BASIC AUTOMORPHISM GROUPS OF CARTAN FOLIATIONS COVERED BY FIBRATIONS

K.I. SHEINA

National Research University Higher School of Economics,
Department of Informatics, Mathematics and Computer Science,
ul. Bolshaja Pecherskaja, 25/12, Nizhny Novgorod, 603155, Russia
E-mail:ksheina@hse.ru

Abstract The basic automorphism group $A_B(M, F)$ of a Cartan foliation (M, F) is the quotient group of the automorphism group of (M, F) by the normal subgroup, which preserves every leaf invariant. For Cartan foliations covered by fibrations, we find sufficient conditions for the existence of a structure of a finite-dimensional Lie group in their basic automorphism groups. Estimates of the dimension of these groups are obtained. For some class of Cartan foliations with integrable an Ehresmann connection, a method for finding of basic automorphism groups is specified.

Keynwords: foliation; Cartan foliation; Lie group; basic automorphism; automorphism group

1 Introduction

One of the main objects associated with a geometric structure on a smooth manifold is its automorphism group. In the introduction to the monograph by S. Kobayashi [11], it was emphasized that the existence of a structure of a finite-dimensional Lie group in the group of automorphisms of a manifold with a geometric structure is one of the central problems in differential geometry.

As is known, the solved Hilbert's 5-th problem is devoted to finding conditions under which a topological group admits the structure of a Lie group [16]. It is known from the numerous works of E. Cartan, R. Mayer, H. Steenrod, K. Nomizu, S. Kabayashi, S. Ehresmann and other authors that the automorphism groups of many geometric structures are Lie groups of transformations (see overview [7]).

The spaces that are now called Cartan geometries were introduced by E. Cartan in the 1920-s. The theory of Cartan geometries is presented in the monographs of A. Čap, J. Slovak [5], R.V. Sharpe [14], M. Krampin and D. Saunders [8]. Currently, Cartan geometries and Cartan foliations are studied by many mathematicians and find application in various physical theories, see, for example, [1], [6], [13] and [9], [17].

Let (M, F) be a smooth foliation. Recall that geometry structure on the manifold M is called transverse to (M, F) if it is a invariant with respect to local holonomic diffeomorphisms. Another, equivalent definition of a transverse geometric structure, which is represented by Cartan geometry, is given in Section 3. Morphisms are understood as local diffeomorphisms mapping leaves onto leaves and preserving transverse geometries (the precise definition see in Section 3). Let us denote by $\mathfrak{CF}$ the category of Cartan foliations.

This paper is devoted to the investigation of automorphism groups of Cartan foliations, i.e. foliations that admit Cartan geometries as transverse structures. The study of Cartan foliations is motivated by the fact that such broad classes of foliations as parabolic, conformal, projective, pseudo-Riemannian, Lorentzian, Weyl, transverse homogeneous foliations and foliations with transverse linear connection belong to Cartan

foliations. Therefore, the investigation of Cartan foliations allows us to study the general properties of these foliations from a single point of view, while many authors study them separately.

Let us denote by $A(M, F)$ the group of all the automorphisms of the Cartan foliation (M, F) in the category $\mathfrak{CF}$. The group

$$A_L(M, F) := \{f \in A(M, F) \mid f(L_\alpha) = L_\alpha \ \forall L_\alpha \in F\}$$

is a normal subgroup of the group $A(M, F)$ and called *the group of leaf automorphisms* of (M, F). The quotient group $A(M, F)/A_L(M, F)$ is called *the basic automorphism group* and denoted by $A_B(M, F)$.

We study the groups of basic automorphisms $A_B(M, F)$ of Cartan foliations (M, F) covered by fibration and find sufficient conditions for the existence of a structure of a finite-dimensional Lie group in the group $A_B(M, F)$. J. Leslie [12] was the first who solved a similar problem for smooth foliations on compact manifolds and considered an application to foliations with transverse G-structures. For foliations with complete transversely projectable affine connection, this problem was raised by I.V. Belko [2]. Foliations (M, F) with effective transverse rigid geometries were investigated by N.I. Zhukova [19] where an algebraic invariant $\mathfrak{g}_0 = \mathfrak{g}_0(M, F)$, called the structural Lie algebra of (M, F), was constructed and it was proved that $\mathfrak{g}_0 = 0$ is a sufficient condition for the existence of a unique Lie group structure in the basic automorphism group of this foliation. In [15], the existence of a Lie group structure was investigated in the basic automorphism groups of Cartan foliations modeled on inefficient Cartan geometries.

2 Main results

Among the Cartan foliations, foliations covered by fibrations are distinguished.

Definition 1. Let $\kappa : \widetilde{M} \to M$ be the universal covering map. We say that a smooth foliation (M, F) *is covered by fibration* if the induced foliation $(\widetilde{M}, \widetilde{F})$ is formed by fibres of a locally trivial fibration $\widetilde{r} : \widetilde{M} \to B$.

The following theorem describes the global structure of Cartan foliations covered by fibrations.

Theorem 1. *Let (M, F) be a Cartan foliation modeled on a Cartan geometry ξ covered by the fibration $\widetilde{r} : \widetilde{M} \to B$, where $\widetilde{\kappa} : \widetilde{M} \to M$ is the universal covering map. Then:*

(1) *there exists a regular covering map $\kappa : \widehat{M} \to M$ such that the induced foliation $\widehat{F}$ is made up of fibres of the locally trivial bundle $r : \widehat{M} \to B$ over a simply connected manifold B, and ξ induces on B a Cartan geometry η that is locally isomorphic to ξ;*

(2) *an epimorphism $\chi : \pi_1(M, x) \to \Psi$, $x \in M$, of the fundamental group $\pi_1(M, x)$ onto a subgroup Ψ of the automorphism Lie group $Aut(B, \eta)$ of the Cartan manifold (B, η) is determined;*

(3) *the group of deck transformations of the covering $\kappa : \widehat{M} \to M$ is isomorphic to the group Ψ.*

Definition 2. The group $\Psi = \Psi(M, F)$ satisfying Theorem 1 is called the *global holonomy group* of the Cartan foliation (M, F) covered by fibration.

We give a detailed proof of the following theorem, formulated without a proof in the work [15, Prop. 8]. Theorem 2 establishes a connection between the basic automorphism group $A_B(M, F)$ of a Cartan foliation (M, F) covered by fibration and its global holonomy group Ψ.

Theorem 2. *Let (M, F) be a Cartan foliation covered by fibration $r : \widehat{M} \to B$, and B is the simply connected Cartan manifold. Suppose that the global holonomy group Ψ is a discrete subgroup of the Lie group $Aut(B, \eta)$. Let $N(\Psi)$ be the normalizer of Ψ in $Aut(B, \eta)$. Then the basic automorphism group $A_B(M, F)$ is a Lie group which is isomorphic to an open-closed subgroup of the Lie quotient group $N(\Psi)/\Psi$ and $\dim(A_B(M, F)) = \dim(N(\Psi)/\Psi)$. The structure of the Lie group in $A_B(M, F)$ is unique.*

The following theorem specifies a method for computing basic automorphism groups for Cartan foliations with an integrable Ehresmann connection.

Theorem 3. *Let (M, F) be Cartan foliation with an integrable Ehresmann connection. Then*

1. There is a regular cover $\kappa : \widetilde{M} \to M$ such that $\widetilde{M} = L_0 \times B$, where L_0 is a manifold diffeomorphic to any leaf with a trivial holonomy group and B is a simply connected manifold, and the induced foliation $\widetilde{F} = \kappa^* F$ is formed by leaves of the canonical projection $r : L_0 \times B \to B$ onto the second factor, and Cartan geometry η is induced on B, with respect to which κ is a morphism of Cartan foliations (M, F) and $(\widetilde{M}, \widetilde{F})$ in the category $\mathfrak{CF}$.

2. The foliation (M, F) is an $(Aut(B, \eta), B)$-foliation.

3. If moreover, the normalizer $N(\Psi)$ of global holonomy group Ψ is equal to the centralizer $Z(\Psi)$ of Ψ in the group $Aut(B, \eta)$, then

$$A_B(M, F) \cong N(\Psi)/\Psi.$$

Using Theorem 3, we construct an example of computing the basic automorphism group of some conformal foliation of an arbitrary codimension q, where $q \geq 3$, on a $(q + 1)$-dimensional manifold in Section 7.2. Some other examples are constructed in [15].

3 The category of Cartan foliations

The category of Cartan geometries Let G and H be Lie groups with the Lie algebras $\mathfrak{g}$ and $\mathfrak{h}$ relatively. Let H be a closed subgroup of G. A *Cartan geometry* of type (G, H) on the smooth manifold N is a principal H-bundle $P(N, H)$ with a $\mathfrak{g}$-valued 1-form ω on P satisfying the following conditions:

(c_1) the map $\omega_u : T_u P \to \mathfrak{g}$ is an isomorphism of vector spaces for every $u \in P$;

(c_2) $\omega(A^*) = A$ for every $A \in \mathfrak{h}$, where A^* is the fundamental vector field determined by A;

(c_3) $R_h^* \omega = Ad_G(h^{-1})\omega$ for every $h \in H$, where $Ad_G : H \to GL(\mathfrak{g})$ is the adjoint representation of the Lie subgroup H of G in the Lie algebra $\mathfrak{g}$.

The $\mathfrak{g}$-valued form ω is called a *Cartan connection form*. This Cartan geometry is denoted by $\xi = (P(N, H), \omega)$. The pair (N, ξ) is called a *Cartan manifold*.

Maximal normal subgroup K of the group G belonging to H is called the *kernel* of pair (G, H). We denote the Lie algebra of the group K by $\mathfrak{k}$. The Cartan geometry $\xi = (P(M, H), \omega)$ of type (G, H) is called *effective* if the kernel K of the pair (G, H) is trivial. Further, we assume that all Cartan geometries under consideration are effective.

Let $\xi = (P(N, H), \omega)$ and $\xi' = (P'(N', H), \omega')$ be two Cartan geometries with the same structure Lie group H. The smooth map $\Gamma : P \to P'$ is called a morphism from ξ to ξ' if $\Gamma^* \omega' = \omega$ and $R_a \circ \Gamma = \Gamma \circ R_a$ $\forall a \in H$. The category of Cartan geometries is denoted by $\mathfrak{Car}$. If $\Gamma \in Mor(\xi, \xi')$, then the projection $\gamma : N \to N'$ is defined such that $p' \circ \Gamma = \gamma \circ p$, where $p : P \to N$ and $p' : P' \to N'$ are the projections of the respective H-bundles.

The projection γ is called *an automorphism of the Cartan manifold* (N, ξ). Denote by $Aut(N, \xi)$ the full automorphism group of a Cartan foliation (N, ξ) and by $Aut(\xi)$ the full automorphism group of a Cartan geometry ξ. Let $A(P, \omega) := \{\Gamma \in Diff(P) \,|\, \Gamma^* \omega = \omega\}$ be the automorphism group of the parallelizable manifold (P, ω), which is known to be a Lie group, and $\dim(A(P, \omega)) \leq \dim P$.

Remark, that $Aut(\xi) = \{\Gamma \in A(P, \omega) \,|\, \Gamma \circ R_a = R_a \circ \Gamma \,\forall a \in H\}$ is a closed Lie subgroup of the Lie group $A(P, \omega)$. Therefore, $Aut(\xi)$ is a Lie group, and due to the effictivity of a Cartan geometry ξ, there exists a Lie group isomorphism

$$\sigma : A^H(P, \omega) \to Aut(N, \xi) : \Gamma \mapsto \gamma$$

mapping $\Gamma \in A^H(P, \omega)$ to its projection γ.

Cartan foliations Let N be a smooth q-dimensional manifold, the connectivity of which is not assumed. Let M be a smooth n-dimensional manifold, where $0 < q < n$. Assume, that $\xi = (P(N, H), \omega)$ is a Cartan geometry of type (G, H) on the manifold N. Let $p : P \to N$ be the projection the principal H-bundle. For every open subset $V \subset N$, the Cartan geometry $\xi_V = (P_V(V, H), \omega_V)$ of the same type (G, H) is induced, where $P_V := p^{-1}(V)$ and $\omega_V := \omega|_{P_V}$. Remind that (N, ξ)-*cocycle* on M is a family $\zeta = \{U_i, f_i, \{\gamma_{ij}\}\}_{ij \in J}$ satisfying the following conditions:
1)the set $\{U_i \,|\, i \in J\}$ is a covering of the manifold M by open connected subsets U_i of M, and every $f_i : U_i \to N$ is a submersion with connected fibres;
2) if $U_i \cap U_j \neq \emptyset$, $i, j \in J$, then there exists an isomorphism $\Gamma_{ij} : \xi_{f_j(U_i \cap U_j)} \to \xi_{f_i(U_i \cap U_j)}$ of the Cartan geometries induced on open subsets $f_j(U_i \cap U_j)$ and $f_i(U_i \cap U_j)$ such that the projection γ_{ij} of the isomorphism Γ_{ij} satisfies the equality $f_i = \gamma_{ij} \circ f_j$ on $U_i \cap U_j$, $i, j \in J$;
3) if $U_i \cap U_j \cap U_k \neq \emptyset$, then $\gamma_{ij} \circ \gamma_{jk} = \gamma_{ik}$ for all $x \in U_i \cap U_j \cap U_k$ and $\gamma_{ii} = id_{U_i}$, $i, j, k \in J$.

Two N-cocycles are called *equivalent* if there exists an N-cocycle containing both of these cocycles. Let $[\{U_i, f_i, \{\gamma_{ij}\}\}_{ij \in J}]$ be the equivalence class of N-cocycles on the manifold M containing the cocycle $\zeta = \{U_i, f_i, \{\gamma_{ij}\}\}_{ij \in J}$. Denote by Σ the set of fibres of all the submersions f_i of this equivalence class. Note, that Σ is the base of some new topology τ on M. The path-connected components of the topological space (M, τ) form a partition $F := \{L_\alpha \,|\, \alpha \in \mathfrak{J}\}$ of the manifold M. The pair (M, F) is called *a Cartan foliation of codimension q* modeled on the Cartan geometry $\xi = (P(N, H), \omega)$ which is called *a transverse Cartan geometry* for (M, F). Subsets L_α, $\alpha \in \mathfrak{J}$ are called *leaves* of this foliation. It is said that (M, F) is given by the (N, ξ)-cocycle ζ.

Morphisms in the category of Cartan foliations Let (M, F) and (M', F') are Cartan foliations defined by an (N, ξ)-cocycle $\zeta = \{U_i, f_i, \{\gamma_{ij}\}\}_{ij \in J}$ and an (N', ξ')-cocycle $\zeta' = \{U'_r, f'_r, \{\gamma'_{rs}\}\}_{i'j' \in J'}$, respectively. All objects belonging to ζ' are distinguished by prime. Let $f : M \to M'$ be a smooth map which is a local isomorphism in the foliation category $\mathfrak{Fol}$. Hence for any $x \in M$ and $y := f(x)$ there exist neighborhoods $U_k \ni x$ and $U'_s \ni y$ from ζ and ζ' respectively, a diffeomorphism $\varphi : V_k \to V'_s$, where $V_k := f_k(U_k)$ and $V'_s := f'_s(U'_s)$, satisfying the relations $f(U_k) = U'_s$ and $\varphi \circ f_k = f'_s \circ f|_{U_k}$.

Further we shall use the following notations: $P_k := P|_{V_k}$, $P'_s := P'|_{V'_s}$ and $p_k := p|_{P_k}$, $p'_s := p|_{P'_s}$

We say that f preserves transverse Cartan geometry if every such diffeomorphism $\varphi \colon V_k \to V'_s$ is an isomorphism of the induced Cartan geometries (V_k, ξ_{V_k}) and $(V'_s, \xi'_{V'_s})$. This means the existence of isomorphism $\Phi : P_k \to P'_s$ in the category $\mathfrak{Car}$ with the projection φ such that the following diagram

$$
\begin{array}{ccc}
& P_k & \\
& {\scriptstyle p_k}\downarrow \quad \searrow {\scriptstyle \Phi} & \\
M \supset U_k \xrightarrow{\; f_k \;} V_k & & P'_s \\
\searrow {\scriptstyle f|_{U_k}} \quad \searrow {\scriptstyle \varphi} & & \downarrow {\scriptstyle p'_s} \\
& M' \supset U'_s \xrightarrow{\; f'_s \;} V'_s &
\end{array}
$$

is commutative. We emphasize that the indicated above isomorphism $\Phi : P_k \to P'_s$ is unique if the transverse Cartan geometries are effective. The introduced concept is well defined, i. e., it does not depend of the choice of neighborhoods U_k and U'_k from the cocycles ζ and ζ'.

Definition 3. By a *morphism of two Cartan foliations (M, F) and (M', F')* we mean a local diffeomorphism $f : M \to M'$ which transforms leaves to leaves and preserves transverse Cartan structure. The category $\mathfrak{CF}$ objects of which are Cartan foliations, morphisms are their morphisms, is called *the category of Cartan foliations.*

4 Ehresmann connections for foliations

R. A. Blumenthal and J. J. Hebda [3] introduced the notion of Ehresmann connection for foliation (M, F) as a natural generalization of Ehresmann connection for submersions.

Let (M, F) be a foliation of codimension q and $\mathfrak{M}$ be a smooth q-dimensional distribution on M that is transverse to the foliation F, i. e. $T_x M = \mathfrak{M}_x \oplus T_x F \; \forall x \in M$. The piecewise smooth integral curves of the distribution $\mathfrak{M}$ are said to be *horizontal,* and the piecewise smooth curves in the leaves are said to be *vertical*. A piecewise smooth mapping H of the square $I_1 \times I_2$ to M is called a *vertical-horizontal homotopy* if the curve $H|_{\{s\}\times I_2}$ is vertical for any fixed $s \in I_1$ and the curve $H|_{I_1 \times \{t\}}$ is horizontal for any fixed $t \in I_2$. In this case, the pair of paths $(H|_{I_1 \times \{0\}}, H|_{\{0\} \times I_2})$ is called the *base* of H. It is well known that there exists at most one vertical-horizontal homotopy with a given base.

A distribution $\mathfrak{M}$ is called an *Ehresmann connection for a foliation (M, F)* (in the sense of R. A. Blumenthal and J. J. Hebda [3]) if, for any pair of paths (σ, h) in M with a common initial point $\sigma(0) = h(0)$, where σ is a horizontal curve and h is a vertical curve, there exists a vertical-horizontal homotopy H with the base (σ, h).

A simple foliation with an Ehresmann connection Let $f : M \to N$ be a submersion with connected fibers. Recall that the foliation $F = \{p^{-1}(z), \, z \in N\}$ formed by the fibers of the submersion is called *a simple foliation.* Let (M, F) be an arbitrary smooth foliation with the Ehresmann connection. It easy to show that, the existens of a covering $\widehat{k} : \widehat{M} \to M$ such that the lifted foliation is simple implies that the foliation (M, F) is covered by fibration.

5 Classes of foliations covered by fibrations

(G, X)-foliations with an Ehresmann connection Let X be a smooth connected manifold and G be the Lie group of diffeomorphisms of X. Recall that the action of a group G on a manifold X is called *quasi-analytically* if for any open subset $U \subset X$ and an element $g \in G$ the equality $g|_U = id_U$ implies $g = e$, where $e = id_X$.

Assume that a Lie group G of diffeomorphisms of a manifold X acts on N quasi-analytically. A foliation (M, F) defined by an X-cocycle $\{U_i, f_i, \{\gamma_{ij}\}\}_{i,j \in J}$ is called a (G, X)-foliation if for any $U_i \cap U_j \neq \emptyset$, $i, j \in J$, there is an element $g \in G$ such that $\gamma_{ij} = g|_{f_j(U_i \cap U_j)}$. If, moreover, (X, ξ) is a Cartan manifold and the group G is a subgroup of the automorphism Lie group $Aut(X, \xi)$, then (M, F) is a Cartan (G, X)-foliation. It follows from [7, Section VI] that every Cartan (G, X)-foliations with Ehresmann connections is a foliation covered by fibration.

Cartan foliation with a vanishing transverse curvature Let (M, F) be a Cartan foliation of type (G, H) with an Ehresmann connection. As is known [1, Section VI], if the transverse curvature of (M, F) vanishes, then foliation (M, F) is covered by fibration. Consequently, all the obtained results are valid for Cartan foliations with zero transverse curvature that admiting an Ehresmann connection.

Conformal foliations of codimension $q, q \geq 3$ According to [20, Thm. 5], any non-Riemannian conformal foliation of codimension $q \geq 3$ with an Ehresmann connection is covered by fibration.

Foliations with an integrable Ehresmann connection Recall that an Ehresmann connection $\mathfrak{M}$ for a foliation (M, F) is called integrable if the distribution $\mathfrak{M}$ is integrable i.e. if there exists the foliation such that $TF^t = \mathfrak{M}$. Accoding to Kashiwabara's theorem [10], foliations with an integrable Ehresmann connection are covered by fibrations.

Suspended foliations The construction of a suspension foliation was proposed by A. Haefliger and described in detail in [18]. Note that suspension foliations form a class of foliations with integrable Ehresmann connection and are covered by fibrations.

Cartan foliation of codimension $q = 1$ Any smooth one-dimensional distribution is integrable, so a Cartan foliation (M, F) of codimension $q = 1$ with an Ehresmann connection is covered by fibration.

6 Proof of Theorems 1 and 2

6.1 Regular covering maps

Definition 4. Let $f : M \to B$ be a submersion. It is said that $\widehat{h} \in Diff(M)$ lying over $h \in Diff(B)$ relatively f if $h \circ f = f \circ \widehat{h}$. In this case $\widehat{h}$ is called *a lift* of h with respect to $f : M \to B$.

Let $\widetilde{\kappa} : \widetilde{K} \to K$ be the universal covering map, where K and $\widetilde{K}$ are smooth manifolds. By analogy with Theorem 28.10 in [4], it is easy to show that for any $h \in Diff(K)$ there exists $\widetilde{h} \in Diff(\widetilde{K})$ lying over h. For an arbitrary covering map the same statement is incorrect, in general. It is not difficult to prove the following

criterion for the existence of lifts of arbitrary diffeomorphisms with respect to regular covers.

Proposition 1. *Let $\kappa : \widehat{K} \to K$ be a smooth regular covering map with the deck transformation group Γ. A diffeomorphism $\widehat{h} \in Diff(\widehat{K})$ lies over some diffeomorphism $h \in Diff(K)$ if and only if it satisfies the equality $\widehat{h} \circ \Gamma = \Gamma \circ \widehat{h}$.*

6.2   Proof of Theorem 1

Suppose that a Cartan foliation (M, F) modeled on an effective Cartan geometry $\xi = (P(N, H), \omega)$ is covered by a fibration $\widetilde{r} : \widetilde{M} \to B$, where $\widetilde{\kappa} : \widetilde{M} \to M$ is the universal covering map. The fibration $\widetilde{r} : \widetilde{M} \to B$ has connected fibres and simply connected space $\widetilde{M}$. Therefore, due to the application of the exact homotopic sequence for this fibration we obtain that the base manifold B is also simply connected.

For an arbitrary point $b \in B$ take $y \in \widetilde{r}^{-1}(b)$ and $x = \widetilde{\kappa}(y)$. Without loss generality, we assume that there is a neighbourhood U_i, $x \in U_i$, from the (N, ξ)-cocycle $\{U_i, f_i, \{\gamma_{ij}\}\}_{i,j \in J}$ which defines (M, F) and a neighbourhood $\widetilde{U}_i$, $y \in \widetilde{U}_i$, such that $\widetilde{\kappa}|_{\widetilde{U}_i} : \widetilde{U}_i \to U_i$ is a diffeomorphism.

Let $\widetilde{V}_i := \widetilde{r}(\widetilde{U}_i)$. Then there exists a diffeomorphism $\phi : \widetilde{V}_i \to V_i$ satisfying the equality $\phi \circ \widetilde{r}|_{\widetilde{U}_i} = f_i \circ \widetilde{\kappa}|_{\widetilde{U}_i}$. The diffeomorphism ϕ induces the Cartan geometry $\eta_{\widetilde{V}_i}$ on $\widetilde{V}_i$ such that ϕ becomes the isomorphism $(\widetilde{V}_i, \eta_{\widetilde{V}_i})$ and (V_i, ξ_{V_i}) in the category $\mathfrak{Car}$ of Cartan geometries. The direct check shows that by this way we define the Cartan geometry η on B, and $\eta|_{\widetilde{V}_i} = \eta_{\widetilde{V}_i}$, $i \in J$. Thus, the statement (1) is proved.

Let us fix points $x_0 \in M$ and $y_0 \in \widetilde{\kappa}^{-1}(x_0) \in \widetilde{M}$. Then the fundamental group $\pi_1(M, x_0)$ acts on the universal covering space $\widetilde{M}$ as the deck transformation group $\widetilde{G} \cong \pi_1(M, x_0)$ of $\widetilde{\kappa}$. Since $\widetilde{G}$ preserves the inducted foliation $(\widetilde{M}, \widetilde{F})$ formed by fibres of the fibration $\widetilde{r} : \widetilde{M} \to B$, then every $\widetilde{\psi} \in \widetilde{G}$ defines $\psi \in Diff(B)$ satisfying the relation $\widetilde{r} \circ \widetilde{\psi} = \psi \circ \widetilde{r}$. The map $\chi : \widetilde{G} \to \Psi : \widetilde{\psi} \to \psi$ is a group epimorphism and the statement (2) is proved.

Observe that $\widetilde{G}$ is a subgroup of the automorphism group $Aut(\widetilde{M}, \widetilde{F})$ of $(\widetilde{M}, \widetilde{F})$ in the category $\mathfrak{CF}$. Therefore Ψ is a subgroup of the automorphism group $Aut(B, \eta)$ in the category of Cartan geometries $\mathfrak{Car}$. The kernel $ker(\chi)$ of χ determines the quotient manifold $\widehat{M} := \widetilde{M}/ker(\chi)$ with the quotient map $\widehat{\kappa} : \widetilde{M} \to \widehat{M}$ and the quotient group $\widehat{G} := \widetilde{G}/ker(\chi)$ such that $M \cong \widehat{M}/\widehat{G}$. The quotient map $\kappa : \widehat{M} \to M$ is the required regular covering map, with $\widehat{G}$ acts on $\widehat{M}$ as a deck transformation group. The map $\widehat{G} \to \Psi : \widetilde{\psi} \cdot ker(\chi) \mapsto \chi(\widetilde{\psi})$, $\widetilde{\psi} \in \widetilde{G}$, is a group isomorphism. Thus the statment (3) is proved.

6.3   The associated foliated bundle

Let (M, F) be a Cartan foliation modeled on Cartan geometry $\xi = (P(\mathcal{N}, H), \omega)$ of type (G, H). Then there exists a principal H-bundle with the projection $\pi : \mathcal{R} \to M$, the H-invariant foliation $(\mathcal{R}, \mathcal{F})$ and the $\mathfrak{g}$-valued H-equivariant 1-form β on $\mathcal{R}$ which satisfy the following conditions:

(i) $\beta(A^*) = A$ for any $A \in \mathfrak{h}$;

(ii) the mapping $\beta_u : T_u\mathcal{R} \to \mathfrak{g}$ $\forall u \in \mathcal{R}$ is surjective, and $ker(\beta_u) = T_u\mathcal{F}$;

(iii) the foliation $(\mathcal{R}, \mathcal{F})$ is transversely parallelizable;

(iv) the Lie derivative $L_X \beta$ is equal to zero for every vector field X tangent to the foliation $(\mathcal{R}, \mathcal{F})$.

Definition 5. The principal H-bundle $\mathcal{R}(M, H)$ is called *the foliated bundle* over the Cartan foliation (M, F). The foliation $(\mathcal{R}, \mathcal{F})$ is called *the lifted foliation* for the Cartan foliation (M, F).

If the lifted foliation $(\mathcal{R}, \mathcal{F})$ is formed by fibres of the locally trivial fibration $\pi_b : \mathcal{R} \to W$, then $W = \mathcal{R}/\mathcal{F}$ is a smooth manifold , and a $\mathfrak{g}$-valued 1-form β such that $\pi_b^* \widetilde{\beta} := \beta$ and locally free action of the Lie group H on W are induced. In this case, $(W, \widetilde{\beta})$ is a parallelizable manifold and $A(W, \widetilde{\beta})$ is the Lie group of its automorphisms that acts freely on W. Further, as above, by $A^H(W, \widetilde{\beta})$ we denote the closed Lie subgroup of $A(W, \widetilde{\beta})$ formed by transformations commuting with the induced action of the Lie group H on W. $\square$

6.4   Proof of Theorem 2

Suppose that a Cartan foliation (M, F) is covered by fibration. By definition 1, the induced foliation $(\widetilde{M}, \widetilde{F})$ on the space of the universal covering $\widetilde{\kappa} : \widetilde{M} \to M$ is defined by a locally trivial fibration $\widetilde{r} : \widetilde{M} \to B$. Due to Theorem 1, the regular covering map $\kappa : \widehat{M} \to M$ and locally trivial fibration $r : \widehat{M} \to B$ are defined, where B is a simply connected manifold with the inducted Cartan geometry η. Let Ψ be the global holonomy group of the foliation (M, F), then Ψ is isomorphic to the deck transformations group G of the covering $\kappa : \widehat{M} \to M$. Since the manifold $\widetilde{M}$ is simply connected, there exists the universal covering map $\widehat{\kappa} : \widetilde{M} \to \widehat{M}$ satisfying the equality $\kappa \circ \widehat{\kappa} = \widetilde{\kappa}$. Let $\widetilde{G}$, G and $\widehat{G}$ be the deck transformation groups of the covering maps $\widetilde{\kappa}$, κ and $\widehat{\kappa}$ respectively, with $\Psi \cong G \cong \widetilde{G}/\widehat{G}$.

Let us consider the following preimages of the H-bundle $\mathcal{R}$ respectively $\widetilde{\kappa}$ and κ

$$\widetilde{\mathcal{R}} := \{(\widetilde{x}, u) \in \widetilde{M} \times \mathcal{R} \,|\, \widetilde{\kappa}(\widetilde{x}) = \pi(u)\} = \widetilde{\kappa}^* \mathcal{R} \;\; and$$

$$\widehat{\mathcal{R}} := \{(\widehat{x}, u) \in \widehat{M} \times \mathcal{R} \,|\, \kappa(\widehat{x}) = \pi(u)\} = \kappa^* \mathcal{R}.$$

Remark that the maps

$$\widetilde{\theta} : \widetilde{\mathcal{R}} \to \mathcal{R} : (\widetilde{x}, u) \mapsto (\widetilde{\kappa}(\widetilde{x}), u) \;\; \forall (\widetilde{x}, u) \in \widetilde{\mathcal{R}},$$

$$\theta : \widehat{\mathcal{R}} \to \mathcal{R} : (\widehat{x}, u) \mapsto (\kappa(\widehat{x}), u) \;\; \forall (\widehat{x}, u) \in \widehat{\mathcal{R}},$$

$$\widehat{\theta} : \widetilde{\mathcal{R}} \to \widehat{\mathcal{R}} : (\widetilde{x}, u) \mapsto (\widehat{\kappa}(\widetilde{x}), u) \;\; \forall (\widetilde{x}, u) \in \widetilde{\mathcal{R}},$$

are regular covering maps with the deck transformation groups $\widetilde{\Gamma}$, Γ and $\widehat{\Gamma}$, respectively, which are isomorphic to the relevant groups $\widetilde{G}$, G and $\widehat{G}$, i.e. $\widetilde{\Gamma} \cong \widetilde{G}$, $\Gamma \cong G$ and $\widehat{\Gamma} \cong \widehat{G}$.

Let $(\widetilde{\mathcal{R}}, \widetilde{\mathcal{F}})$ and $(\widehat{\mathcal{R}}, \widehat{\mathcal{F}})$ be the corresponding lifted foliations for (M, F). Since $(\widetilde{M}, \widetilde{F})$ and $(\widehat{M}, \widehat{F})$ are simple foliations, then $(\widetilde{\mathcal{R}}, \widetilde{\mathcal{F}})$ and $(\widehat{\mathcal{R}}, \widehat{\mathcal{F}})$ are also simple foliations, which are formed by locally trivial fibrations $\widetilde{\pi}_b : \widetilde{\mathcal{R}} \to \widetilde{W}$ and $\widehat{\pi}_b : \widehat{\mathcal{R}} \to \widehat{W}$. Hence $\mathfrak{g}_0 (\widetilde{\mathcal{R}}, \widetilde{\mathcal{F}}) = 0$, $\mathfrak{g}_0(\widehat{\mathcal{R}}, \widehat{\mathcal{F}}) = 0$, and $\widetilde{W} = \widetilde{\mathcal{R}}/\widetilde{\mathcal{F}}$, $\widehat{W} = \widehat{\mathcal{R}}/\widehat{\mathcal{F}}$ are manifolds.

Since the fibrations $\widetilde{r} : \widetilde{M} \to B$ and $r : \widehat{M} \to B$ have the same base B, each leaf of the foliation $(\widetilde{M}, \widetilde{F})$ is invariant respectively the group $\widehat{G}$, i.e. $\widehat{G} \subset A_L(\widetilde{M}, \widetilde{F})$. Therefore $\widehat{\Gamma} \subset A_L(\widetilde{\mathcal{R}}, \widetilde{\mathcal{F}})$ and the leaf spaces $\widetilde{\mathcal{R}}/\widetilde{\mathcal{F}} = \widetilde{W}$ and $\widehat{\mathcal{R}}/\widehat{\mathcal{F}} = \widehat{W}$ are coincided, i.e. $\widetilde{W} = \widehat{W}$. Consequently, basic automorphism groups $A_B(\widetilde{\mathcal{R}}, \widetilde{\mathcal{F}})$ and $A_B(\widehat{\mathcal{R}}, \widehat{\mathcal{F}})$ may be identified. Further we put $A_B(\widetilde{\mathcal{R}}, \widetilde{\mathcal{F}}) = A_B(\widehat{\mathcal{R}}, \widehat{\mathcal{F}})$.

According to the conditions of Theorem 2, Ψ is a discrete subgroup of the Lie group $Aut(B,\eta)$. Let $N(\Psi)$ be the normalizer of Ψ in the Lie group $Aut(B,\eta) \cong A^H(W,\widetilde{\beta})$. Hence, $N(\Psi)$ is a closed Lie subgroup of the Lie group $Aut(B,\eta)$ and the quotient group $N(\Psi)/\Psi$ is also a Lie group.

Let $\pi : \mathcal{R} \to M$ be the projection of the foliated bundle over (M,F). Due to the discreteness of the global holonomy group Ψ, the lifted foliation $(\mathcal{R},\mathcal{F})$ is formed by fibres of some locally trivial fibration $\pi_b : \mathcal{R} \to W$, which is called the basic fibration.

Observe that there exists a map $\tau : \widehat{W} \to W$ satisfying the equality $\tau \circ \widehat{\pi}_b = \theta \circ \pi_b$. It is easy to show that $\tau : \widehat{W} \to W$ is a regular covering map with the deck transformations group Φ, $\Phi \subset A^H(\widehat{W},\widehat{\beta})$, which is naturally isomorphic to each of the groups Ψ, G and Γ.

Denote by $\eta = (P(B,H),\omega)$ the Cartan geometry with the projection $p : P \to B$ onto B determined in the proof of Theorem 1. Remark that $\widehat{W} = P$ is the space of the H-bundle of the Cartan geometry η.

Since $\kappa : \widehat{M} \to M$, $\theta : \widehat{\mathcal{R}} \to \mathcal{R}$ and $\pi : \mathcal{R} \to M$ are morphisms of the following foliations $\kappa : (\widehat{M},\widehat{F}) \to (M,F)$, $\theta : (\widehat{\mathcal{R}},\widehat{\mathcal{F}}) \to (\mathcal{R},\mathcal{F})$ и $\pi : (\mathcal{R},\mathcal{F}) \to (M,F)$ in the category of the foliations $\mathfrak{Fol}$, then maps $\widehat{\tau} : B \to M/F$ and $s : W \to W/H \cong M/F$ are defined, and the following diagram

is commutative.

Due to Proposition [15, Thm. 1] there are the Lie group isomorphisms

$$\varepsilon : A_B(M,F) \to im(\varepsilon) \subset A^H(W,\widetilde{\beta}) \; and$$

$$\widehat{\varepsilon} : A_B(\widetilde{M},\widetilde{F}) = A_B(\widehat{M},\widehat{F}) \to im(\widehat{\varepsilon}) \subset A^H(\widehat{W},\widehat{\beta}).$$

Let us define a map $\Theta : im(\varepsilon) \to N(\Phi)/\Phi$ by the following a way. Take any $h \in im(\varepsilon) \subset A^H(W,\widetilde{\beta})$. Denote the element $\varepsilon^{-1}(h) \in A_B(M,F)$ by $f \cdot A_L(M,F) \in A_B(M,F)$, where $f \in A(M,F)$. Since $\widetilde{\kappa} : \widetilde{M} \to M$ is the universal covering map there exists $\widetilde{f} \in Diff(\widetilde{M})$ lying over f relatively $\widetilde{\kappa}$. It not difficult to see that $\widetilde{f} \in A(\widetilde{M},\widetilde{F})$. Hence $\widetilde{f} \cdot A_L(\widetilde{M},\widetilde{F}) \in A_B(\widetilde{M},\widetilde{F})$. Consider $\widehat{h} := \widehat{\varepsilon}(\widetilde{f} \cdot A_L(\widetilde{M},\widetilde{F})) \in im(\widehat{\varepsilon}) \subset A^H(\widehat{W},\widehat{\beta})$. The direct check shows that $\widehat{h}$ lies over h respectively τ. Remind that Φ is the deck transformation group of the covering map $\tau : \widehat{W} \to W$. Applying the Proposition 1, we get that $\widehat{h} \in N(\Phi)$, hence the set of all automorphisms in $im(\widehat{\varepsilon})$ lying over h is equal to the set of transformations from the class $\widehat{h} \cdot \Phi$. Let us put $\Theta(h) := \widehat{h} \cdot \Phi \in N(\Phi)/\Phi$. It is easy to check that the map $\Theta : im(\varepsilon) \to N(\Phi)/\Phi$ is a group monomorphism.

The effectiveness of the Cartan geometry $\eta = (P(B,H),\omega)$ on B, where $P = \widehat{W}$, implies the existence of the Lie group isomorphism $\sigma : A^H(\widehat{W},\widehat{\beta}) \to Aut(B,\eta)$. Observe that $\sigma(\Phi) = \Psi$ and $\sigma(N(\widetilde{\Phi})) = N(\Psi)$, hence there exists the inducted Lie group isomorphism $\widetilde{\sigma} : N(\Phi)/\Phi \to N(\Psi)/\Psi$. Thus, the composition of the Lie group

monomorphisms

$$\delta := \widetilde{\sigma} \circ \Theta \circ \varepsilon : A_B(M, F) \to N(\Psi)/\Psi$$

is the required Lie group monomorphism. Due to uniqueness of the Lie group structure in $A_B(M, F)$, in conforming with [15, Thm. 1], the image $im(\delta)$ is an open-closed subgroup of the Lie group $N(\Psi)/\Psi$. $\square$

7 Basic automorphism groups of Cartan foliations with an integrable Ehresmann connection

7.1 Proof of Theorem 3

1. According to the conditions of the theorem being proved, (M, F) has an intagrable Ehresmann connection $\mathfrak{M}$. In this case, distribution $\mathfrak{M}$ is integrable. In this case, there is q-dimensional foliation (M, F^t) such that $TF^t = \mathfrak{M}$.

Let $\widetilde{\kappa} : \widetilde{M} \to M$ be the universal covering map. According to the decomposition theorem belonging to S. Kashiwabara [10], the universal covering manifold $\widetilde{M}$ is equal to the product of manifolds $\widetilde{M} = \widetilde{Q} \times B$, where $\widetilde{Q}$ is the universal covering manifold for any leaf of the foliation (M, F), and B is the universal covering manifold for any leaf of the foliation (M, F^t). The induced foliations $\widetilde{F} = \widetilde{\kappa}^* F = \{\widetilde{Q} \times \{y\} \,|\, y \in B\}$, $\widetilde{F}^t = \widetilde{\kappa}^* F^t = \{\{z\} \times B \,|\, z \in \widetilde{Q}\}$ are defined. Therefore, (M, F) is covered by fibration $\widetilde{s} : \widetilde{Q} \times B \to B$. In this case, by the same way as in the proof of Theorem 1, the Cartan geometry η is induced on B such that (M, F) becomes an $(Aut(B, \eta), B)$-foliation.

2. Let Ψ be the global holonomy group of this foliation. Suppose now that the normalizer $N(\Psi)$ is equal to the centralizer $Z(\Psi)$ of the group Ψ in the group $Aut(B, \eta)$.

Let us fix points $x_0 \in M$ and $(z_0, y_0) \in \widetilde{\kappa}^{-1}(x_0) \in \widetilde{M}$. Then the fundamental group $\pi_1(M, x_0)$ acts on the universal covering space $\widetilde{M} = \widetilde{Q} \times B$ as the deck transformation group $\widetilde{G} \cong \pi_1(M, x_0)$ of $\widetilde{\kappa}$. Since $\widetilde{G}$ preserves both the inducted foliations $(\widetilde{M}, \widetilde{F})$ and $(\widetilde{M}, \widetilde{F}^t)$, then every $\widetilde{g} \in \widetilde{G}$ may be written in the form $\widetilde{g} = (\psi^t, \psi)$, where ψ^t generates a subgroup Ψ^t in $Diff(\widetilde{Q})$, $\psi \in \Psi$, and $\widetilde{g}(z, y) = (\psi^t(z), \psi(y))$, $(z, y) \in \widetilde{Q} \times B$. The maps $\widetilde{\chi} : \widetilde{G} \to \Psi : \widetilde{g} \to \psi$ and $\widetilde{\chi}^t : \widetilde{G} \to \Psi^t : \widetilde{g} \to \psi^t$ are the group epimorphisms.

Let h be any element from $N(\Psi)/\Psi$. Since $N(\Psi) = Z(\Psi)$, we have the following chain of equalities

$$\widetilde{g} \circ (id_{\widetilde{Q}}, h) = (\psi^t, \psi) \circ (id_{\widetilde{Q}}, h) = (\psi^t \circ id_{\widetilde{Q}}, \psi \circ h) =$$

$$(id_{\widetilde{Q}} \circ \psi^t, h \circ \psi) = (id_{\widetilde{Q}}, h) \circ (\psi^t, \psi) = (id_{\widetilde{Q}}, h) \circ \widetilde{g}$$

for any $\widetilde{g} = (\psi^t, \psi) \in \widetilde{G}$, i.e. $\widetilde{G} \cdot (id_{\widetilde{Q}}, h) = (id_{\widetilde{Q}}, h) \cdot \widetilde{G}$. Therefore, by the Proposition 1, for the deck transformation group $\widetilde{G}$, there exists $\widetilde{h} \in Diff(M)$ such that $(id_{\widetilde{Q}}, h)$ lies over $\widetilde{h}$ respectively to $\widetilde{\kappa} : \widetilde{M} \to M$.

Taking into account that $(id_{\widehat{Q}}, h) \in A(\widetilde{M}, \widetilde{F})$, it is not difficult to check that $\widetilde{h} \in A(M, F)$. Hence, $\varepsilon(\widetilde{h} \cdot A_L(M, F)) = h$. This means that $\varepsilon : A_B(M, F) \to N(\Psi)/\Psi$ is surjective. Thus, ε is a the group isomorphism.

Since $N(\Psi)$ is a closed Lie subgroup of the automorphism Lie group $Aut(B, \eta)$, and Ψ is a discrete subgroup of $N(\Psi)$, the quotient group $N(\Psi)/\Psi$ is a Lie group. Therefore, the group isomorphism ε induces a Lie group structure in $A_B(M, F)$ such that $\varepsilon : A_B(M, F) \to N(\Psi)/\Psi$ becomes a Lie group isomorphism. According to [15, Thm. 1], the Lie group structure in $A_B(M, F)$ is unique. $\square$

7.2 Example of finding a basic automorphism group

Let $\mathbb{S}^q$ be a q-dimensional standard sphere, where $q \geq 3$. We identify $\mathbb{S}^q$ with $\mathbb{R}^q \cup \{\infty\}$, where $\{\infty\}$ is the point at infinity. Define the transformation $\psi : \mathbb{S}^q \cong \mathbb{R}^q \cup \{\infty\} \to \mathbb{S}^q$ by equality $\psi(z) = \lambda z$ for any $z \in \mathbb{S}^q \cong \mathbb{R}^q \cup \{\infty\}$, where λ is a real number, and $0 < \lambda < 1$. We denote by $Conf(\mathbb{S}^q)$ the Lie group of all conformal transformations of the sphere $\mathbb{S}^q$.

Let $\Psi = <\psi>$ be the subgroup of the group $Conf(\mathbb{S}^q)$ generated by ψ, and Ψ is isomorphic to the group of integers $\mathbb{Z}$. Define the action of the group $\mathbb{Z}$ on the product of manifolds $\mathbb{R}^1 \times \mathbb{S}^q$ by the equality $n(t, z) = (t - n, \psi^n(z))$ for any $n \in \mathbb{Z}$, $(t, z) \in \mathbb{R}^1 \times \mathbb{Z}$. This action is free and properly discontinuous. Therefore, the manifold of orbits $M = \mathbb{R}^1 \times_{\mathbb{Z}} \mathbb{S}^q$ is defined. Denote by $f : \mathbb{R}^1 \times \mathbb{S}^q \to M$ the quotient map. Fix a point $(t_0, z_0) \in \mathbb{R}^1 \times \mathbb{S}^q$, put $x_0 = f(t_0, z_0) \in M$. Then the fundamental group $\pi_1(M, x_0)$ acts on the universal covering space $\mathbb{R}^1 \times \mathbb{S}^q$ as the deck transformation group $\widetilde{G} \cong \pi_1(M, x_0)$ of f. Since the action $\widetilde{G}$ preserves the structure of the product $\mathbb{R}^1 \times \mathbb{S}^q$, then two foliations (M, F) and (M, F^t), covered by trivial fibrations $pr_2 : \mathbb{R}^1 \times \mathbb{S}^q \to \mathbb{S}^q$ and $pr_1 : \mathbb{R}^1 \times \mathbb{S}^q \to \mathbb{R}^1$ respectively, are defined. Let us denote by $\chi : \mathbb{R}^1 \to \mathbb{S}^1 = \mathbb{R}^1 / \mathbb{Z}$ and $\nu : \mathbb{S}^q \to \mathbb{S}^q / \Psi$ the quotient maps onto the orbit spaces. Let $r : M \to M/F$ be the quotient map onto the leaf space. Observations show that the topological spaces M/F and $\mathbb{S}^q / \Psi$ are homeomorphic and satisfy the commutative diagram

$$
\begin{array}{ccccc}
\mathbb{R}^1 & \xleftarrow{pr_1} & \mathbb{R}^1 \times \mathbb{S}^q & \xrightarrow{pr_2} & \mathbb{S}^q \\
\downarrow{\scriptstyle\chi} & & \downarrow{\scriptstyle f} & & \downarrow{\scriptstyle\nu} \\
\mathbb{S}^1 & \xleftarrow{p} & M & \xrightarrow{r} & \mathbb{S}^q / \Psi \cong M/F,
\end{array}
$$

where $p : M \to \mathbb{S}^1$ is the projection of the locally trivial fibration transforming the orbit $\mathbb{Z}.(t, z)$ of a point $(t, z) \in \mathbb{R}^1 \times \mathbb{S}^q$, considered as a point from M, into the orbit $\mathbb{Z}.t$ of a point $t \in \mathbb{R}^1$, considered as a point of the circle $\mathbb{S}^1$. Since the manifold M is the space of a locally trivial fibration $p : M \to \mathbb{S}^1$ over the circle $\mathbb{S}^1$ with a compact standard fiber $\mathbb{S}^q$, then M is compact.

The distribution $\mathfrak{M}$ tangent to (M, F^t), is an integrable Ehresmann connection for the foliation (M, F). The foliation (M, F) has two compact leaves L_1 and L_2 which are diffeomorphic to the circle $\mathbb{S}^1$. Every other leaf L of (M, F) is diffeomorphic to $\mathbb{R}^1$, and its closure $\overline{L}$ is equal to the union $L \cup L_1 \cup L_2$. We emphasize that (M, F) is a proper conformal foliation, which can be regarded as a Cartan foliation of type (G, H), where $G = Conf(\mathbb{S}^q)$ and H is a stationary subgroup of the group $Conf(\mathbb{S}^q)$ at some point in $\mathbb{S}^q$.

As is known, $H \cong CO(q) \ltimes \mathbb{R}^q$ is a semidirect product of a conformal group $CO(q) \cong \mathbb{R}^+ \times O(q)$ and a normal abelian subgroup $\mathbb{R}^q$. Note that Ψ is the global holonomy group of the foliation (M, F), and Ψ is a discrete subgroup of the Lie group $Conf(\mathbb{S}^q)$.

The direct check shows that the normalizer of the group Ψ is equal to $N(\Psi) = \mathbb{R}^+ \times O(q)$, and $N(\Psi)$ coincides with the centralizer $Z(\Psi)$. Applying Theorem 3, we obtain that the group of basic automorphisms $A_B(M, F)$ is a Lie group isomorphic to the quotient group $N(\Psi)/\Psi \cong U(1) \times O(q)$, where $U(1) \cong \mathbb{S}^1$. Thus, the Lie group of basic automorphisms $A_B(M, F)$ is isomorphic to the product of Lie groups $U(1) \times O(q)$.

Список литературы

[1] Y.V. Bazaikin, A.S. Galaev, N.I. Zhukova, Chaos in Cartan foliations *Chaos.* **30**:10 (2020). https://aip.scitation.org/doi/10.1063/5.0021596

[2] I.V. Belko, Affine transformations of a transversal projectable connection on a foliated manifold, *Mathematics of the USSR-Sbornik* **45**:2 (1983), 191–203.

[3] R.A. Blumenthal, J.J. Hebda, Ehresmann connections for foliations, *Indiana Univ. Math. J.* **33**:4 (1984), 597–611.

[4] H. Busemann, *The geometry of geodesics.* Academic Press, New York, 2011.

[5] A. Čap, J. Slovak, *Parabolic Geometries I: Background and General Theory,* Mathematical Surveys and Monographs **154**. AMS: Publishing House, 2009.

[6] A. Čap, A. R. Gover, and M. Hammerl, Holonomy reductions of Cartan geometries and curved orbit decompositions *Duke Math. J.* **163** (2014), 1035–1070.

[7] H. Chu, S. Kobayashi, The automorphism group of a geometric structure. *Trans. Amer. Math. Soc.* **113** (1964), 141–150.

[8] M. Crampin, D. Saunders, Cartan Geometries and their Symmetries, A Lie Algebroid Approach. *ATLANTIS Press Atlantis Studies in Variational Geometry.* **4** (2016).

[9] H. Jennen, Cartan geometry of spacetimes with a nonconstant cosmological function A. *arXiv:1406. 2621v2[gr-qc]. Physics.* REV. D 90, 084046. (2014).

[10] S. Kashiwabara, The decomposition of differential manifolds and its applications, *Tohoku Math. J.* **11**:1 (1959), 43–53.

[11] S. Kobayashi, Transformation group in differential geometry. *Springer-Verlag,* New York, 1995.

[12] J. Leslie, A remark on the group of automorphisms a foliation having a dense leaf, *J. Differ. Geom.* **7** (1972), 597–601.

[13] V. Pecastaing, Om two theorems about local automorphisms of geometric structures. *Ann. Int. Fourier,* Grenoble. **66**:1 (2016), 175–208.

[14] R.W. Sharpe, Differential Geometry: Cartan's Generalization of Klein's Erlangen Progpam. *Graduate Texts in Mathematics Springer-Verlag,* New York, **166** (1997).

[15] K.I. Sheina, N.I. Zhukova, The Groups of Basic Automorphisms of Complete Cartan Foliations. *Lobachevskii Journal of Mathematics.* **39**:2 (2018), 271–280.

[16] E.G. Skljarenko, To Hilbert's fifth problem. In the book: *Hilbert's Problems, edited by P. S. Alexandrov.* (1969), 101–115.

[17] H.F. Westman, T. G. Zlosnik, Cartan gravity, matter fields, and the gauge principle. *arXiv:1209.5358v2* [gr-qc].

[18] N.I. Zhukova, Minimal sets of Cartan foliations, *Proc. of the Steklov Inst. of Math.* **256** (2007), 105–135.

[19] N.I. Zhukova, Complete foliations with transverse rigid geometries and their basic automorphisms, *Bulletin of Peoples' Friendship University of Russia. Ser. Math. Information Sci. Phys.* **2** (2009), 14–35.

[20] N.I. Zhukova, Global attractors of complete conformal foliations, Sb. Math. **203**:3 (2012), 380–405.