Measurement of D^0-$\bar{D}^0$ mixing parameters using semileptonic decays of neutral kaon

P. Pakhlov,[1] and V. Popov[1]

[1]*P.N. Lebedev Physical Institute of the RAS, Moscow, Russia*[∗]

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Abstract

We propose a new method to extract D^0-$\bar{D}^0$ mixing parameters using the $D^0 \to \bar{K}^0 \pi^0$ decay with the $\bar{K}^0$ reconstructed in the semileptonic mode. Although a $K^0 \to \pi^{\pm} \ell^{\mp} \nu_\ell$ decay suffers from low statistics and complexity of the secondary vertex reconstruction in comparison to the standard $K_S^0 \to \pi^+ \pi^-$ vertex, it provides much richer, sometimes unique information about the initial state of a K^0-meson produced in a heavy-flavor hadron decay. In this paper it is shown that the reconstruction of the chain $D^0 \to K^0(\pi^{\pm} \ell^{\mp} \nu_\ell) \pi^0$ allows one to extract the strong phase difference between the doubly Cabibbo-suppressed and Cabibbo-favored decay amplitudes, which is of key importance for determination of the D^0-$\bar{D}^0$-mixing parameters.

[∗]Electronic address: popovve@lebedev.ru

I. INTRODUCTION

Neutral meson oscillations and CP violation provide one of the most powerful probes in searching for New Physics (NP), setting bounds on flavor structure of NP at the TeV scale and above and severely constraining possible extensions of the Standard Model (SM) [1]. While oscillations in the K^0 and $B^0_{(s)}$ systems have been well studied for a long time, the D^0-$\bar{D}^0$ mixing was established in 2007 only [2, 3]. The D^0 system is unique in circulating internal d-type quarks in the underlying mixing box diagram and thus provides complimentary information on possible NP effects. The recent observation of the direct CP violation in charm sector by LHCb attracts even more interest in this field [4].

From the theory side, the disadvantage of the D^0-$\bar{D}^0$ mixing is caused by large long-distance effects. Some progress in calculations of the mixing parameters can be expected from quickly progressing lattice computations [5–7]. Thus, one can hope that there will be reliable predictions with which experimental measurements can be compared.

The largest sensitivity in mixing measurements has been achieved from the measurements of the time-dependent decay rates in the $D^0 \to K^+\pi^-$ process, referred to as "wrong-sign" decay. However, in spite of the unprecedental high significance of the D^0-$\bar{D}^0$ mixing observation and precise measurement of its strength [8–10], the fundamental mixing parameters still need to be extracted from these measurements and compared to the SM predictions. It is sufficient to mention that the parameter x remains consistent with 0 within a 2σ uncertainty [11]. The main problem arises from the unknown strong-phase difference, δ, between doubly Cabibbo-suppressed (DCS) and Cabibbo-favored (CF) decays, that rotates the measured mixing parameters relative to their true values.

In this paper we discuss a possibility of using of $D^0 \to K^0\pi^0$ decays with K^0 reconstructed in the semileptonic mode to measure δ. While the semileptonic mode, $K^0 \to \pi^\pm\ell^\mp\nu$, is rather rare for K^0_S, it can be more informative relative to the standard $K^0_S \to \pi\pi$ decay mode. The proposed method can be applied in the already running LHCb [12] and Belle II [13] experiments as well as at the future Super c-τ factory [14, 15], where the expected high integrated luminosity compensates the smallness of $\mathcal{B}(K^0_S \to \pi^\pm\ell^\mp\nu)$.

The key idea of this paper is that D^0 decays to $|\bar{K}^0\rangle$ via CF and $|K^0\rangle$ via DCS mechanisms (Fig. 1), thus producing $a|K^0\rangle + b|\bar{K}^0\rangle$ state, where a relative complex phase between the a and b is the strong phase difference between $\bar{K}^0\pi^0$ and $K^0\pi^0$, which is equal to the sought δ due to isospin symmetry. The evolution of the $a|K^0\rangle + b|\bar{K}^0\rangle$ state to pure $|K^0\rangle$ or $|\bar{K}^0\rangle$ (that are fixed at the time of semileptonic K^0 decay by the lepton sign) provides a more powerful tool to measure a and b as will be shown below.
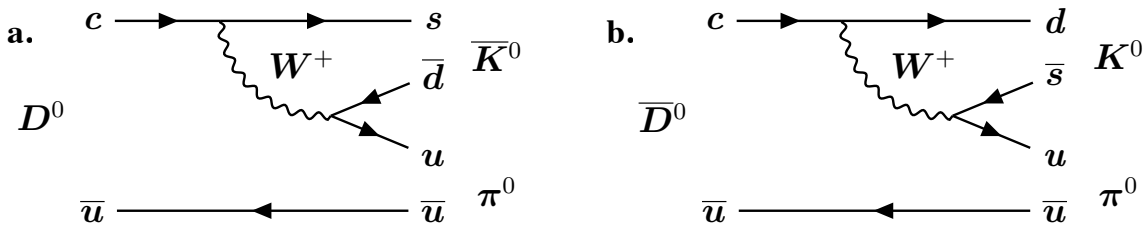

FIG. 1: Feynman diagrams for **a** CF and **b** DCS $D^0 \to \overset{(-)}{K}{}^0\pi^0$ decays.

II. MIXING PARAMETERS IN THE D^0 SYSTEM

We briefly remind the basic mixing formalism. Solving the Schrödinger equation in the effective Hamiltonian approach one could obtain the following evolution equations for physical states produced as pure $D^0 (\bar{D}^0)$:

$$|D^0_{phys}(t)\rangle = g_+(t)|D^0\rangle - \left(\frac{q}{p}\right)_D g_-(t)|\bar{D}^0\rangle,$$

$$|\bar{D}^0_{phys}(t)\rangle = g_+(t)|\bar{D}^0\rangle - \left(\frac{p}{q}\right)_D g_-(t)|D^0\rangle, \tag{1}$$

where $g_\pm = \frac{1}{2}\left(e^{-i\lambda_2 t} \pm e^{-i\lambda_1 t}\right)$ for eigenvalues $\lambda_{1,2} = m_{1,2} - \frac{i}{2}\Gamma_{1,2}$. It is conventional to describe evolution with dimensionless mixing parameters x, y:

$$x \equiv \frac{\Delta m}{\Gamma}, \qquad y \equiv \frac{\Delta\Gamma}{2\Gamma}, \tag{2}$$

where $\Gamma = (\Gamma_1 + \Gamma_2)/2$.

The main problem in x, y determination arises because of the contribution of DCS decay in addition to the mixing to the "wrong sign" final state. While the mixing contribution can be disentangled from the DCS one by D^0 decay time study [16]:

$$R^+(t) = \left(r_D + \left|\left(\frac{q}{p}\right)_D\right|\sqrt{r_D}\Gamma t(y'\cos\phi_D - x'\sin\phi_D) + \left|\left(\frac{q}{p}\right)_D\right|^2 \frac{(\Gamma t)^2}{4}(x^2 + y^2)\right)e^{-\Gamma t},$$

$$R^-(t) = \left(\bar{r}_D + \left|\left(\frac{p}{q}\right)_D\right|\sqrt{\bar{r}_D}\Gamma t(y'\cos\phi_D + x'\sin\phi_D) + \left|\left(\frac{p}{q}\right)_D\right|^2 \frac{(\Gamma t)^2}{4}(x^2 + y^2)\right)e^{-\Gamma t}, \tag{3}$$

the measured parameters remain biased, as in the measured time-dependent decay rates x, y enter as linear combinations,

$$\begin{aligned} x' &= x\cos\delta_{K\pi} + y\sin\delta_{K\pi}, \\ y' &= y\cos\delta_{K\pi} - x\sin\delta_{K\pi}, \end{aligned} \tag{4}$$

where $\delta_{K\pi}$ is a strong phase difference between the CF and DCS D decays. In Eq. 3 ϕ_D is the mixing weak phase (with a high degree of accuracy equal to 0 in SM) and $r_D, \bar{r}_D$ are given by

$$r_D = \left|\frac{\langle \bar{K}\pi|\mathcal{H}|D^0\rangle}{\langle K\pi|\mathcal{H}|D^0\rangle}\right|^2, \qquad \bar{r}_D = \left|\frac{\langle K\pi|\mathcal{H}|\bar{D}^0\rangle}{\langle \bar{K}\pi|\mathcal{H}|\bar{D}^0\rangle}\right|^2. \tag{5}$$

These ratios are proportional to $|V_{cd}V^*_{us}/V_{cd}V^*_{ud}|^2 \sim \mathcal{O}(\tan^4\theta_c)$.

To obtain the true values of the mixing parameters x, y, knowledge of amplitude ratios r_D and strong phase difference $\delta_{K\pi}$ is crucial. While $|r_D|$ can be determined from the fit to the t-dependent rate (Eq. 3), it is not possible to extract the exact value of the strong phase without supplemental studies. One method is to measure $\delta_{K\pi}$ using decays of coherent $D^0 - \bar{D}^0$ pairs produced from the $\psi(3770)$ decay. This way $\cos\delta$ could be extracted from the interference between the $K^-\pi^+$ and CP eigenstate. The CP eigenstate tags the $K^-\pi^+$ to be the eigenstate with an opposite eigenvalue which represents a linear combination of D^0 and $\bar{D}^0$. The resulting decay rate for the second D-meson is modulated by the relative

strong phase between $D^0 \to K^-\pi^+$ and $\bar{D}^0 \to K^-\pi^+$. Up to now the only measurement of $\sin\delta$ has been performed by the CLEO-c collaboration using $D \to K_S^0\pi^+\pi^-$ as a tagging decay [17]. However, this measurement suffers from the overall sign ambiguity that could not be resolved without external inputs and also leads to non-Gaussian uncertainties of δ.

The method proposed in this paper allows one to measure the strong phase difference between DCS and CF decays with better accuracy and both $\cos\delta$ and $\sin\delta$ are measured simultaneously, thus achieving uniform sensitivity in the whole δ range and resolving the ambiguity.

III. $\mathbf{K^0}$ EVOLUTION

The time evolution of the K^0–$\bar{K}^0$ system is described by the Schrödinger equation:

$$i\partial_t \begin{pmatrix} K^0(t) \\ \bar{K}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma} \right) \begin{pmatrix} K^0(t) \\ \bar{K}^0(t) \end{pmatrix}, \tag{6}$$

where the $\mathbf{M}$ and $\mathbf{\Gamma}$ matrices are Hermitian, and CPT invariance requires $M_{11} = M_{22} \equiv M$ and $\Gamma_{11} = \Gamma_{22} \equiv \Gamma$.

The Hamiltonian eigenvalues could be written as follows:

$$m_{1,2} - i\frac{\Gamma_{1,2}}{2} = \left(M_{11} - i\frac{\Gamma_{11}}{2} \right) \pm \left(\frac{p}{q} \right)_K \left(M_{12} - i\frac{\Gamma_{12}}{2} \right), \tag{7}$$

where $m_{1,2}$ are masses, $\Gamma_{1,2}$ are widths of the Hamiltonian eigenstates and parameters p, q which correspond to the flavor admixtures of flavor states are defined by

$$\left(\frac{p}{q} \right)_K^2 = \frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}. \tag{8}$$

For K^0 from the studied decays the following boundary conditions are met:

$$\begin{pmatrix} K^0(t) \\ \bar{K}^0(t) \end{pmatrix}\bigg|_{t=0} = \begin{pmatrix} a \\ b \end{pmatrix}, \tag{9}$$

where $a = 1$, $b = \sqrt{r_D}e^{i\delta}$ ignoring D^0-$\bar{D}^0$ mixing which introduces just a tiny bias.

From Eq. (6) one can obtain the time evolution of the K^0-meson produced as a linear combination $a|K^0\rangle + b|\bar{K}^0\rangle$ into K^0 and $\bar{K}^0$:

$$|K^0(t)\rangle = \frac{1}{2}e^{-imt}e^{-\frac{1}{2}\Gamma t} \left[a\left(1 + e^{-i\Delta mt}e^{\frac{1}{2}\Delta\Gamma t} \right) + b\left(\frac{p}{q} \right)_K \left(1 - e^{-i\Delta mt}e^{\frac{1}{2}\Delta\Gamma t} \right) \right],$$

$$|\bar{K}^0(t)\rangle = \frac{1}{2}e^{-imt}e^{-\frac{1}{2}\Gamma t} \left[b\left(1 + e^{-i\Delta mt}e^{\frac{1}{2}\Delta\Gamma t} \right) + a\left(\frac{q}{p} \right)_K \left(1 - e^{-i\Delta mt}e^{\frac{1}{2}\Delta\Gamma t} \right) \right], \tag{10}$$

where m and Γ are K_S^0 mass and width respectively, $\Delta m = m_1 - m_2 > 0$, and $\Delta\Gamma = \Gamma_1 - \Gamma_2$. In this paper we consider only semileptonic final states and denote corresponding decay amplitudes as

$$A_{\ell^+} = \langle \pi^-\ell^+\nu|\mathcal{H}|K^0\rangle, \qquad A_{\ell^-} = \langle \pi^+\ell^-\bar{\nu}|\mathcal{H}|\bar{K}^0\rangle \tag{11}$$

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The corresponding decay rates could be expressed as

$$N_{\ell+}(t) = \frac{1}{4}e^{-\Gamma t}|A_{\ell+}|^2 \left[|a|^2 K_+(t) + \left| b\left(\frac{p}{q}\right)_K \right|^2 K_-(t) + 2Re\left\{ ab\left(\frac{p}{q}\right)_K K_i(t) \right\} \right],$$

$$N_{\ell-}(t) = \frac{1}{4}e^{-\Gamma t}|A_{\ell-}|^2 \left[|a|^2 K_-(t) + \left| b\left(\frac{q}{p}\right)_K \right|^2 K_+(t) + 2Re\left\{ ab\left(\frac{q}{p}\right)_K K_i(t) \right\} \right], \quad (12)$$

where $K_{\pm,i}(t)$ are defined as

$$K_\pm(t) = 1 \pm 2e^{\frac{1}{2}\Delta\Gamma t}\cos(\Delta m t) + e^{\Delta\Gamma t}, \quad K_i(t) = 1 + 2ie^{\frac{1}{2}\Delta\Gamma t}\sin(\Delta m t) - e^{\Delta\Gamma t}. \quad (13)$$

The strong-phase difference δ enters the last term of each rate, and thus can be extracted from the measured $N_{\ell\pm}(t)$. To illustrate effects induced by δ better, we form an "asymmetry" from these decay rates. Assuming that we have successfully tagged the flavor of D^0 in its decay time (e. g. by a charge of a slow pion from the $D^{*+} \to D^0\pi^+$) decay, we are able to use the lepton charge to form the asymmetry in the following way:

$$\mathcal{A}(t) \equiv \frac{N_{\ell+}(t) - N_{\ell-}(t)}{N_{\ell+}(t) + N_{\ell-}(t)}. \quad (14)$$

These asymmetries with different parameters a and b are shown in Fig. 2. From this figure it can be seen that the most sensitive interval for the strong-phase δ lies in the range $\sim [0.5, 7]$ K_S^0 lifetimes.

IV. FEASIBILITY STUDY

In this section we test the proposed method, obtain its potential efficiency and compare the accuracy with results from the CLEO-c [17] and BESIII [18] experiments. This method could be applied in the LHCb and Belle II experiments or at the future c-τ factory.

A. Reconstruction

In order to perform time-dependent analysis, proper kaon vertex and momentum reconstruction is required. Because of the missing neutrino in the final state the direct kaon-momentum reconstruction is not possible. However, the kaon flight direction can be obtained from the primary and secondary vertices, while the momentum magnitude can be extracted from the measured pion and lepton momenta relying on the four-momentum conservation ($(P_K - P_{\pi\ell})^2 = P_\nu^2 = 0$) by constructing the following equation:

$$m_K^2 - 2E_{\pi\ell}\sqrt{\mathbf{p}_K^2 + m_K^2} + 2|\mathbf{p}_K||\mathbf{p}_{\pi\ell}|\cos\theta + m_{\pi\ell}^2 = 0, \quad (15)$$

where $\mathbf{p}_K$, m_K are kaon three-momentum and mass, $E_{\pi\ell}$, $\mathbf{p}_{\pi\ell}$, $m_{\pi\ell}$ are energy, three-momentum and mass of the reconstructed pion and lepton combination, and θ is the angle between the K^0 direction obtained from the vertex information and measured momentum of the $\pi\ell$ combination. The magnitude of the kaon momentum, $|\mathbf{p}_K|$, is the only unknown,
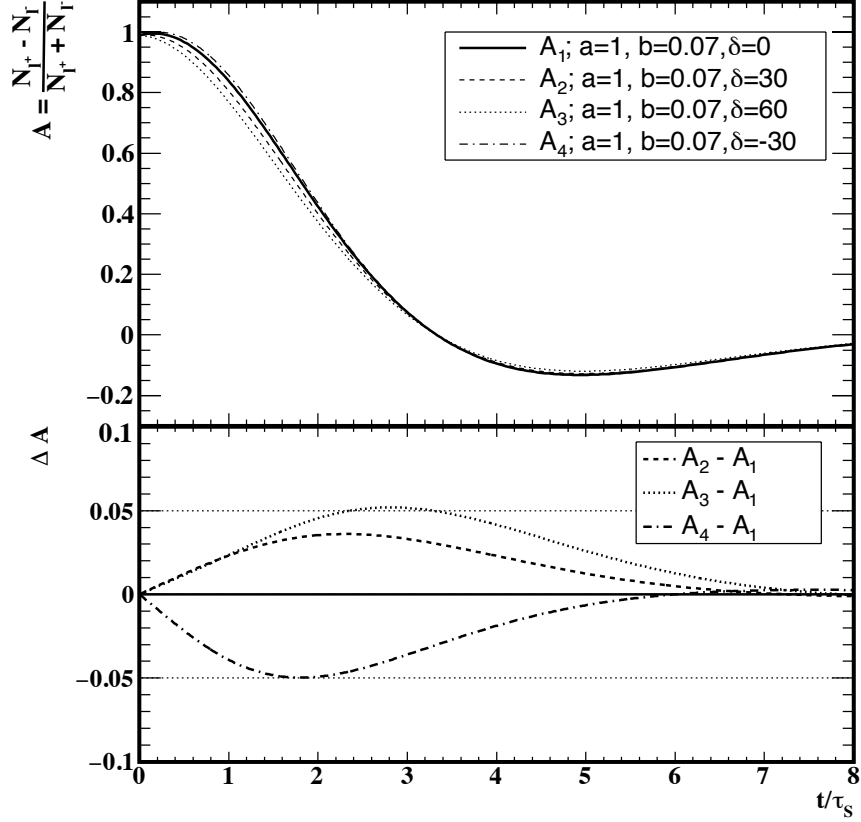
FIG. 2: *(top)* Lines show the lepton charge asymmetry for different initial admixtures of $a|K^0\rangle + be^{i\delta}|\bar{K}^0\rangle$. The solid line corresponds to the case of exact $SU(3)$ symmetry and so $\delta = 0$. The dashed lines illustrate the obtained asymmetry for the values of the strong phase 30°, 90°, 180°; *(bottom)* Here the dashed lines show the differences between asymmetries and $\delta = 0$ case.

while all other terms in the equation are measured. Solving the quadratic equation (16) one can obtain two solutions for the kaon momentum

$$|\mathbf{p}_K|_{(1,2)} = \frac{|p_{\pi\ell}|\cos\theta(m_K^2 + m_{\pi l}^2) \pm \sqrt{w}}{2(E_{\pi\ell}^2 - p_{\pi l}^2\cos^2\theta)}, \tag{16}$$

where w is given by

$$w = E_{\pi\ell}^2 \left(4m_K^2 p_{\pi\ell}^2\cos^2\theta - 4E_{\pi\ell}^2 m_K^2 + m_{\pi l}^2(m_K^2 + m_{\pi l}^2)\right). \tag{17}$$

B. Feasibility

While the method can be applied in all high-luminosity experiments, in this section we consider only Belle II. At LHCb a measurement in the channel $D^0 \to K^0\pi^0$ is somewhat problematic due to huge background in the neutral mode, however, one could select the kinematic region in $D^0 \to K^0\pi^0$ decay at the expense of losing statistics, where π^0 is energetic while K^0 is relatively soft. Both factors are favorable for this study, as a requirement of energetic π^0 results in much smaller background, while soft K^0 has a smaller boost, thus

its decay length is smaller, which increases the range of the accepted K^0 proper time in the relatively short LHCb tracking system. For estimates of the LHCb potential the toy Monte Carlo is not sufficient. For the charm-tau factory a high data sample for $D^{*+} \to D^0 \pi^+$ is anticipated, as part of the data will be taken above the $D^{*\pm} D^{\mp}$ threshold. However, a huge data sample is expected at the $\psi(3770)$ resonance. If tagging is performed using semileptonic decays of the second D-meson in the event, the situation is equivalent to the D^{*+} tagging. The interesting effects arise in case of hadronic tagging due to quantum correlations. A brief discussion of this case and its features could be found in Section VI.

Good tracking performance provides a sufficient vertex resolution ($\sim 100\,\mu$m) at Belle II, that results in the angular resolution of ~ 2 mrad for a kaon with $t \gtrsim \tau_{K_S}$ and typical $\beta\gamma \sim 2$. In $\sim 35\%$ cases the discriminant is negative due to detector smearing; setting $w \equiv 0$ in this case does not lead to degradation of the $|\mathbf{p}_K|$ resolution but eliminates the ambiguity. In $\sim 30\%$ of positive w, when two-fold ambiguity arises, only one solution is physical, while the second one could be rejected because of the negative value obtained for the magnitude of the kaon momentum or the D^0 momentum exceeding a kinematic limit. In the remaining cases the correct solution can be selected by choosing those giving the D^0 mass closer to the expected one.
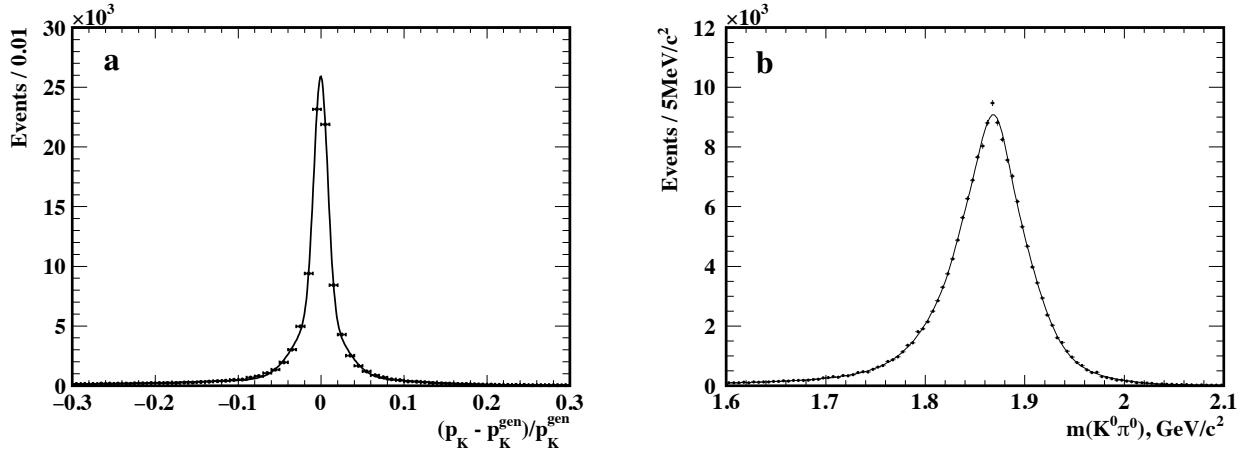

FIG. 3: (**a**) Relative K^0 momentum resolution, (**b**) distribution of the $K^0\pi^0$ invariant mass for selected candidates.

For the selected candidates the resolution of K^0 momenta is shown in Fig. 3 **a** as estimated from the toy Monte Carlo (MC) simulation with the actual detector tracking performance [13]. The resolution is estimated to be $\sigma_{p_K}/p_K \sim 0.02$. The $D^0 \to K^0\pi^0$ mass resolution ($\sim 40\,\text{MeV}/c^2$) is dominated by π^0, rather than K^0 momentum resolution (Fig. 3 **b**).

It is expected that backgrounds can be suppressed to a level at which they are dominated by combinations of real $K^0 \to \pi^{\pm}\ell^{\mp}\nu$ with π^0. Rejecting $\pi^{\pm}$ or $\ell^{\mp}$ coming from the primary vertex, suppresses fake secondary vertices formed by random intersection of primary tracks. While this requirement suppresses a signal with a short kaon lifetime, the affected part of the signal is not interesting for our study. The true secondary vertices can be produced by such kaon decays as $K_S^0 \to \pi^+\pi^-$, $K_L^0 \to \pi^+\pi^-\pi^0$, $K^+ \to \pi^+\pi^+\pi^-$, or by secondary interactions. $K_S^0 \to \pi^+\pi^-$ is initially a huge background source that exceeds the signal by two orders of magnitude. However, it can be efficiently vetoed by requiring the mass of the $\pi^{\pm}\ell^{\mp}$ combination in the "pion" mass hypothesis (when the $\pi^{\pm}$ mass is ascribed to both tracks from the secondary vertex in a calculation of the mass of the combination) to be
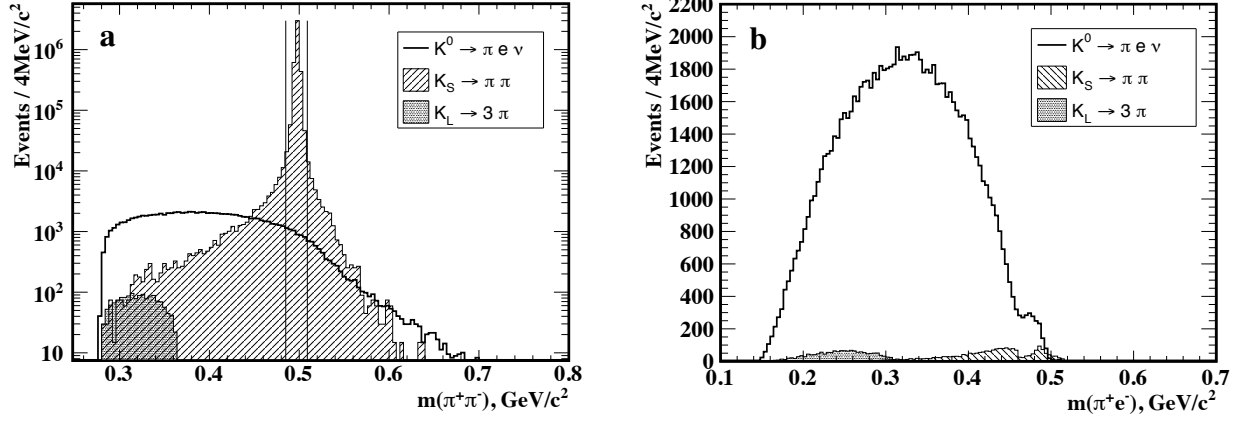
FIG. 4: (**a**) Distribution of the $\pi^+\pi^-$ invariant mass. The contributions from three processes are drawn. Here the pion mass is also assigned to an electron from $K^0 \to \pi e \nu_e$ decay. The vertical lines show the proposed K_S^0 veto. Applying this veto one can obtain the $e\pi$ invariant mass distribution shown in the right (**b**) where hatched histograms represent the background contribution.

outside the nominal K_S^0 mass window. Figure 4 **a** shows the secondary vertex mass in the "pion" mass hypothesis for the signal and different backgrounds, where a huge $K_S^0 \to \pi^+\pi^-$ signal can be easily rejected at the expense of the $\sim (2-3)\%$ loss of the signal efficiency. After the K_S^0 veto all real strange vertices becomes smaller than the signal. The $\pi^\pm \ell^\mp$ mass spectrum remaining after the K_S^0 veto is shown in Fig. 4 **b**. Lepton identification suppresses the remaining non-$K^0 \to \pi^\pm \ell^\mp \nu$ background down to a negligible level, taking into account that the typical misidentification rate is $(0.5 - 2)\%$.
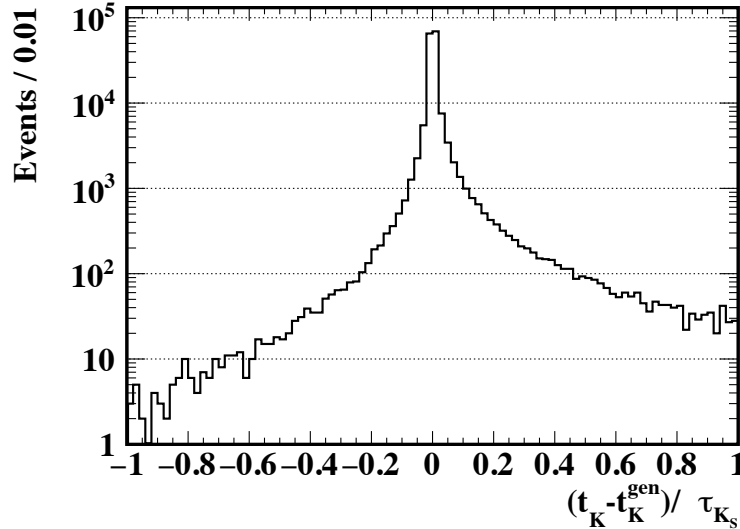

FIG. 5: Resulting kaon lifetime resolution.

To test potential accuracy of the proposed method and to confirm that there is no bias, we generate 200 MC samples of 10^5 events, each with a value of the angle δ in the $[-90°, 90°]$ interval with a step of $10°$. An estimation of the approximate number of events is based on

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a full data sample that will be collected in the Belle II experiment (50ab^{-1}), corresponding branching fractions (10^{-5}) and reconstruction efficiency ($\sim 20\%$). In order to extract δ we perform a simultaneous fit to both "right-" and "wrong-sign" histograms. We limit the fit range to the [0.5, 8] of kaon lifetimes where the highest sensitivity could be achieved.

Each pair of histograms is fitted 20 times with different initial values of the δ parameter to ensure that a fit is converged to a global rather than a local minimum independently of the starting value. The value obtained with the best χ^2 is chosen. Figure 6 **a** illustrates the fit results for a MC sample with $\delta = 20°$ which is close to the central value published by PDG [19], and Fig. 6 **b** shows the asymmetry with the fitted asymmetry superimposed. The results for the whole range of generated δ values is illustrated in Fig. 7 **a**.
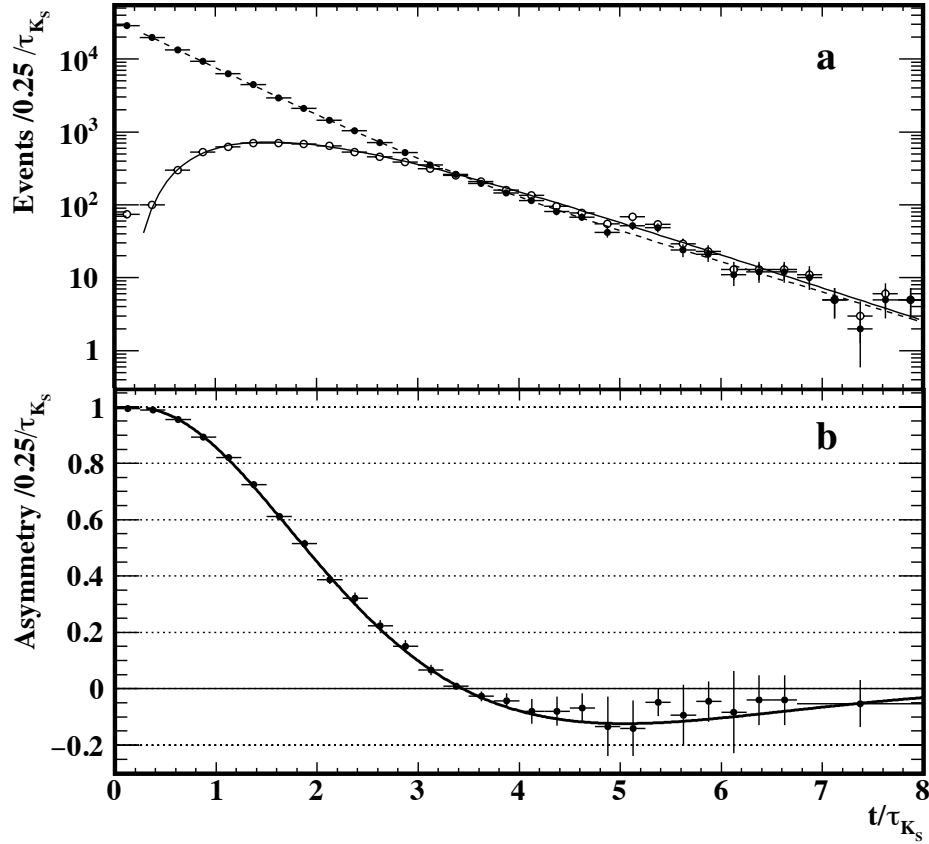

FIG. 6: (**a**) "Toy" MC generated time-dependent decay rates. The histogram with filled black circles corresponds to "right sign" and with open circles correspond to the "wrong sign" decay rates. The solid and dashed lines correspond to the result of the simultaneous fit. (**b**) The resulting decay rate asymmetry with the fit result superimposed.

The resulting distribution for the uncertainty is shown in Fig. 7 **b**. In the region of small values of the strong phase (which are consistent with the previous measurements of CLEO-c and BESIII) the obtained uncertainty basically is under $4°$. This fact makes the proposed method comparable to the measurement at BESIII with a $20\,\text{fb}^{-1}$ data sample at the $\psi(3770)$ proposed in Ref. [20].

The procedure mentioned was performed with 200 MC samples. The resulting distribution of obtained values for the strong phase was fitted and the Gaussian mean and standard deviation were extracted. As an example, we present here the obtained distribution for
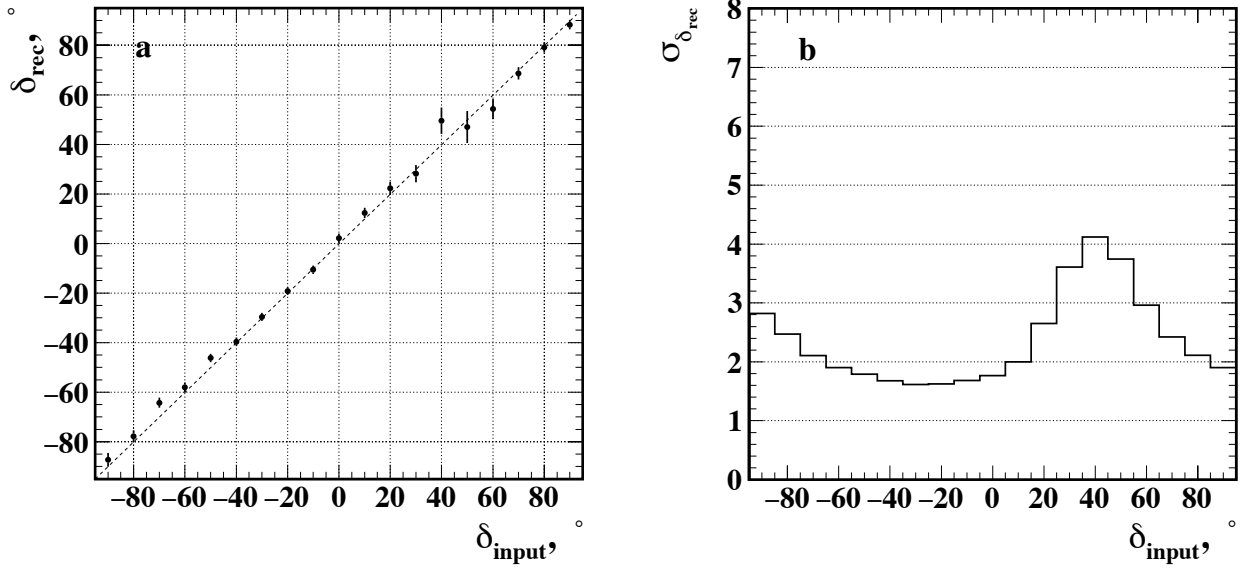
FIG. 7: (**a**) Resulting δ values extracted from the fit with the obtained uncertainty (**b**).

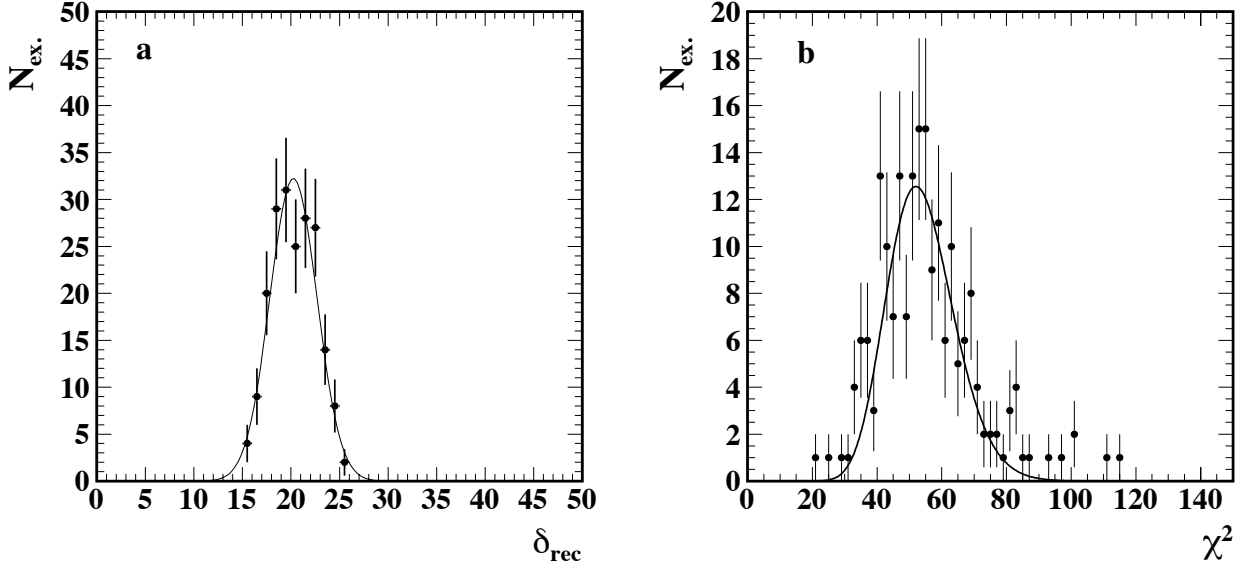$\delta = 20°$. The extracted mean and standard deviation are consistent with results of individual fits.

FIG. 8: (**a**) Distribution of δ values obtained in 200 pseudoexperiments and (**b**) distribution of chi-square values. The solid lines are fit results with Gaussian and chi-square distribution functions, respectively.

V. CONCLUSION

In this paper we have performed a phenomenological study of the evolution of the neutral kaons produced in the decay $D^0 \to \bar{K}^0 \pi^0$. Usage of a semileptonic final state allows us to

tag the kaon-flavor final state, hence providing sensitivity to a strong-phase measurement. As it was shown in Section III, both $\sin\delta$ and $\cos\delta$ contribute to the resulting decay rate. This way it is possible to measure δ without trigonometrical ambiguity. It was shown that the effect induced by the strong phase in the decay rate asymmetry depends on the δ value. Fortunately, the interval most sensitive to the measurement allows us to effectively cut off the background coming from the interaction region. Using MC simulation background produced by true secondary vertexes has been studied. In Section IV it was shown that these types of background could be suppressed down to a negligible level.

To test the proposed method, a feasibility study was performed. The full data sample of $50ab^{-1}$ that will be collected in the Belle II experiment was used as an approximate amount of data. The feasibility study showed that potential accuracy of the proposed method is comparable to the classic method based on the $\psi(3770) \to D^0\bar{D}^0$ study [20].

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VI.   APPENDIX A: CORRELATED $D^0\bar{D}^0$ PAIR CASE

For the future c-τ factory it seems more reasonable to use correlated D-mesons from the $\psi(3770) \to D^0\bar{D}^0$ decay, since the c-τ factory will be able to produce $\sim 2 \times 10^9$ $\psi(3770)$ mesons per year [14, 15]. The pair of D-mesons is produced in the 1^{--} state and is described by the following wave function

$$\Psi_{D\bar{D}} = \frac{1}{\sqrt{2}}[|D^0_{phys}(t)\rangle|\bar{D}^0_{phys}(t)\rangle - |\bar{D}^0_{phys}(t)\rangle|D^0_{phys}(t)\rangle], \qquad (18)$$

where $|D^0_{phys}(t)\rangle$, $|\bar{D}^0_{phys}(t)\rangle$ describe time evolution of initially pure states defined in (1). The time-dependent decay rate for the given system could be expressed as [21]

$$R(f_1,t_1,f_2,t_2) \propto |A_{f_1}|^2|A_{f_2}|^2 e^{-\Gamma(t_1+t_2)}\left[\frac{1}{2}|\xi+\zeta|^2 e^{-\Delta\Gamma/2(t_2-t_1)} + \frac{1}{2}|\xi-\zeta|^2 e^{\Delta\Gamma/2(t_2-t_1)} - \right.$$
$$\left. - (|\xi|^2 - |\zeta|^2)\cos(\Delta m(t_2-t_1)) + 2Im(\xi^*\zeta)\sin(\Delta m(t_2-t_1))\right], \quad (19)$$

where A_{f_1}, A_{f_2} – amplitudes of the D-meson decay to the final states f_1 and f_2, respevtively, t_1 and t_2 – proper decay times for D-mesons, Δm and $\Delta\Gamma$ – mass and width difference for states D_1 and D_2, and parameters ξ and ζ are given by

$$\xi = \left(\frac{p}{q}\right)_D - \left(\frac{q}{p}\right)_D \frac{\overline{A}_{f_1}}{A_{f_1}}\frac{\overline{A}_{f_2}}{A_{f_2}}, \qquad (20)$$

$$\zeta = \frac{\overline{A}_{f_2}}{A_{f_2}} - \frac{\overline{A}_{f_1}}{A_{f_1}}, \qquad (21)$$

where
$$A_{f_i} = \langle f_i|H|D^0\rangle, \quad \overline{A}_{f_i} = \langle f_i|H|\bar{D}^0\rangle. \tag{22}$$

Eq. 19 could be also written in the following way:

$$R(f_1, t_1, f_2, t_2) \propto |A_{f_1}|^2|A_{f_2}|^2 e^{-\Gamma(t_1+t_2)} \Big[(|\xi|^2 + |\zeta|^2) \cosh{(\Delta\Gamma\Delta t/2)} +$$

$$+ 2Re(\xi\zeta)\sinh{(\Delta\Gamma\Delta t/2)} - (|\xi|^2 - |\zeta|^2)\cos{(\Delta m\Delta t)} + 2Im(\xi^*\zeta)\sin{(\Delta m\Delta t)}\Big], \tag{23}$$

If we assume here that $\Delta\Gamma \ll 1$ and $\Delta m \ll 1$, the decay rate could be expressed in terms of x and y as follows:

$$R(f_1, t_1, f_2, t_2) \propto |A_{f_1}|^2|A_{f_2}|^2 e^{-\Gamma(t_1+t_2)} \Big[2|\zeta|^2 + \frac{1}{2}|\xi|^2 (\Gamma\Delta t)^2 (x^2 + y^2) +$$

$$+ \frac{1}{2}|\zeta|^2 (\Gamma\Delta t)^2 (y^2 - x^2) + 2\Gamma\Delta t \{yRe(\xi\zeta) + xIm(\xi^*\zeta)\}\Big], \tag{24}$$

An extensive study of different combinations of D-meson decay can be found in Ref. [22]. However, the decay modes like $D \to K^0\pi^0$ were not considered there. Here we consider the following combinations of the $\{K^-\pi^+; \bar{K}^0\pi^0\}$ and $\{X\ell^-\nu; K^0\pi^0\}$ final states:

(1) The final state $\{K^-\pi^+; \bar{K}^0\pi^0\}$ is particularly interesting because both strong phases δ^0 and δ^- are present in the resulting amplitude. Parameters ξ and ζ will be given by

$$\xi = \left(\frac{p}{q}\right)_D - \left(\frac{q}{p}\right)_D \sqrt{r_D^0}\sqrt{r_D^-}e^{i(\delta^- + \delta^0)}, \tag{25}$$

$$\zeta = \sqrt{r_D^0}e^{i\delta^0} - \sqrt{r_D^-e^{i\delta^-}}, \tag{26}$$

And the expression for the decay rate is

$$R(t_1, t_2) \propto 2|A_{K^-\pi^+}|^2|A_{\bar{K}^0\pi^0}|^2 e^{-\Gamma(t_1+t_2)} \Bigg[\left(r_D^0 + r_D^- - 2\sqrt{r_D^0 r_D^-}\cos{(\delta^0 - \delta^-)}\right) +$$

$$+ \left(\left|\frac{p}{q}\right|_D^2 + \left|\frac{q}{p}\right|_D^2 r_D^0 r_D^- - 2\sqrt{r_D^0}\sqrt{r_D^-}\cos{(\delta^0 + \delta^-)}\right) \frac{(\Gamma\Delta t)^2}{4}(x^2 + y^2) +$$

$$+ \Gamma\Delta t\Delta_{12} \Bigg], \tag{27}$$

where we have neglected the term $\propto (y^2 - x^2)$ and defined a new variable

$$\Delta_{12} = \sqrt{r_D^-}\left(\left(\frac{p}{q}\right)_D x' + \left(\frac{q}{p}\right)_D r_D^0 y'\right) - \sqrt{r_D^0}\left(\left(\frac{p}{q}\right)_D x'' + \left(\frac{p}{q}\right)_D r_D^- y''\right). \tag{28}$$

Comparing the given result to the amplitude of the final state $\{K^-\pi^+; K^-\pi^+\}$ which is accessible only through mixing, one can notice that a new component not proportional to mixing arises. Relying on the isospin $SU(2)$ symmetry we assumed previously that $\delta^0 \simeq \delta^-$. The amplitude ratios r_D^0, r_D^-, however, could not be assumed equal even in the strict $SU(2)$ limit. A difference in internal and external W-boson emission as well as color suppression implies that $r_D^0 \neq r_D^-$. The latter guarantees that the resulting decay rate will have a non-vanishing component independent of $D^0 - \bar{D}^0$ mixing.

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(2) A semileptonic final state for one of the D-mesons implies that $\overline{A}_{f_1}/A_{f_1} = 0$. In such a case we could expect the amplitude to be similar to the case $\{X\ell^-\nu; K^+\pi^-\}$ with substitution $r_D^- \to r_D^0$ and $\delta^- \to \delta^0$. In this case one can obtain

$$\xi = \left(\frac{p}{q}\right)_D,$$
(29)

$$\zeta = \sqrt{r_D^0}\, e^{i\delta^-}$$
(30)

$$R(t_1, t_2) \propto 2|A_{Xl\nu}|^2 |A_{\bar{K}^0\pi^0}|^2 e^{-\Gamma(t_1+t_2)} \left[r_D^0 + \left|\frac{p}{q}\right|_D^2 \frac{(\Gamma\Delta t)^2}{4}(x^2 + y^2) + \right.$$

$$\left. + \left(\frac{p}{q}\right)_D \Gamma\Delta t \sqrt{r_D^0}\, y'' \right], \quad (31)$$