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Testing dynamic consistency and consequentialism under ambiguity[☆]

Han Bleichrodt[a,b], Jürgen Eichberger[c], Simon Grant[b], David Kelsey[d], Chen Li[a,*]

[a] *Erasmus School of Economics, Rotterdam, the Netherlands*
[b] *Research School of Economics, Australian National University, Australia*
[c] *Alfred Weber Institut, Universität Heidelberg, Germany*
[d] *Nottingham University Business School England, United Kingdom*

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Accounting for ambiguity aversion in dynamic decisions generally implies that either dynamic consistency or consequentialism must be given up. To gain insight into which of these principles better describes people's preferences, we tested them using a variation of Ellsberg's three-color urn experiment. Subjects were asked to make a choice both before and after they received a signal. We found that most ambiguity neutral subjects satisfied both dynamic consistency and consequentialism and behaved consistent with subjective expected utility with Bayesian updating. The majority of ambiguity averse subjects satisfied consequentialism, but violated dynamic consistency.

1. Introduction

Many real-world decisions involve ambiguity, where outcomes are uncertain and their probabilities unknown. Moreover, most of these decisions are dynamic in the sense that information becomes available over time, probabilities are updated, and optimal strategies or policies may be revised. Examples are the threats from a new disease and climate change. The classic model to analyze such decisions is subjective expected utility, which assumes that decision makers are ambiguity neutral with updating of probabilities according to Bayes' rule. However, since Ellsberg (1961), empirical research has shown that people do not behave according to subjective expected utility, but are averse to ambiguity and update in non-Bayesian manners.

There are by now a number of well-established theories to capture such ambiguity aversion, but they are essentially static in nature.[1] To extend these theories to dynamic decisions raises an interesting problem. Several authors have shown

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* Corresponding author.
E-mail address: c.li@ese.eur.nl (C. Li).

[1] In these models beliefs either cannot be represented by a (single) probability measure or the probability measure is a second-order belief and the corresponding induced preferences do not satisfy reduction of compound lotteries. Examples of the former are the multiple priors model of

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that modeling ambiguity aversion in dynamic decisions implies giving up one of two principles that are usually considered rational: *consequentialism*, the requirement that preferences only depend on outcomes on states that are still possible, and *dynamic consistency*, the requirement that decision makers stick to their optimal contingent plans.[2] Subjective expected utility maximizers who update their beliefs according to Bayes' rule satisfy both consequentialism and dynamic consistency, but ambiguity averse decision makers cannot. Hence, dynamic ambiguity models must choose which of these principles to retain.[3]

The purpose of our paper is to inform this choice by providing experimental evidence on the descriptive validity of these two principles. The question which of these principles has more descriptive appeal is also of interest for applied economic research (standard dynamic optimization techniques require dynamic consistency) and for policy. For example, consequentialism may need to be given up if policy makers care not only about the outcome of decisions, but also about the process by which they are arrived at. This has been emphasized for example in the provision of health care.

Our experiment is a variation of the Ellsberg three-color urn problem (Ellsberg, 1961). We used 200 cards that were both numbered and colored. The color composition was the same for even- and odd-numbered cards: for each parity there were 33 red cards while the remaining 67 cards were blue and yellow in unknown proportions. Subjects chose a color to bet on for both odd and even numbered cards. This allowed them to hedge against ambiguity. For instance, if a subject chose to bet on blue for odd cards and yellow for even cards then, even though they did not know the exact number of blue odd cards or yellow even cards, they knew that this combination gave a 67/200 chance to win. Subjects then received a signal whether the winning card was odd or even. This signal removed the possibility to hedge against ambiguity: conditional on knowing whether the cards was even or odd, the choice became similar to the standard Ellsberg three-color problem.

By comparing subjects' choices before and after the signal, we could test dynamic consistency and consequentialism. An ambiguity averse subject who satisfied dynamic consistency would make the same choice before and after the signal, whereas an ambiguity averse subject who satisfied consequentialism would switch choices after the signal. To distinguish ambiguity averse subjects clearly and to detect violations of dynamic consistency, we included a small cost for being ambiguity averse. This also allowed us to test to what extent subjects' preferences were monotonic in probability and satisfied (first-order) stochastic dominance.

We observed that 42% of our subjects were ambiguity neutral. Around two thirds of these ambiguity neutral subjects satisfied both consequentialism and dynamic consistency and behaved consistent with subjective expected utility with Bayesian updating. Around one third of our subjects were ambiguity averse. Most of these behaved according to consequentialism and violated dynamic consistency: violations of dynamic consistency occurred about twice as often as those of consequentialism. This provides support for dropping dynamic consistency in dynamic decision problems when accounting for ambiguity aversion.

Dominiak et al. (2012) also tested dynamic consistency and consequentialism in a dynamic Ellsberg experiment.[4] Like us, they also found more violations of dynamic consistency than of consequentialism. In fact, the proportion of subjects satisfying consequentialism in their study was the same as the proportion we observed (73%). Their test was not based on actual information, but they asked their subjects to imagine that some information was revealed to them. By contrast, in our experiment subjects made their choices based on the actual information that was revealed to them. Another difference between our study and Dominiak et al. (2012) is the treatment of indifference. They used unincentivised verbal statements about strength of preference to distinguish between ambiguity averse and ambiguity neutral subjects. We derive this distinction from incentivised choices by incurring a small cost to be ambiguity averse. This small cost might be responsible for the lower proportion of ambiguity averse subjects in our study. It is reassuring that in spite of these differences in design our conclusions are similar: if given the choice, ambiguity averse subjects are more likely to satisfy consequentialism than dynamic consistency.

The following Section 2 presents the design of the experiment. Section 3 defines consequentialism and dynamic consistency and it explains how our experiment tested them. Section 4 presents our results. Section 5 discusses the results in the context of the literature. Section 6 concludes.

Gilboa and Schmeidler, 1989 and the Choquet expected utility model of Schmeidler (1989). Examples of the latter include the two-stage lotteries without reduction model of Segal (1990), the second-order beliefs models of Klibanoff et al. (2005), Nau (2006) and Seo (2009), and the subjective compound lottery model of Ergin and Gul, 2009.

[2] Ghirardato (2002) showed that in a Savage framework without the sure-thing principle and with a more intuitive version of Savage's axiom P3. Epstein and Le Breton (1993) showed in a framework without Savage's P2 that dynamic consistency implies probabilistic sophistication (Machina and Schmeidler, 1992). See also Siniscalchi (2011).

[3] Machina (1989) argued to give up consequentialism. Studies that followed his recommendation are Hanany and Klibanoff (2007), Hanany and Klibanoff (2009), and Klibanoff et al. (2009). Examples of studies that give up dynamic consistency and retain consequentialism are Karni and Safra (1990), Gilboa and Schmeidler (1993), and Siniscalchi (2011). Dropping dynamic consistency is also common in the literature on hyperbolic discounting (O'Donoghue and Rabin, 1999).

[4] A related paper on decision-making under risk is Cubitt et al. (1998).

2. Experiment

We conducted a lab experiment to elicit subjects' ambiguity attitudes and to test whether they satisfied consequentialism and dynamic consistency. Screenshots of the experimental instructions and all decision situations are in the online appendix.[5]

2.1. Subjects and incentives

The experiment was computer-run and conducted in the ESE-Econlab at Erasmus University Rotterdam in March 2018. The experiment consisted of 7 sessions, with 23 to 27 subjects per session. In each session, two subjects were randomly assigned to be implementers. Their role was to generate ambiguity by determining the color composition of bags with cards, as explained below. A total of 171 subjects were recruited from the ESE-Econlab subject pool of whom 14 were assigned to be implementers. Data were collected from the remaining 157 subjects.

The experiment was incentivized using the prior incentive system (Prince; Johnson et al., 2021). Upon entering the lab, every subject drew an envelope from a pile of n sealed envelopes ($n =$ the number of participants in each session). A subject ID was written on each envelope. Subjects who drew an ID starting with "m" became the implementers.

We told the subjects in the main experiment that their envelope contained one of the decision situations they would encounter during the experiment. Each experimental decision situation had the same chance of being drawn. Subjects could only open their envelopes at the end of the experiment when the experimenters told them so. They would be paid for real according to their choice in the decision situation contained in their envelope, which varied across subjects. The implementers received a flat participation fee of € 10. The subjects in the main experiment received a participation fee of € 5 and a variable amount, which depended on their choices in the decision situation in their envelope. The average payment was € 9.62.

2.2. Stimuli

Subjects faced four decision situations. The first two situations measured subjects' ambiguity attitudes and were based on Ellsberg's three-color problem using a bag containing 100 *unnumbered* cards colored red, blue, or yellow. The final two decision situations were based on the draw of a card from a second bag containing 200 cards that were both colored (red, blue, or yellow) and *numbered.* Per experimental session, there were two implementers, one to determine the color composition of the cards in the first bag and one to determine (independently) the color composition of the cards in the second bag. The implementers did not know how the composition they determined might affect the other subjects' payoffs and the subjects were told that the implementers did not know this.

Decision situations 1 and 2: Test of ambiguity attitude

The first implementer was asked to write down a number N between 1 and 67 and then to put 33 red cards, N blue cards, and $67 - N$ yellow cards into a bag. Subjects knew this procedure but they did not know the number N that the implementer had written down. After having filled the bag, the implementer drew the winning card.

Subjects could bet on their winning colors. If a card with a winning color was drawn, subjects won € 10. Otherwise, they won nothing. In decision situation 1, subjects could bet on red, blue, or yellow. In the second decision situation, subjects could bet on two colors depending on which winning color they had selected in decision situation 1. If they had chosen either red or blue in decision situation 1, then they were offered a choice between "red and yellow" and "blue and yellow". If they had chosen yellow in decision situation 1, then they were offered a choice between "red and blue" and "blue and yellow". We let subjects choose their wining color to avoid suspicion.

In decision situation 1, "red" was unambiguous and "blue" and "yellow" were ambiguous. In decision situation 2, "red and yellow" and "red and blue" were ambiguous whereas "blue and yellow" was unambiguous. Consequently, we expected an ambiguity averse decision maker to choose "red" in the first choice and "blue and yellow" in the second choice. The expected probability of drawing a blue or yellow card in decision situation 1 was 0.335, which was slightly higher than the 0.33 probability of drawing a red card. Hence, the expected loss of ambiguity aversion was 5 cents. We put a small price on ambiguity aversion to avoid indifference. Kelsey and LeRoux (2017) showed that even small prices can substantially affect the number of ambiguity averse choices. A limitation of our approach was that some (very weakly) ambiguity averse decision makers might choose B or Y. Consequently, the proportion of ambiguity averse choices that we observed may be a lower estimate. Given the low price of 5 cents, this downward bias is probably small to negligible.

Decision situations 3 and 4: dynamic decisions

The second implementer wrote down a number M between 1 and 67 and then put 66 red cards (numbered from 135 to 200), $2M$ blue cards (numbered from 1 to $2M$), and 134-$2M$ yellow cards (numbered from $2M$+1 to 134) into a bag. Again,

[5] https://www.dropbox.com/s/2d5wr5fg7vb1y0i/online-appendix.pdf?dl=0

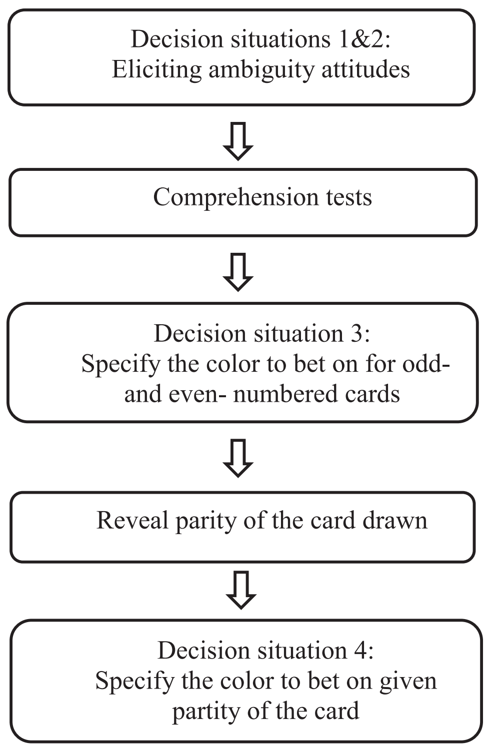

Fig. 1. Structure of the experiment.

the subjects were aware of this procedure, but they did not know the number M that the second implementer had written down. After having filled the bag, the implementer drew the winning card.

Subjects first answered three comprehension questions. They could only proceed to the main experiment after having answered all of these correctly.

In decision situation 3, subjects had to specify their winning color for both odd- and even-numbered cards.

After they had made their choices in decision situation 3, implementer 2 revealed the parity of the number on the card drawn. Then in decision situation 4, subjects indicated which color they would like to bet on given this signal about the parity of the card drawn.

At the end of the experiment, the two implementers revealed the color and the number of the drawn cards to the other participants.

The structure of the experiment is summarized in Fig. 1.

3. Theory: consequentialism versus dynamic inconsistency

This section presents our theoretical analysis of subjects' choices in the experiment. Uncertainty is modeled by a state space S. Single states are denoted $s_1, s_2, \ldots$.. An *event* E is a subset of the set S. In our experiment, the events in decision situations 1 and 2 are drawing a red, blue, or yellow card, and in decision situations 3 and 4 they are all combinations of color and parity of the card, such as, *drawing a red card with an even number on it* or *drawing a yellow card with an odd number on it.* We denote the complement of an event E by E^c. A *strategy* defines at each decision node what to do. For example, in decision situation 3 the strategy Y_oR_e stands for *bet on yellow when the number on the card is odd, bet on red when it is even.* We follow Sarin and Wakker (1998), by defining preferences $\succsim$ over the set of strategies.

Fig. 2 shows the decision situations of our experiment that we used to test for consequentialism and dynamic consistency.[6] Squares denote decision nodes and circles denote chance nodes. Fig.2(b) shows decision situation 3, which is a *single-stage decision* in which decision makers must commit beforehand to their choices, that is before the uncertainty about the parity of the winning card is resolved. Fig. 2(a) shows decision situation 4, which is an example of a *dynamic decision*. In decision situation 3, subjects had specified on which color to bet when the number were odd and when it were even. They now receive information whether the winning card has an odd or an even number on it and are asked to make another choice. These decisions are made after the uncertainty about the parity of the card has been resolved. Finally, Fig. 2(c) shows decision situation 1, which can be interpreted as a *no-history decision,* and which is similar to the upper or lower branch of Fig. 2(a) once the parity of the card is known.

Dynamic consistency is the requirement that choices in the dynamic decision agree with those in the single-stage decision. So if strategy Y_oR_e is chosen in (the single-stage) decision situation 3 (Fig. 2(b)) then in the (dynamic) decision situation 4, Y

[6] Decision situation 2 was only relevant for testing ambiguity aversion and we, therefore, do not display it in the figure.

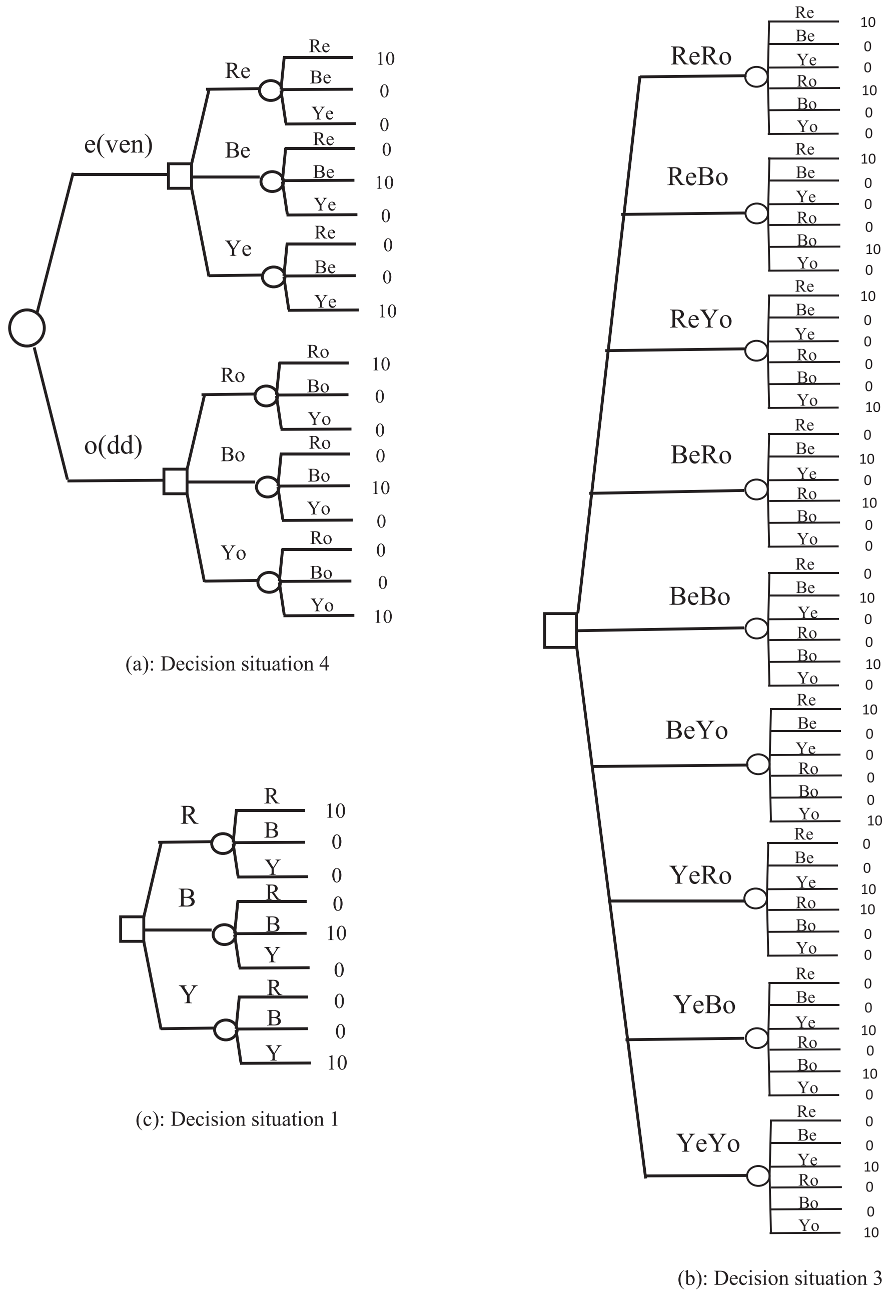

Fig. 2. Representation of decision situations in our experiment. Squares represent choice nodes and circles represent chance nodes.

must be chosen if the number on the drawn card is odd and *R* if the number on the card is even. In other words, dynamic consistency requires that decision makers do not change their minds after the uncertainty resolves at the chance nodes.

By the design of our experiment, we gave subjects real information (whether the number on the drawn card was even or odd). Hence, we could only observe subjects' conditional choices for the event that actually occurred and not those for the counterfactual event. For instance, when an odd-numbered card was drawn, we could only observe subjects' choices in the lower branch of Fig. 2(a), but not in the upper branch. Therefore, at the individual subject level, we did not have a full test of dynamic consistency. Nevertheless, since the two conditional decision situations (for odd- and even-numbered cards) are informationally indistinguishable, we believe that it is reasonable to assume that subjects' choices would not differ in these two situations. Our data support this assumption. In the aggregate, roughly half of the subjects faced each conditional bet and the color choices did not differ between the case where the card was even and the case where it was odd. Statistical tests could not reject the null that the choice distributions were equal in the two conditional decision situations.[7]

[7] In decision situation 4, of the decision makers who made an "odd-numbered contingent" choice, 51% preferred red, 30% preferred blue, and 19% preferred yellow, whereas for those who made an "even-numbered contingent" choice, 46% preferred red, 30% preferred blue, and 24% preferred yellow ($p = .68$ in Fisher's Exact test).

Consequentialism means that the choices made in dynamic decision situations are independent of risks born in the past, see Hammond (1988).[8] In our experiment, it means that the choices in the upper and lower branches of Fig. 2(a) agree with those in the no-history decision situation 1, which isolates these decisions. Consequentialism requires that choices made after the resolution of uncertainty are not affected by what would have happened in the counterfactual event. Hence, choices are history-independent, and coincide with those made in "no-history" decision situations as depicted in Fig. 2(c). Comparing the choice in Fig. 2(c) with that in Fig. 2(a) constitutes our test of consequentialism. This test requires the additional assumption that subjects perceived the two decision situations as similar even though the uncertainty was resolved by different implementers. If subjects did not consider the two decision situations as similar, for instance because they held different beliefs about the implementers, then they might fail our test of consequentialism even though their underlying preferences satisfied consequentialism and, vice versa, they might pass our test of consequentialism even though their underlying preference relation did not satisfy consequentialism. We believe that the assumption is plausible given that the two conditional decision situations and the no-history decision situation are informationally indistinguishable and that subjects typically did not know the implementers. The similarity between the violation rates of consequentialism in our experiment and those in Dominiak et al., 2012 lends empirical credence to ourconjecture.

Consequentialist decision makers with non-neutral attitudes toward ambiguity may be dynamically inconsistent in our problems as the partial resolution of uncertainty (revealing whether the drawn card had an even or an odd number on it) changes ambiguous strategies into *conditionally* unambiguous strategies and vice versa. To illustrate this observation, consider the strategies in 2(b). Given the composition of the bag, decision-makers know the strategy R_oR_e has 66 winning cards (33 odd-numbered red cards and 33 even-numbered red cards). Thus choosing R_oR_e amounts to selecting the unambiguous strategy $10_{R_oR_e}0$ (which stands for winning € 10 if the selected card is red and odd or red and even and nothing otherwise) with $p(R_oR_e) = 66/200$. The strategy B_oY_e has 67 winning cards (M odd-numbered blue cards and $67 - M$ even-numbered yellow cards). Thus the strategy B_oY_e corresponds to the unambiguous strategy $10_{B_oY_e}0$ with $p(B_oY_e) = 67/200$.

By stochastic dominance, we expect decision makers to prefer B_oY_e to R_oR_e. Similarly, we also expect them to prefer Y_oB_e to R_oR_e. However, the information about the parity of the number on the card drawn changes the ambiguity of some of the options. The originally undominated unambiguous strategies B_oY_e and Y_oB_e become ambiguous, whereas the originally dominated unambiguous strategy R_oR_e remains unambiguous.

Consequentialist decision makers who are ambiguity seeking will initially choose either B_oB_e or Y_oY_e. Without loss of generality, suppose that they initially select B_oB_e. Regardless of the parity of the number on the card drawn, they will strictly prefer the ambiguous bet on blue over the unambiguous bet on red and weakly prefer it over the bet on yellow. So there is no reason for ambiguity seeking consequentialists to reverse their initial preference.

For ambiguity averse consequentialists, however, this is different. They initially choose one of the undominated unambiguous strategies B_oY_e and Y_oB_e. Suppose that they initially choose B_oY_e and that they are told that the number on the card drawn is odd. Since consequentialists no longer pay attention to what would have happened for even-numbered cards, they will be (conditionally) indifferent between R_oR_e and R_oY_e. Moreover, if they are sufficiently ambiguity averse then they will (strictly) prefer a bet on red over a bet on blue. Thus, they will conditionally prefer R_oY_e to B_oY_e. By transitivity, it follows that R_oR_e is strictly preferred to B_oY_e *reversing* their initial preference of B_oY_e over R_oR_e. By an analogous argument, they will prefer R_oR_e to B_oY_e when they learn that the drawn ball is even.

The behavior of ambiguity seeking dynamically consistent decision makers will be indistinguishable from that of ambiguity loving consequentialists. They will initially select either B_oB_e or Y_oY_e and have no reason to change their choice after being informed about the parity of the number of the card drawn.

Ambiguity averse dynamically consistent decision makers, however, unconditionally prefer B_oY_e over R_oR_e and R_oR_e over R_oY_e and so, by transitivity, they prefer B_oY_e over R_oY_e. If they know that an odd-numbered ball has been drawn then, by dynamic consistency, they will prefer betting on blue over red. By a similar line of reasoning, if they know that an even-numbered ball has been drawn they will prefer betting on yellow over red.

Consequently, we have derived the following observation.

Observation 1. If stochastic dominance holds then ambiguity averse decision makers whose preferences satisfy

- consequentialism will violate dynamic consistency by choosing B_oY_e over R_oR_e in decision situation 3 (Fig. 2b) and red over blue in decision situation 4 (Fig. 2a).
- dynamic consistency will violate consequentialism by choosing B_oY_e over R_oR_e in decision situation 3 (Fig. 2b) and blue over red in decision situation 3 (Fig. 2a).

[8] A more technical definition of consequentialism is included in the appendix.

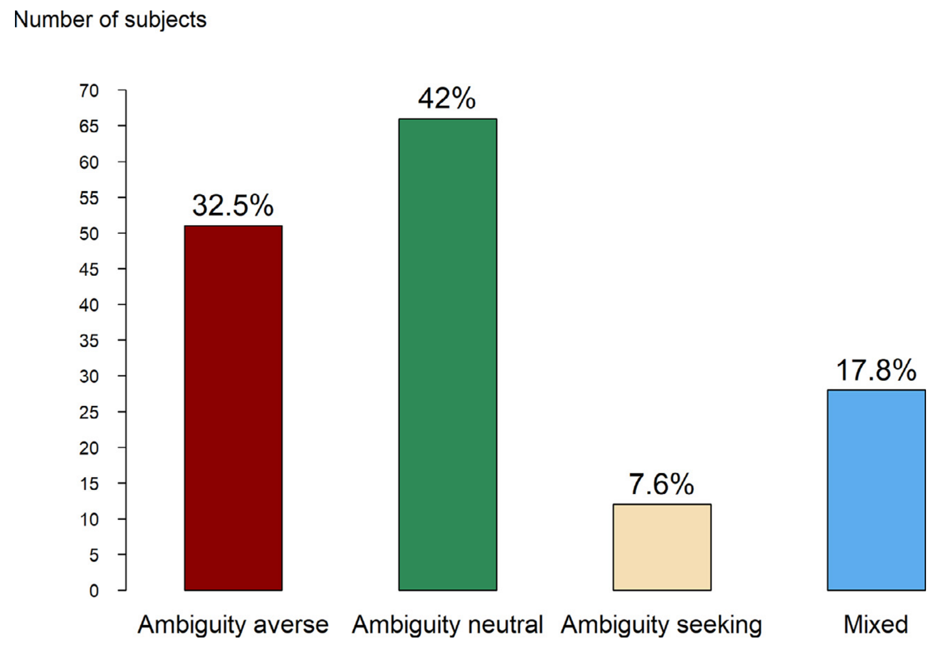

Fig. 3. Ambiguity attitudes.

The following picture summarizes the tests in our experiment.

Tests of the experiment:

(1) Ambiguity attitude: decision situations 1 and 2
(2) Consequentialism: decision situations 1 and 4
(3) Dynamic Consistency: decision situations 3 and 4
(4) Stochastic Dominance: decision situation 3

4. Results

We report the results using the responses from all subjects.[9]

4.1. Ambiguity attitudes

We classified subjects who chose red in decision situation 1 and blue and yellow in decision situation 2 as ambiguity averse and subjects who chose blue or yellow in decision situation 1 and red and yellow or red and blue in decision situation 2 as ambiguity seeking. Because we put a small price on ambiguity aversion and the expected probability of blue or yellow card was slightly higher than that of red, an ambiguity neutral subject would choose blue or yellow in decision situation 1 and blue and yellow in decision situation 2. The remaining subjects (who chose red in decision situation 1 and red and yellow in decision situation 2) were classified as mixed.

Fig. 3 shows that ambiguity neutrality was the modal pattern, followed by ambiguity aversion. Few subjects were ambiguity seeking. Our finding of limited ambiguity aversion is not uncommon, especially if subjects have to pay a price to pay to be ambiguity averse (see Trautmann and van de Kuilen, 2015 for a review of the empirical literature). In addition, it might be that some of the subjects classified as mixed were actually ambiguity averse, but failed to notice that blue and yellow in decision situation 2 was unambiguous.[10]

[9] We performed several robustness tests. We also analyzed the data removing the subjects who (i) violated stochastic dominance; (ii) failed the comprehension tests at least twice. This did not affect our conclusions. Details are in the online appendix.

[10] Their answers in decision situation 3 provided support for this conjecture. 70% of the mixed subjects chose one of the ambiguity averse options in decision situation 3 (R_oR_e,B_oY_e, or Y_oB_e). We found no evidence that the mixed subjects had more difficulty understanding the tasks and that their answers reflected confusion. There was no relation between the number of failures in the comprehension tests and ambiguity attitude (χ^2-test, $p = .29$).

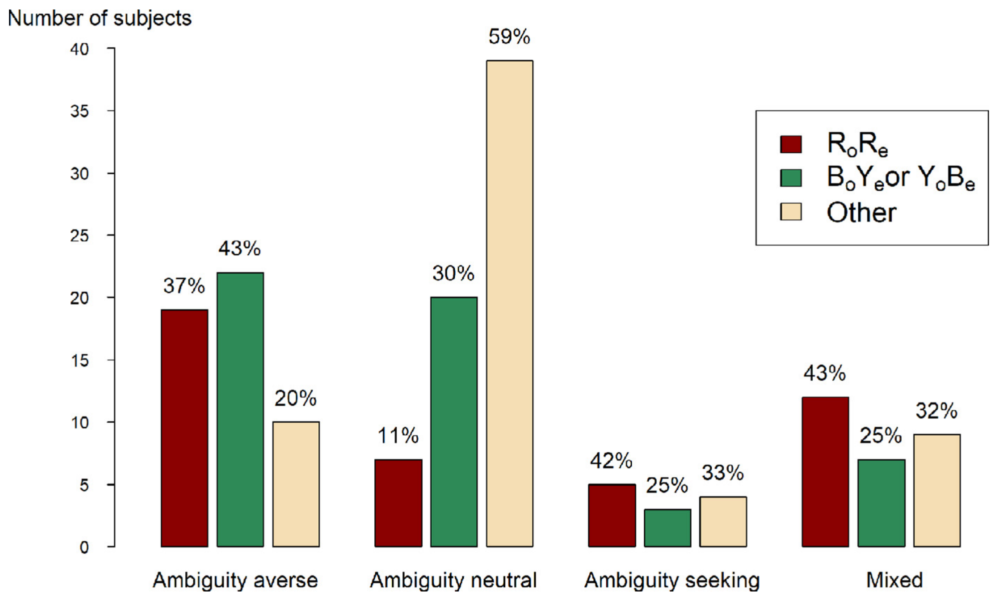

Fig. 4. Results of decision situation 3.

4.2. Dynamic decisions

Fig. 4 breaks down the choice pattern in decision situation 3 by ambiguity attitude. The proportions above the bars show for each category which share displayed which choice pattern.

Among the ambiguity averse subjects, 80% chose one of the three unambiguous options (R_oR_e, B_oY_e and Y_oB_e). This is more than the proportion (41%) among ambiguity neutral subjects (two-tailed proportion test, $p < .01$, $N_A = 51$, $N_N = 66$). However, it is not different from the proportion among subjects classified as "mixed" (two-tailed proportion test, $p = .33$, $N_A = 51$, $N_B = 28$).

Forty-three subjects (27.4%) chose R_oR_e over B_oY_e and Y_oB_e, violating stochastic dominance. Most of these subjects were ambiguity averse or mixed. Relatively few ambiguity neutral subjects chose R_oR_e.[11] The proportion violating stochastic dominance may appear high, but it should be kept in mind that the cost of violating stochastic dominance was low. It was also easier to detect that $R_oR_e$was unambiguous than that B_oY_e and Y_oB_e were unambiguous by providing a hedge against ambiguity. Some subjects may have considered the cost of 5 cents of violating stochastic dominance too low to take the cognitive effort of understanding why B_oY_e and Y_oB_e provided a hedge against ambiguity.[12] We discuss the violations of stochastic dominance in Section 5.

Fig. 5 shows how many subjects behaved in line with dynamic consistency and consequentialism, split out by ambiguity attitudes. In the figure's legend, *DC* and *C* correspond to a subject satisfying the respective property, while $\overline{DC}$ and $\overline{C}$ mean that the subject's choices violated the respective property.

One-hundred and four subjects, (66.2%) were dynamically consistent. Of the remaining fifty-three dynamically inconsistent subjects, twenty-six (49%) revised their initial choice of the *unconditionally* unambiguous option B_oY_e or Y_oB_e to the *conditionally* unambiguous option of choosing red.

One-hundred and fifteen subjects, (73.2%) satisfied consequentialism. In computing this number, we also included the seventeen subjects who switched between blue and yellow in decision situations 1 and 4 since we could not exclude the possibility that they were indifferent between betting on blue and beting on yellow.

Fig. 5 also shows that ambiguity averse subjects were more prone to violate dynamic consistency than consequentialism (two-tailed proportion test, $p = .04$, $N = 51$). Most ambiguity neutral subjects satisfied dynamic consistency and consequentialism and behaved according to subjective expected utility with Bayesian updating. The remaining ambiguity neutral subjects, who behaved according to subjective expected utility in decision situations 1 and 2, did not update in a Bayesian manner in decision situations 3 and 4. Among the ambiguity neutral subjects who violated at least one of the dynamic principles, there was no difference between the violation rates (two-tailed proportion test, $p = 1$, $N = 66$). The violation rates also did not differ among the ambiguity seeking subjects (two-tailed proportion test, $p = .40$,$N = 12$) or the mixed subjects (two-tailed proportion test, $p = .58$, $N = 28$). The proportions of ambiguity averse and ambiguity neutral subjects violating consequentialism were approximately equal (26% versus 21%). However, whereas 47% of the ambiguity averse subjects vio-

[11] Excluding the subjects who violated stochastic dominance did not affect our main conclusions. However, it led to an increase in support for subjective expected utility with Bayesian updating: after the exclusion 35.2% of the subjects behaved according to subjective expected utility with Bayesian updating. See the online appendix for details.

[12] On the other hand, the third comprehension test explained how to hedge against ambiguity.

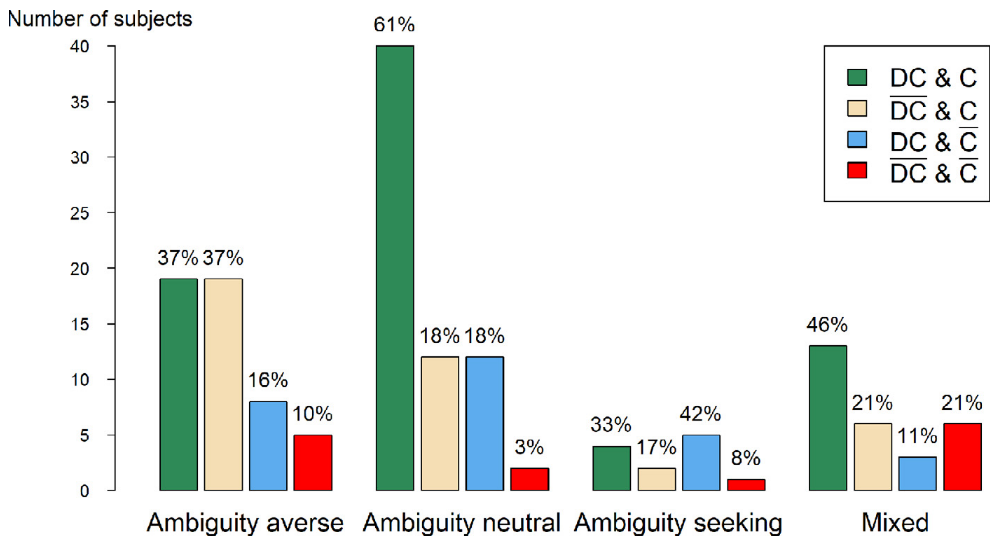

Fig. 5. Subjects satisfying dynamic consistency and consequentialism split out by ambiguity attitude.

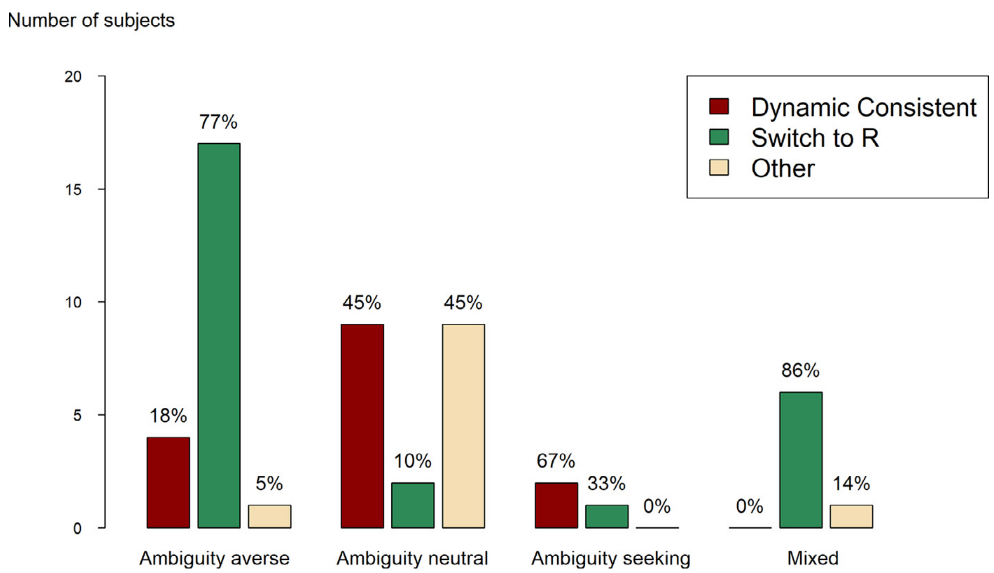

Fig. 6. Revision pattern of subjects choosing B_oY_e or Y_oB_e split out by ambiguity attitude.

lated dynamic consistency, only 21% of the ambiguity neutral subjects violated dynamic consistency. Two-tailed Fisher Exact tests showed that the violations of dynamic consistency were associated with ambiguity attitudes ($p = .02$, $N = 157$), but the violations of consequentialism were not ($p = .19$, $N = 157$).

In light of Observation 1, it may seem surprising that nineteen ambiguity averse subjects satisfied both dynamic consistency and consequentialism. That observation assumes, however, hat the decision maker's preferences satisfy stochastic dominance. Indeed, eighteen out of the nineteen ambiguity averse subjects who satisfied both dynamic consistency and consequentialism violated stochastic dominance by choosing R_oR_e.[13]

Fig. 6 presents the revision pattern of subjects who chose B_oY_e or Y_oB_e in decision situation 3. Among these subjects, only four (18%) ambiguity averse subjects satisfied dynamic consistency, whereas seventeen (77%) switched to the unambiguous option R_o or R_e in decision situation 4, in accordance with Observation 1. This pattern is also observed for the mixed subjects. However, the ambiguity neutral subjects exhibited different patterns. Only two (10%) switched to the unambiguous option. Half of the others satisfied dynamic consistency, whereas the other half switched from blue to yellow or vice versa.

[13] The other subject opted for R_oY_e in decision situation 3 and, when informed that the parity of the card drawn was odd, then selected red in decision situation 4. If the parity of the card drawn had been even, however, then in decision situation 4, opting for yellow or blue would have constituted a violation of consequentialism and opting for red a violation of dynamic consistency, again illustrating the point made in Observation 1 that in our study an ambiguity averse decision maker has to give up at least one of stochastic dominance, dynamic consistency, and consequentialism.

5. Discussion

We have performed an experimental test of dynamic consistency and consequentialism, two key principles of dynamic decision making, and studied how these are related to ambiguity attitudes. We used a variation of Ellsbergs three-color urn problem in which the cards (used instead of balls) differed not only by color, but were also numbered. This allowed for a comparison between unconditional and conditional preferences, that is required to test consequentialism and dynamic consistency. We found that ambiguity averse subjects were nearly twice as likely to violate dynamic consistency as consequentialism. Sarin and Wakker (1998) showed that the only ambiguity model that can account for consequentialism and dynamic consistency simultaneously is maxmin expected utility. However, empirical tests provide little support for this model (e.g. Baillon and Bleichrodt 2015; Chew et al. 2017). Hence, ambiguity models that strive for empirical realism should give up either dynamic consistency or consequentialism. Our results suggest that from a descriptive point of view, giving up dynamic consistency is the better choice.

Our test of consequentialism assumes that the decision makers view the two conditional decision situations and our no-history version of this decision situation as indistinguishable. We believe this extra assumption is plausible, but it is stronger than the definition in, for example, Ghirardato (2002) and, perhaps, our test may be interpreted as putting a lower bound on the support for consequentialism. In spite of this, we found that ambiguity averse decision makers were more likely to violate dynamic consistency than consequentialism.

In our tests, subjects did not know that after their initial choice they would receive a signal and would be asked to make another choice. One way to extend our research would be to make subjects aware of the sequential nature of the experiment. This would allow for a test of whether subjects are naive planners and use the same preference functional at each decision node, resolute planners, in which case they remain with their initial choice regardless of how uncertainty is resolved, or consistent planners, in which case they realize that they will not follow through some plans due to ambiguity aversion and, consequently, delete these plans from their feasible sets.

Another way to extend our research is to use larger incentives. The cost for subjects of violating stochastic dominance, ambiguity neutrality, consequentialism and dynamic consistency was low, only 5 cents. If subjects considered these costs too low to take the questions seriously, then we would expect choices to be random. However, we found no evidence of this, suggesting that subjects took the questions seriously. It would be interesting to replicate our study with a larger penalty for ambiguity aversion and a larger cost of violating stochastic dominance. A limitation of using larger costs is that they may bias the results. If we put, for instance, a larger penalty on being ambiguity averse, then a substantial fraction of ambiguity averse subjects (who would like to choose R in decision situation 1 of our experiment if there were no costs) may choose ambiguity neutral or seeking (B or Y). This would deflate the support for ambiguity aversion. Hence, there appears to be a trade-off between the size of the penalty and the precision with which the degree of ambiguity aversion can be estimated. It is hard to say what the optimal balance is. Hopefully, future research can shed light on this question.

We found no relation between violations of consequentialism and ambiguity attitudes. It might be of interest to explore whether this also holds for settings other than decision under ambiguity. That is, do ambiguity neutral and ambiguity averse subjects who violate consequentialism in decisions under ambiguity also do so in other decisions? This opens up a research agenda of the deeper causes of violations of consequentialism that we leave for future research.

Even though the comprehension test ensured that the subjects understood that choosing B_oY_e or Y_oB_e gave a higher objective winning chance than choosing R_oR_e, we nevertheless observed that a non-negligible proportion of subjects preferred R_oR_e in choice 3. This might be due to the relatively small cost of violating stochastic dominance. It might also have been caused by an aversion to complexity. Options where the outcome depends on both the number and color of the card may appear more complex. In a similar situation, Dominiak and Schnedler, 2011 report that subjects view a coin flip between two complementary ambiguous options as worse than either option on its own. This can be interpreted as evidence of complexity aversion in a related setting.

As we mentioned in the Introduction, we found the same proportion of subjects satisfying consequentialism as Dominiak et al. (2012) in spite of the differences in experimental design. We found more support for dynamic consistency than they did, but this was to a large extent driven by the subjects choosing the stochastically dominated option R_oR_e. As mentioned before, we believe these responses reflect more a cognitive shortcut to ambiguity aversion than an intrinsic preference for dynamic consistency. If we do not count these choices as reflecting dynamic consistency, then the proportion of subjects satisfying dynamic consistency drops to 38.9%, which is close to the 32.2% observed by Dominiak et al. (2012). Moreover, among the ambiguity averse subjects 17.6% satisfy dynamic consistency, which, again, is very close to the 14.5% observed by Dominiak et al. (2012). The similarity between our findings and those of Dominiak et al. (2012) provides support for their robustness.

6. Conclusion

We have performed empirical tests of consequentialism and dynamic consistency, two key principles of rational dynamic choice. Most models of ambiguity aversion have to give up one of these two principles. We used a variation of the Ellsberg three-color urn decision problem to test these principles in the lab. Around 25% of our subjects behaved in line with subjective expected utility with Bayesian updating: they were ambiguity neutral and satisfied both dynamic consistency and consequentialism. Ambiguity averse subjects typically violated one of these principles, with violations of dynamic consistency about twice as common as violations of consequentialism.

Appendix A

This appendix presents the definition of consequentialism, which we test in our experiment.

Consequentialism is usually defined as an axiom about choices in decision trees. Hammond (1988) takes choices in decision trees as a primitive and derives preferences from these choices. An exception is Ghirardato (2002), who defines consequentialism in terms of preferences. We shall present a definition of consequentialism as it applies to our experiments. For a definition in more general decision trees see Hammond (1988).

Consequentialism requires that subjects make the same choice in different trees with the same consequences. This applies to continuation trees, such as in our decision situation 4, as well as whole trees, such as in our decision situation 1.

Let T denote a decision tree and let $C(T)$ denote the consequences which the subject would choose from tree T. Denote the decision tree starting from an odd (resp. even) node in the second experiment (i.e. decision situation 4 in Fig. 2a), by T_o (resp. T_e). Let T_1 denote the decision tree in Fig. 2c, where implementer 1 puts N blue cards into a bag which contains 33 red cards and $67 - N$ yellow cards. Similarly, let T_2 denote the decision tree for the case where implementer 2 determines the composition of colors in the bag.

Consequentialism implies that

$$C(T_o) = C(T_e) = C(T_2).$$

We make the auxiliary hypothesis that

$$C(T_1) = C(T_2).$$

This says that subjects make the same choice when the bag is filled by implementer 1 as when it is filled by implementer 2. This can be justified by an appeal to symmetry of information.

We may thus derive

$$C(T_1) = C(T_o) = C(T_e). \tag{A.1}$$

This is our Observation 1.

Ambiguity averse subjects will choose red in decision problem 1. If in addition we assume consequentialism then Eq. (A.1) implies that they will also choose red in decision problem 4.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.euroecorev.2021.103687

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