(1 - \frac{1-\alpha}{\alpha} q_2\right)$ holds. Then $r_2 = q_2 - T(q_2)$ implies $r_2 < q_2$, and $\lambda_2 = 0$ follows.

With all the assumptions on parameters made, the set of (possibly inactive) constraints in the designer's optimization problem reduces to:

$$\begin{cases} \hat{r}_2(q_2) \geq 0 & (1.1) \\ \hat{r}_2(q_2) \geq 2q_2 - 1 & (1.2) \end{cases}$$

Next, we will use condition (1.1) and condition (1.2) to define the upper and lower bounds on feasible values of q_2. Then we will select the actual value of q_2 from that interval. It will be equal to an extremum of $J^M(\cdot)$ (i.e., to a solution of the first order-conditions) when it

[34] This local optimum turns out to be global, as confirmed by simulations in Supplementary Appendix 1.

[35] As before, $v^H(q_1, q_2)$ and $v^L(q_1, q_2)$ denote the contestant's types defined in Section 4.2.

is strictly inside the interval. Otherwise it will be equal to the upper bound of the interval (since we will show that J^M is higher at the upper than at the lower bound).

First, let us take a closer look at inequality (1.1). The underlying equation has two real roots with respect to q_2:

$$q_2 = 0 \,\text{and}\, q_2 = \begin{cases} \frac{H-3-3\alpha(H-1)}{2(1-2\alpha)(H-1)} \equiv \underline{q}_2, & \alpha \neq \frac{1}{2} \\ 0, & \alpha = \frac{1}{2} \end{cases}$$

The only case when $\underline{q}_2$ turns out to be feasible (namely, $\underline{q}_2 \in [0,\, 1]$) corresponds to $H > 3$ and $\alpha \in \left[0,\, \frac{H-3}{3(H-1)}\right]$. Otherwise, we obtain either (1) $\underline{q}_2 < 0$ and $\hat{r}_2\left(q_2\right) \geq 0$ for any $q_2 \in [0,\, 1]$ or (2) $\underline{q}_2 > 1$ and $\hat{r}_2\left(q_2\right) \geq 0$ for any $q_2 \in [0,\, 1]$. Thus, we can define the lower bound on q_2 as follows:

- If $H > 3$ and $\alpha \in \left[0,\, \frac{H-3}{3(H-1)}\right]$, then it must be that $q_2 \geq \underline{q}_2$ for $\hat{r}_2\left(q_2\right) \geq 0$ to hold.
- Otherwise, $r_2 = \hat{r}_2\left(q_2\right) \geq 0$ is true for any $q_2 \geq 0$.

Thus, fixing $\alpha \geq \max\left\{0,\, \frac{H-3}{3(H-1)}\right\}$ ensures that the lower bound on q_2 is zero, i.e., $q_2 \geq 0$ must hold in the optimum.

Further, take inequality (1.2) and find the roots of $\{\hat{r}_2\left(q_2\right) \geq 2q_2 - 1\}$ with respect to q_2:

$$q_2 = \begin{cases} \frac{H-3-5\alpha(H-1)\pm\sqrt{D}}{4(H-1)(1-2\alpha)} \equiv q_2^{1,\,2}, & \alpha \neq \frac{1}{2} \\ \frac{2(H+1)}{3H+1}, & \alpha = \frac{1}{2} \end{cases}$$

where

$$D = -7\left(H-1\right)\alpha^2 + 6\left(H^2 - 4H + 3\right)\alpha + H^2 + 10H - 7 > 0\,\forall\,\alpha \in [0,\, 1]$$

For $\alpha = \frac{1}{2}$, there is only one root with respect to q_2, and $\hat{r}_2\left(q_2\right) \geq 2q_2 - 1$ holds for any $q_2 \leq \frac{2(H+1)}{3H+1} < 1$. Next, take the case of $\alpha \neq \frac{1}{2}$. Here, q_2^1 belongs to the $[0,\, 1]$ interval for any $\alpha \in [0,\, 1]$. At the same time, q_2^2 turns out to be negative for $\alpha \in \left[0,\, \frac{1}{2}\right)$ and exceeds 1 for $\alpha \in \left(\frac{1}{2},\, 1\right]$, that is, q_2^2 can never be feasible. Hence, the inequality $\hat{r}_2\left(q_2\right) \geq 2q_2 - 1$ holds if and only if $q_2 \in [0,\, q_2^1]$, and we get the upper bound on q_2:

$$q_2 \leq \bar{q}_2 = \begin{cases} q_2^1, & \alpha \neq \frac{1}{2} \\ \frac{2(H+1)}{3H+1}, & \alpha = \frac{1}{2} \end{cases}$$

Here $\bar{q}_2 > \frac{1}{2}$ for any $\alpha \in [0,\, 1]$. Thus, feasible q_2 satisfies $q_2 \in [0,\, \bar{q}_2]$.

Suppose first that $q_2 \in (0,\, \bar{q}_2)$ in the optimum, i.e., it is strictly inside the interval, so that constraints $q_2 \geq 0$ (or $\hat{r}_2\left(q_2\right) \geq 0$) and $q_2 \leq \bar{q}_2$ (or $\hat{r}_2\left(q_2\right) \geq 2q_2 - 1$) do not bind. Due to slackness conditions, this can be supported with $\gamma_2 = 0$.

Under the fixed parameter values, the system of the first-order conditions that need to be solved for q_2 and r_2 reduces to $\frac{\partial L(\cdot)}{\partial q_2} = \frac{\partial L(\cdot)}{\partial r_2} = 0$. Combined with $\eta_1 \geq 0$ and $\gamma_2 = 0$, as well as the definition of r_2 above, this produces:

$$\begin{cases} \eta_1 = -\frac{\partial J^M(\cdot)}{\partial r_2} / \frac{\partial\left(v^H P_{L|H} - v^L P_{L|L}\right)}{\partial r_2} = -\frac{\partial J^M(\cdot)}{\partial q_2} / \frac{\partial\left(v^H P_{L|H} - v^L P_{L|L}\right)}{\partial q_2} \\ r_2 = \hat{r}_2\left(q_2\right) \end{cases}$$

Solving this system, we obtain two real roots for q_2:

$$q_2 = \begin{cases} \frac{\left(\alpha \pm \sqrt{\alpha(1-\alpha)}\right)(1+\alpha(H-1))}{\alpha(2\alpha-1)(H-1)} \equiv \hat{q}_2^{1,\,2}, & \alpha \neq \frac{1}{2} \\ \frac{H+1}{2(H-1)}, & \alpha = \frac{1}{2} \end{cases}$$

where

$$\hat{q}_2^1 < 0 \,\forall\, \alpha \in \left[0,\, \frac{1}{2}\right) \text{ and } \hat{q}_2^1 > 1 \,\forall\, \alpha \in \left(\frac{1}{2},\, 1\right]$$

$$\hat{q}_2^2 > 0 \,\forall\, \alpha \in [0,\, 1] \text{ and}$$

$$\hat{q}_2^2 \leq 1 \Leftrightarrow (H \geq 3) \text{ and } \alpha \in [\alpha_1,\, \alpha_2] \text{ where}$$

$$\alpha_{1,\,2} = \frac{H-1 \mp \sqrt{(H-3)(H+1)}}{2(H-1)};\; \frac{H+1}{2(H-1)} \leq 1 \Leftrightarrow H \geq 3$$

Thus, when $\alpha \neq \frac{1}{2}$, the only root that can be feasible under certain conditions, is q_2^2. Then we define the ultimate solution for q_2, denoted by $\hat{q}_2$, as follows:

$$\hat{q}_2 = \begin{cases} \frac{\alpha - \sqrt{\alpha(1-\alpha)}(1+\alpha(H-1))}{\alpha(2\alpha-1)(H-1)}, & \alpha \neq \frac{1}{2} \\ \frac{H+1}{2(H-1)}, & \alpha = \frac{1}{2} \end{cases}$$

Now, we need to verify that this solution satisfies the assumed condition, $\hat{q}_2 \in (0,\, \bar{q}_2)$. With $\alpha \geq \max\left\{0,\, \frac{H-3L}{3(H-L)}\right\}$, the inequality $\hat{q}_2 > 0$ is always satisfied. Next, we solve $\hat{q}_2 < \bar{q}_2$ for $\alpha \neq \frac{1}{2}$. The corresponding equation has two roots:

$$\hat{\alpha}_{1,\,2} = \frac{H-1 \mp \sqrt{(H+3)(H-5)}}{2(H-1)}$$

When $H \in (1,\, 5)$, these roots are complex, and we have $\{\hat{q}_2 > \bar{q}_2\}$ for any α. If $H \geq 5$, then both $\hat{\alpha}_1$ and $\hat{\alpha}_2$ become real, and $\alpha \in (\hat{\alpha}_1,\, \hat{\alpha}_2)$ supports $\{\hat{q}_2 < \bar{q}_2\}$, where $\hat{\alpha}_1 > \alpha_1$ and $\hat{\alpha}_2 < \alpha_2$ are true for any $H > 1$. For $\alpha = \frac{1}{2}$, the inequality $\{\hat{q}_2 < \bar{q}_2\}$ holds if and only if $H > 5$. Putting things together, we obtain:

- For $H \in (1,\, 5)$, the condition $\hat{q}_2 < \bar{q}_2$ does not hold, and thus, the constraint $q_2 \leq \bar{q}_2$ or $q_2 \geq 0$ must bind in the optimum, implying $\gamma_2 \geq 0$ (not necessarily 0).

- For $H \geq 5$, the condition $\hat{q}_2 < \bar{q}_2$ always holds for $\alpha = \frac{1}{2}$ and requires $\alpha \in (\hat{\alpha}_1,\, \hat{\alpha}_2)$ for $\alpha \neq \frac{1}{2}$.

Thus, the derived solution for q_2, $\hat{q}_2$, is feasible for $H \geq 5$ and either $\alpha = \frac{1}{2}$ or $\alpha \in (\hat{\alpha}_1,\, \hat{\alpha}_2)$. Otherwise, $q_2 = \bar{q}_2$ or $q_2 = 0$ must hold.

Now, observe that $\hat{q}_2$ is the only extremum point of J^M in the interval $[0,\, \bar{q}_2]$. To make sure that $q_2 = \hat{q}_2$ is a maximum, we compare the values of the objective function $J^M(\cdot)$ at $q_2 = 0$, $q_2 = \bar{q}_2$, and $q_2 = \hat{q}_2$:

- $J^M(\cdot,\, \hat{r}_2(0)\,,\, 0) < J^M(\cdot,\, \hat{r}_2(\hat{q}_2)\,,\, \hat{q}_2)$ holds for any $\alpha \in [0,\, 1]$ and

- $J^M(\cdot,\, \hat{r}_2(\bar{q}_2)\,,\, \bar{q}_2) < J^M(\cdot,\, \hat{r}_2(\hat{q}_2)\,,\, \hat{q}_2)$ is true if and only if $\alpha \in (\hat{\alpha}_1,\, \hat{\alpha}_2)$.

Hence, if $\hat{q}_2$ is feasible, it constitutes a maximum. Otherwise, $q_2 = \bar{q}_2$ must be chosen:

$$J^M\left(\cdot,\, \hat{r}_2\left(0\right),\, 0\right) < J^M\left(\cdot,\, \hat{r}_2\left(\bar{q}_2\right),\, \bar{q}_2\right) \,\forall\, \alpha\, [0,\, 1]$$

Finally, we restore the set of Kuhn-Tucker multipliers that support the optimality of $q_1 = r_1 = q_3 = r_3 = 1$, $q_2 = \hat{q}_2$ (when $\hat{q}_2$ is feasible) or $q_2 = \bar{q}_2$ (when $\hat{q}_2$ is not feasible) and $r_2 = \hat{r}_2\left(q_2\right)$ for $\alpha \geq \max\left\{0,\, \frac{H-3}{3(H-1)}\right\}$. First, consider the case when $\hat{q}_2$ is feasible, i.e., $\gamma_2 = 0$ holds. Then, the following set of Kuhn-Tucker multipliers solves the system of first-order conditions:

$$\eta_1 = \begin{cases} \frac{3\alpha^2 - \alpha - \alpha\sqrt{\alpha(1-\alpha)}}{2\alpha - 1}, & \alpha \neq \frac{1}{2} \\ \frac{3}{4}, & \alpha = \frac{1}{2} \end{cases}$$

$$\lambda_i = \gamma_i = 0,\, i \in \{1,\, 2,\, 3\}$$

where $\eta_1 > 0$ for any $\alpha \in [0,\, 1]$.

Second, we solve for the multipliers that support $q_2 = \bar{q}_2$ as the optimum:

$$\eta_1 = \left(-\frac{\partial J^M\left(\cdot\right)}{\partial r_2} - \frac{1}{2}\frac{\partial J^M\left(\cdot\right)}{\partial q_2}\right) \Bigg/ \left(\frac{\partial\left(v^H P_{L|H} - v^L P_{L|L}\right)}{\partial r_2} + \frac{1}{2}\frac{\partial\left(v^H P_{L|H} - v^L P_{L|L}\right)}{\partial q_2}\right)\Bigg|_{\left(\bar{q}_2,\, \hat{r}_2\left(\bar{q}_2\right)\right)}$$

$$\gamma_2 = \left(-\frac{\partial J^M\left(\cdot\right)}{\partial r_2} - \eta_1\frac{\partial\left(v^H P_{L|H} - v^L P_{L|L}\right)}{\partial r_2}\right)\Bigg|_{\left(\bar{q}_2,\, \hat{r}_2\left(\bar{q}_2\right)\right)}$$

$$\gamma_1 = \lambda_2 = 0$$

$$\lambda_1 = \left(\frac{\partial J^M\left(\cdot\right)}{\partial r_1} + \eta_1\frac{\partial\left(v^H P_{L|H} - v^L P_{L|L}\right)}{\partial r_1}\right)\Bigg|_{\left(\bar{q}_2,\, \hat{r}_2\left(\bar{q}_2\right)\right)}$$

$$\gamma_3 = \frac{\left(1-\alpha\right)^2}{\alpha^2}\left(\frac{\partial J^M\left(\cdot\right)}{\partial r_1} + \eta_1\frac{\partial\left(v^H P_{L|H} - v^L P_{L|L}\right)}{\partial r_1} + \right.$$

$$\left.\frac{\alpha^2}{\left(1-\alpha\right)^2}\left(\frac{\partial J^M\left(\cdot\right)}{\partial r_3} + \eta_1\frac{\partial\left(v^H P_{L|H} - v^L P_{L|L}\right)}{\partial r_3}\right)\right)\Bigg|_{\left(\bar{q}_2,\, \hat{r}_2\left(\bar{q}_2\right)\right)}$$

$$\lambda_3 = \left(\frac{\partial J^M\left(\cdot\right)}{\partial r_3} + \eta_1\frac{\partial\left(v^H P_{L|H} - v^L P_{L|L}\right)}{\partial r_3} + \gamma_3\right)\Bigg|_{\left(\bar{q}_2,\, \hat{r}_2\left(\bar{q}_2\right)\right)}$$

Checking when $\eta_1 \geq 0$, $\gamma_2 \geq 0$, $\gamma_3 \geq 0$, $\lambda_1 \geq 0$ and $\lambda_3 \geq 0$, we obtain:

$$\eta_1 \geq 0 \,\forall\, \alpha \in [0,\, 1]$$

$$\gamma_2 \geq 0,\, \gamma_3 \geq 0,\, \lambda_1 \geq 0 \,\text{and}\, \lambda_3 \geq 0 \Leftrightarrow \begin{cases} H < 5 \\ \alpha \in [0,\, 1] \end{cases} \quad \text{or} \quad \begin{cases} H \geq 5 \\ \alpha \in [0,\, \hat{\alpha}_1] \cup [\hat{\alpha}_2,\, 1] \end{cases}$$

where $\hat{\alpha}_1$ and $\hat{\alpha}_2$ were introduced above. Thus, if $\hat{q}_2$ is not feasible, the Kuhn-Tucker multipliers that support the optimality of $q_1 = r_1 = q_3 = r_3 = 1$, $q_2 = \bar{q}_2$, and $r_2 = \hat{r}_2\left(\bar{q}_2\right)$, are always well defined. □

Proof. [Proof of **Proposition 5**] To support the claim, we refer to the proof of Proposition

4. Fix $H \geq 3$ and $\alpha \in \left[0,\, \frac{H-3}{3(H-1)}\right]$, where the upper bound on α never exceeds $\frac{1}{2}$. For $q_1 = r_1 = q_3 = r_3 = 1$ and the binding monotonicity constraint, we can rewrite the original optimization program in terms of q_2:[36]

$$\max_{q_2 \in [0,\,1]} \left\{ (1+\alpha)\, P_{L|L} v^L + \alpha P_{H|H} v^H \right\}$$

$$s.t.\, q_2 \in \left[\underline{q}_2,\, \bar{q}_2\right] \neq \emptyset$$

$$\underline{q}_2 = \frac{H - 3 - 3\alpha\,(H-1)}{2\,(1-2\alpha)\,(H-1)}$$

$$\underline{q}_2 = \frac{H - 3 - 5\alpha\,(H-1) + \sqrt{D}}{4\,(H-1)\,(1-2\alpha)}$$

$$D = -7\,(H-1)\,\alpha^2 + 6\left(H^2 - 4H + 3\right)\alpha + H^2 + 10H - 7 > 0\,\forall\,\alpha \in [0,\,1]$$

where $H \geq 3$ and $\alpha \in \left[0,\, \frac{H-3}{3(H-1)}\right]$ ensure that the lower bound on q_2 is positive, $\underline{q_2}$ solves $\hat{r}_2\,(q_2) = 0$ and $\bar{q}_2$ solves $\hat{r}_2\,(q_2) = 2q_2 - 1$. As we showed in the proof of Proposition 4, the interior solution of the designer's program for $\alpha \neq \frac{1}{2}$ corresponds to:

$$\hat{q}_2 = \frac{\alpha - \sqrt{\alpha\,(1-\alpha)}\,(1 + \alpha\,(H-1))}{\alpha\,(2\alpha - 1)\,(H-1)}$$

First, we check if there exist α's such that $\left\{\hat{q}_2 < \underline{q}_2\right\}$ holds, that is, the lower bound constraint binds. The underlying equation solved for α has two roots:

$$\tilde{\alpha}_{1,\,2} = \frac{H - 9 \mp \sqrt{H^2 - 18H + 1}}{10\,(H-1)}$$

For $H \in \left(1,\, \left(9 + 4\sqrt{5}\right)\right)$, both roots are complex, and it is $\left\{\hat{q}_2 > \underline{q}_2\right\}$ for any feasible α. If $H \geq \left(9 + 4\sqrt{5}\right)$ holds, then $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ become real, and $\alpha \in (\tilde{\alpha}_1,\, \tilde{\alpha}_2)$ supports $\left\{\hat{q}_2 < \underline{q}_2\right\}$. As $\hat{q}_2$ is always positive and it is $\left\{\hat{q}_2 < \underline{q}_2\right\}$, the $\alpha \in (\tilde{\alpha}_1,\, \tilde{\alpha}_2)$ set must have a non-empty intersection with $\alpha \in \left[0,\, \frac{H-3}{3(H-1)}\right]$ for $H \geq \left(9 + 4\sqrt{5}\right)$.

Second, we verify if the value of the objective function $J^m\,(\cdot)$ at the lower bound of q_2 is greater than $J^M\,(\cdot)$ at the upper bound when $\left\{\hat{q}_2 < \underline{q}_2\right\}$ holds. Here, we solve the next condition for α:[37]

$$J^M\left(\cdot,\, \underline{q}_2,\, 0\right) > J^M\left(\cdot,\, \bar{q}_2,\, \hat{r}_2\,(\bar{q}_2)\right)$$

The underlying equation has four roots – $\bar{\alpha}_1$, $\bar{\alpha}_2$, $\bar{\alpha}_3$, and $\bar{\alpha}_4$ – with the following properties:

- $\bar{\alpha}_1 = -\frac{2}{H-1} < 0$,
- $\bar{\alpha}_2 > \tilde{\alpha}_2$ for any $H > 1$,
- $\bar{\alpha}_3 < 0$ for any $H > 1$, and

[36] See the proof of Proposition 4 for more detail.
[37] All calculations were performed with Matlab symbolic toolbox.

- $0 < \bar{\alpha}_4 < \tilde{\alpha}_1$ for any $H > 1$.

Thus, $\left\{J^M\left(\cdot,\, \underline{q}_2,\, 0\right) - J^M\left(\cdot,\, \bar{q}_2,\, \hat{r}_2\left(\bar{q}_2\right)\right)\right\}$ does not change its sign in the interval where $\left\{\hat{q}_2 < \underline{q}_2\right\}$ is satisfied. Finally, we take the value of $H = 22$ that does not violate the $\left\{H \geq \left(9 + 4\sqrt{5}\right)\right\}$ constraint, and show that the inequality of interest has a positive sign. Hence, $\left\{J^M\left(\cdot,\, \underline{q}_2,\, 0\right) > J^M\left(\cdot,\, \bar{q}_2,\, \hat{r}_2\left(\bar{q}_2\right)\right)\right\}$ holds when the interior solution $\hat{q}_2$ is located to the left of $\underline{q}_2$. □

Proof. [Proof of **Proposition 6**] Non-optimality of private disclosure that induces the non-monotonic equilibrium with strong negative correlation, is shown in the main text. Then it remains to consider the case with strong positive correlation.

An optimal signal distribution under private disclosure that supports the non-monotonic equilibrium with strong positive correlation solves:

$$\max_{\pi=\{q_j,\, r_j\}_{j=1}^3} \left\{ J^{NM,+} \equiv \frac{v^H\left(v^L\left(P_{H|H} - P_{H|L}\right) + \alpha\left(v^H - v^L\right)\right)}{P_{H|H}v^H - P_{H|L}v^L} \right\} \tag{A-1}$$

$$\text{s. t. } \alpha^2 q_1 + \alpha\left(1-\alpha\right) + \left(1-\alpha\right)^2\left(1-q_3\right) = \alpha \tag{A-2}$$

$$q_1 \geq 1 - \frac{1-\alpha}{\alpha} q_2 \tag{A-3}$$

$$v^H P_{L|H} - v^L P_{L|L} < 0 \tag{A-4}$$

$$q_j \geq r_j \tag{A-5}$$

$$r_j \geq \max\left\{0,\, 2q_j - 1\right\} \tag{A-6}$$

$$q_j \leq 1 \tag{A-7}$$

As in case of the monotonic equilibrium, (A-2) is the consistency requirement, (A-3) is the inequality stating that $v^H \geq v^L$, (A-4) sustains an equilibrium with strong positive correlation, and conditions (A-5)–(A-7) ensure that the signal distribution π is well defined.

Let us write down the Lagrangian for this optimization program:

$$\begin{gathered} L\left(\{q_j,\, r_j\}_{j=1}^3,\, \mathscr{M}\right) = J^{NM,+}\left(\{q_j,\, r_j\}_{j=1}^3\right) + \eta\left(-v^H P_{L|H} + v^L P_{L|L}\right) + \\ \Sigma_{j=1}^3 \lambda_j\left(q_j - r_j\right) + \Sigma_{j=1}^3 \gamma_j\left(r_j - \max\left\{0,\, 2q_j - 1\right\}\right) + \Sigma_{j=1}^3 \phi_j\left(1 - q_j\right) + \\ \chi\left(q_1 - \left(1 - \frac{1-\alpha}{\alpha} q_2\right)\right) + \omega\left(\alpha^2\left(q_1 - 1\right) + \left(1-\alpha\right)^2\left(1 - q_3\right)\right), \end{gathered}$$

where $\mathscr{M} = \left\{\eta,\, \{\lambda_j,\, \gamma_j,\, \phi_j\}_{j=1}^3,\, \chi,\, \omega\right\}$ is the set of non-negative Kuhn-Tucker multipliers. The system of first-order conditions with respect to $\{q_j,\, r_j\}_{j=1}^3$ looks as follows:

$$\begin{cases} \frac{\partial L(\cdot)}{\partial r_j} = \frac{\partial J^{NM,-}(\cdot)}{\partial r_j} + \eta\frac{\partial\left(-v^H P_{L|H} + v^L P_{L|L}\right)}{\partial r_j} - \lambda_j + \gamma_j = 0,\, j \in \{1,\, 2,\, 3\} \\ \frac{\partial L(\cdot)}{\partial q_j} = \frac{\partial J^{NM,-}(\cdot)}{\partial q_j} + \eta\frac{\partial\left(-v^H P_{L|H} + v^L P_{L|L}\right)}{\partial q_j} + \lambda_j - 2\gamma_j I\left\{q_j \geq \frac{1}{2}\right\} - \phi_j + \\ \chi\left(I\{j=1\} + \frac{1-\alpha}{\alpha} I\{j=2\}\right) + \omega\left(\alpha^2 I\{j=1\} - \left(1-\alpha\right)^2 I\{j=3\}\right) = 0 \end{cases}$$

The slackness conditions are:

$$\omega\left(\alpha^2\left(q_1-1\right)+\left(1-\alpha\right)^2\left(1-q_3\right)\right)=0$$
$$\chi\left(q_1-\left(1-\frac{1-\alpha}{\alpha}q_2\right)\right)=0$$
$$\eta\left(v^H P_{L|H}-v^L P_{L|L}\right)=0$$
$$\lambda_j\left(q_j-r_j\right)=0$$
$$\gamma_j\left(r_j-\max\left\{0,\,2q_j-1\right\}\right)=0$$
$$\phi_j\left(1-q_j\right)=0$$

From the slackness conditions it follows that $\eta=0$ because inequality (A-4), supporting positive signal correlation, is strict.

Let us consider how the value of the designer's objective function, $J^{NM,+}(\cdot)$, depends on r_j, $j\in\{1,\,2,\,3\}$:

$$\frac{\partial J^{NM,+}(\cdot)}{\partial r_1}=-\frac{\alpha^2\left(v^H-v^L\right)^2}{\left(P_{H|H}v^H-P_{H|L}v^L\right)^2}\leq 0\quad\forall\,v^H\geq v^L$$

$$\frac{\partial J^{NM,-}(\cdot)}{\partial r_2}=\frac{2v^H\left(v^H-v^L\right)}{\left(P_{H|H}v^H-P_{H|L}v^L\right)^2}\geq 0\quad\forall\,v^H\geq v^L$$

$$\frac{\partial J^{NM,-}(\cdot)}{\partial r_3}=-\frac{\left(1-\alpha\right)^2\left(v^H-v^L\right)^2}{\left(P_{H|H}v^H-P_{H|L}v^L\right)^2}\leq 0\quad\forall\,v^H\geq v^L$$

Note that all inequalities are strict if $v^H>v^L$.

Suppose that indeed, $v^H>v^L$, that is, condition (A-3) does not bind in the optimum. In view of the slackness conditions, this means that $\chi=0$. A simple argument below demonstrates that in this case we should also have $\gamma_1>0$, $\gamma_3>0$ and $\lambda_2>0$, so that constraint (A-6) binds for $j\in\{1,\,3\}$ and constraint (A-7) binds for $j=2$.

First, if $\gamma_j=0$ for $j\in\{1,\,3\}$, then using the observation regarding the sign of $\frac{\partial J^{NM,+}(\cdot)}{\partial r_j}$ in the first-order conditions, we obtain a contradiction with $\lambda_j\geq 0$:

$$\frac{\partial L(\cdot)}{\partial r_1}=\frac{\partial J^{NM,+}(\cdot)}{\partial r_1}-\lambda_1=0\Leftrightarrow\lambda_1=\frac{\partial J^{NM,+}(\cdot)}{\partial r_1}<0$$
$$\frac{\partial L(\cdot)}{\partial r_3}=\frac{\partial J^{NM,+}(\cdot)}{\partial r_3}-\lambda_3=0\Leftrightarrow\lambda_3=\frac{\partial J^{NM,+}(\cdot)}{\partial r_3}<0$$

Hence, in any optimum with $v^H>v^L$ (if it exists), $\gamma_1>0$ and $\gamma_3>0$, so that conditions $r_1=\max\{0,\,2q_3-1\}$ and $r_3=\max\{0,\,2q_3-1\}$ must hold. Similarly, if $\lambda_2=0$, then we obtain a contradiction with $\gamma_2\geq 0$:

$$\frac{\partial L(\cdot)}{\partial r_2}=\frac{\partial J^{NM,+}(\cdot)}{\partial r_2}+\gamma_2=0\Leftrightarrow\gamma_2=-\frac{\partial J^{NM,+}(\cdot)}{\partial r_2}<0.$$

So, it must be that $\lambda_2>0$, i.e., $q_2=r_2$.

Next, let us rewrite condition (A-4) as follows:

$$v^H P_{L|H} - v^L P_{L|L} < 0 \Leftrightarrow \frac{v^H}{v^L} < \frac{P_{L|L}}{P_{L|H}},$$

which due to $v^H > v^L$ requires $P_{L|L} > P_{L|H}$, or equivalently, $P_{H|H} > P_{H|L}$. Using the definition of $P_{H|H}$ and $P_{H|L}$ in (4) - (5) and the just established equalities for r_1, r_2 and r_3, consider all possible combinations of the values of r_1 and r_3, and show that none of these combinations is possible in the optimum:

1. $r_1 = r_3 = 0$. This requires $q_1 < \frac{1}{2}$ and $q_3 < \frac{1}{2}$. Then inequality $P_{H|H} > P_{H|L}$ holds if and only if:
$$q_1 < \frac{2\alpha - 1}{2\alpha^2}.$$
For $\alpha < \frac{1}{2}$, this results in a contradiction since $q_1 \geq 0$. For $\alpha \geq \frac{1}{2}$, this condition is only consistent with condition (A-3) if the following is true:
$$\frac{2\alpha - 1}{2\alpha^2} > 1 - \frac{1-\alpha}{\alpha} q_2 \Leftrightarrow q_2 > \frac{2\alpha^2 - 2\alpha + 1}{2\alpha(1-\alpha)} > 1 \;\; \forall \alpha \in (0, 1)$$
Again, this is a contradiction since $q_2 \leq 1$.

2. $r_1 = 0$ and $r_3 = 2q_3 - 1$. This implies $q_1 < \frac{1}{2}$ and $q_3 \geq \frac{1}{2}$. Then $P_{H|H} > P_{H|L}$ reduces to $-\frac{\alpha}{1-\alpha} > 0$, which is a contradiction.

3. $r_1 = 2q_1 - 1$ and $r_3 = 0$. This means that $q_1 \geq \frac{1}{2}$ and $q_3 < \frac{1}{2}$. In this case, inequality $P_{H|H} > P_{H|L}$ simplifies to $-\frac{1-\alpha}{\alpha} > 0$, a contradiction.

4. $r_1 = 2q_1 - 1$ and $r_3 = 2q_3 - 1$. This implies that $q_1 \geq \frac{1}{2}$ and $q_3 \geq \frac{1}{2}$. Here, $P_{H|H} > P_{H|L}$ holds if and only if:
$$-\frac{2\alpha(1-q_1)}{1-\alpha} > 0,$$
which is, once again, a contradiction.

Thus, if $v^H > v^L$, there is no solution of the designer's optimization program. Then, $v^H = v^L$ must hold in the optimum. In this case the designer's objective function becomes:
$$J^{NM,+}|_{\left(q_1 = 1 - \frac{1-\alpha}{\alpha} q_2\right)} = v^H|_{\left(q_1 = 1 - \frac{1-\alpha}{\alpha} q_2\right)} = \alpha H + 1 - \alpha,$$

and this is equal to J^{pub}, the value of the ex ante expected aggregate effort under optimal public disclosure policy. As we have shown in the proof of Proposition 3, there exists a signal distribution with private and mildly positively correlated messages that induces a strictly higher ex ante expected aggregate effort than J^{pub}. Then it is also strictly higher than $J^{NM,+}$, which establishes the non-optimality of private disclosure with strong positive correlation. □

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