Published for SISSA by Springer

Received: *July 13, 2021*
Revised: *August 20, 2021*
Accepted: *August 25, 2021*
Published: *September 16, 2021*

JHEP09(2021)092

Time-dependent study of $K_S \to \pi^+\pi^-$ decays for flavour physics measurements

P. Pakhlov[a,b] and V. Popov[a]

[a] *Laboratory of heavy quarks and leptons, P.N. Lebedev Physical Institute of RAS,*
53 Leninskiy Prospekt, Moscow 119991, Russia
[b] *International Laboratory of Elementary Particle Physics,*
National Research University Higher School of Economics,
20 Myasnitskaya Street, Moscow 101000, Russia

E-mail: `pakhlov@lebedev.ru`, `popovve@lebedev.ru`

Abstract: Nowadays High Energy Physics experiments can accumulate unprecedented statistics of heavy flavour decays that allows to apply new methods, based on the study of very rare phenomena, which used to be just desperate. In this paper we propose a new method to measure composition of K^0-$\overline{K}^0$, produced in a decay of heavy hadrons. This composition contains important information, in particular about weak and strong phases between amplitudes of the produced K^0 and $\overline{K}^0$. We consider possibility to measure these parameters with time-dependent $K^0 \to \pi^+\pi^-$ analysis. Due to CP-violation in kaon mixing time-dependent decay rates of K^0 and $\overline{K}^0$ differ, and the initial amplitudes revealed in the CP-violating decay pattern. We perform phenomenological study of K^0 decay evolution initially produced as a combination $a\left|K^0(t)\right\rangle + b\left|\overline{K}^0(t)\right\rangle$, where a and b, complex amplitudes, could also be dependent on decay time of heavy mother particle. In particular we consider cases of charmed hadrons decays: $D^+ \to K^0\pi^+$, $D_s^+ \to K^0K^+$, $\Lambda \to pK^0$ and with some assumptions $D^0 \to K^0\pi^0$. This can be used to test the sum rule for charmed mesons and to obtain input for the full constraint of the two body amplitudes of D-mesons.

Keywords: Phenomenological Models

ArXiv ePrint: 2107.05062

https://doi.org/10.1007/JHEP09(2021)092

Contents

JHEP09(2021)092

1 Introduction

The final states of heavy hadron decays sometimes differ only by the replacement of the strange K^0 meson by its antiparticle due to the contribution of different diagrams or neutral meson mixing, thus representing a superposition of the strange and antistrange states. An example is the decay $D^+ \to \overline{K}^0(K^0)\pi^+$, where both $\overline{K}^0$ and K^0 are produced due to the presence of both Cabibbo favourite (CF) and doubly Cabibbo suppressed (DCS) amplitudes.

Considering charm hadron decays from the point of view of flavour $\mathrm{SU}(3)_f$ symmetry one can obtain the following sum rules [1]:

$$\sqrt{2}A_{D^0\to\overline{K}^0\pi^0} + A_{D^0\to K^-\pi^+} - A_{D^+\to\overline{K}^0\pi^+} = 0, \tag{1.1}$$

$$\sqrt{2}A_{D^0\to K^0\pi^0} + A_{D^0\to K^+\pi^-} + \sqrt{2}A_{D^+\to K^+\pi^0} - A_{D^+\to K^0\pi^+} = 0. \tag{1.2}$$

These relations are the pure isospin sum rules and both are broken at the same level of $\mathcal{O}\left((m_u - m_d)/\Lambda_{QCD}\right) \sim 1\%$. Amplitudes involved in eq. (1.1), (1.2) are of the same order of Cabibbo suppression and correspond to CF and DCS decay amplitudes, respectively. These sum rules could be illustrated as shown in figure 1.

There is another sum based on $\mathrm{SU}(3)_f$ that is particularly interesting since here CF and DCS amplitudes are mixed together:

$$A_{D^+\to\overline{K}^0\pi^+} - A_{D_s^+\to\overline{K}^0K^+} - \frac{A_{D^+\to\overline{K}^0K^+}}{\lambda} + \frac{A_{D_s^+\to\overline{K}^0\pi^+}}{\lambda} + \frac{A_{D^+\to K^0\pi^+}}{\lambda^2} - \frac{A_{D_s^+\to K^0K^+}}{\lambda^2} = 0, \tag{1.3}$$

where $\lambda = \sin\theta_{12}$ — is the CKM parameter and θ_{12} is the Cabibbo angle.

Recently, the question of validity of $\mathrm{SU}(3)_f$ rules and accuracy of their approximation has become urgent as they are widely used to explain the anomalously large CP violation observed in D^0 decays by LHCb experiment [2]:

$$\Delta A_{CP} = A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = (-15.4 \pm 2.9) \times 10^{-4}, \tag{1.4}$$

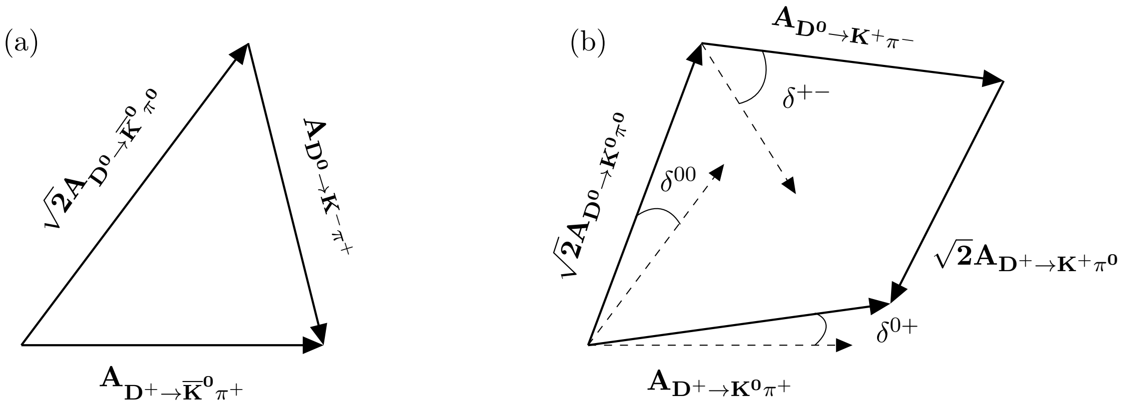

Figure 1. $SU_f(3)$ sum rules for CF *(a)* and DCS *(b)* decay amplitudes.

Many papers trying to reconcile LHCb result with the Standard Model suppose that such discrepancy could be explained by the enhanced penguin amplitudes [3–6]. And it was argued that FSI play an important role in the enhancement of penguin contributions. For all these studies $\mathrm{SU}(3)_f$ or at least isospin symmetry play important role and probe of FSI generated phases corresponding to different ΔI amplitudes is an important task.

Structure of charm decay amplitudes could be described in terms of Wick contractions of operators of effective Hamiltonian. In this work we are particularly interested in amplitudes involving K^0 in the final state. In the following we adopt results for amplitude expressions obtained in [7], for CF decays:

$$\begin{aligned}
A_{D^0\to K^-\pi^+} &= \frac{1}{5}(3T-2C-K)e^{i\delta_{1/2}} + \frac{2}{5}(T+C+\kappa) \\
A_{D^0\to\overline{K}^0\pi^0} &= -\frac{1}{5\sqrt{2}}(3T-2C-K)e^{i\delta_{1/2}} + \frac{3}{5\sqrt{2}}(T+C+\kappa) \\
A_{D^+\to\overline{K}^0\pi^0} &= (T+C+\kappa) \\
A_{D_s^+\to\overline{K}^0K^+} &= -\frac{1}{5}(2T-3C+\Delta)e^{i\delta_1'} + \frac{2}{5}(T+C+\kappa),
\end{aligned} \tag{1.5}$$

and for DCS decays (to be multiplied by λ^2):

$$\begin{aligned}
A_{D^0\to K^+\pi^-} &= -\frac{1}{5}(3T-2C+K)e^{i\delta_{1/2}} - \frac{2}{5}(T+C+\kappa') \\
A_{D^0\to K^0\pi^0} &= \frac{1}{5\sqrt{2}}(3T-2C+K)e^{i\delta_{1/2}} - \frac{3}{5\sqrt{2}}(T+C+\kappa') \\
A_{D^+\to K^0\pi^+} &= \frac{1}{5}(2T-3C+\Delta-K')e^{i\delta_{1/2}} - \frac{2}{5}(T+C+\kappa') \\
A_{D_s^+\to K^0K^+} &= -(T+C+\kappa').
\end{aligned} \tag{1.6}$$

Here T and C correspond to "tree"-level color-connected and color-suppressed amplitudes, K and K' are parameters corresponding to non-conservation of strangeness changing currents, κ and κ' — parameters allowing $\mathrm{SU}(3)_f$-breaking in CF and DCS amplitudes. Also two phases are present $\delta_{1/2}$, δ_1' corresponding to $I=\frac{1}{2}$ and $I=1$ amplitudes respectively.

Buccella et al. [7] performed fit to the above mentioned amplitude parameters based on observed values of CP asymmetries and branching ratios for charm hadron decays, and made predictions for the strong phase difference in the $D^0 \to K^\pm\pi^\mp$ decay channel:

$$\delta^{+-} = (3.14 \pm 5.69)^\circ. \tag{1.7}$$

Relying on the results for different topologies contributions (see table III in [7]) and using equations (1.5), (1.6) we are able to estimate other strong phase differences as following:

$$\delta^{00} = (-3 \pm 6)^\circ; \tag{1.8}$$

$$\delta^{0+} = (-76 \pm 4)^\circ; \tag{1.9}$$

$$\delta_s^{0+} = (108 \pm 4)^\circ. \tag{1.10}$$

In this paper we propose a method that allows to measure strong phase differences and amplitude ratios for final states with K^0-meson, hence probe the validity of approaches used to explain LHCb result and in general the sum rules. Such measurements will possibly allow to identify the source and scale of $\mathrm{SU}(3)_f$ breaking in charm hadron decays.

In our previous paper [8] we suggested to use semileptonic K^0 decays and study their time evolution to measure the complex phase between K^0 and $\overline{K}^0$ at the production point. The method is based on the disentanglement of the production phase from the (known) $K^0-\overline{K}^0$ mixing phase, using the later as a reference. While the method can work properly, its experimental application is a challenge due to unobserved neutrino in kaon decay. We realized that the similar sensitivity could be achieved using the standard $K^0 \to \pi^+\pi^-$ decays, which are easy to reconstruct. In this paper we show that time evolution of the state $a\left|K^0(t)\right\rangle + b\left|\overline{K}^0(t)\right\rangle$ decaying into $\pi^+\pi^-$ allows to extract the complex parameters a and b as well. In this case we utilize the CP-violating phase ϕ_{+-} as a reference for the proposed measurements. Both methods are based on the rare neutral kaon effects: rare decays (semileptonic for short lived kaon component) or rare interference (between short and long lived components).

2 Formalism

Time evolution of the neutral kaon system could be described by Shrödinger equation

$$i\partial_t \begin{pmatrix} K^0(t) \\ \overline{K}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right) \begin{pmatrix} K^0(t) \\ \overline{K}^0(t) \end{pmatrix}, \tag{2.1}$$

where effective Hamiltonian is a sum of absorptive and dispersive parts, $\mathbf{M}$ and $\mathbf{\Gamma}$ are 2×2 hermitian matrices. The Hamiltonian eigenvalues could be written as follows:

$$\lambda_{S,L} \equiv m_{S,L} - i\frac{\Gamma_{S,L}}{2} = \left(M_{11} - i\frac{\Gamma_{11}}{2}\right) \pm \left(\frac{p}{q}\right)_K \left(M_{12} - i\frac{\Gamma_{12}}{2}\right), \tag{2.2}$$

where $m_{S,L}$, $\Gamma_{S,L}$ are masses and widths of the Hamiltonian eigenstates K_S^0 and K_L^0, and parameters p, q correspond to the flavour admixtures of eigenstates defined by

$$\left(\frac{p}{q}\right)_K^2 = \frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}. \tag{2.3}$$

Since we consider only $\pi^+\pi^-$ final state it is convenient to use $p/q = (1+\varepsilon)/(1-\varepsilon)$, where ε describes the CP-even component in K^0_L. Then amplitudes describing evolution of initially pure flavour eigenstates in terms of K^0_S/K^0_L could be written as

$$\left|K^0(t)\right\rangle = \frac{(1-\varepsilon)}{\sqrt{2}}\left[e^{-i\lambda_S t}\left|K_S\right\rangle + e^{-i\lambda_L t}\left|K_L\right\rangle\right], \tag{2.4}$$

$$\left|\overline{K}^0(t)\right\rangle = \frac{(1+\varepsilon)}{\sqrt{2}}\left[e^{-i\lambda_S t}\left|K_S\right\rangle - e^{-i\lambda_L t}\left|K_L\right\rangle\right]. \tag{2.5}$$

Using these equations one can obtain for time-dependent decay rates

$$\mathcal{R}(t) = \frac{1 \mp 2\mathrm{Re}(\varepsilon)}{2}\left|A_{K_S\to\pi\pi}\right|^2\Big[e^{-\Gamma_S t} + |\eta_{+-}|^2 e^{-\Gamma_L t} \\ \pm 2|\eta_{+-}|e^{-\frac{1}{2}(\Gamma_L+\Gamma_S)t}\cos\left(\Delta m t - \phi_{+-}\right)\Big], \tag{2.6}$$

where the upper (lower) sign corresponds to initial pure K^0 ($\overline{K}^0$), $\Delta m = m_L - m_S$ is a mass difference and we introduced for the amplitude ratio parameter

$$\frac{\left\langle \pi^+\pi^-\left|H\right|K^0_L\right\rangle}{\left\langle \pi^+\pi^-\left|H\right|K^0_S\right\rangle} = \eta_{+-} = |\eta_{+-}|e^{i\phi_{+-}}. \tag{2.7}$$

The third interference term in eq. (2.6) basically allows to distinguish the initial flavour of neutral kaon. Parameters η_{+-} and ϕ_{+-} have been measured with great precision and current world averages (assuming CPT invariance) are [9]: $\eta_{+-} = (2.232 \pm 0.011) \times 10^{-3}$, $\phi_{+-} = (43.51 \pm 0.05)^\circ$. In the following calculations we neglect the direct CPV in kaons and assume $\varepsilon = \eta_{+-}$.

Despite the smallness of indirect CPV in kaons, itself it opens interesting possibilities for flavour physics measurements [10–13]. The recent papers [12, 13] pointed out the importance of CP violation in neutral kaon decays for charm studies. For example in [12] it was shown that the interference between charm and kaon mixing could create a interesting opportunity to measure CP observables through analysis of both charm and neutral kaon decay times. And in [13] a few cases of charged charm mesons we studied and it was shown that analysis of kaon decay time reveals the interference between kaon mixing and DCS decays thus provides more clear direct CPV estimation. However, prospects of neutral kaon studies are not limited by CP searches. As future experiments aim to accumulate unprecedented statistics the wide range of kaon lifetimes would be available. Big kaon lifetimes are especially useful for different complex phases measurements. In the following section we consider few cases of charm hadron decays that are particularly interesting for the SU$(3)_f$ probe.

3 Strong phase difference between CF and DCS decays

Here we consider a set of two-body decays, where both Cabibbo-favourite and doubly Cabibbo-suppressed amplitudes contribute to the final states. The following analysis could be applied to the decays $D^+ \to K^0_S\pi^+$, $D^+_s \to K^0_S K^+$ and $\Lambda_c \to pK^0_S$ (the later is not related to $\mathrm{SU}(3)_f$ sum rules, but is interesting on its own). Evolution of K^0-$\overline{K}^0$ system produced in such decays could be described than:

$$\Psi^+(t) = |\overline{K}^0(t)\rangle + \sqrt{r_f}e^{i\delta}|K^0(t)\rangle, \tag{3.1}$$

$$\Psi^-(t) = |K^0(t)\rangle + \sqrt{\overline{r_f}}e^{i\delta}|\overline{K}^0(t)\rangle, \tag{3.2}$$

where δ, r_f — strong phase difference and branching fractions ratio for CF and DCS decay modes defined as

$$r_f \equiv \left|\frac{\langle K^0h|H|D\rangle}{\langle \overline{K}^0h|H|D\rangle}\right|^2 \simeq \left|\frac{V_{cd}V^*_{us}}{V_{cs}V^*_{ud}}\right|^2. \tag{3.3}$$

In general case $r_f \neq \overline{r_f}$ and present itself direct CPV in charm decays. Current world averages on A_{CP} for D^+ and D^+_s mesons are [14]:

$$\begin{aligned} A_{CP}^{D_s\to K^0_S K^+} &= (8 \pm 26)\times 10^{-4},\\ A_{CP}^{D^+\to K^0_S\pi^+} &= (-41\pm 9)\times 10^{-4}. \end{aligned} \tag{3.4}$$

While non-zero effect was observed for D^+, it was noted [15] that after subtraction K^0-$\overline{K}^0$-mixing contribution asymmetry is consistent with zero. So we assume no CPV thereafter $r_f = \overline{r_f}$.

Time-dependent decay rates could be obtained by substituting $K^0(\overline{K}^0)$ decay amplitudes in (3.1), (3.2) with evolution equations (2.4), (2.5).

$$\begin{aligned} \mathcal{R}^+(t) = {} & \overline{\mathcal{R}}(t) + r_f\mathcal{R}(t) + \sqrt{r_f}\,(\cos\delta + 2|\eta_{+-}|\sin\delta\sin\phi_{+-})\times\left(e^{-\Gamma_S t} - |\eta_{+-}|^2 e^{-\Gamma_L t}\right) \\ & + 2\sqrt{r_f}|\eta_{+-}|\Big(\sin\delta + 2|\eta_{+-}|\cos\delta\sin\phi_{+-}\Big)e^{-\frac{1}{2}(\Gamma_L+\Gamma_S)t}\sin(\Delta m t - \phi_{+-}), \end{aligned} \tag{3.5}$$

$$\begin{aligned} \mathcal{R}^-(t) = {} & \mathcal{R}(t) + r_f\overline{\mathcal{R}}(t) + \sqrt{r_f}\,(\cos\delta - 2|\eta_{+-}|\sin\delta\sin\phi_{+-})\times\left(e^{-\Gamma_S t} - |\eta_{+-}|^2 e^{-\Gamma_L t}\right) \\ & - 2\sqrt{r_f}|\eta_{+-}|\Big(\sin\delta - 2|\eta_{+-}|\cos\delta\sin\phi_{+-}\Big)e^{-\frac{1}{2}(\Gamma_L+\Gamma_S)t}\sin(\Delta m t - \phi_{+-}). \end{aligned} \tag{3.6}$$

These formulas demonstrate that the $K^0 \to \pi^+\pi^-$ time-dependent decay rates depend on the initial strong phase, moreover, both sine and cosine of the strong phase enter the formula, therefore there are no trigonometrical ambiguities in this measurement. For illustrative purposes its is convenient to construct an asymmetry in the following way:

$$\mathcal{A} = \frac{\mathcal{R}^+(t) - \mathcal{R}^-(t)}{\mathcal{R}^+(t) + \mathcal{R}^-(t)}. \tag{3.7}$$

The decay rates along with asymmetry are illustrated in figure 2. One could see that the largest impact produced by the strong phase on resulting asymmetry falls on big lifetimes $\sim [6, 14]\,\tau_{K_S}$.

For the D^0-mesons situation gets complicates because of mixing. Time evolution of flavour states is described by

$$\begin{aligned} |D^0_{phys}(t')\rangle &= g_+(t')\left|D^0\right\rangle - \left(\frac{q}{p}\right)_D g_-(t')\left|\overline{D}^0\right\rangle,\\ |\overline{D}^0_{phys}(t')\rangle &= g_+(t')\left|\overline{D}^0\right\rangle - \left(\frac{p}{q}\right)_D g_-(t')\left|D^0\right\rangle, \end{aligned} \tag{3.8}$$

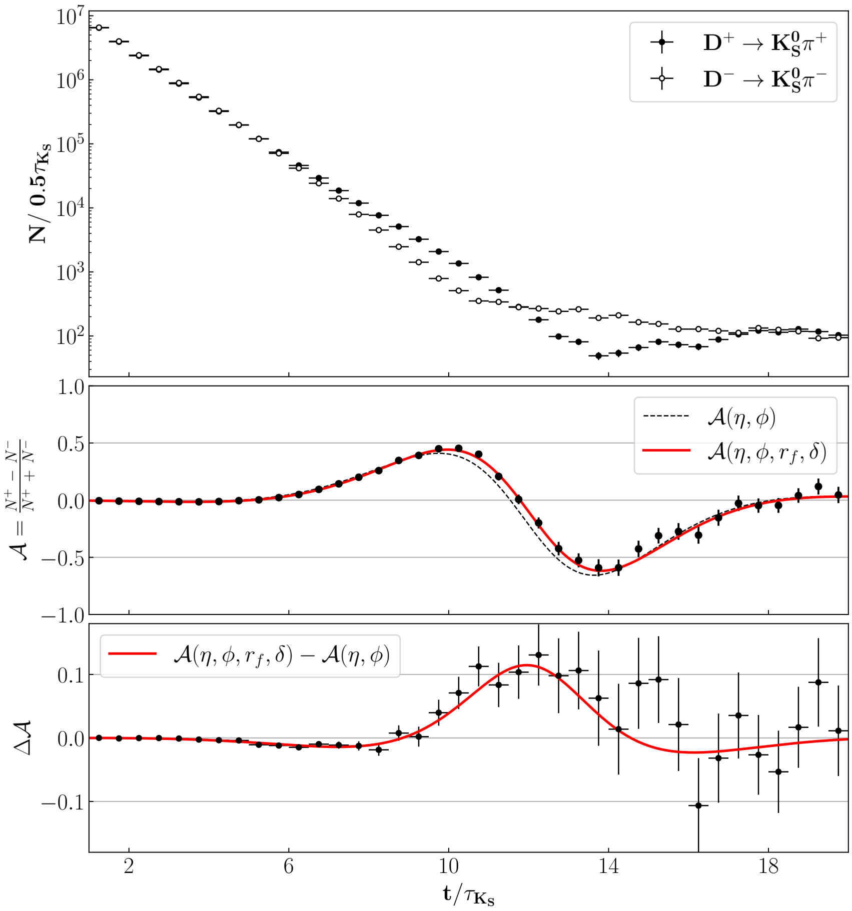

Figure 2. At the top plot time-dependent decay rates obtained with MC simulation are shown. Black and open circles represent data simulated according to decay rates (3.5), (3.6) respectively. In the middle plot the asymmetry defined as (3.7) is shown. The bottom plot illustrates the difference between asymmetries with and without DCS contribution. For these plots we used $\sqrt{r_f} = 0.06$ and $\delta^{0+} = -76°$.

where $g_\pm = \frac{1}{2}\left(e^{-i\lambda_2 t'} \pm e^{-i\lambda_1 t'}\right)$, $\lambda_{1,2}$ — are Hamiltonian eigenvalues defined likewise eq. (2.2) and t' — D^0 decay time. Here we assume no CPV in mixing, thus $|q/p| = 1$. Amplitudes (3.1), (3.2) can be rewritten as follows

$$A_f(t, t') = a^+(t')\langle\pi\pi|H|\overline{K}^0\rangle + b^+(t')\langle\pi\pi|H|K^0\rangle, \tag{3.9}$$

$$\overline{A_f}(t, t') = a^-(t')\langle\pi\pi|H|K^0\rangle + b^-(t')\langle\pi\pi|H|\overline{K}^0\rangle, \tag{3.10}$$

where time-dependent coefficients are given by

$$\begin{aligned} a^+(t') &\equiv \langle\overline{K}^0\pi^0|H|D^0_{phys}(t')\rangle = A_{D^0}\left[g_+(t') - \sqrt{r_f}e^{i(\delta+\phi)}g_-(t')\right], \\ b^+(t') &\equiv \langle K^0\pi^0|H|D^0_{phys}(t')\rangle = A_{D^0}\left[\sqrt{r_f}e^{i(\delta-\phi)}g_+(t') - g_-(t')\right], \end{aligned} \tag{3.11}$$

and a^-, b^- could be obtained from a^+, b^+ by substitution $\phi \to -\phi$. Combined measurements of D^0-$\overline{D}^0$-mixing yielded following values for mixing parameters [9]:

$$\begin{aligned} x &\equiv \frac{\Delta M}{\Gamma} = (0.43^{+0.10}_{-0.11})\%, \\ y &\equiv \frac{\Delta\Gamma}{2\Gamma} = (0.60 \pm 0.06)\%, \\ \phi &\equiv Arg\left(\frac{q}{p}\right)_D = (0.08 \pm 0.31)^\circ. \end{aligned} \tag{3.12}$$

In [12] it was demonstrated that 2-dimensional distribution for the $D^0 \to K^0_S\pi^0$ decay is sensitive to a set of CP observables. However, it is unlikely that sufficient D^0-lifetime resolution will be achieved in a neutral decay mode. Given the smallness of mixing parameters in D^0-$\overline{D}^0$-system in the next section we consider a possibility to integrate over D^0 lifetime and to use eq. (3.5), (3.6) for $D^0/\overline{D}^0$ as well.

4 Feasibility study

In this section we estimate potential precision of proposed measurement in future experiments. It was shown in previous section that one would expect the most sensitivity for strong phase could be achieved at big lifetimes — $[6, 14]\,\tau_{K_S}$. Based on this one could conclude that it is essential that experiment should possess large tracking detector and/or produce soft kaons. Also proper charged hadron identification is needed, since some of the final states differ by K/π interchange.

We consider the most promising experiments the project of future Super c–τ factory (SCTF) [18] and Belle II experiment [16] that is already taking data. Both experiments possess large drift chambers ($R \sim$ 1m) and produce relatively soft kaons, $\beta\gamma \sim 1..4$. Hadron identification in Belle II provided with TOP in barrel part and ARICH in endcaps and for the c–τ factory identification will be provided with FARICH detector that covers almost full solid angle. Given the spatial resolution of drift chambers $\sim 100\mu m$, kaon life time resolution could be expected at the level of a few percent that is more than enough to perform proposed measurement.

We perform feasibility study for the decay channels listed in table 1. Future Super c–τ factory is aiming to accumulate 10ab^{-1} data varying energies in c.m.s. from 3.097 GeV to 4.650 GeV. In particular 3ab^{-1} will be taken at $\psi(3770)$-resonance, 1ab^{-1} at $\psi(4160)$ and 1ab^{-1} near $\Lambda_c^+\Lambda_c^-$ threshold. For the Belle II experiment main goal is 50ab^{-1}. To estimate potential yield of charm hadrons we use $\sigma(ee \to c\bar{c}) = 1.1nb$ and fragmentation-fractions obtained in [19, 20]. There is of course ambiguity due to event selection criteria in each experiment and each particular channel. Here we used conservative estimations for number of events, assuming only 30% of event will pass the selection for Belle II experiment and 70% for c–τ factory, since much cleaner environment is expected there. For the D^0 studies we assumed $D^{*\pm}$ tagging in Belle II and semileptonic tag-side decays for $\psi(3770) \to D^0\overline{D}^0$ case at Super c–τ factory. Results are summarized in table 1.

To confirm that there is no bias, we generate 100 MC samples of 40×10^6 events, which correspond to $D^+ \to K^0_S\pi^+$ decay, each with a value of the angle δ in the $[-90^\circ, 90^\circ]$ interval

Channel	Branching fraction, % [9]	Estimated yield, ×10^6, Belle II/SCTF	Uncertainty in δ, Belle II/SCTF
$D^+ \to K_S^0 \pi^+$	1.56 ± 0.03	40/50	5°/3°
$D_s^+ \to K_S^0 K^+$	1.46 ± 0.04	20/40	7°/5°
$\Lambda_c^+ \to K_S^0 p$	1.59 ± 0.08	15/10	8°/10°
$D^0 \to K_S^0 \pi^0$	1.23 ± 0.02	30/20	6°/7°

Table 1. Branching fractions and production yields for considered channels. In the brackets estimated statistical uncertainty in δ measurement is specified.

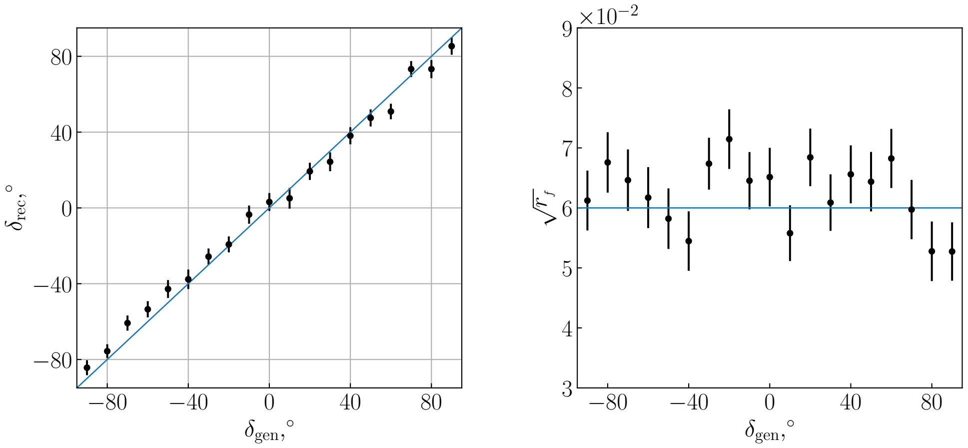

Figure 3. Results of feasibility study for the decay $D^+ \to K_S^0 \pi^+$.

with a step of 10°. For the two-body decay modes with K^0 DCS/CF amplitude ratios have not been measured yet, and could be approximated as $r_f \sim \mathcal{O}(\tan^4 \theta_c)$. Analogous estimation for $D^0 \to K^+\pi^-$ slightly differs from experimental result, $r_D = (0.344 \pm 0.002)\%$ [14]. SU(3)$_f$-breaking terms K and K' introduced in (1.5), (1.6) aiming to fix this small discrepancy on the amplitude level. Based on both experimental and theoretical data it is reasonable to assume other DCS/CF ratios to be of the same order $\sim \mathcal{O}(10^{-3})$. For this test we use $\sqrt{r_f} = 0.06$ which is very close to value measured for $D^0 \to K^+\pi^-$ decay. Each sample of MC contains time-dependent decay rates for both particle and antiparticle (see example in figure 2). For each sample we perform simultaneous unbinned maximum-likelihood fit for both time-dependent decay rates. In the fit we consider events with $t/\tau_{K_S} > 1$, since kaon mixing does not contribute at low lifetimes. Fit results of one of the samples presented in figure 3. Obtained results are in good agreement with generated values of strong phase difference and amplitude ratio.

Since amplitude ratios $\sqrt{r_f}$ were not previously measured we perform a scan for its values in the interval $[0.01, 0.11]$ with step 2.5×10^{-3} simultaneously varying strong phase in the interval $[-90°, 90°]$. Obtained uncertainties for 20 and 50 million events, corresponding to $D^0 \to K_S^0 \pi^0$ and $D^+ \to K_S^0 \pi^+$ decays shown in figure 4. As one could expect we observe increasing sensitivity for strong phase with higher values of $\sqrt{r_f}$. For the given amplitude

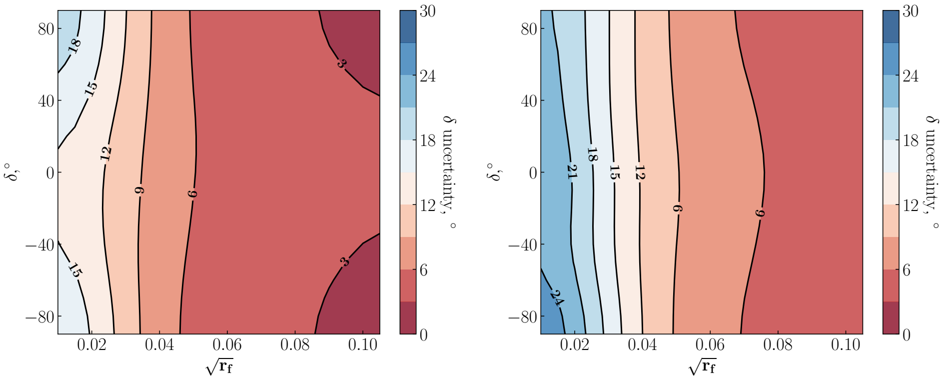

Figure 4. Obtained uncertainties for the strong phase difference — δ in D^+ decay *(left)* and in D^0 decay *(right)*.

ratio uncertainty in δ varies insignificantly over the range $[-90°, 90°]$, which is certainly the advantage of the method comparing it to usage of semileptonic K^0 decays [8].

Measurements for the decay $D^0 \to K^0_S\pi^0$ are of great importance for the $\mathrm{SU}(3)_f$ probe. Since achieving proper D^0 lifetime resolution is hardly feasible we consider integration over D^0 lifetime — t'. For the purpose of the test MC generated distributions take into account mixing effects, but fitting *p.d.f.* are not. 1000 pseudo experiments we performed and we found that amplitude ratio distribution turn out to be shifted at about 1σ to the higher values. Such shift in general is expected due to excess of "wrong"-flavour kaons arised from mixing. On the other hand strong phase measurements still proved to be in good agreement with generated values. We observed a 2° bias in δ, whereas the statistical uncertainty is 6°.

5 Kaon regeneration

Proposed method based on the time-dependent study of $K^0 \to \pi^+\pi^-$ decays, however beside flavour physics parameters some other effects could contribute to decay rates. For example studies carries out in [21] showed that regeneration in the environment of today's experiments could induce a bias in the A_{CP} measurement up to the level of 10^{-3}.

In order to describe kaon propagation through matter the Hamiltonian in Schrödinger equation (2.1) should be modified in the following way [22]

$$i\partial_t \begin{pmatrix} K^0(t) \\ \overline{K}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right) \begin{pmatrix} K^0(t) \\ \overline{K}^0(t) \end{pmatrix} - \begin{pmatrix} \chi & 0 \\ 0 & \overline{\chi} \end{pmatrix} \begin{pmatrix} K^0(t) \\ \overline{K}^0(t) \end{pmatrix}, \tag{5.1}$$

where the second matrix describe nuclear scattering and coefficients defined as

$$\chi = -\frac{2\pi N}{m} f \quad \text{and} \quad \overline{\chi} = -\frac{2\pi N}{m} \overline{f}, \tag{5.2}$$

where $f(\overline{f})$ — are forward scattering amplitudes for $K^0(\overline{K}^0)$, m — K^0 mass, $N = (\rho N_A)/M$ — volume density of the material, N_A — Avogadro's number, ρ — mass density, M — molar mass. Strangeness conservation in strong interactions leads to inequality forward scattering amplitudes $\Delta f \equiv f - \overline{f} \neq 0$. The evolution of the $K^0_S(K^0_L)$ beam could be expressed than

$$\alpha_{S,L} = e^{-i\Sigma t}\left[\alpha^0_{S,L}\cos\left(\frac{\Delta\lambda}{2}\sqrt{1+4r^2t}\right) \pm i\frac{\alpha^0_{S,L} \mp 2r\alpha^0_{L,S}}{\sqrt{1+4r^2}}\sin\left(\frac{\Delta\lambda}{2}\sqrt{1+4r^2t}\right)\right], \tag{5.3}$$

where

$$\begin{aligned}\Sigma &= \frac{1}{2}\left(\lambda_S + \lambda_L + \chi + \overline{\chi}\right),\\ \Delta\lambda &= \lambda_S - \lambda_L,\\ \Delta\chi &= \chi - \overline{\chi},\\ r &= \frac{1}{2}\frac{\Delta\chi}{\Delta\lambda}.\end{aligned} \tag{5.4}$$

Regeneration parameter — r is typically of the order of 10^{-2}, so in the following calculations we use the expansion for $\alpha_{S,L}$ to the lowest order of r (details could be found in ref. [23]). It is conventional to introduce the geometrical regeneration parameter:

$$\zeta = r\left(1 - e^{i\Delta\lambda\frac{Lm}{p}}\right), \tag{5.5}$$

where p — is kaon momentum and L — regenerator thickness. Amplitudes (5.3) could be expressed than in the form:

$$\begin{aligned}\alpha_S(t) &= e^{\frac{1}{2}(\chi+\overline{\chi})t}e^{-i\lambda_S t}(\alpha^0_S + \zeta\alpha^0_L e^{-i\Delta\lambda t}),\\ \alpha_L(t) &= e^{\frac{1}{2}(\chi+\overline{\chi})t}e^{-i\lambda_L t}(\alpha^0_L + \zeta\alpha^0_S).\end{aligned} \tag{5.6}$$

Applying equations (5.6) recursively for each passage through matter one could account for kaon regeneration.

While accurate estimation of bias induced by regeneration should be performed for each particular experiment, here we present an estimation based on typical configurations. Since this study is mostly concerned with big kaon lifetimes, we assume that neutral kaon have to pass through a beryllium beam pipe ($\sim$ 1mm) and a number of silicon layers of vertex detector. As a reference Belle II configuration was used, where silicon vertex detector consists of 6 layers ($L_{1,2} \simeq 50\mu$m and $L_{3-6} \simeq 300\mu$m).

For this test we considered only leading regeneration contribution to CF decay modes, since DCS/CF interference term is $\mathcal{O}(10^{-2})$ suppressed and DCS term $\mathcal{O}(10^{-3})$ suppressed. We used the cross sections and differences of forward scattering amplitudes obtained in [24].[1] Using MC simulation we found that for 1GeV/c kaons bias in the strong phase measurement is under 4°. Obtained value is comparable with potential statistical uncertainty, however regeneration could be the main source of systematic uncertainty and for each particular environment studies are required.

---

[1] Regeneration studies in CPLEAR experiment [25] showed good agreement between optical model predictions and experimental results.

6 Summary

In this paper we presented a method to measure strong phase differences in charm hadron decay with K^0-meson in the final state. It was shown that CPV in K^0-$\overline{K}^0$ mixing allows us to disentangle initial combination $a\left|K^0(t)\right\rangle + b\left|\overline{K}^0(t)\right\rangle$ that arises in the presence of CF and DCS decays. In order to perform such measurement the experiment should satisfy following requirements: large tracking detector that allows reconstruct K_S^0 decays even after $10\tau_{K_S}$, sufficient statistics of charm decays — $\mathcal{O}(10^6..10^7)$ and proper charged hadron identification. The Belle II experiment and future Super c–τ factory are good candidates for such measurement. While LHC experiments have huge data samples of charmed mesons, some of the features of the detectors significantly reduce the possibilities of such measurements: LHCb has too short tracker, while CMS and Atlas have no particle identification.

To estimate potential precision of the method feasibility study was performed. Expected number of events was calculated for each particular channel for both experiments. Proposed measurements proved to be unbiased and free of trigonometrical ambiguity. Obtained results for statistical uncertainty (assuming $r_f \sim \mathcal{O}(10^{-3})$) are comparable with current theoretical uncertainties and uncertainties that could be obtained with semileptonic kaon decays. We also presented an estimation of regeneration contributions to proposed measurement.

Acknowledgments

The work of P. Pakhlov was conducted within the framework of the Basic Research Program at the National Research University Higher School of Economics (HSE). The reported study of V. Popov was funded by RFBR, project number 19-32-90104.

References

[1] Y. Grossman and D.J. Robinson, SU(3) *Sum Rules for Charm Decay*, *JHEP* **04** (2013) 067 [`arXiv:1211.3361`] [INSPIRE].

[2] LHCb collaboration, *Observation of CP-violation in Charm Decays*, *Phys. Rev. Lett.* **122** (2019) 211803 [`arXiv:1903.08726`] [INSPIRE].

[3] M. Golden and B. Grinstein, *Enhanced CP-violations in Hadronic Charm Decays*, *Phys. Lett. B* **222** (1989) 501 [INSPIRE].

[4] G. Hiller, M. Jung and S. Schacht, SU(3)*-flavor anatomy of nonleptonic charm decays*, *Phys. Rev. D* **87** (2013) 014024 [`arXiv:1211.3734`] [INSPIRE].

[5] J. Brod, Y. Grossman, A.L. Kagan and J. Zupan, *A Consistent Picture for Large Penguins in* $D \to \pi^+\pi^-$, $K? + K^-$, *JHEP* **10** (2012) 161 [`arXiv:1203.6659`] [INSPIRE].

[6] E. Franco, S. Mishima and L. Silvestrini, *The Standard Model confronts CP-violation in* $D^0 \to \pi^+\pi^-$ *and* $D^0 \to K^+K^-$, *JHEP* **05** (2012) 140 [`arXiv:1203.3131`] [INSPIRE].

[7] F. Buccella, A. Paul and P. Santorelli, $\mathrm{SU}(3)_F$ *breaking through final state interactions and* CP *asymmetries in* $D \to PP$ *decays*, *Phys. Rev. D* **99** (2019) 113001 [`arXiv:1902.05564`] [INSPIRE].

[8] P. Pakhlov and V. Popov, *Measurement of* $D^0 - \bar{D}^0$ *mixing parameters using semileptonic decays of neutral kaon*, *JHEP* **02** (2020) 160 [`arXiv:1912.04955`] [INSPIRE].

[9] Particle Data Group collaboration, *Review of Particle Physics*, *PTEP* **2020** (2020) 083C01 [INSPIRE].

[10] Y. Grossman and Y. Nir, *CP Violation in* $\tau^\pm \to \pi^\pm K_S \nu$ *and* $D^\pm \to \pi^\pm K_S$*: The Importance of* $K_S - K_L$ *Interference*, *JHEP* **04** (2012) 002 [`arXiv:1110.3790`] [INSPIRE].

[11] I.I.Y. Bigi and H. Yamamoto, *Interference between Cabibbo allowed and doubly forbidden transitions in* $D \to K_{S,L} + \pi$*'s decays*, *Phys. Lett. B* **349** (1995) 363 [`hep-ph/9502238`] [INSPIRE].

[12] A.L. Kagan and L. Silvestrini, *Dispersive and absorptive* CP *violation in* $D^0 - \overline{D^0}$ *mixing*, *Phys. Rev. D* **103** (2021) 053008 [`arXiv:2001.07207`] [INSPIRE].

[13] F.-S. Yu, D. Wang and H.-n. Li, CP *asymmetries in charm decays into neutral kaons*, *Phys. Rev. Lett.* **119** (2017) 181802 [`arXiv:1707.09297`] [INSPIRE].

[14] HFLAV collaboration, *Averages of b-hadron, c-hadron, and* τ*-lepton properties as of 2018*, *Eur. Phys. J. C* **81** (2021) 226 [`arXiv:1909.12524`] [INSPIRE].

[15] Belle collaboration, *Evidence for CP-violation in the Decay* $D^+ \to K^0_S \pi^+$, *Phys. Rev. Lett.* **109** (2012) 021601 [*Erratum ibid.* **109** (2012) 119903] [`arXiv:1203.6409`] [INSPIRE].

[16] Belle-II collaboration, *Belle II Technical Design Report*, `arXiv:1011.0352` [INSPIRE].

[17] Belle-II collaboration, *The Belle II Physics Book*, *PTEP* **2019** (2019) 123C01 [*Erratum ibid.* **2020** (2020) 029201] [`arXiv:1808.10567`] [INSPIRE].

[18] Charm-Tau Factory collaboration, *Project of a Super Charm-Tau factory at the Budker Institute of Nuclear Physics in Novosibirsk*, *Phys. Atom. Nucl.* **76** (2013) 1072 [INSPIRE].

[19] M. Lisovyi, A. Verbytskyi and O. Zenaiev, *Combined analysis of charm-quark fragmentation-fraction measurements*, *Eur. Phys. J. C* **76** (2016) 397 [`arXiv:1509.01061`] [INSPIRE].

[20] L. Gladilin, *Fragmentation fractions of c and b quarks into charmed hadrons at LEP*, *Eur. Phys. J. C* **75** (2015) 19 [`arXiv:1404.3888`] [INSPIRE].

[21] B.R. Ko, E. Won, B. Golob and P. Pakhlov, *Effect of nuclear interactions of neutral kaons on CP asymmetry measurements*, *Phys. Rev. D* **84** (2011) 111501 [`arXiv:1006.1938`] [INSPIRE].

[22] M.L. Good, *Relation between Scattering and Absorption in the Pais-Piccioni Phenomenon*, *Phys. Rev.* **106** (1957) 591 [INSPIRE].

[23] W. Fetscher, P. Kokkas, P. Pavlopoulos, T. Schietinger and T. Ruf, *Regeneration of arbitrary coherent neutral kaon states: A new method for measuring the K0 anti-K0 forward scattering amplitude*, *Z. Phys. C* **72** (1996) 543 [INSPIRE].

[24] P.H. Eberhard and F. Uchiyama, *Neutral kaon regeneration probabilities at asymmetric phi factory energies*, *Nucl. Instrum. Meth. A* **350** (1994) 144 [INSPIRE].

[25] CPLEAR collaboration, *Measurement of the neutral kaon regeneration amplitude in carbon at momenta below 1-GeV/c*, *Phys. Lett. B* **413** (1997) 422 [INSPIRE].