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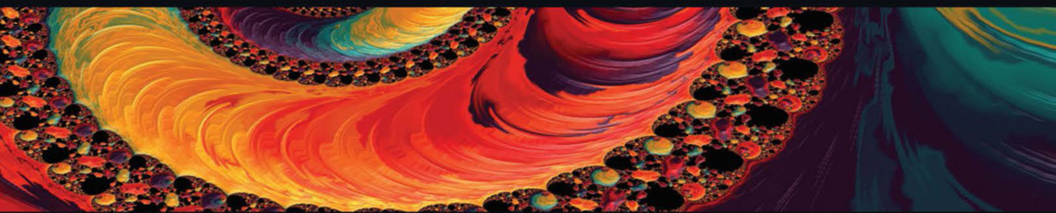

NEW CENTRALITY MEASURES IN NETWORKS

How to Take into Account the Parameters of the Nodes and Group Influence of Nodes to Nodes

Fuad Aleskerov

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A CHAPMAN & HALL BOOK

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About the Authors

Fuad Aleskerov graduated from the Mathematical Department of Moscow State University in 1974. He is the Head of the International Center of Decision Choice and Analysis, Head of the Department of Mathematics in the Faculty of Economic Sciences, National Research University Higher School of Economics (HSE University); he also holds the Mark Aizerman Chair on Choice Theory and Decision Analysis, Trapeznikov Institute of Control Sciences of the Russian Academy of Sciences. Aleskerov is on the editorial board for 16 journals, has published ten books and more than 150 articles in peer-reviewed journals, and has presented invited papers at more than 160 international conferences. He is a member of several international scientific societies including Academia Europaea.

Sergey Shvydun earned his PhD degree (*cum laude*) in Applied Mathematics from the National Research University Higher School of Economics (HSE University) in 2020. He is a senior research fellow at HSE's International Center of Decision Choice and Analysis and an associate professor at the Department of Mathematics in the HSE Faculty of Economic Sciences. He is also a senior research fellow at the Laboratory on Choice Theory and Decision Analysis of the Trapeznikov Institute of Control Sciences of the Russian Academy of Sciences. His research interests are in data analysis, social choice theory, and social networks analysis.

Natalia Meshcheryakova is a PhD student in Computer Science at the National Research University Higher School of Economics (HSE University). She received a master's degree in Computer Science from the HSE University in 2018. She is a research fellow at HSE's International Center of Decision Choice and Analysis and a lecturer at the Department of Mathematics in the HSE Faculty of Economic Sciences. She is also a research fellow at the Laboratory on Choice Theory and Decision Analysis of the Trapeznikov Institute of Control Sciences of the Russian Academy of Sciences. Her research interests include social networks, machine learning, and data analysis.

Introduction

Today, networks are used to represent socio-economic processes, human relations, biological and physical processes, etc. (e.g., Newman 2010, Jackson 2008). Usually, the main, and probably the first, problem studied in networks analysis has been the detection of the most important elements in a network. This very problem was investigated from the first publications on social networks, such as relations between schoolchildren in classes in the 1930s (see Newman 2010). There is also interest nowadays in these topics in Russia (e.g., Gubanov et al. 2011, Gubanov et al. 2019, Kireyev & Leonidov 2018). A very interesting survey was done by Kalinina et al. (2018).

However, in all these models, the vertices in networks are considered to be objects of similar type, i.e., the parameters of the vertices are not taken into account, although in some very important publications this shortage is emphasized openly (e.g., Newman 2010). To attract attention to this problem, let us discuss an example.

Consider a loan of $1 million, borrowed from a bank, and assume that the loan is not repaid in time. If the bank is large it can survive, but for a small, say regional, bank, it might be a cause of bankruptcy. Thus, in the analysis of the loans network, the parameters of the banks should be taken into account.

Let us extend this example to the case of two borrowers who take $500,000 each from the same small regional bank. If one of them repays the loan, the bank will survive, however, if neither borrower returns the loan, it will be the same as the $1 million, and again, the bank will announce bankruptcy.

This last example shows that in the analysis of influence in networks we should take into account the group influence of nodes (players in networks) on an individual node. To the best of our knowledge, this very concept has been discussed in few works (e.g., Myerson 1977).

We can see many examples of this kind. For instance, in the network of international conflicts, the parameters of the countries might be the level of armaments in the countries involved, and the group influence might be evaluated by the military blocs to which the countries belong.

In this book we introduce a class of indices, incorporating these new ideas, and we illustrate the use of these indices via many examples.

THE STRUCTURE OF THE BOOK

In Chapter 1, we introduce the notion of networks, discuss the main classic centrality indices in networks, and introduce new indices – short-range interaction centrality (SRIC) and long-range interaction centrality (LRIC), which differ in the lengths of the path taken into account in the analysis of networks.

In this chapter we also discuss new concepts in the analysis of the power of nodes, based on their interdependence and the impact of indirect connections in network structures. Both these concepts use ideas we have developed before.

Chapter 2 widely illustrates applications of the new indices. It contains the analysis of a global financial market where the countries are key borrowers in the market. Next, we study the networks of international migration, world trade, the global food network, the network of global arms transfers, the network of terrorist groups, and the network of international economic journals. In each case we discuss how to take the parameters of the vertices into account, as well as how to define the group influence in each case.

CHAPTER 1

Centrality Indices in Network Analysis

In this chapter, we present well-known and widely used classical centrality indices such as different forms of the in- and out-degree indices, centralities based on the eigenvector evaluation, centralities based on the idea of the shortest path, and a centrality index based on cooperative game theory. There are more known indices, and we mention them shortly without presenting their formal definitions. Then, we discuss the shortcomings of classical indices and provide an example showing the necessity of taking into account the parameters of nodes in networks and the possibility of the group influence of nodes to a node in the network. Hence, we propose new classes of indices introduced by our team – short- and long-range interaction centralities (SRIC and LRIC). They take into account not only the features mentioned above but also indirect influence among nodes. Additionally, we have extended the LRIC index for the evaluation of influence in a network where a flow in the network may result in nodes becoming too interdependent on each other, and consequently have some power against each other using the same flow. This measure

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is called the interdependence index. Finally, we propose several measures of the edge importance assessment.

1.1 CLASSICAL CENTRALITY MEASURES

Standard methods for the detection of the most influential elements in networks are based on the evaluation of centrality indices for each node, and ranking nodes according to these centrality values (Newman 2010). The higher the centrality of the node, the more important the node is in a network.

There have been many indices developed to measure the centrality level of each node. The measures used have different natures and interpretation and include information about the position of a node and its neighbors in a network. Some of these are based on the number of links to other nodes. Others consider how closely each node is located to other nodes in a network, in terms of distance, or how many times it is on the shortest paths connecting any given pairs of nodes. There are also some indices based on ideas from cooperative game theory and voting theory. In this chapter, we consider the most popular centrality measures known in the literature.

We operate with a graph $G = (V,E)$, where $V = \{1,\ldots,n\}$ is a set of nodes, and $E \subseteq V \times V$ is a set of edges (edge $(i,j) \in E$). We consider undirected and directed graphs. For the latter, the existence of edge (i,j) does not imply the existence of edge (j,i). To describe a graph, we use adjacency matrix $A = [a_{ij}]$, where $a_{ij} = 1$ if there is edge (i,j), and $a_{ij} = 0$ otherwise. Additionally, if connections between nodes are associated with some numerical values, representing the intensity of connections, the graph can be described by a weighted adjacency matrix $W = [w_{ij}]$ that stores the weights of the edges.

1.1.1 Degree Centralities

The simplest centrality measure for undirected graphs is the degree centrality, which is calculated as the total number of neighbors for each node i (Freeman 1979):

$$C_i^{deg} = \sum_{j=1}^{n} a_{ij} = \sum_{j=1}^{n} a_{ji} \tag{1.1}$$

High values of the degree centrality identify nodes with the highest number of connections to other nodes, i.e., nodes for which it is easier to gain access to and/or influence over other nodes locally.

For directed unweighted graphs, four versions of degree centrality measure are possible: in-degree centrality, out-degree centrality, degree centrality, and degree difference. These measures take into account the direction of connections. Additionally, a degree centrality can be adapted to directed or undirected weighted networks. A description of each measure is provided in Table 1.1.

TABLE 1.1 Degree Centrality Measures

Name		Equation	Description
Unweighted graph	In-degree centrality	$C_i^{in-deg} = \sum_{j=1}^{n} a_{ji}$	The number of incoming edges
	Out-degree centrality	$C_i^{out-deg} = \sum_{j=1}^{n} a_{ij}$	The number of outgoing edges
	Degree centrality	$C_i^{deg} = \sum_{j=1}^{n} \left(a_{ij} + a_{ji}\right)$	The total number of *i*'s connections
	Degree difference	$C_i^{diff} = \sum_{j=1}^{n} \left(a_{ij} - a_{ji}\right)$	The difference between the number of outgoing and incoming edges
Weighted graphs	Weighted in-degree centrality	$C_i^{w\,in-deg} = \sum_{j=1}^{n} w_{ji}$	The total weight of incoming edges
	Weighted out-degree centrality	$C_i^{w\,out-deg} = \sum_{j=1}^{n} w_{ij}$	The total weight of outgoing edges
	Weighted degree centrality	$C_i^{w\ deg} = \sum_{j=1}^{n} \left(w_{ij} + w_{ji}\right)$	The total weight of *i*'s connections
	Weighted degree difference	$C_i^{w\,diff} = \sum_{j=1}^{n} \left(w_{ij} - w_{ji}\right)$	The difference between the total weight of outgoing and incoming edges

In-degree and out-degree centralities show how a node is affected by its neighbors. For instance, the higher the out-degree centrality of a particular node is, the more nodes are under its control. Degree centrality identifies the most active nodes in different parts of a network, while degree difference is used to evaluate the relative influence of a node on its neighbors. The interpretation of weighted degree centralities is practically the same as for unweighted degree centralities, but weighted measures are more representative than unweighted ones, due to the fact that weighted networks consider the intensities of connections. One should note here that normalized versions of these measures also exist.

1.1.2 Eigenvector Centralities

Since the degree centrality measures do not consider the importance of adjacent nodes, i.e., the information about the degree centrality of its neighbors, several indices have been developed which take this feature into account. An eigenvector centrality considers not only neighboring, but also long-distance connections. The eigenvector centrality (C^{ev}) assigns relative scores to all nodes in a network based on the concept that connections to high-scoring nodes contribute more to the score of the node in question than connections to low-scoring nodes. The idea is that the importance of node i depends on the importance of its neighbors, which, in turn, depends on the importance (degree) of its neighbors, and so on, i.e.,

$$C_i^{ev} = \frac{1}{\lambda} \cdot \sum_{(i,j)\in E} C_j^{ev} = \frac{1}{\lambda} \cdot \sum_{j=1}^{n} C_j^{ev} \cdot A_{ij} \tag{1.2}$$

The calculation of the centrality measure for each node is related to an eigenvalue problem with respect to adjacency matrix A of a graph: a vector of relative centrality C^{ev} is an eigenvector of the adjacency matrix, i.e.,

$$A \cdot C^{ev} = \lambda \cdot C^{ev}. \tag{1.3}$$

Generally, all eigenvectors of matrix A can be considered as a centrality measure. However, an eigenvector that corresponds to a maximal eigenvalue is preferable: by the Perron–Frobenious theorem, this vector (and only this, except its co-directional vectors) is positive and real for irreducible non-negative matrix A (Gantmacher 2000) which, by definition, can be presented as a strongly connected graph.

Another generalization of a degree centrality is the Katz centrality (Katz 1953). It measures the weighted count of all paths coming from the node, while a contribution of path of length n is counted with respect to attenuation factor β^n, i.e.,

$$C_i^{Katz} = \beta \sum_{j=1}^{n} a_{ij} + \beta^2 \sum_{j=1}^{n} \left(A^2\right)_{ij} + \ldots = \sum_{k=1}^{\infty} \sum_{j=1}^{n} \beta^k \left(A^k\right)_{ij} \tag{1.4}$$

or in a matrix form

$$C^{Katz} = \left(\left(I - \beta A\right)^{-1} - I\right) \cdot \vec{e}, \tag{1.5}$$

where $\vec{e}$ is the unit vector, I is the identity vector.

Basically, this measure is applicable to symmetric graphs since the computation of eigenvectors is more difficult for non-symmetric matrices and can produce complex or zero eigenvalues.

Other measures have been introduced to overcome this shortage. An example of these measures is α-centrality (Bonacich 2001). This centrality is defined as the solution of the two-parameter equation

$$C_i^{\alpha}\left(\alpha, \beta\right) = \alpha \cdot \textstyle\sum_j C_j^{\alpha}\left(\alpha, \beta\right) \cdot A_{ij} + \beta \tag{1.6}$$

or in a matrix form

$$C^{\alpha}\left(\alpha, \beta\right) = \left(I - \alpha \cdot A\right)^{-1} \cdot \beta. \tag{1.7}$$

The introduction of parameter β, which corresponds to the initial value of centralities, precludes the possibility of a solution with zero components. In practice, parameter α is selected so that $\alpha < 1/\lambda_{max}$, where λ_{max} is the largest eigenvalue of matrix A.

Another example of centrality that can be applied to directed graphs is the PageRank centrality (Brin & Page 1998). According to the model, the importance of a particular node depends on the probability that it be visited by a random walker, i.e.,

$$C_i^{PageRank} = \alpha \cdot \sum_j \frac{C_j^{PageRank}}{C_i^{out-deg}} \cdot a_{ij} + \frac{1-\alpha}{n} \tag{1.8}$$

or in a matrix form

$$C^{PageRank} = \frac{1-\alpha}{n} \cdot \left[I - \alpha \cdot A \cdot \left(I \cdot C^{out-deg} \right)^{-1} \right]^{-1} \cdot \vec{e}, \tag{1.9}$$

where α is the probability of continuing the walk (in general, $\alpha = 0.85$).

Many other measures exist which are based on the idea of eigenvector calculation: Bonacich centrality (Bonacich 1987), hubs and authorities (Kleinberg 1999), subgraph centrality (Estrada & Rodriguez-Velazquez 2005), etc. Note that these measures can be easily adapted to weighted matrix W.

1.1.3 Centralities Based on the Shortest Paths

Another class of centralities is based on the shortest paths between nodes. Two of the most well-known measures are closeness (Bavelas 1950) and betweenness (Freeman 1977) centralities.

Closeness centrality considers how closely each node is located to other nodes of a network in terms of distance. As a result, it elucidates nodes that are the closest to other nodes, and it is usually calculated as

$$C_i^{cl} = \frac{1}{\sum_{j=1}^{n} d_{ij}}, \tag{1.10}$$

where d_{ij} is the length of the shortest path from node i to node j.

A similar idea to closeness centrality was proposed by Rochat (2009), and is called harmonic centrality. This centrality is calculated as the sum of inverse distances between pairs of nodes. If there is no path between a pair of nodes, then the corresponding summand is equal to zero. Thus, this measure performs better on disconnected graphs.

Another way to consider the lengths of the shortest paths is a decay centrality (Jackson 2008). The idea of this measure is to summarize some coefficient $\delta \in (0,1)$ to the power of the lengths of considered paths. A generalized measure of centrality based on closeness was also proposed by Agneessens et al. (2017). The author introduced a tuning parameter δ that measures the importance of geodesic distances and showed that using the parameter, degree-centrality and closeness centrality are two specific instances of their more general measure.

Betweenness centrality detects nodes that lie on the shortest paths between any other two nodes most of the time. It is defined as

$$C_i^{btw} = \sum_{jk} \frac{\sigma_{jk}(i)}{\sigma_{jk}}, \tag{1.11}$$

where σ_{jk} is the number of the shortest paths from node j to node k, $\sigma_{jk}(i)$ is the number of the shortest paths from node j to node k going through the node i. High betweenness centrality identifies nodes that are crucial hubs and/or bridges between disparate clusters in a network.

A measure similar to betweenness centrality is stress centrality (Shimbel 1953). The main difference is that coefficients of stress centrality are not normalized to the total number of the shortest paths between considered nodes. The concept of betweenness centrality was extended to a group level in Everett & Bogatti (1999). There is also percolation centrality (Piraveenan et al. 2013) which is based on the idea that each considered path is weighted to the contribution of this path to a percolation process, while the weights of paths depend on the percolation level of a source node and the total percolation state of a network. Contrary to the centralities that are described above, percolation centrality requires initial conditions of the level of percolation of each node.

1.1.4 Centralities from Cooperative Game Theory

An influence in networks is also evaluated in the field of game theory and mechanism design. There are various power indices that are applied to the network theory. In this case, a network is interpreted as a set of interacting individuals that contribute to a total productive value of a network, and the problem is how to share this generated value among them.

Myerson (1977) proposed a value that is based on the Shapley–Shubik index (Shapley & Shubik 1954). The Myerson value shows an average contribution for each node, where the contribution is a function v generated by the network, with and without this individual, i.e.,

$$C_i^{MV}(G,v) = \sum_{S \in V} \frac{(|S|-1)!(|N|-|S|)!}{|N|!}\left(v(S) - v(S \setminus \{i\})\right), \qquad (1.12)$$

The main disadvantages of this approach are its large computational complexity (since it requires consideration of all possible subgraphs) and uncertainty (about how the value of a subgraph should be assigned). Partial rankings of nodes based on the neighborhood-inclusion principle were also discussed by Schoch (2018).

Many other approaches to key nodes detection exist. For instance, a different perspective of the central nodes' estimation was proposed by Kang et al. (2012) which is called diffusion centrality. This centrality takes into account the attributes of nodes and their properties, and measures the quality of diffusion of a property p starting from a node v. Despite the completeness of the proposed measure, some prior conditions should be known as conditional probabilities of the 'infection' or so-called 'diffusion rules'.

Many real networks are complex, and their elements are not homogenous. The nodes of a network may have various individual attributes that characterize their size, importance, level of influence, etc. This possibility was mentioned by Newman (2003): '[…] and vertices or edges may have a variety of properties, numerical or otherwise, associated with them'. For instance, the threshold of influence, which indicates the level when this node becomes affected, may give the result that even connections with the same weight w can be influential for node i, and not influential for some other node, j, depending on the attributes of these nodes. The size or importance of nodes may lead to the situation that influence on a group of nodes may, in total, contribute less than influence on a single node. Finally, some nodes can influence other nodes only in collaboration with some other members.

All these aspects show that power distribution in networks should be evaluated with respect to both the individual attributes of nodes and the connections between them. Unfortunately, centrality measures based on degree or the shortest paths do not take into account the nodes' features, and consequently cannot be applied since initial connections of a network do not represent the actual picture of the nodes' influence. Diffusion centrality considers particular attributes of nodes, but it lacks such features as individual thresholds of influence, nodes' importance, and the possibility of group influence. Moreover, the way to define diffusion rules in various applications is unclear. On the other hand, the influence measure proposed by Myerson (1977) considers group influence but does not take into consideration the individual characteristics of nodes, or the possibility of indirect influence. Therefore, we consider new centrality measures that consider the individual attributes of nodes, as well as their group and indirect influence.

1.2 SHORT- AND LONG-RANGE INTERACTION CENTRALITY INDICES

1.2.1 Individual and Group Influence

First, we introduce parameter q_j that each node can possess in an explicit form. This possibility was mentioned by Newman (2003).

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