Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 49-31 (2016) 137–142

A Novel Numerical Approach to the MCLP Based Resilent Supply Chain Optimization

V. Azhmyakov [1], **J. P. Fernández-Gutiérrez** [1], **S. K. Gadi** [2], **St. Pickl** [3]

1 Departamento de Ciencias Basicas, Universidad de Medellin, Medellin, Colombia (e-mail: vazhmyakov@udem.edu.co; jpfernandez@udem.edu.co)
2 Facultad de Ingenieria Mecanica y Electrica, Universidad Autonoma de Coahuila, Torreon, Mexico (email: research@skgadi.com)
3 Department of Computer Science, Universität der Bundeswehr München, München, Germany (email: stefan.pickl@unibw.de)

Abstract: This paper deals with the Maximal Covering Location Problem (MCLP) for Supply Chain optimization in the presence of incomplete information. A specific linear-integer structure of a generic mathematical model for Resilient Supply Chain Management System (RSCMS) makes it possible to reduce the originally given MCLP to two auxiliary optimization Knapsack-type problems. The equivalent transformation (separation) we propose provides a useful tool for an effective numerical treatment of the original MCLP and reduces the complexity of algorithms. The computational methodology we follow involves a specific Lagrange relaxation procedure. We give a rigorous formal analysis of the resulting algorithm and apply it to a practically oriented example of an optimal RSCMS design.

1. INTRODUCTION

Constructive optimization of complex technological processes and the corresponding computer oriented methods and software are nowadays a usual and efficient methodology for the practical development of several real-world Management Systems (see e.g., [1,5-7,9,10,11,15,18,23,24]). Our paper studies mathematical aspects of a particular RSCMS model that involves incomplete information. The requested optimal design of a RSCMS can be formalized as a specific "disturbed" MCLP [10]. Recall that the celebrated Maximal Covering Location Problem is a challenging optimization problem with numerous applications in practice. It has a decisive role in the success of supply chains, with applications including location of industrial plants, landfills, hubs, cross-docks, etc (see e.g., [1,3,8-10,12-15,18,20,22,24]). A well-known MCLP and the related supply chain activity involve the delivery of a manufactured product to the end customer or/and to a warehouse. In a classical MCLP, one seeks the location of a number of facilities on a network in such a way that the covered "population" is maximized [14,24].

MCLP was first introduced by Church and ReVelle [14] on a network, and since then, several extensions to the original problem have been made. A variety of numerical approaches have been proposed to the practical treatment of distinct MCLPs. Let us mention here exact, heuristic and metaheuristic families of methods and also refer to [8-10,12-15,18,20,22] for some necessary details, concrete solution algorithms and further references. Note that heuristics and metaheuristics have usually been employed in order to solve large size MCLPs (see e.g., [3,13,18,20]). A recent interest to MCLPs has arisen out the uncertainty of model parameters, such as demands or/and locations of demand nodes [9,10,24].

The main aim of our contribution is with a strong theoretic foundation of the newly elaborated separation method. The optimization approach we propose includes an equivalent transformation of the original MCLP that finally involves a common Knapsack problem (see e.g., [16] and references therein). The developed approach reduces the complexity of an initially given MCPL and makes it possible to apply various exact methods to the original MCLP. We concretely use the well-known Lagrange relaxation scheme for this purpose [12,16]. And, it should be noted already at this point that the MCLP based optimization algorithm we propose can be effectively implemented (at the optimization stage) in a concrete RSCMS.

The remainder of our paper is organized as follows: Section 2 contains a formal problem statement and some necessary concepts. In Section 3 we prove our main separation result, namely, Theorem 1 and give a constructive characterization of the obtained auxiliary problems. Section 4 deals with the celebrated Lagrange relaxation scheme applied to the initially given MCLP as well as to the auxiliary Knapsack problem. We use our main theoretic results and propose a self-closed algorithm for an effective numerical treatment of the initially given MCLP. Section 5 contains a simplified computational example of an optimal RSCMS design. This practically oriented example illustretes the applicability of the proposed numerical scheme. Section 6 summarizes our paper.

2. PROBLEM FORMULATION AND SEPARATION

An optimal design of a complex logistics network can generally be implemented in two steps. Firstly one solves the location problem and next considers the corresponding demand allocation problem. Note that a conventional

10.1016/j.ifacol.2016.12.175

MCLP does not constitute a "universal" solution approach under assumption of possible process disruptions (technical faults, maintenance and so on). This is specifically true with respect to the second sub-problem mentioned above. We next introduce a suitable analytic extension of the conventional MCLP that includes the possible updates of the demand allocation for the same location distribution. The extended modelling approach we propose can be expressed in the form of a (specific) linear integer program

$$\text{maximize} \quad J(z(y)) := \sum_{i=1}^{n} w_j z_j$$
$$\text{subject to} \quad \begin{cases} \sum_{i=1}^{l} y_i = k \in \mathbb{N}, \ l > k, \\ z_j \le \sum_{i=1}^{l} a_{ij} y_i, \\ z \in \mathbb{B}^n, \ y \in \mathbb{B}^l \end{cases} \tag{1}$$

Here $w_j \in \mathbb{R}_+, \ j = 1, ..., n$ are given nonnegative objective "weights" and variables $z_j, \ j = 1, ..., n$ determine the "facilities to be served". By y_i, where $i = 1, ..., l$, we define the generic decision variables of the problem under consideration and $k \in \mathbb{R}_+$ in (1) describes the total amount of the facilities to be located. Elements a_{ij}, where

$$1 \ge a_{ij} \ge 0, \quad \sum_{i=1,...,l} a_{ij} \ge 1,$$

are components of the so called "eligibility matrix"

$$A := (a_{ij})_{j=1,...,n}^{i=1,...,l}$$

associated with the eligible sites that provide a resilient covering of the demand points indexed by $j = 1, ..., n$. Note that the second index in (1), namely, $i = 1, ..., l$ is related to the given "facilities sites". Finally, the admissible sets $\mathbb{B}^n$ and $\mathbb{B}^l$ in the main problem (1) are defined as follows:

$$\mathbb{B}^n := \{0,1\}^n, \quad \mathbb{B}^l := \{0,1\}^l.$$

Note that the objective functional $J(\cdot)$ from (1) has a linear structure. We use the following natural notation $z := (z_1, ..., z_n)^T$ and $y := (y_1, ..., y_l)^T$. The implicit dependence

$$J(z(y)) = \langle w, z \rangle, \ w := (w_1, ..., w_n)^T$$

of the objective functional J on the vector y is given by the corresponding (componentwise) inequalities constraints $z \le A^T y$ in (1). By $\langle \cdot, \cdot \rangle$ we denote here the scalar product in the corresponding Euclidean space. A vector pair (z, y) that satisfies all the constraints in (1) is next called an admissible pair for the main problem (1).

The abstract optimization framework (1) provides a constructive and modelling approach for various practically oriented problems (see e.g., [1,9,11,13,18,22,24]). Following [14] we next call the main optimization problem (1) a Maximal Covering Location Problem (MCLP). Let us also refer to [24] for a detailed discussion on the applied interpretation of the MCLP (1). Note that the main MCLP is formulated under the general (non-binary) assumption related to the elements a_{ij} of the eligibility matrix A. This corresponds to a suitable modelling approach under incomplete information (see e.g., [10] and references therein). Roughly speaking every selection of an admissible parameter a_{ij} in (1) has a "fuzzy" nature (similar to [8]). This fuzzy characterization of the MCLP under consideration provide an adequate modelling framework for the RSCMS (see Section 5).

The mathematical characterization of (1) can evidently be given in terms of the classic integer programming (see e., g. [11,16,19] for mathematical details). Let us note that (1) posesses an optimal solution (an optimal pair)

$$(z^{opt}, y^{opt}) \in \mathbb{B}^n \bigotimes \mathbb{B}^l,$$

where

$$z^{opt} := (z_1^{opt}, ..., z_n^{opt})^T, \quad y^{opt} := (y_1^{opt}, ..., y_l^{opt})^T.$$

This fact is a direct consequence of the basic results from [11,16,19]. Our aim is to develop a simple and effective numerical approach to the sophisticated MCLP (1). We firstly "separate" the original optimization problem and introduce two auxiliary optimization problems. These formal constructions provide a necessary basis for the future numerical development we propose. The first auxiliary problem can be formulated as follows

$$\text{maximize} \quad \sum_{j=1}^{n} \mu_j \sum_{i=1}^{l} a_{ij} y_i$$
$$\text{subject to} \quad \begin{cases} \sum_{i=1}^{l} y_i = k, \quad y \in \mathbb{B}^l, \\ \mu_j \in [0,1] \quad \forall j = 1, ..., n \end{cases} \tag{2}$$

The second auxiliary problem has the following specific form:

$$\text{maximize} \quad J(z) := \sum_{j=1}^{n} w_j z_j$$
$$\text{subject to} \quad \begin{cases} z_j \le \sum_{i=1}^{l} a_{ij} \hat{y}_i \\ z \in \mathbb{B}^n \end{cases} \tag{3}$$

where $\hat{y} \in \mathbb{B}^l$ is optimal solution of problem (2). The components of $\hat{y}$ are denoted as $\hat{y}_i, \ i = 1, ..., l$. The existency of an optimal solution for (2) is a direct consequence of the results from [11,19]. The same is also true with respect to the auxiliary problem (3). Let $\hat{z} \in \mathbb{B}^n$, $\hat{z} := (\hat{z}_1, ..., \hat{z}_n)^T$ be an optimal solution to (3). Evidently, problem (3) coincides with the originally given MCLP (1) in a specific case of a fixed variable $y = \hat{y}$. Let us note that in general $\hat{y} \neq y^{opt}$.

The first auxiliary problem, namely, problem (2) is a usual linear scalarization of the following multiobjective optimization problem (vector optimization):

$$\text{maximize} \quad \{\sum_{i=1}^{l} a_{i1} y_i, ..., \sum_{i=1}^{l} a_{in} y_i\}$$
$$\text{subject to} \quad \begin{cases} \sum_{i=1}^{l} y_i = k, \\ y \in \mathbb{B}^l \end{cases} \tag{4}$$

Recall that a scalarizing of a multi-objective optimization problem is an adequate numerical approach, which means formulating a single-objective optimization problem such that optimal solutions to the single-objective optimization problem are Pareto optimal solutions to the multi-objective optimization problem. We next assume that the multipliers $\mu_j, \ j = 1, ..., n$ in (2) are chosen by such a way that problems (2) and (4) are equivalent (see e.g., [2,11,19] for necessary details). In this particular case we call (2) an adequate scalarizing of (4). Moreover, problems (2) and (3) have a structure of a so-called Knapsack problem (see [16] and references therein). Various efficient numerical algorithms are recently proposed for a generic Knapsack problem. We refer to [16] for a comprehensive overwiev about the modern implementable numerical approaches to this basic optimization problem.

3. THE SEPARATION BASED SOLUTION APPROACH

The relevance and main motivation of the auxiliary optimization problems (2) and (3) introduced in Section 2 can be stated by the following abstract result.

Theorem 1. Assume (z^{opt}, y^{opt}) is an optimal solution of (1) and (2) is an adequate scalarizing of (4). Let $\hat{y}$ be an optimal solutions of (2) and $\hat{z}$ be an optimal solution of the auxiliary problem (3). Then (1) and (3) possess the same optimal values, that is

$$J(z^{opt}(y^{opt})) = J(\hat{z}). \tag{5}$$

Moreover, in the case problems (1), (2), and (3) possess unique solutions we additionally have $(z^{opt}, y^{opt}) = (\hat{z}, \hat{y})$.

Proof: Since

$$\sum_{i=1}^{l} \hat{y}_i = k, \quad \hat{z}_j \leq \sum_{i=1}^{l} a_{ij} \hat{y}_i,$$

we conclude that $(\hat{z}, \hat{y})$ is an admissible pair for the original MCLP (1). Taking into account the definition of an optimal pair for problem (1), we next deduce

$$J(\hat{z}(\hat{y})) \leq J(z^{opt}(y^{opt})). \tag{6}$$

Let

$$\Gamma = \Gamma_z \bigotimes \Gamma_y \subset \mathbb{B}^n \bigotimes \mathbb{B}^l$$

be a solutions set (the set of all optimal solutions) for problem (1). We also define the solutions sets $\Gamma_{(2.2)} \subset \mathbb{B}^l$ and $\Gamma_{(2.3)} \subset \mathbb{B}^n$ of problems (2) and (3), respectively. From (6) it follows that

$$\Gamma_{(2.3)} \bigotimes \Gamma_{(2.2)} \subset \Gamma. \tag{7}$$

Taking into account the restrictions associated with the variable y in (1) and (2), we next obtain

$$\Gamma_y \equiv \Gamma_{(2.2)}. \tag{8}$$

Since (2) is an adequate scalarization of the multi-objective maximization problem (4), we deduce

$$z_j \leq \max_{\begin{cases} \sum_{i=1}^{l} y_i = k, \\ y \in \mathbb{B}^l \end{cases}} \sum_{i=1}^{l} a_{ij} y_i.$$

This fact implies

$$\Gamma_z \subset \Gamma_{(2.3)}. \tag{9}$$

Inclusions (7), (9) and the basic equivalence (8) now imply the following crucial equivalence

$$\Gamma_{(2.3)} \bigotimes \Gamma_{(2.2)} \equiv \Gamma. \tag{10}$$

Taking into account the same form of the objective functionals in (1) and (2.3), we immediately obtain the basic relation (5). In a specific case of one point sets Γ, $\Gamma_{(2.3)}$ and $\Gamma_{(2.2)}$ the expected relation $(z^{opt}, y^{opt}) = (\hat{z}, \hat{y})$ is a direct consequence of (10). The proof is completed. □

Theorem 1 makes it possible to separate (equivalently) the originally given sophisticated problem (1) into two simple optimization problems. It provides a theoretical basis for effective numerical approaches to the abstract MCLPs and to corresponding applications.

We now observe that the first auxiliary optimization problem, namely, problem (2) has a trivial combinatorial structure and can be easily solved:

$$\hat{y}_i = 1 \text{ if } i \in \hat{I}; \quad \hat{y}_i = 0 \text{ if } i \in \{1, ..., l\} \setminus \hat{I}, \tag{11}$$

where

$$\hat{I} := \{1 \leq i \leq l \mid S_{\mathcal{A}_i} \in \max_k \{S_{\mathcal{A}_1}, ..., S_{\mathcal{A}_l}\}\},$$
$$S_{\mathcal{A}_i} := \sum_{j=1}^{n} \mu_j a_{ij}, \; \mathcal{A}_i := (a_{i1}, ..., a_{in})^T. \tag{12}$$

Here $\mathcal{A}_i$ is a vector of i-row of the eligibility matrix A and operator $\max_k$ determines an array of k-largest numbers from the given array. Evidently, the choice (11)-(12) determines an optimal solution of (2). Roughly speaking the combinatorial algorithm (11)-(12) assigns the maximal value $\hat{y}_i = 1$ for all vectors $\mathcal{A}_i$ which sum of components $S_{\mathcal{A}_i}$ belongs to the array of k-largest sums of compoents of all vectors $\mathcal{A}_i$, $i = 1, ..., l$. It is easy to see that for the given eligibility matrix A with the specific elements a_{ij} (determined in Section 2) the sum of components $S_{\mathcal{A}_i}$ constitutes a specific norm of the given vector $\mathcal{A}_i$. Let us also note that the total complexity of the combinatorial algorithm (11)-(12) is equal to

$$O(l \times \log k) + O(k)$$

(see e.g., [16] for details).

Let us denote

$$c := \sum_{j=1}^{n} \sum_{i=1}^{l} a_{ij} \hat{y}_i.$$

Then the inequality constraints in (3) imply the generic Knapsack-type constraint with uniform weights

$$\sum_{j=1}^{n} z_j \leq c.$$

We now present a fundamental solvability result for the second auxiliary optimization problem, namely, the Knapsack problem (3).

Theorem 2. The Knapsack problem (3) can be solved in $O(nc)$ time and $O(n + c)$ space.

The formal proof of Theorem 2 can be forund in [16].

4. LAGRANGE RELAXATION AND CONSTRUCTIVE NUMERICAL TREATMENT OF THE ORIGINAL MCLP

Our main analytic results, namely, Theorem 1, the combinatorial choice algorithm (11)-(12) and Theorem 2 provide a theoretic basis for a novel exact solution scheme for the originally given MCLP (1). Finally we need to define a suitable and implementable procedure for an effective numerical treatment of (3). This auxiliary optimization problem, which is $\mathcal{NP}$-hard, has been comprehensively studied in the last few decades and several exact algorithms for its solution can be found in the literature (see [16] and the references therein). Constructive algorithms for Knapsack problems are mainly based on two basic approaches: branch-and-bound and dynamic programming. Let us also mention here the celebrated "combined" approach.

In this paper we apply the well-known Lagrange relaxation scheme to the second auxiliary problem (problem (3)). "Relaxing a problem" has various meanings in applied mathematics, depending on the areas where it is defined, depending also on what one relaxes (a functional, the underlying space, etc.). We refer to [2,4-7,12, 21] for various implementable relaxation techniques. Introducing the Lagrange function

$$\mathcal{L}(z,\lambda) := \sum_{j=1}^{n} w_j z_j - \sum_{j=1}^{n} \lambda_j \big(z_j - \sum_{i=1}^{l} a_{ij}\hat{y}_i\big)$$

associated with the Knapsack problem (3), we next consider the following relaxed problem

$$\begin{array}{ll} \text{maximize} & \mathcal{L}(z,\lambda) \\ \text{subject to} & z \in \mathbb{B}^n \end{array} \qquad (13)$$

The relaxed problem (13) does not contain the unpleasant inequality constraints which are included in the objective function (3.17) as a penalty term

$$\sum_{j=1}^{n} \lambda_j \big(z_j - \sum_{i=1}^{l} a_{ij}\hat{y}_i\big).$$

Recall that all feasible solutions to (3) are also feasible solutions to (13). The objective value of feasible solutions to (3) is not larger than the objective value in (13) (see [16] for the necessary proofs). Thus, the optimal solution value to the relaxed problem (13) is an upper bound to the original problem (3) for any vector of nonnegative multipliers $\lambda := (\lambda_1, ..., \lambda_n)^T$, $\lambda_j \geq 0$. For a concrete numerical solution of the relaxed problem (13) we use here the classic branch-and-bound method (see e.g., [11,16]). In a branch-and-bound algorithm we are interested in achieving the tightest upper bound in (13). Hence, we would like to choose a vector of nonnegative multipliers

$$\hat{\lambda}^{\mathcal{L}} := (\hat{\lambda}_1^{\mathcal{L}}, ..., \hat{\lambda}_n^{\mathcal{L}})^T, \ \hat{\lambda}_j^{\mathcal{L}} \geq 0$$

such that (13) is minimized. This evidently leads to the generic Lagrangian dual problem

$$\begin{array}{ll} \text{minimize} & \mathcal{L}(z,\lambda) \\ \text{subject to} & \lambda \geq 0 \end{array} \qquad (14)$$

It is well-known that the Lagrangian dual problem (14) yields the least upper bound available from all possible Lagrangian relaxations. The problem of finding an optimal vector of multipliers $\hat{\lambda}^{\mathcal{L}} \geq 0$ in (14) is in fact a linear programming problem [11,19]. In a typic branch-and-bound algorithm one will often be satisfied with a sub-optimal choice of multipliers $\lambda \geq 0$ if only the bound can be derived quickly. In this case subgradient optimization techniques can be applied [19]. The following analytic result is an immediate consequence of our main Theorem 1 and of the basic properties of the primal-dual system (13)-(14).

Theorem 3. Let $(\hat{z}^{\mathcal{L}}, \hat{\lambda}^{\mathcal{L}})$ be an optimal solution of the primal-dual system (13)-(14) associated with the auxiliary problem (3). Assume that all conditions of Theorem 1 be satisfied. Then

$$J(z^{opt}(y^{opt})) \leq J(\hat{z}^{\mathcal{L}}). \qquad (15)$$

and the obtained inequality (15) constitutes a tightest upper bound.

We are now ready to formulate a complete algorithm for an effective numerical treatment of the basic MCLP (1).

Algorithm 1.

I. Given an initial MCLP (1) separate it into two auxiliary problems (2) and (3);
II. Apply the combinatorial algorithm (11)-(12) and compute $\hat{y}$;
III. Using $\hat{y}$, construct the Lagrange function $\mathcal{L}(z,\lambda)$ and solve the primal-dual system (13)-(14).

The numerical consistency of the proposed Algorithm 1 is established by our main theoretic results, namely, by Theorem 1 - Theorem 3.

Finally let us note that the Lagrange relaxation scheme is usually applied to the original problem (1) (see e.g., [12,16]). In that case the resulting (relaxed) problem and the corresponding Lagrangian dual problem possess a higher complexity in comparison with the proposed "partial" Lagrange relaxation (13)-(14) of the original MCLP (1). This is an immediate consequence of the proposed separation method (Section 3) that reduces the initial problem (1) to two auxiliary optimization problem (2)-(3).

5. APPLICATION TO THE OPTIMAL DESIGN OF A RESILENT SUPPLY CHAIN MANAGEMENT SYSTEM

This section is devoted to a practical application of the proposed novel numerical approach to the MCLP (1). We use the basic MCLP model and optimize a Resilient Supply Chain for a family of manufacturing plants - warehouses. Note that the "resilience" of a Supply Chain Management

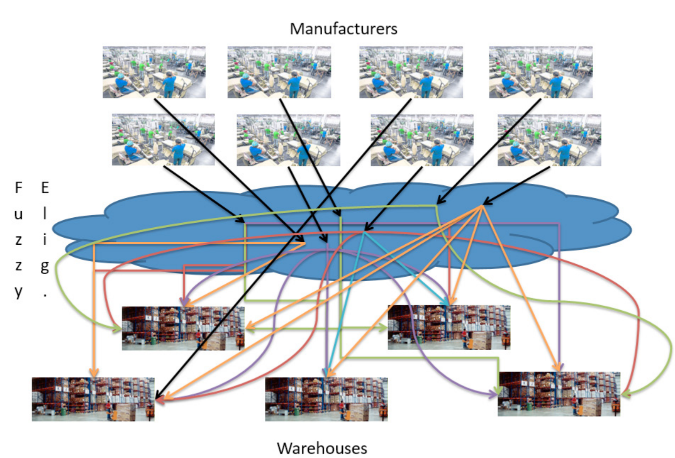

Fig. 1. Fuzzy eligibility model

System is modelled here by an eligibility matrix A with the fuzzy-type components a_{ij} (see Section 2). The conceptual Supply Chain scheme that include $l = 8$ manufacturing plants and $n = 5$ warehouses is indicated on Fig. 1.

Here i' is an index that corresponds to a "resilient" cover of demand point. We also assume that $a_{ij} + a_{i'j} \geq 1$ for $i = 1, ...5$ $j = 1, ...8$. The last condition means that at least two feasible facilities (warehouse) cover a given demand point (the manufacturing plants). The corresponding (transposed) eligibility matrix A is given as follows:

$$A^T = \begin{pmatrix} 0.81286 & 0.0 & 0.0 & 0.62968 & 0.0 \\ 0.25123 & 0.58108 & 0.32049 & 0.89444 & 0.79300 \\ 0.0 & 0.0 & 0.64850 & 0.91921 & 0.94740 \\ 0.54893 & 0.90309 & 0.74559 & 0.50869 & 0.99279 \\ 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.77105 & 0.27081 & 0.65883 & 0.60434 & 0.23595 \\ 0.0 & 0.51569 & 0.0 & 0.0 & 0.57810 \\ 0.64741 & 0.91733 & 0.60562 & 0.63874 & 0.71511 \end{pmatrix}$$

Recall that the objective weights $w_j \in \mathbb{R}_+,\ j = 1, ..., n$ indicates a priority and are assumed to be equal to

$$w^T = (32.0,\ 19.0,\ 41.0,\ 26.0\ 37.0\ 49.0\ 50.0\ 11.0)^T$$

Note that the fifth demand point in this example has no "resilient" character (only one facility covers this point). We assume that the Supply Chain decision maker is interested opens $k = 2$ facilities. Moreover, we also calculate from (12)

$$S_{\mathcal{A}_1} = 8.06295\ S_{\mathcal{A}_2} = 5.86033\ S_{\mathcal{A}_3} = 5.30955$$
$$S_{\mathcal{A}_4} = 7.47098\ S_{\mathcal{A}_5} = 6.99921$$

Application of the basic *Algorithm 1* leads to the following computational results:

$$z^{opt} = (1,\ 1,\ 0,\ 1,\ 1,\ 1,\ 0,\ 1)^T, \quad y^{opt} = (1,\ 0,\ 0,\ 1,\ 0)^T, \tag{16}$$

The corresponding (maximal) value of the objective functional is equal to

$$J(z^{opt}(y^{opt})) = \max_{Problem(1)} J(z(y)) = 174.0$$

Let us also note that the computed scalarizing multiplier μ in the auxiliary problem (2) for the given problem data is equal to

$$\mu = (2.0,\ 2.0\ 1.0\ 2.0\ 2.0\ 2.0\ 1.0\ 2.0)^T.$$

The practical implementation of the computationall *Algorithm 1* was carried out by using the standard Python package and an author-written program.

For comparison, the given MCLP problem was also solved by a direct application of the standard CPLEX optimization package. We use the concrete problem parameters given above and obtain the same optimal pair as in (16). The CPLEX integer programming solver proceeds with 6 MIP simplex iterations and 0 branch-and-bound nodes for in total 13 binary variables and 9 linear constraints. Let us finally note that all the customers (except the fifth) are covered and moreover, could still be covered if one of the facilities is closed.

6. CONCLUDING REMARKS

In this contribution, we proposed a conceptually new numerical approach to a wide class of Maximal Covering Location Problems with the fuzzy-type eligibility matrices. This computational algorithm is next applied to the optimal design of a practically motivated Resilient Supply Chain Management System. The developed computational scheme is based on a novel separation approach to the initially given maximization problem. The separation scheme we propose makes it possible to reduce the original sophisticated problem to two Knapsack-type optimization problems. The first one constitutes a generic linear scalarization of a multiobjective optimization problem and the second auxiliary problem is a simple version of the classic Knapsack formulation. Application of the conventional Lagrange relaxation in combination with a specific combinatorial algorithm leads to an implementable algorithm for the given Maximal Covering Location Problem as well as for the optimal design of a Resilient Supply Chain.

Theoretical and computational methodologies we present in this contribution can be applied to various generalizations and extensions of the basic MCLP and also to several optimization problems associated with the RSCMS design. One can combine the elaborated separation scheme with the conventional branch-and-bound method, with the celebrated dynamic programming approach or/and with an alternative exact or heuristic numerical algorithm. Let us finally note that we discussed here only theoretic aspects of the newly elaborated approach and presented the corresponding conceptual solution procedure. The basic methodology we developed needs a comprehensively numerical examination that includes solutions of several MCLPs and simulations of the corresponding optimal RSCMSs.

REFERENCES

[1] G. Alexandris, I. Giannikos, A new model for maximal coverage exploiting GIS capabilities, European Journal of Operational Research, vol. 202(2), 2010, pp. 328 – 338.

[2] K. Atkinson, W. Han, *Theoretical Numerical Analysis*. Springer, New York 2005.

[3] H. Aytug, C. Saydam, Solving large-scale maximum expected covering location problems by genetic algorithms: a comparative study, *European Journal of Operational Research*, vol. 141(3), 2002, pp. 480 – 494.

[4] V. Azhmyakov and W. Schmidt, Approximations of relaxed optimal control problems, *Journal of Optimization Theory and Applications*, vol. 130, 2006, pp. 61 - 77,

[5] V. Azhmyakov, M. Basin, C. Reincke-Collon, Optimal LQ-type switched control design for a class of linear systems with piecewise constant inputs, in: *Proceedings of the 19th IFAC World Congress*, Cape Town, South Africa, 2014, pp. 6976 – 6981.

[6] V. Azhmyakov, J. Cabrera, A. Poznyak, Optimal fixed - levels control for non - linear systems with quadratic cost functionals, *Optimal Control Applications and Methods*, to appear in 2016.

[7] V. Azhmyakov, R. Juarez, On the projected gradient method for switched - mode systems optimization, in: *Proceedings of the 5th IFAC Conference on Analysis and Design of Hybrid Systems*, Atlanta, USA, 2015, .

[8] V. Batanovic, D. Petrovic, R. Petrovic, Fuzzy logic based algorithms for maximum covering location

problems, *Information Sciences*, vol. 179(12), 2009, pp. 120 – 129.

[9] O. Berman, J. Kalcsics, D. Krass, S. Nickel, The ordered gradual covering location problem on a network, *Discrete Applied Mathematics*, vol. 157(18), 2009, pp. 3689 – 3707.

[10] O. Berman, J. Wang, The minmax regret gradual covering location problem on a network with incomplete information of demand weights, *European Journal of Operational Research*, vol. 208(3), 2011, pp. 233 – 238.

[11] D. Bertsekas, *Nonlinear Programming*, Athena Scientific, Belmont, USA, 1995.

[12] R.D. Galvao, L.G. Acosta Espejo, B. Boffey, A comparison of Lagrangean and surrogate relaxations for the maximal covering location problem, *European Journal of Operational Research*, vol. 124(2), 2000, pp. 377 – 389.

[13] M.S. Canbolat, M. von Massow, Planar maximal covering with ellipses, *Computers and Industrial Engineering*, vol. 57(1), 2009, pp. 201 – 208.

[14] R.L. Church, C.S ReVelle, The maximal covering location problem, in *Papers of the Regional Science Association*, vol. 32, 1974, pp. 101 – 118.

[15] G. Ji, S. Han, A strategy analysis in dual-channel supply chain based on effort levels, in: *Proceedings of the 1th International Conference on Service Systems and Service Management*, Beijing, China, 2014, pp. 2161 – 1890.

[16] H. Kellerer, U. Pferschy, D. Pisinger, *Knapsack Problem*, Springer, Berlin, 2004.

[17] A. Mitsos, B. Chachuat and P.I. Barton, McCormic - based relaxation algorithm, *SIAM Journal on Optimization*, vol. 20, 2009, pp. 573 – 601.

[18] G.C. Moore, C.S. ReVelle, The hierarchical service location problem, *Management Science*, vol. 28, 1982, pp. 775 – 780.

[19] E. Polak, *Optimization*, Springer-Verlag, New York, USA, 1997.

[20] C. ReVelle, M. Scholssberg, J. Williams, Solving the maximal covering location problem with heuristic concentration, *Computers and Operations Research*, vol. 35(2), 2008, pp. 427 – 435.

[21] T. Roubicek, *Relaxation in Optimization Theory and Variational Calculus*, De Gruyter, Berlin, 1997.

[22] H. Shavandi, H. Mahlooji, A fuzzy queuing location model with a genetic algorithm for congested systems, *Applied Mathematics and Computation*, vol. 181(1), 2006, pp. 440 – 456.

[23] P. Sitek, J. Wikarek, A hybrid approach to modeling and optimization for supply chain management with multimodal transport, in: *Proceedings of the 18th International Conference on Methods and Models in Automation and Robotics*, Miedzyzdroje, Poland, 2013, pp. 777 – 782.

[24] F. Zarandi, A. Haddad Sisakht, S. Davari, Design of a closed-loop supply chain (CLSC) model using an interactive fuzzy goal programming, *The International Journal of Advanced Manufacturing Technology*, vol. 56, 2011, pp. 809 – 821.

[25] G. Ghiani, G. Laporte, R. Musmanno *Introduction to Logistics Systems Planning and Control*, John Wiley & Sons Ltd, Chichester, England, 2004.