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Modelling the Economic Policy of a Tax Shield in the Context of Overcoming Crises in the Economy

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ABSTRACT

The tax shield is defined as the amount of possible payments, which in turn can increase or decrease by the difference in planned changes and market fluctuations in the supply of products. The main purpose of the study is to outline the main aspects of the approach to assessing the tax burden on capital investments. A leading method of investigating this problem is the method of uncertain coefficients, which was used to obtain difference equations of high accuracy. The novelty of the research is determined by the fact that the tax shield and its tools are determined according to price fluctuations on the main products of the enterprise. For a formal expression of the influence of incentives to invest an indicator called "tax shield" was developed. It has been revealed that at the theoretical and practical levels, the impact of tax mechanisms on the development of the national economy is assessed. The practical significance of the research is determined by the fact that for the first time such tools as a stagnating economic situation are considered. This will allow predicting tax revenues and adjusting the growth of the economy in the conditions of crisis.

Keywords: Tax Regulation, Economic Activity, Investment, Tax Rates, Crisis Conditions

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1. INTRODUCTION

Despite the fact that the study of the effect of taxation on economic growth and, in particular, on investment activity, acquires the status of a field with extremely active scientific research, an amount of empirical research accumulated so far has been insignificant (moreover, its results do not allow to draw firm conclusions). Thus, certain approaches to calculating the tax burden provide for the application of generalizing indicators of added value or newly created value, to which the level of tax burden is brought (Kruschwitz and Löffler, 2018). The specified general indicators include individual elements of sources of tax payment and, at the same time, strictly targeted financial resources — the wage fund and depreciation resource. Such indicators can be used for statistical comparisons, but are ineffective from the standpoint of assessing the tax burden on investments. For a formal expression of the influence of incentives to invest created by the tax system, including tax rates on personal and entrepreneurial income, deduction of depreciation expenses from the tax base, investment tax credits and discounts, and other elements of taxes and tax incentives, an indicator called "tax shield" was developed (Arnold *et al*., 2018).

Despite the objective necessity of introducing indicators to calculate values of the economic condition of the state, the “tax shield” value has not been given proper attention: it is neither a subject matter of theoretical and empirical research, nor is it used by government and statistics authorities to assess the level of investment attractiveness of the national economy or individual industries and activities (Valaskova and Bakes, 2018).

In accordance with the basic principles of modern economic theory, investment is the main source of economic growth (Evers *et al.*, 2015). Therefore, increasing the level of investment attractiveness is included in the list of leading objectives of economic policy, which is of particular importance in the context of the implementation of the national sustainable development course (Menichini, 2017). The level of investment attractiveness is determined by many factors, including the tax factor (Zhang, 2009). To indicate the level of tax impact on all types of economic and business activity, as well as to analyse the possible economic consequences of introducing amendments to tax legislation, a concept of “tax burden” is employed, which, in particular, is widely used in the economic literature upon assessment of tax incentives to invest (Streitferdt, 2010). The matters of taxation, including the issues of tax incentives for investment activity, occupy a leading place in modern research of foreign and Russian scientists and a significant contribution to the study of the theoretical foundations of the main directions of tax regulation improvement was made by leading scientists (Baldenius and Ziv, 2003). It has been revealed that at the theoretical and practical levels, the impact of tax mechanisms on the development of the national economy is assessed (Krause and Lahmann, 2016; Mishchenko *et al.*, 2016; Atanelishvili and Silagadze, 2018; Silagadze, 2019).

An analysis of the potential for implementing supranational anti-crisis tax regulation in the WTO is being performed (Dothan, 2013). The subject matter of research is, in particular, the conditions for the provision of tax benefits (Schäfer, 2018). The indicators of fiscal and economic efficiency of providing tax benefits are researched, indicating their contradictory nature as an instrument of state regulation of the economy (Zhang, 2013). A theoretical justification of the main forms of fiscal policy is carried out (Bogner *et al.,* 2004). Budgetary tax leverage over economic development is also in the center of scholars’ and practitioners’ attention (Maßbaum and Sureth, 2009). The state and dynamics of capital of enterprises is researched, factoring in the effect of fiscal leverage on these processes (Couch *et al.,* 2012). The role of public investment in solving infrastructure issues of economic development is determined (Scholze, 2010). Upon research of the effects of taxes, special attention is given to the issues of tax pressure on current assets and financial resources of enterprises (Dwenger and Steiner, 2014).

Therefore, the purpose of this scientific article is to outline the main aspects of the approach to assessing the tax burden on capital investments on the basis of effective marginal tax rates and balancing the tax burden amid a crisis.

2. MATERIALS AND METHODS

To reduce the number of difference equations while maintaining the required accuracy of the results, it is prudent to use approximations that factor in a greater number of expansion terms of the desired solution in the Taylor series (Shaviro, 2008). The coefficients of such approximations are found by the method of indefinite coefficients (Qi *et al.,* 2012). Let us zero in on the features of the use of discrete time (Grier and Zychowicz, 1992). To replace a differential equation by a difference equation, two steps must be taken: 1) replace the area of continuous variation of the argument with the area of its discrete variation; 2) replace the differential operators with difference operators.

Thus, for a differential equation

$$dy / dt = f(y, t) \tag{1}$$

A simple difference operator obtained from the definition of the derivative is used

$$(dy / dt)_{t_j} = (y_{m+1} - y_m) / h \tag{2}$$

$$h = t_{m=1} - t_m \tag{3}$$

where h is a time step. The difference approximation (2) is the simplest, but at the same time the least accurate, because it searches for information at the point $m = 1$ only on the basis of information at the point m. For application of more accurate operators on the basis of the method of indefinite coefficients, difference equations of increased accuracy were obtained. The most general way of constructing finite-difference equations is that not each derivative in particular is approximated by the corresponding difference relation, but the entire differential operator. A given set of nodes is made up of a finite-difference equation that approximates the differential equation at the m^th^ nodal point, which lies in the middle of the set of nodes with numbers $m-k, \ldots, m, \ldots,$ $m+k (k = 1, 2, \ldots)$ and which can be written as follows:

$$\begin{aligned}&\alpha_{m-k}Y_{m-k} + \ldots + \alpha_m Y_m + \ldots + \alpha_{m+k}Y_{m+k}\\&= h\left(b_{m-k}Y'_{m-k} + \ldots + b_m Y'_m + .. + b_{m+k}Y'_{m+k}\right) + R_k\end{aligned} \tag{4}$$

The number k is called the order of this equation, and the number p is called its degree. The remainder term R_k means the difference between the left and right sides of expression (4) and determines the approximation error.

3. RESULTS AND DISCUSSION

The point functions $Y_{m-k},\ldots,Y_m,\ldots,Y_{m+k}$ and their derivatives $Y'_{m-k},\ldots,Y'_m,\ldots,Y'_{m+k}$ are expanded according to the Taylor formula to terms with derivatives of degree $p+1$. After substituting (5) and (6) in (4), the coefficients of the derivatives on the right-hand side of expression (4) coincide with the coefficients of the corresponding derivatives of the left-hand side. As a result, a system of algebraic equations is obtained

$$Y_{m+k} = Y_m + khY'_m + \frac{(kh)^2}{2!}Y''_m + \ldots + \frac{(kh)^p}{p!}Y^p_m + \frac{(kh)^{p+1}}{(p+1)!}Y^{p+1}_m + 0\left(h^{p+1}\right) \quad (5)$$

$$Y_{m+k} = Y'_m + khY''_m + \frac{(kh)^2}{2!}Y''_m + \ldots + \frac{(kh)^{p-1}}{(p-1)!}Y^p_m + \frac{(kh)^p}{p!}Y^{p+1}_m + 0\left(h^p\right) \quad (6)$$

$$\sum\nolimits_0^K \alpha_K = 0 \quad (7)$$

$$\sum\nolimits_1^k \left(\sum_1^K \alpha_K K^S - Sb_K K^{S-1} \right) = 0, (S = 2,3,\ldots,p) \quad (8)$$

$$\sum\nolimits_1^K \alpha_K K^S - \sum\nolimits_0^K b_K = 0 \text{ W} \quad (9)$$

In total, there is $p+1$ homogeneous linear algebraic equations for $2(K+1)$ unknown $\alpha_{m\pm K}, b_{m\mp K}$. If this system of equations has a solution, then the problem of constructing a finite-difference equation, which approximates the given differential equation, can be considered solved. Using the method of indefinite coefficients, the values of the coefficients a and b are obtained for equations of different orders K. Let $K=1$. Then the equations (7), (8), (9) will be as follows:

$$\alpha_{m-1} + \alpha_{m+1} + \alpha_m = 0 \quad (10)$$

$$-\alpha_{m-1} + \alpha_{m+1} = h1\left(b_{m-1} + b_{m+1} + b_m\right) \quad (11)$$

$$\alpha_{m-1} + \alpha_{m+1} = h2\left(-b_{m-1} + b_{m+1}\right) \quad (12)$$

$$-\alpha_{m-1} + \alpha_{m+1} = h3\left(b_{m-1} + b_{m+1}\right) \quad (13)$$

Considering that for a given discretization of the argument, one can construct many difference schemes equivalent in approximation order and without affecting the universality of the result in the system of equations (10-13), $\alpha_m = 1,\ b_m = 0$. are taken. The result of the solution is formula (14). Difference equations of increased accuracy are obtained. Information at a point is obtained on the basis of information at points $m-1$ and $m+1$. The difference equation for the approximation order $k=1$ is considered with a fifth-order error (15).

$$2y_{m-1} - 4y_m + 2y_{m+1} = h\left(y'_{m-1} + y'_{m+1}\right) + \frac{1}{24}h^4 y^4_m \quad (14)$$

$$-3y_{m-1} + 3y_{m+1} = h\left(y'_{m-1} + 4y'_m + y'_{m+1}\right) + \frac{1}{30}h^5 y^{(5)}_m \quad (15)$$

where m is a nodal point number; h is a sampling frequency; y_{m-1}, y_m, y_{m+1} are network functions; y'_{m-1}, y'_m, y'_m are their derivatives. The finite-difference formula (15) connects the desired function at the $(m-1)^{\text{th}}$ and $(m+1)^{\text{th}}$ nodes due to the value of its derivatives at the $(m-1)^{\text{th}}$, m^{th}, $(m+1)^{\text{th}}$ nodes. Approximating formulas solved with respect to functions shall be obtained, that is, defining a function in the m^{th} node due to the value of its derivatives in three other nodes. The method of obtaining such expressions shall be considered by the example of equation (15). For economic processes that are characterized by the periodicity of sample (16) as a repeatability interval, it is prudent to take a half-period, which will reduce the time it takes to solve the problem.

$$y_m(t) = y_m\left(t + 180^\circ\right) \quad (16)$$

The minimum number of nodes in the period for a three-node approximation is four $(n=4)$. Equation (15) is written for all the nodal points of the period, taking into account the boundary conditions, which will be such for periodic economic processes (17). The following system of finite-difference equations is arrived at (18-21).

$$y_{n+1} = -y_1 \quad (17)$$

$$-y_1 + y_3 = \frac{h}{3}\left(y'_1 + 4y'_2 + y'_3\right) \quad (18)$$

$$-y_2 + y_4 = \frac{h}{3}\left(y'_2 + 4y'_3 + y'_4\right) \quad (19)$$

$$-y_3 + y_1 = \frac{h}{3}\left(y'_3 + 4y'_4 + y'_1\right) \quad (20)$$

$$-y_4 + y_2 = \frac{h}{3}\left(y'_4 + 4y'_1 + y'_2\right) \quad (21)$$

As a result of solving the system of difference equations (17-21) with respect to nodal functions

$$y_1 = \frac{h}{3}\left(-2y'_2 - y'_3 - 2y'_4\right) \quad (22)$$

$$y_2 = \frac{h}{3}\left(2y'_1 - 2y'_3 + y'_4\right) \quad (23)$$

$$y_3 = \frac{h}{3}\left(y'_1 + 2y'_2 - 2y'_4\right) \quad (24)$$

Table 1. Matrix form for y_1, y_2, y_3, y_4

y_1			-2	-1	-2	y_1'
y_2	$=\frac{h}{3}$	2		-2	-1	y_2'
y_3		1	2		-2	y_3'
y_4		2	1	2		y_4'

$$y_4=\frac{h}{3}\left(2y_1'+4y_2'+2y_3'\right) \tag{25}$$

The number of nodes in the period shall be doubled, that is, take $n=8$. The system of finite-difference equations for all the nodal points of the period will be as follows:

$$-y_1+y_3=\frac{h}{3}\left(y_1'+4y_2'+y_3'\right) \tag{26}$$

$$-y_2+y_4=\frac{h}{3}\left(y_2'+4y_3'+y_4'\right) \tag{27}$$

$$-y_3+y_5=\frac{h}{3}\left(y_3'+4y_4'+y_5'\right) \tag{28}$$

$$-y_4+y_6=\frac{h}{3}\left(y_4'+4y_5'+y_6'\right) \tag{29}$$

$$-y_5+y_7=\frac{h}{3}\left(y_5'+4y_6'+y_7'\right) \tag{30}$$

$$-y_6+y_8=\frac{h}{3}\left(y_6'+4y_7'+y_8'\right) \tag{31}$$

$$-y_7+y_1=\frac{h}{3}\left(y_7'+4y_8'+y_1'\right) \tag{32}$$

$$-y_8+y_2=\frac{h}{3}\left(y_8'+4y_1'+y_2'\right) \tag{33}$$

Upon determining the number of nodes on the period (34), where $k=0,1,2,\ldots,$ the difference equation in the vector-matrix form shall be obtained (35) where equations (36) and (37) are transposed matrices; Table 3 is square matrix of dimension n.

Table 2. The expanded matrix form for the system of equations (26-33)

y_1			-2	-1	-2	-1	-2	-1	-2	y_1'
y_2		2		-2	-1	-2	-1	-2	-1	y_2'
y_3		1	2		-2	-1	-2	-1	-2	y_3'
y_4	$=\frac{h}{3}$	2	1	2		-2	-1	-2	-1	y_4'
y_5		1	2	1	2		-2	-1	-2	y_5'
y_6		2	1	2	1	2		-2	-1	y_6'
y_7		1	2	1	2	1	2		-2	y_7'
y_8		2	1	2	1	2	1	2		y_8'

Table 3. Square matrix of dimension n

		-2	-1	-2
g =	2		-2	-1
	1	2		-2
	2	1	2	

$$n=4(k+1) \tag{34}$$

$$\frac{h}{3}\bar{Y}=g\bar{Y}' \tag{35}$$

$$\bar{Y}=\left(y_1, y_2, \ldots, y_n\right)_t \tag{36}$$

$$\bar{Y}'=\left(y_1', y_2', \ldots, y_n'\right)_t \tag{37}$$

For the model of formation of the valuation of the tax shield of economic objects and its solution using difference equations of increased accuracy, a three-point template and a system of finite-difference equations (26-33) shall be used. q, q' and p, p' shall be substituted, respectively (38). The matrix form for the variables that determine the level of asset is expanded.

$$\begin{cases}-q_1+q_3=\frac{h}{3}\left(q_1'+4q_2'+q_3'\right)\\ -q_2+q_4=\frac{h}{3}\left(q_2'+4q_3'+q_4'\right)\\ -q_3+q_5=\frac{h}{3}\left(q_3'+4q_4'+q_5'\right)\\ -q_4+q_6=\frac{h}{3}\left(q_4'+4q_5'+q_6'\right)\\ -q_5+q_7=\frac{h}{3}\left(q_5'+4q_6'+q_7'\right)\\ -q_6+q_8=\frac{h}{3}\left(q_6'+4q_7'+q_8'\right)\\ -q_7+q_1=\frac{h}{3}\left(q_7'+4q_8'+q_1'\right)\\ -q_8+q_2=\frac{h}{3}\left(q_8'+4q_1'+q_2'\right)\end{cases} \tag{38}$$

Similarly, the following system is obtained (39). After solving the given systems of equations with respect to

Table 4. The expanded matrix form for the variables that determine the level of asset q

q_1			-2	-1	-2	-1	-2	-1	-2	q_1'
q_2		2		-2	-1	-2	-1	-2	-1	q_2'
q_3		1	2		-2	-1	-2	-1	-2	q_3'
q_4	$=\frac{h}{3}$	2	1	2		-2	-1	-2	-1	q_4'
q_5		1	2	1	2		-2	-1	-2	q_5'
q_6		2	1	2	1	2		-2	-1	q_6'
q_7		1	2	1	2	1	2		-2	q_7'
q_8		2	1	2	1	2	1	2		q_8'

the model for the formation of the valuation of the tax shield of economic objects, the following results are obtained and presented graphically (Figure 1). Initial conditions are in equations (40), (41).

$$\begin{cases} -p_1+p_3=\frac{h}{3}\left(p_1'+4p_2'+p_3'\right) \\ -p_2+p_4=\frac{h}{3}\left(p_2'+4p_3'+p_4'\right) \\ -p_3+p_5=\frac{h}{3}\left(p_3'+4p_4'+p_5'\right) \\ -p_4+p_6=\frac{h}{3}\left(p_4'+4p_5'+p_6'\right) \\ -p_5+p_7=\frac{h}{3}\left(p_5'+4p_6'+p_7'\right) \\ -p_6+p_8=\frac{h}{3}\left(p_6'+4p_7'+p_8'\right) \\ -p_7+p_1=\frac{h}{3}\left(p_7'+4p_8'+p_1'\right) \\ -p_8+p_2=\frac{h}{3}\left(p_8'+4p_1'+p_2'\right) \end{cases} \quad (39)$$

$$q_{(0)}=19 \quad (40)$$

Table 5. A matrix form for a system of equations (39) for variables determining the price p

p_1			-2	-1	-2	-1	-2	-1	-2	p_1'
p_2		2		-2	-1	-2	-1	-2	-1	p_2'
p_3		1	2		-2	-1	-2	-1	-2	p_3'
p_4	$=\frac{h}{3}$	2	1	2		-2	-1	-2	-1	p_4'
p_5		1	2	1	2		-2	-1	-2	p_5'
p_6		2	1	2	1	2		-2	-1	p_6'
p_7		1	2	1	2	1	2		-2	p_7'
p_8		2	1	2	1	2	1	2		p_8'

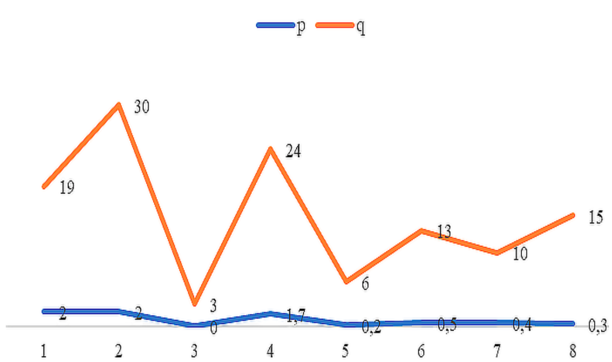

Figure 1. The result of modelling the formation of the valuation of the tax shield of economic objects (solution using equations with increased accuracy).

$$p_{(0)}=2 \quad (41)$$

Economic science includes mathematical methods and models as necessary tools. Their use facilitates the formalization of the most important bundles of economical systems and their further analysis on this basis, performance of forecasting and optimization. Mathematical and econometric methods allow to obtain new knowledge about the economic object and its behaviour, evaluate the form and parameters of the dependencies of its variables. The objectives solved by the economic science and practice are distinguished, depending on the inclusion of the time factor, into static and dynamic. Static objectives study the state of economic objects at a certain point in time without consideration of the changes in their parameters over time. Dynamic objectives cover not only the dependence of variables on time, but also their relationship in time. As an example, the dependence of the dynamics of the value of fixed capital on the dynamics of investment is used, which, in turn, leads to a change in the volume of output.

In economic dynamics, continuous and discrete time is used. Continuous time is convenient for modelling, as it allows you to use the apparatus of differential calculus and differential equations. Discrete time is convenient for solving applied problems, as statistics are always discrete and relate to specific units of time. For discrete time, the apparatus of difference equations is applied. By the way, well-known models of economic dynamics exist both in continuous and in discrete versions. In both cases, they have approximately the same accuracy and the complexity level of the models themselves is almost the same. Determination of the next value, unknown due to the previous value, upon given initial conditions (40), (41) is the transient analysis.

$$\begin{cases} \frac{q_{n+1}-q_n}{\Delta t}=9p_n-12; \\ \frac{p_{n+1}-p_n}{\Delta t}=2-0{,}1q_n; \end{cases} \quad (42)$$

$$\begin{cases} q_{n+1}=q_n+(9p_n-12)\Delta t; \\ p_{n+1}=p_n+(2-0{,}1q_n)\Delta t. \end{cases} \quad (43)$$

If the period T is not fixed, then the graphic images of the results of the calculation of expressions (43) under the initial conditions (40), (41) Δtwill be as shown in Figure 2. From the above results the following can be concluded about the transient process.

The solutions of the boundary value problem based on the model for the formation of the valuation of the tax shield of economic objects in the context of the formation of the tax shield are determined by several methods. Model based on a linear system of differentiations (44) which have initial conditions (40), (41).

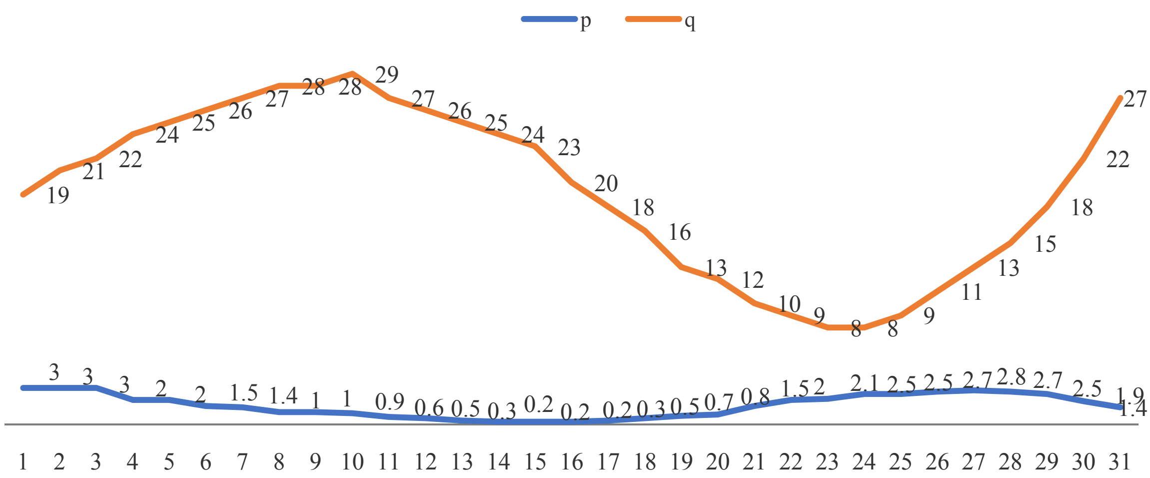

Figure 2. The result of modelling the formation of the valuation of the tax shield of economic objects (using finite-difference method).

$$\begin{cases} \dfrac{dq}{dt} = 9p - 12; \\ \dfrac{dp}{dt} = 2 - 0{,}1q; \end{cases} \tag{44}$$

A boundary value problem based on a linear system of differentiations. Three-point difference pattern for function (45) wherey is q or p will be as follows in equations (46), (47) or (48). Then (49), (50).

$$\frac{dy}{dt} = f(y;t) \tag{45}$$

$$\frac{-3y_{m-1} + 3y_{m+1}}{h} = y_{m-1} + 4y_m + y_{m+1} \tag{46}$$

$$-3y_{m-1} + 3y_{m+1} = h\left(y_{m-1} + 4y_m + y_{m+1}\right) \tag{47}$$

$$-y_{m-1} + y_{m+1} = \frac{h}{3}\left(y'_{m-1} + 4y'_m + y'_{m+1}\right) \tag{48}$$

$$-q_{m-1} + q_{m+1} = \frac{h}{3}[9p_{m-1} - 12 + 4(9p_m - 12) + 9p_{m+1} - 12]; \tag{49}$$

$$-p_{m-1} + p_{m+1} = \frac{h}{3}[2 - 0{,}1q_{m+1} + 4(2 - 0{,}1q_m) + 2 - 0{,}1q_{m+1}] \tag{50}$$

Ten points should be taken on the period, where $n = 10$ is the number of nodes on the period; $N = 2$ is the number of differential equations; $nN = 20$ is the order of the system of difference equations; then the difference equations (49) and (50) are transformed into a system of difference equations for a three-point pattern and they will be as follows:

$$\begin{cases} -p_1 + p_3 = \frac{h}{3}(12 - 0{,}1q_1 - 0{,}4q_2 - 0.1q_3); \\ -p_2 + p_4 = \frac{h}{3}(12 - 0{,}1q_2 - 0{,}4q_3 - 0{,}1q_4); \\ -p_3 + p_5 = \frac{h}{3}(12 - 0{,}1q_3 - 0{,}4q_4 - 0{,}1q_5); \\ -p_4 + p_6 = \frac{h}{3}(12 - 0{,}1q_4 - 0{,}4q_5 - 0{,}1q_6); \\ -p_5 + p_7 = \frac{h}{3}(12 - 0{,}1q_5 - 0{,}4q_6 - 0{,}1q_7); \\ -p_6 + p_8 = \frac{h}{3}(12 - 0{,}1q_6 - 0{,}4q_7 - 0{,}1q_8); \\ -p_7 + p_9 = \frac{h}{3}(12 - 0{,}1q_7 - 0{,}4q_8 - 0{,}1q_9); \\ -p_8 + p_{10} = \frac{h}{3}(12 - 0{,}1q_8 - 0{,}4q_9 - 0{,}1q_{10}); \\ -p_9 + p_{11} = \frac{h}{3}(12 - 0{,}1q_9 - 0{,}4q_{10} - 0{,}1q_{11}); \\ -p_{10} + p_{12} = \frac{h}{3}(12 - 0{,}1q_{10} - 0{,}4q_{11} - 0{,}1q_{12}); \end{cases} \tag{51}$$

$$\begin{cases} -q_1 + q_3 = h(3p_1 + 12p_2 + 3p_3 - 24); \\ -q_2 + q_4 = h(3p_2 + 12p_3 + 3p_4 - 24); \\ -q_3 + q_5 = h(3p_3 + 12p_4 + 3p_5 - 24); \\ -q_4 + q_6 = h(3p_4 + 12p_5 + 3p_6 - 24); \\ -q_5 + q_7 = h(3p_5 + 12p_6 + 3p_7 - 24); \\ -q_6 + q_8 = h(3p_6 + 12p_7 + 3p_8 - 24); \\ -q_7 + q_9 = h(3p_7 + 12p_8 + 3p_9 - 24); \\ -q_8 + q_{10} = h(3p_8 + 12p_9 + 3p_{10} - 24); \\ -q_9 + q_{11} = h(3p_9 + 12p_{10} + 3p_{11} - 24); \\ -q_{10} + q_{12} = h(3p_{10} + 12p_{11} + 3p_{12} - 24); \end{cases} \tag{52}$$

The calculation of the determining quantities $Y = (q_1, q_3, p_1, p_3)^t$ should be performed. The interchanges are written according to recurrence formulas:

$$\begin{cases} p_2 = 2 - \frac{(q_1 - q_3)}{12h} - \frac{(p_1 + p_3)}{4}; \\ p_3 = 2 - \frac{(q_2 - q_4)}{12h} - \frac{(p_2 + p_4)}{4}; \\ p_4 = 2 - \frac{(q_3 - q_5)}{12h} - \frac{(p_3 + p_5)}{4}; \\ p_5 = 2 - \frac{(q_4 - q_6)}{12h} - \frac{(p_4 + p_6)}{4}; \\ p_6 = 2 - \frac{(q_5 - q_7)}{12h} - \frac{(p_5 + p_7)}{4}; \\ p_7 = 2 - \frac{(q_6 - q_8)}{12h} - \frac{(p_6 + p_8)}{4}; \\ p_8 = 2 - \frac{(q_7 - q_9)}{12h} - \frac{(p_7 + p_9)}{4}; \\ p_9 = 2 - \frac{(q_8 - q_{10})}{12h} - \frac{(p_8 + p_{10})}{4}; \\ p_{10} = 2 - \frac{(q_9 - q_{11})}{12h} - \frac{(p_9 + p_{11})}{4}; \\ p_{11} = 2 - \frac{(q_{10} - q_{12})}{12h} - \frac{(p_{10} + p_{12})}{4}; \end{cases} \quad (53)$$

$$\begin{cases} q_2 = 30 - \frac{(q_1 + q_3)}{4} + \frac{15(p_1 - p_3)}{2h}; \\ q_3 = 30 - \frac{(q_2 + q_4)}{4} + \frac{15(p_2 - p_4)}{2h}; \\ q_4 = 30 - \frac{(q_3 + q_5)}{4} + \frac{15(p_3 - p_5)}{2h}; \\ q_5 = 30 - \frac{(q_4 + q_6)}{4} + \frac{15(p_4 - p_6)}{2h}; \\ q_6 = 30 - \frac{(q_5 + q_7)}{4} + \frac{15(p_5 - p_7)}{2h}; \\ q_7 = 30 - \frac{(q_6 + q_8)}{4} + \frac{15(p_6 - p_8)}{2h}; \\ q_8 = 30 - \frac{(q_7 + q_9)}{4} + \frac{15(p_7 - p_9)}{2h}; \\ q_9 = 30 - \frac{(q_8 + q_{10})}{4} + \frac{15(p_8 - p_{10})}{2h}; \\ q_{10} = 30 - \frac{(q_9 + q_{11})}{4} + \frac{15(p_9 - p_{11})}{2h}; \\ q_{11} = 30 - \frac{(q_{10} + q_{12})}{4} + \frac{15(p_{10} - p_{12})}{2h}; \end{cases} \quad (54)$$

Further it is accepted that the determining variables are equal (Table 6). Then, after the calculation, the data presented in the Table 7 should be obtained.

Table 6. The determining variables

p_1	p_3	q_1	q_3	h
0	0	0	0	7,00

Table 7. Data after calculation

No.	p	q
2	2,000000	30,000000
3	0,000000	0,000000
4	1,307692	21,692308
5	0,724260	7,029586
6	0,831797	16,853410
7	0,958728	10,395446
8	0,751018	14,445899
9	0,960835	12,089449
10	0,782081	13,314742
11	0,920342	12,792530
12	0,820753	12,914861
13	0,888117	12,993431
14	0,845358	12,834724

Now, the two equations per each of the systems (51) and (52), that is, the ninth and tenth equations, determine the remainder of the connected parameters, that is, the unconnected parameters of the tax shield (55), (56). Zero unrelated parameters (57), after solving the system of recurrence equations, will be as follows (58).

$$\begin{cases} \Delta_{11}^0 = h(3p_9 + 12p_{10} + 3p_{11} - 24) + q_9 - q_{11}; \\ \Delta_{12}^0 = h(3p_{10} + 12p_{11} + 3p_{12} - 24) + q_{10} - q_{12}; \end{cases} \quad (55)$$

$$\begin{cases} \Delta_{21}^0 = \frac{h}{3}(12 - 0{,}1q_9 - 0{,}4q_{10} - 0{,}1q_{11}) + p_9 - p_{11}; \\ \Delta_{22}^0 = \frac{h}{3}(12 - 0{,}1q_{10} - 0{,}4q_{11} - 0{,}1q_{12}) + p_{10} - p_{12}; \end{cases} \quad (56)$$

$$\Delta_0 = \left(\Delta_{11}^0, \Delta_{12}^0, \Delta_{21}^0, \Delta_{22}^0\right) \quad (57)$$

$$\Delta_0 = \begin{pmatrix} -61{,}28006144; -58{,}32972217; \\ -9{,}961630512; -9{,}839955372 \end{pmatrix} \quad (58)$$

Next, a successive change in the determining variables (Table 8) should be taken. Accordingly, the following is written in the Table 9.

Unrelated parameters will equate to (-61,15323431; -58,39456157; 9,950261811; 9,84962326). Similarly, values for the following variables are taken (Table 10). As a result, the Table 11 should be obtained.

Table 8. Change in defining values

p_1	p_3	q_1	q_3	h
1	0	0	0	7,00

Table 9. Change dynamics in determining variables

No.	p	q
2	1,750000	31,071429
3	0,000000	0,000000
4	1,348116	21,151884
5	0,692489	7,565753
6	0,854065	16,497242
7	0,941541	10,588179
8	0,764839	14,357569
9	0,950149	12,120419
10	0,789787	13,312561
11	0,915179	12,781815
12	0,823964	12,929866
13	0,886279	12,978602
14	0,846298	12,847335

Table 10. Values for unrelated parameters

p_1	p_3	q_1	q_3	h
0	1	0	0	7,00

Table 11. Summary parameters for changing unrelated parameters

No.	p	q
2	1,750000	28,928571
3	0,000000	0,000000
4	1,377159	21,622841
5	0,660850	7,157831
6	0,874688	16,776498
7	0,932123	10,400502
8	0,767141	14,485896
9	0,951322	12,033847
10	0,787348	13,368160
11	0,917777	12,748698
12	0,821688	12,947637
13	0,888075	12,970544
14	0,844982	12,849749

Table 12. Change dynamics in determining variables

p_1	p_3	q_1	q_3	h
0	0	1	0	7,00

(-61,27715629; -58,30375986; 9,944084083; 9,85147455). Similarly, the value for q is obtained (Table 12). The numerical values for q and p are given in Table 13.

(-61,29143014;-58,32005429; 9,960221322; 9,840675809). Finally, for q_3 (Table 14). The resulting unrelated parameters will have the values indicated in the Table 15.

Table 13. Modelling numerical values

No.	p	q
2	0,988095	29,750000
3	0,000000	0,000000
4	1,313697	21,732732
5	0,718303	6,997814
6	0,835754	16,875679
7	0,956586	10,378260
8	0,752000	14,459720
9	0,960491	12,078762
10	0,782105	13,322449
11	0,920461	12,787368
12	0,820586	12,918072
13	0,888282	12,991593
14	0,845218	12,835664

Table 14. Final values of the parameters

p_1	p_3	q_1	q_3	h
0	0	0	1	7,00

Table 15. The final values of unrelated parameters

No.	p	q
2	2,011905	29,750000
3	0,000000	0,000000
4	1,308464	21,761774
5	0,722835	6,966176
6	0,832651	16,896302
7	0,958671	10,368841
8	0,750574	14,462022
9	0,961453	12,079935
10	0,781487	13,320009
11	0,920829	12,789965
12	0,820388	12,915796
13	0,888371	12,993389
14	0,845191	12,831348

(-61,29760787; -58318203; 9,961598232; 9,839666901). Now, form the system of equations of unrelated parameters is formed

$$\Delta_0 = \Delta Y \quad (58)$$

$$\Delta_0 = \begin{matrix} -61,2801 \\ -58,3297 \\ 9,961631 \\ 9,839955 \end{matrix} \quad (59)$$

$$Y = \begin{matrix} p_1 \\ p_3 \\ q_1 \\ q_3 \end{matrix} \quad (60)$$

$$\Delta = \begin{matrix} -61,153234 & -61,2772 & -61,2914 & -61,2976 \\ -58,394562 & -58,3038 & -58,3201 & -58,3182 \\ 9,950261181 & 9,944084 & 9,960221 & 9,961598 \\ 9,84962326 & 9,851475 & 9,840676 & 9,839667 \end{matrix} \qquad (61)$$

For a compact representation of the above system of equations, the matrix image is used (62). Hence the solution is obtained (Table 16). The values in the system of recurrence equations are substituted (Table 17). The results are illustrated in Figure 3.

$$Y = \Delta^{-1}\Delta_0 \qquad (62)$$

Table 16. Solution of unrelated parameters

p_1	0,03125
p_2	-0,15625
q_1	1,5
q_2	-0,5

Table 17. Final values of recurrence equations

No.	p	q	h
1	0,03125	1,5	7,00
2	2,007440	29,950893	
3	-0,156250	-0,500000	
4	1,456082	22,029484	
5	0,624945	6,774620	
6	0,888905	17,103985	
7	0,928725	10,151755	
8	0,765265	14,657739	
9	0,955409	11,924017	
10	0,782785	13,433258	
11	0,921911	12,713519	
12	0,818353	12,964053	
13	0,890553	12,965087	
14	0,843261	12,849398	

If the boundary conditions are accepted (63-66) then the results presented in the Tables 18, 19 should be obtained.

$$q_8 = -q_2 \qquad (63)$$

$$q_9 = -q_3 \qquad (64)$$

$$p_8 = -p_2 \qquad (65)$$

$$p_9 = -p_3 \qquad (66)$$

Table 18. Solution of a boundary value problem with unrelated parameters

p_7	$p_9 = -p_3$	q_7	$q_9 = -q_3$
0,928725	0,15625	10,15175533	0,5

Table 19. Final data of the solution of boundary conditions

No.	p	q	h
7	0,928725	10,15175533	7,00
8	1,613854	28,164713	
9	0,156250	0,500000	
10	1,268074	21,307347	
11	0,723662	7,429064	
12	0,842177	16,520390	
13	0,947279	10,623904	
14	0,761475	14,310300	
15	0,951994	12,160094	
16	0,789058	13,283926	
17	0,915203	12,801104	
18	0,824292	12,917486	
19	0,885836	12,986123	
20	0,846725	12,843105	

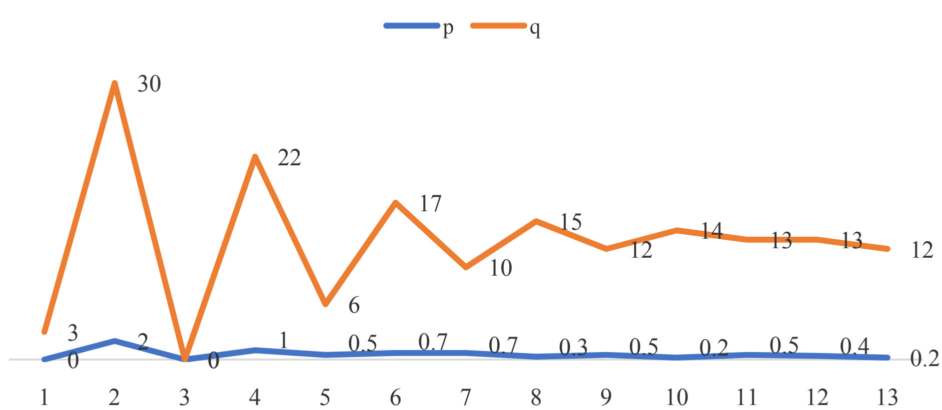

Figure 3. The result of modelling the formation of the valuation of the tax shield of economic objects (solution of the boundary value problem).

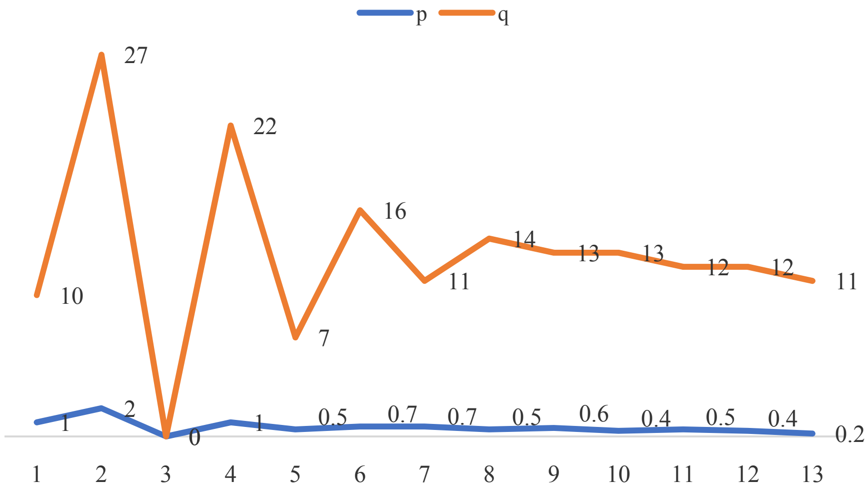

Figure 4. The result of modelling the formation of the valuation of the tax shield of economic objects (solutions with imposition of boundary conditions).

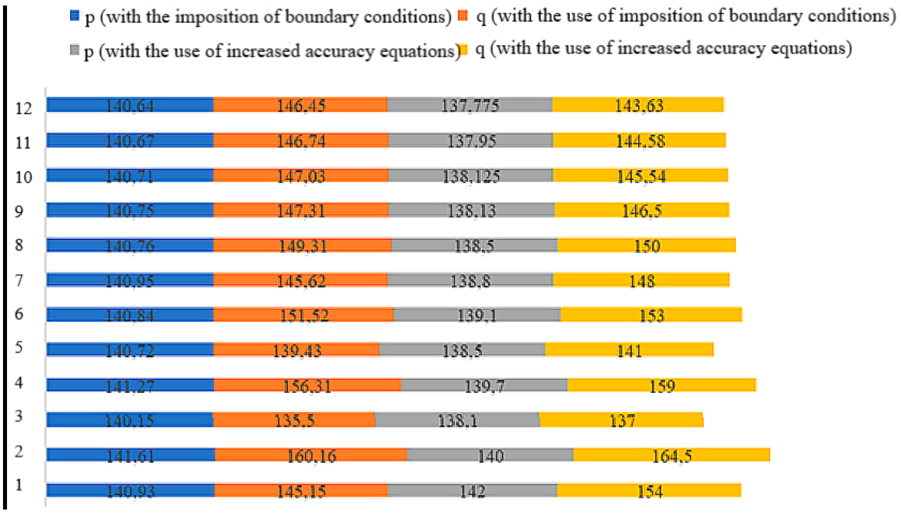

Figure 5. The result of modelling the price equalization of economic objects.

Table 20. The results of the formation of the valuation of the tax shield for LLC "Vektor"

No.	p (with the imposition of boundary conditions)	q (with the imposition of boundary conditions)	p (with the use of increased accuracy equations)	q (with the use of increased accuracy equations)
1	140,93	145,15	142	154
2	141,61	160,16	140	164,5
3	140,15	135,5	138,1	137
4	141,27	156,31	139,7	159
5	140,72	139,43	138,5	141
6	140,84	151,52	139,1	153
7	140,95	145,62	138,8	148
8	140,76	149,31	138,5	150
9	140,75	147,31	138,13	146,5
10	140,71	147,03	138,125	145,54
11	140,67	146,74	137,95	144,58
12	140,64	146,45	137,775	143,63

The result of modelling the formation of the valuation of the tax shield of economic objects with imposition of boundary conditions will be as follows (Figure 4).

A comparison of the results of solving the model for the formation of the valuation of the tax shield of economic objects using equations of increased accuracy and with the imposition of boundary conditions is displayed in Figure 5.

A numerical experiment of solving the model for the formation of the valuation of the tax shield of economic objects by the method of the boundary value problem and the method of solving equations of high accuracy shows a higher accuracy obtained by the first method. The accuracy of the calculation lies in the range of 5-10%. When solving the model for the formation of the valuation of the tax shield for LLC "Vektor" on the example of one of the types of products, the following results were obtained (Table 20).

After solving the model for the formation of the valuation of the tax shield for LLC "Vektor" for other types of products, the expected economic effect amounted to 76.5 thousand dollars, and in the long term – 78 thousand dollars. The total expected economic effect from the implementation of the results was 154.5 thousand dollars.

4. CONCLUSIONS

The paper describes how to reduce the number of difference equations while maintaining the required accuracy of the results, it is prudent to use approximations that factor in the greater number of expansion terms of the desired solution in a Taylor series. The coefficients of such approximations are found using the method of indefinite coefficients. A method for determining difference equations that approximate differential equations is proposed. Mathematical modelling and computer simulation of periodic economic processes, which are discrete in time, are considered. The methodology of replacing differential equations with difference equations is provided. The model of the formation of the valuation of the tax shield by asset level is researched. The solution and analysis were performed in two numerical ways: by the method of indefinite coefficients and by solving a boundary value problem. Rational methods were developed for approximating differential equations with difference equations upon modelling economic processes that are discrete in time. Difference equations of increased accuracy are obtained, which make it possible, at the expense of a slight complication of the calculation formulas, to significantly reduce the total number of calculable nodes and ultimately require less computational cost.

The solution of finite-difference equations of increased accuracy with respect to nodal functions greatly simplifies and facilitates the procedure for approximating differential equations of the economic process with difference equations. The proposed method for obtaining difference equations of high accuracy is general and can be extended to any number of nodes of a discrete network. The numerical solution of the model for the formation of the valuation of the tax shield by asset level confirmed the high accuracy of the proposed finite-difference equations.

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