The Relations Between Grit, Math Determination,

and Persistence on Challenging Math Problems

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Author Note

Abstract

Three studies conducted in the U.S. and Russia examine a novel pathway by which the perseverance component of the personality trait *grit* might predict college students' behavioral persistence when solving challenging math problems. Specifically, we focus on the intervening role of what we refer to as *math determination*, which captures students' perceived tendency to persevere on challenging math problems. Across studies, we found that math determination was correlated with behavioral math persistence, whereas grit was not. Despite there being no correlation between grit and math persistence, we found statistically significant *indirect* effects of grit on persistence through math determination in all three studies: the grittier students were, the more they viewed themselves as capable of persevering on challenging math problems, which ultimately predicted their math persistence at a later time point.

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Over the last few decades, much concern has been expressed about the underperformance of U.S. students in mathematics relative to students from other developed nations (e.g., Hanushek, Peterson, & Woessmann, 2010). Although, traditionally, researchers have attempted to account for this underperformance in terms of mathematical knowledge and skills (e.g., Geary, Bow-Thomas, Liu, & Siegler, 1996; Vasilyeva et al., 2015), they have recently begun to offer explanations that focus more on non-cognitive factors, such as effort and persistence (e.g., Boe, May, & Boruch, 2002). This focus appears to have informed the efforts of policy makers, such that recent initiatives aimed at improving the mathematics standards employed in U.S. schools have stressed the importance of fostering students' persistence during problem solving. In fact, the very first Standard for Mathematical Practice advanced by the Common Core State Standards (CCSS) is for students to "make sense of problems and *persevere* in solving them" (CCSS, 2010, p. 6; emphasis added). In addition, the Problem Solving Process Standard proposed by the National Council of Teachers of Mathematics (NCTM) states that, "Students require frequent opportunities to formulate, grapple with, and solve complex problems that involve a significant amount of *effort*… By solving mathematical problems, students acquire ways of thinking, habits of *persistence* and curiosity, and confidence in unfamiliar situations that serve them well outside the mathematics classroom" (NCTM, 2000, p. 4; emphasis added; see also Star, 2015).

In order to explain why some students tend to persevere on challenging math problems more than others, we examine a general personality trait that has been shown to predict persistence in variety of educational contexts (i.e., grit), as well as students' more specific beliefs about how they typically respond to challenge in the math domain (what we refer to as math determination). We examined these constructs in a series of exploratory analyses across three

studies. In each study, college students first completed measures of general grit and math determination and then, days or weeks later, completed a set of challenging math problems. Our aims here are two-fold. First, we seek to advance prior research by examining whether grit and math determination predict persistence on *challenging* math problems. Second, we extend these initial analyses by exploring the possibility that there is an indirect association between grit (a general personality factor) and behavioral persistence in math through students' math-specific beliefs.

Grit as a Predictor of Persistence on Math Tasks

Defined as "*trait-level perseverance* and passion for long-term goals" (Duckworth & Quinn, 2009, p. 166, emphasis added; see also Duckworth, Peterson, Matthews, & Kelly, 2007), grit has been widely touted as an important predictor of positive academic outcomes (such as grade point average, retention, and intention to persist in college) across multiple academic domains (Credé, Tynan, & Harms, 2017; Eskreis-Winkler, Duckworth, Shulman, & Beal, 2014; Eskreis-Winkler et al., 2016; Muenks, Wigfield, Yang, & O'Neal, 2017; Muenks, Yang, & Wigfield, 2018). Although most studies of grit have focused on broad academic outcomes, such as grades and retention (Credé et al., 2017), a number of studies have examined the association between grit and specific on-task behaviors that are believed to contribute to academic performance. For instance, Duckworth, Kirby, Tsukayama, Berstein, and Ericsson (2011) showed that the effect of grit on spelling bee performance was mediated by the total number of hours competitors engaged in deliberate practice.

Other studies assessing on-task behaviors have been conducted specifically in a math context. For instance, Galla et al. (2014) found that grit predicted the amount of time high school students spent working on one-digit subtraction problems versus playing a game or watching a movie. Similarly, Lucas, Gratch, Cheng, and Marsella (2015) found that grit predicted adult

participants' decision to continue with a task that involved competing against another individual to solve as many three-digit addition problems as possible in a limited time; but only when the participants thought they were losing. Finally, Eskreis-Winkler et al. (2016) had middle school students solve standard textbook math problems on the computer during a normal class period, and gave them the option to take breaks by surfing the Internet. Each time the students correctly answered five consecutive problems they advanced to more difficult problems. The results showed that students' grit was associated with how much time they spent working on math problems versus surfing the Internet.

Critically, however, such studies have generally not assessed persistence on the kinds of challenging math tasks that are believed to foster deep conceptual learning. Specifically, the rote arithmetic tasks used in the studies by Galla et al. (2014) and Lucas et al. (2015) required participants to overcome feelings of boredom and time pressure, rather than difficulties in finding a problem solution. In fact, Galla et al. explicitly distinguished between "diligence on facile but tedious work tasks," like the one used in their study, and "perseverance in the face of extreme difficulty" (p. 316), which is what we hoped to capture with the math tasks used in the present studies. And while the adaptive feature of Eskreis-Winkler et al.'s (2016) math task may have ensured that most students were presented with at least some difficult problems, the extent to which students were *consistently* challenged may have varied in accordance with their math abilities (because it presumably took the higher ability students longer to reach a set of problems that they found challenging). Thus, in that study, time-on-task was not necessarily a uniform measure of how much time the students spent persisting on *challenging* problems (though the authors partly addressed this concern by controlling for participants' standardized math test achievement).

Exploring Math Determination as a Potential Predictor of Behavioral Persistence on

Challenging Math Tasks

Although grit has been shown to predict behavioral persistence on certain types of math tasks (e.g., Eskreis-Winkler et al., 2016; Galla et al., 2014; Lucas et al., 2015), more research is needed to understand how trait-level individual differences in students' general propensity to persevere are related to math-specific motivational factors. As some researchers have argued about other traits, such as self-esteem, it is "simply not enough to measure global attitudes and traits while ignoring the large number of variables that mediate and moderate the links between predictor and outcome variables" (Swann, Chang-Schneider, & McClarty, 2007, p. 85). In other words, a broad personality factor like grit is likely to exert its influence on domain-specific outcomes like math persistence through several indirect paths involving distinct intervening variables.

One potential intervening variable pertains to students' beliefs about their perseverance and determination when solving math problems. That is, in addition to the domain-general beliefs about perseverance captured by the grit scale (e.g., "I am a hard worker"), students likely hold domain-specific beliefs about how they typically respond to challenging math problems and how capable they are of completing problems that are difficult or that take a long time to complete. We refer to this construct as *math determination* (Schmidt, Fleckenstein, Retelsdorf, Eskreis-Winkler, & Möller [in press] referred to a similar construct as "subject-specific perseverance of effort"; see also Cormier, Dunn, & Dunn, 2019).

It is important to note that we conceptualize students' *perceived* math determination as being distinct from their *behavioral* persistence on specific math tasks. That is, we do not view these two variables as separate indicators of a single underlying construct. Our measure of math determination captures students' beliefs about their *general* tendency to persevere on challenging

math problems (e.g., "When I don't understand a problem, I keep working until I find the answer"), as well as their *general* sense of efficacy for completing problems that involve high levels of effort (e.g., "I feel I can do math problems that take a long time to complete"). In contrast, behavioral measures assess students' persistence on a *particular* task, in a particular context. Such persistence likely reflects a number of situation-specific considerations, in addition to more stable motivational factors, such as math determination.

Though we believe that math determination and behavioral persistence on math tasks are separate constructs, it is likely that they become reciprocally related over time. On the one hand, a student's belief that she is the kind of person who can persevere when faced with difficult math problems should lead her to spend more time working on a challenging math task. On the other hand, when a student repeatedly observes herself persisting on challenging math tasks, she should ultimately come to see herself as having a high level of math determination. This is parallel to claims made by self-concept researchers, who posit reciprocal relations between self-perception and behavior (Marsh & Martin, 2011; Shavelson, Hubner, & Stanton, 1976).

The Relation Between Grit, Math Determination, and Behavioral Math Persistence

Although past problem-solving success may lead students to draw inferences about their math ability in a bottom-up manner, this is only one factor contributing to their math determination. Students' broad traits, such as grit, are likely to exert a more top-down influence on their self-perceptions in the context of specific domains, such as math. For instance, students who are generally inclined to persist in the face of difficulty (i.e., who possess high levels of grit) may infer that this trait will enable them to persevere and overcome challenges when solving math problems.

What remains to be seen, however, is whether the association between grit and math determination can help us to understand how grit relates to students' behavioral persistence on

challenging math problems. The answer to this question could be useful to both researchers and educators in making decisions about which aspects of personality and motivation to assess in order to best predict students' behavior on math assignments. The present research may also be informative to researchers seeking to understand the paths by which grit influences important long-term academic outcomes, such as grade point average and retention (see Abuhassàn & Bates, 2015).

Overview of the Present Studies

As mentioned above, this paper has two aims. The primary aim is to investigate grit and math determination as predictors of college students' behavioral persistence on challenging math problems. Furthermore, as discussed above, it is also important to enhance understanding of how trait-level individual differences in students' general propensity to persevere (i.e., grit) are related to persistence on the kinds of challenging math tasks that are believed to foster deep conceptual learning. The second aim of the paper is therefore to explore whether grit may exert an effect on behavioral persistence in part through math determination.

Toward these aims, we discuss the results of three studies that were originally conceived as part of separate projects designed to investigate multiple motivational and personality factors that might influence persistence on challenging math problems. Though the studies were not designed with the current framing in mind, because of the conceptual similarity of their methods and the striking consistency in the pattern of findings, we feel that it is informative to discuss these exploratory analyses together.[1] Study 1, which was conducted with a sample of U.S. college students, examined grit and math determination in relation to a *time*-based measure of

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[1] For a full list of measures and procedures for all three studies, along with some additional analyses, see the supplementary materials. For more information about the exploratory and post hoc nature of our analyses and about the evolution of our conceptual framing, also see the supplementary materials. A set of analyses for Study 1 pertaining to participants' growth mindsets were published in Authors (2016).

behavioral math persistence. Specifically, we recorded how much time the students spent trying to complete two highly challenging math problems (e.g., Aspinwall & Richter, 1999; Roney, Higgins, & Shah, 1995; Lucas et al., 2015). Study 2a, which was conducted with a sample of U.S. college students, examined grit and math determination in relation to a *preference*-based measure of persistence. Specifically, after an initial attempt to complete a challenging task, participants were offered a choice between continuing to work on the same math task or switching to an unrelated verbal task (e.g., Hong, Chiu, Dweck, Lin & Wan, 1999; Lucas et al., 2015; Tulis & Fulmer, 2013). Study 2b, which was conducted with a sample of Russian college students, examined grit and math determination in relation to the same preference-based measured used in Study 2a. Thus, across our three studies, we investigate the relations between grit, math determination, and math problem-solving persistence using two different measures of behavioral persistence and with two different populations.

Study 1

In Study 1, we examined whether grit and math determination (assessed during Session 1) predicted the amount of time students spent trying to solve two challenging math problems during a separate later session.

Method

Participants. A total of 188 students enrolled in sections of a course at the school of education at a private research university in Massachusetts participated for course credit. Of these, 50 either did not complete Session 2 or completed a preliminary version of the study materials that did not contain all of the items from the final version. Another participant was missing data due to computer error. Therefore, the final sample included 137 college students who completed both parts of the study (83.9% female; age: $M = 19.01$, $SD = 1.20$, range = 18-24;

70.6% White, 14.0% Asian, 6.6% Hispanic/Latinx, 4.4% Black, 4.4% multiracial) – one

participant did not report ethnicity/race during Session 1.[2]

      Procedure and materials. The study consisted of two lab-based sessions, held 15.7 days

apart ($SD = 3.7$; range: 10 to 33 days). In Session 1, we assessed a number of individual

differences in personality and motivation, including grit and math determination.[3] The measures

were presented in a randomized order and the items within the two focal measures were

randomized. At the end of Session 1, participants completed a demographic questionnaire. In

Session 2, we administered the behavioral persistence task and a second demographic

questionnaire on computers. As mentioned above, these sessions were part of a larger project

centered on identifying variables that predict math problem-solving persistence and, as such,

included two experimental conditions and several other measures that are not of primary interest

given the present aims (see supplementary materials for details).[4] Descriptive statistics and zero-

order correlations for all measures described below are presented in Table 1.

      Grit (Session 1). To assess grit, Duckworth and colleagues developed a self-report scale

which (along with its short-form version) represents the primary means by which grit has been

assessed in the literature (see Credé et al., 2017). The original scale comprises two distinct

subscales: perseverance of effort and consistency of interest. However, a large meta-analysis of

the grit literature ($k = 88$, $N = 66,807$; Credé et al., 2017), suggests both that these subscales do

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[2] The participants who completed Session 1, but not Session 2, did not vary significantly from the final sample in terms grit, math determination, or age. There was a marginal difference between these groups in terms of gender composition (see the supplementary materials for details).

[3] Math confidence—another component of students' math self-concept (see Marsh, Byrne, & Shavelson, 1988; Pajares & Miller, 1994; West, Fish, & Stevens, 1980) that overlaps with math determination to some extent—was also assessed in Studies 1 and 2, but a consistent relation with grit did not emerge. See the supplementary materials for details regarding the measurement of and analyses including this construct.

[4] All three studies involved a feedback/challenge manipulation that was administered prior to the persistence measure (see Authors, 2016). As detailed in the supplementary materials, there were no main effects of the manipulations on the primary outcome measures in any of the studies, nor did the manipulations moderate the relation between any of the primary predictors in the present paper and the outcome measures. This is contrast to findings by Lucas et al. (2015), who found that grit was only associated with persistence in the failure condition.

not always load onto a single second-order or general factor (see also Muenks, et al., 2017; Muenks et al., 2018; Wolters & Hussain, 2015) and that perseverance is generally a better predictor of academic outcomes compared to consistency of interest *and* to the grit scale as a whole. This led Credé et al. (2017) to suggest that "the primary utility of the grit construct [as a predictor of academic outcomes] may lie in the perseverance facet" (p. 492).

Consistent with this suggestion, we assessed only perseverance of effort in the present research using the six-item subscale from Duckworth and colleagues' (2007) original measure. Items included "I am a hard worker" and "I have achieved a goal that took months of work"; note that for the latter item, the word "years" in the original scale was changed to months (in all three studies), in order to better reflect the kinds of goals students pursue in their math classes. Participants responded using a 1 ("Not like me at all") to 5 ("Very much like me") scale. We computed a grit index by averaging participants' ratings across all six items.

Math determination (Session 1). Math determination was assessed using an 8-item scale that assessed students' beliefs about how they tend to respond to challenging math problems. It consisted of the 6-item "I can solve time-consuming mathematics problems" subscale of the Indiana Mathematics Beliefs Scales (e.g., "I find I can do hard math problems if I just hang in there"; Kloosterman & Stage, 1992) with the addition of two complementary items: one from the Motivation/Persistence subscale of the Computer Attitude Questionnaire ("When I don't understand a problem, I keep working until I find the answer"; Knezek & Christensen, 1996) and one that we created ourselves ("If I am struggling to solve a problem, I try to think of alternative ways to solve it"). Participants responded using a 1 = ("Not like me at all") to 5 = ("Very much like me") scale, and we computed an index of math determination by reverse-coding three negatively-worded items and then averaging participants' ratings across all eight items.

Behavioral persistence in math (Session 2). We recorded how much time the students spent trying to complete a highly challenging math task as our index of persistence. It was critical that the problems in the task be extremely challenging, such that they require substantial time and effort even from highly competent individuals; otherwise, it would be unclear whether a particular student finished a task quickly because she gave up prematurely or because she is highly competent. Based on intensive piloting, we selected one problem that was very unlikely to be solved within the allotted time and another that was unsolvable (see Appendix).

Before starting the task, participants were informed that they could stop and move on to the next problem at any point, but would not be able to click back to revisit the previous problem. They were also told that, after a certain amount of time, the computer would automatically advance them to the next problem. Participants were given 16 minutes to solve each problem, though they were not informed about this time limit. If a participant was still working on a problem at the end of the 16-minute period, the program automatically advanced him or her to the next screen. As in prior work (e.g., Aspinwall & Richter, 1999; Battle, 1965; Roney, Higgins, & Shah, 1995), the amount of time participants spent on each problem was used as the measure of behavioral persistence.

Results

Factor analyses. To assess whether the items in the grit and math determination scales measured two distinct constructs, we first conducted an exploratory factor analysis (EFA) using principal axis factoring, an initial eigenvalue threshold of 1 (i.e., the Kaiser rule), and direct oblimin rotation. We found that all items loaded cleanly onto the expected two factors (loadings $\geq .3$ in the pattern matrix), with no cross-loadings. We then conducted a confirmatory factor analysis (CFA) using maximum likelihood estimation and allowed the two factors to covary. The model showed acceptable fit $\chi^2(76) = 116.31$, CFI $= .94$, RMSEA $= .06$, SRMR $= .05$, and fit the

data significantly better than a model with only one factor or a two-factor model in which the

factors were *not* allowed to covary, $\chi^2(77) > 124.72$, $ps < .001$. See the supplementary materials

for more details about these analyses.

Descriptive statistics regarding behavioral persistence in math. Next, we examined

whether the two math problems students worked on were as challenging as intended. Indeed, we

found that only 1.5% of students correctly solved the solvable problem and only 2.9% correctly

recognized that the other problem was unsolvable (in contrast, 9.5% of students suspected that

the solvable problem might be unsolvable). With respect to the amount of time participants spent

solving each of the two problems, there was a great deal of variability (from ~22 seconds to 16

minutes). The mean time spent on the solvable problem was approximately 332.23 seconds (*SD*

= 266.00) and the mean time for the unsolvable problem was approximately 437.04 seconds (*SD*

= 290.46). Time was positively skewed for both problems. That is, most participants spent less

than 400 seconds (73.0% for the solvable problem, and 53.3% for the unsolvable problem), but

several used the maximum amount of time allowed (8.8% for the solvable problem and 11.7%

for the unsolvable problem).

To address this skewness, we conducted square-root transformations of both variables

(Ratcliff, 1993). Both before and after transformation, there were no outliers greater than 3 *SD*

above or below the mean for either variable. Analyses conducted with these transformed

variables showed that the zero-order associations between behavioral persistence and the primary

Session 1 variables[5] were not moderated by problem type (solvable vs. unsolvable) or problem

order (unsolvable first vs. solvable first), *F*s < .84 , *p*s > .36. Thus, we created a single

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[5] This includes the math confidence variable examined in the supplementary materials.

persistence variable to use in subsequent analyses by averaging the raw times for the two

problems and then computing a square-root transformation of the mean.

       Grit and math determination as predictors of behavioral persistence in math. The

correlations between the three main constructs assessed in Study 1 appear in Table 1. As shown,

math determination was significantly correlated with grit. That is, the stronger students' trait-

level tendency to persevere, the more they viewed themselves as being able to persevere on

challenging math problems. More critically, math determination was also significantly correlated

with persistence. In other words, the more confident students were in their math abilities and the

more they perceived themselves as capable of persevering on challenging math problems (in

Session 1), the more time they spent working on the math task in Session 2.[6]

       In contrast with prior research, which did not involve consistently challenging math

problems (e.g., Eskreis-Winkler et al., 2016; Galla et al., 2014; Lucas et al., 2015), the

correlation between grit and math persistence was *not* statistically significant. That is, we did not

find a direct association between students' trait-level propensity for perseverance and the time

they spent working on challenging math problems.

       Although we did not find a direct association between grit and behavioral persistence, we

were still interested in whether grit may exert an *indirect* effect on persistence through math

determination. As Hayes and Rockwood (2017, p. 43-44) explain, the association between the

predictor and the outcome "is the sum of the direct effect and all possible indirect effects, of

which there may be many, and various combinations of sizes of direct and indirect effects can

produce a total effect equal to zero… [Thus] if [the direct association between X and Y] is

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[6] It may be of interest to some readers to note that openness to experience, which we assessed in Session 1 of the present study (but not in the other studies), exhibited significant correlations with grit and math determination and a near significant correlation with behavioral persistence (see the supplementary materials).

zero… this does not mean X doesn't affect Y." Rather, "if there are theoretical reasons to predict the presence of an indirect effect… researchers should explore these effects regardless of the significance of the total or direct effect" (Rucker, Preacher, Tormala, & Petty, 2011, p. 368; see also MacKinnon, Krull, & Lockwood, 2000; Shrout & Bolger, 2002; Zhao, Lynch Jr., & Chen, 2010).

As discussed throughout the introduction, there is good theoretical reason and empirical precedent behind the notion that a broad, general factor like grit may exert an indirect effect on a domain-specific outcome like persistence through more proximal, domain-specific factors like math determination (see e.g., Swann et al., 2007). We therefore tested an intervening variable model (see Hayes, 2009; MacKinnon et al., 2000; Mathieu & Taylor, 2007), with grit included as the predictor, math determination as the intervening variable, and task persistence as the outcome. This analysis was conducted using Hayes' (2013) PROCESS macro for SPSS (v2.13.2), which randomly selected 5,000 samples with replacement from the complete data, estimated regression coefficients for each of the bootstrap samples, and averaged them across all samples. As shown in Figure 1a, the 95% confidence interval for the indirect effect excluded zero, suggesting that grit had a statistically significant positive indirect effect on task persistence via math determination. Note that when this direct effect was accounted for, the remaining association between grit and task persistence became negative. We discuss this finding in the General Discussion; however, it did not emerge in Studies 2a-2b. See the supplementary materials for additional analyses of intervening variable models.

The results of Study 1 therefore indicate that math determination is a better predictor than grit of students' behavioral persistence on challenging math problems. In fact, while math determination was significantly correlated with behavioral persistence on our difficult math problems, grit was not (contrary to prior work examining grit in less challenging math problem-

solving contexts; e.g., Eskreis-Winkler et al., 2016; Galla et al., 2014; Lucas et al., 2015). Although grit was not correlated with math persistence, we did find a positive indirect effect of grit on math persistence via math determination, suggesting that the grittier students were, the more they perceived themselves as capable of persevering on challenging math problems, which in turn predicted how much time they actually spent working on the math task.

Studies 2a and 2b

The results of Study 1 provide some initial support for one pathway by which general grit may exert an influence on behavioral persistence on domain-specific tasks like challenging math problems (i.e., via math determination). Given the novelty and exploratory nature of these findings, we use the remaining two studies to both replicate Study 1's results and to demonstrate the generalizability of this path to different measure of persistence and a distinct population.

As opposed to assessing behavioral persistence in terms of time-on-task (as in Study 1), Studies 2a and 2b assessed students' preference for continuing to work on a challenging math task. We conceptualize the difference between the time-based measure used in Study 1 and the preference-based measure used in the present studies in terms of the distinction made by theorists between volitional and motivational processes (e.g., Corno, 1993, 2001; Kuhl, 1984). Time-on-task captures students' persistence in terms of working toward a goal that they are currently committed to achieving and is, thus, volitional in nature. In contrast, our task preference measure captures the students' persistence in terms of their decision to recommit themselves to their original goal and is, thus, motivational in nature. Given that "different information processing principles seem to account for performance before and after the point of commitment" (Corno, 1993, p. 14), it is possible that the effects of grit and math determination on behavioral persistence may vary depending on *when* the latter is assessed.

In addition, we present evidence that this pathway between grit and behavioral persistence generalizes across distinct populations. Specifically, while Study 2a was conducted with a sample of American college students, in Study 2b, we present a close replication of Study 2a with a sample of non-American (Russian) college students.

Method

Participants

Study 2a. A total of 204 students enrolled in sections of two courses (occurring in separate semesters) participated for course credit. The courses were taught at the school of education at a private research university in Massachusetts. The data from some participants were excluded for the following reasons: 33 did not complete Session 2, 1 previously completed Study 1, 3 did not complete Session 2 within the stipulated amount of time following Session 1, 5 were exposed to key information about Session 2 of the study before completing it, and 1 did not indicate consent on the online consent form during Session 1. Therefore, the final sample included 161 college students who completed both parts of the study (87.0% female; age: $M =$ 19.64, $SD = 1.05$, range = 18-22).

Study 2b. Participants were undergraduate students recruited from communication and sociology courses at a Russian research university. A total of 134 students participated in the study, but 24 missed Session 1 and 14 missed Session 2. Our final sample therefore consisted of 96 students (85.4% female). The mean age was approximately 19.4 years old ($SD = 1.4$).[7]

Procedure and materials.

Study 2a. Study 2a consisted of two sessions held a minimum of 3 days apart ($M = 6.1$, $SD = 3.2$; range: 3 to 22 days). In Session 1, which was conducted online, we again assessed a

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[7] Demographic information on race/ethnicity were not collected for either of these studies.

number of individual differences in personality and motivation, including the same measures of

math determination and grit (in that order) as in Study 1. The order of items within each scale

was not randomized. Note that math determination was administered with a 1 ("Strongly

disagree") to 6 ("Strongly agree") scale in this study. In Session 2, which was conducted in the

lab using paper and pencil, participants completed a number of measures including the

preference-based math persistence task described below. These sessions (along with the sessions

in Study 2b) were part of a larger project centered on identifying variables that predict math

problem-solving persistence and, as such, included two experimental conditions and several

other measures that are not of primary interest given the present aims (see supplementary

materials for details). Descriptive statistics and zero-order correlations for all measures are

presented in Table 2a.

   Study 2b. Study 2b consisted of sessions, held one week apart for all participants, that

were very similar in design to Study 2a. Both sessions were administered using paper and pencil

at the end of regular class time. In Session 1, participants completed measures of grit and math

determination, along with several other scales. In Session 2, participants completed the same

preference-based math persistence task used in Study 2a (for additional measures, which differed

somewhat from Study 2a, see the supplementary materials). The same math determination scale,

math persistence task, and instructions used in Study 2a were translated into Russian by the fifth

author and then back-translated by the fourth author to check for accuracy. Grit was measured

using a 5-item Russian version of Duckworth and colleagues' perseverance subscale that has

been validated in recent research (Tyumeneva, Kardanova, & Kuzmina, in press). Note that the

translation of the Russian scale back to English indicated that the wording of some of the items

varied from the original scale (see Table S1 in the supplementary materials). Participants

responded to both the grit and math determination measures using 1 ("not like me at all") to 5

("very much like me") scales. Two participants did not rate the last item on the grit scale; they were excluded from all analyses involving this measure. Descriptive statistics and zero-order correlations for all measures are presented in Table 2b.

Behavioral persistence in math (Session 2 of both studies). For our preference-based measure of math persistence, we looked for a set of problems that could be framed as a math task, would be very difficult to complete in the allotted time, and would leave participants feeling uncertain about their performance. We ended up selecting 12 Raven's Advanced Progressive Matrices (Raven & Court, 1998) that varied in difficulty. The problems were presented as a "non-verbal math reasoning test." Before the matrices were presented, participants were given instructions about how to go about solving the problems and completed a practice problem with the experimenter. They were told that the problems on the actual test ranged from relatively easy to really hard, that they had 5 minutes to complete the problems, and that most people are not able to complete all 12 in that amount of time. Finally, they were informed that their score would be based on how many problems they solved correctly and that we wanted them to do their very best. Participants' scores on the task reflect the percentage of the 12 problems that they solved correctly.

After the 5 minutes had expired, participants were told that they had a "choice of what task to do" next: they could either (A) "continue working on the items that [they] didn't finish or that [they] were unsure about from the previous test," or (B) complete "a verbal test that is not related to math reasoning." Participants were first asked to indicate their choice of what task to do next by circling A or B. Circling A indicated participant's decision to *persist* in completing the math-framed reasoning task and was coded as 1; the choice of Task B was coded as 0. After making their dichotomous choice, participants were asked to indicate how much they preferred one option over the other using a 7-point scale (1 = "Strongly Prefer A"; 4 = "No preference"; 7

= "Strongly Prefer B").[8] This item was included to increase the sensitivity with which we could

measure task preferences. For all analyses including this variable, participants' ratings were

reverse coded, such that higher values indicated a greater preference to persist on the math task.

Note that 5 participants in Study 2a and 5 in Study 2b provided a rating for the second

item that conflicted with their choice from the first item—for example, they circled B as the task

they wanted to complete next, but then expressed a preference for A on the subsequent rating

scale. Because the contradiction could not be resolved based on the recorded data, we present

analyses of task persistence that were conducted both with these participants excluded and

without them excluded (where indicated). In addition, there were 4 participants in Study 2a and

12 in Study 2b[9] who did not circle A or B, but who did go on to rate their task preference. These

participants were excluded from the analyses of task choice, but were included in the analyses of

task preference.

Results

Factor analyses. To assess whether the items in the grit and math determination scales

measured two distinct constructs, we conducted an EFA for each study. When we ignored

extraneous above-threshold factors and forced a two-factor solution in Study 2a, all items loaded

cleanly onto the appropriate factors, with no cross-loadings. When we did the same in Study 2b,

most items tended to load cleanly onto the appropriate two factors, although one of the grit items

cross-loaded and one of the math determination items did not load onto either factor.

Next, we conducted CFAs for each study. In both cases, the two-factor model did not fit

as well as it did in Study 1: $\chi^2(76) = 174.60$, CFI = .90, RMSEA = .09, SRMR = .08 [Study 2a]

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[8] In Study 2a, one participant's rating was coded as 2.5 because he/she marked both 2 and 3.
[9] The greater percentage of participants who skipped the choice item (compared to Study 2a) can be attributed to a difference in formatting.

and $\chi^2(64) = 114.88$, CFI = .82, RMSEA = .09, SRMR = .09 [Study 2b]. However, for both

studies, the model fit the data significantly better than a model with only one factor or a two-

factor model in which the factors were *not* allowed to covary, $\chi^2 > 125.48$, *p*s < .001. For more

details about both sets of analyses, including our factor retention decisions for the EFAs, see the

supplementary materials.

      Descriptive statistics regarding math persistence. We first examined whether the math

task students worked on was as challenging as intended. Indeed, we found that of the 161

participants who completed both sessions in Study 2a and the 96 participants who completed

both session in Study 2b, the mean percentages of correct answer were 26.4% (*SD* = 11.3%) and

24.3% (*SD* = 11.8%), respectively. These low scores reflect the fact that, on average, participants

in both studies only gave responses for a little more than half of the problems (Study 2a: *M* =

7.27, *SD* = 2.09; Study 2b: *M* = 5.88, *SD* = 2.14) in the allotted time. Furthermore, the mean

percentages of problems solved correctly out of problems attempted were 46.9% (*SD* = 23.2%)

and 53.3% (*SD* = 24.1), respectively. These findings confirm that our math task was experienced

as challenging by both American and Russian participants.

      Of the 152 participants in Study 2a who indicated which task they would like to complete

next (i.e., circled A or B) and who did not provide a conflicting response on the subsequent

Likert preference item, 56.6% chose to "continue working on… the previous test," thus

demonstrating a desire to persist on the math task. For the preference rating, the mean of the 156

participants who did not give a conflicting response was 4.19 (*SD* = 1.75), indicating the lack of

a preference for the A or B task on average, *t*(155) = 1.35, *p* = .18.

      By contrast, of the 79 participants in Study 2b who indicated which task they would like

to complete next (i.e., circled A or B) and who did not provide a conflicting response on the

preference item, approximately 69.6% chose B (i.e., "a verbal test that is not related to math

GRIT AS A PREDICTOR OF MATH PERSISTENCE

reasoning"), thus demonstrating a *lack* of persistence on the math-framed reasoning task. For the preference rating, the mean of the 91 participants who did not give a conflicting response was 3.46 (*SD* = 1.68), indicating a slight, but significant preference for the B task on average, $t(90)$ = 3.07, p = .003.

Grit and math determination as predictors of behavioral persistence in math. In Table 2, correlations are reported for the 156 participants in Study 2a and the 91 who did not give conflicting responses on the persistence measure. In both studies, we found that grit was significantly correlated with math determination, replicating the results of Study 1. That is, the stronger students' trait-level tendency to persevere, the more they viewed themselves as being able to persevere on challenging math problems.

Most critically, we found that math determination significantly predicted the continuous measure of behavioral persistence in both studies (see Table 2). Note that initial analyses of these variables that included the participants who gave conflicting results yielded a significant association in Study 2a, $r(159)$ = .22, p = .005, but only a marginal association in Study 2b, $r(94)$ = .18, p = .08. In addition, because some participants did not make a choice (i.e., circle A or B) before rating their task preference and because these participants may have misinterpreted the preference rating scale (like the participants who gave conflicting responses), we conducted another set of analyses that excluded both sets of participants. In this case, the correlation between math determination and task preference remained significant in Study 2a, $r(150)$ = .26, p = .001, and marginally significant in Study 2b, $r(77)$ = .21, p = .07. Finally, with respect to the association between math determination and the dichotomous measure of behavioral persistence (which was presumably less sensitive than the continuous measure), the point-biserial correlation was significant in Study 2a, but not Study 2b (see Table 2). Overall, the results of both studies replicated those of Study 1, finding that the more students perceived themselves as capable of

persevering on challenging math problems (Session 1), the more they preferred to continue

working on the challenging math task in Session 2.[10]

Also replicating Study 1, but in contrast with prior research on less challenging math

contexts (e.g., Eskreis-Winkler et al., 2016; Galla et al., 2014; Lucas et al., 2015), the

correlations between grit and the two indexes of behavioral math persistence were not

statistically significant in either study (see Tables 2a and 2b). In other words, we did not find a

direct relation between students' trait-level propensity for perseverance and their preference to

continue working on a challenging math task.

Although we did not find a direct association between grit and behavioral persistence in

either study, we were again interested in whether grit indirectly predicted persistence through

math determination. We therefore conducted tests of intervening variable models as in Study 1.

Both studies replicated Study 1: the 95% confidence intervals for the indirect effects excluded

zero, suggesting that grit had an effect on task persistence via math determination, even though

there was no direct effect (see Figures 1b and 1c). Taken together, the results of Studies 2a and

2b therefore provide further evidence (with a different dependent measure and across two

populations) that math determination may be a better predictor than grit of students' behavioral

persistence on challenging math problems.[11]

General Discussion

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[10] It may be of interest to some readers that math *value*, which we assessed in Session 1 of Study 2a (but not in
Session 1 of the other studies), was strongly correlated with math determination and significantly predicted
behavioral persistence (a similar pattern to what we observed with math confidence; see the supplementary
materials).

[11] For all three studies, we conducted additional regression analyses to examine the potential interaction effects of
math determination × task/test performance and/or math determination × math confidence on persistence. These
analyses yielded inconsistent results. For the results of these analyses and for a description of other regression
models we examined across the studies, see the supplementary materials.

Across three studies conducted with distinct measures and with different populations, we found that math determination was significantly correlated with participants' behavioral persistence on challenging math problem-solving tasks. In contrast, the trait-level perseverance component of grit did *not* directly predict behavioral persistence, contrary to our expectations. Although there was no direct association between grit and persistence, we consistently found a significant indirect effect of grit on persistence through math determination. These findings suggest that the grittier students were (at a trait level), the more they perceived themselves as capable of persevering on challenging math problems, which in turn predicted how much they actually persisted on a challenging math task. Importantly, this pattern held regardless of whether persistence was measured in terms of time-on-task (Study 1) or in terms of participants' preference for continuing with the task (Studies 2a-2b), and whether participants were American (Studies 1-2a) or Russian (Study 2b).

General Trait versus Domain-Specific Predictors of Behavioral Persistence

Our finding that grit only indirectly predicted behavioral persistence in the math domain through math determination is consistent with Bandura's (2012) assertion that personality traits are sometimes too general to adequately predict specific behaviors—that "the convenience of all-purpose global tests of personal attributes is gained at the cost of explanatory and predictive power" (p. 358). Thus, for researchers and educators interested in predicting students' levels of effort and persistence on tasks in a particular domain, they may be better off assessing domain-specific factors rather than grit. This is in line with Wigfield's (1997) discussion of "the importance of looking at motivation within particular domains" (p. 65).

This raises important questions about the role of grit in predicting and producing different kinds of outcomes. As discussed, grit has been widely touted as an important predictor of positive academic outcomes (such as grade point average, retention, and intention to persist in

college) across multiple academic domains (Credé et al., 2017; Eskreis-Winkler et al., 2014;

Eskreis-Winkler et al., 2016; Muenks et al., 2017, 2018). The present findings, however, join

more recent contributions in suggesting that specific outcomes—like math persistence (Studies

1-2b), performance in a specific course (Schmidt et al., in press; Muenks et al., 2018), and even

overall academic performance (Cormier et al., 2019; Schmidt et al., in press)—might be better

predicted by more domain-specific factors—like math determination, course- or subject-specific

grit and self-efficacy, and school-specific grit[12], respectively—than by domain-general grit.

More critically, the present findings—especially the indirect association between grit and

behavioral persistence in math via math determination—advance these recent investigations by

suggesting that, when examining narrow or specific outcomes, grit's predictive power may run

through more proximal domain-specific factors. Future work should seek to further explore

whether the direct predictive power of grit wanes at greater levels of outcome specificity; and, if

so, it should aim to identify other intervening variables that may account for indirect relations

between grit and specific outcomes.

Persistence on Challenging Versus Easier Math Problems

The lack of a direct association between grit and math persistence contrasts with prior

work by Eskreis-Winkler et al. (2016), Galla et al. (2014), and Lucas et al. (2015). However, as

discussed, these studies used tasks (i.e., standard textbook math problems, one-digit subtraction,

and three-digit addition problems, respectively) that did not consistently pose a high level of

mathematical challenge for the middle school, high school, and adult participants. Indeed, in

Eskreis-Winkler et al.'s (2016) study, participants began with relatively easy problems and only

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[12] Given its similarities with school- and subject-specific grit (e.g., "In school/in mathematics I finish whatever I begin."; Cormier et al., 2019; Schmidt et al., in press), we note that math determination (e.g., "When I don't understand a problem, I keep working until I find the answer") could perhaps also be considered a domain-specific version of grit.

advanced to a more challenging level when they correctly answered five problems in a row. And, in the other studies, the problems were not inherently challenging from a mathematical perspective. Thus, what may have created the perception of challenge for participants were the situational demands of the various tasks—namely, the fact that participants had to sustain their attention over a 45-minute period (Eskreis-Winkler et al., 2016), complete a tedious and boring task (Galla et al., 2014), or solve as many addition problems as possible in a short amount of time while being outperformed by a competitor (Lucas et al., 2015).

In these cases, participants may have viewed the tasks as reflecting a general challenge of having to remain attentive, overcome boredom, work within strict time constraints, or outperform a more skilled opponent. Participants' persistence in the face of such challenges may have reflected their trait-level perseverance (i.e., grit). In contrast, the participants in our studies were presented with tasks that were challenging due to the nature of their content (i.e., challenging math word problems in Study 1 and challenging math-framed reasoning problems in Studies 2a-2b) and our results confirmed that they were in fact experienced as challenging. Thus, students' decisions to persist were more likely to reflect a domain-specific determinant of math persistence—specifically, their determination to persevere on challenging math problems.

Limitations and Future Directions

While the studies presented here provide fairly consistent results regarding the relations between grit, math determination, and behavioral persistence in math, it is important to acknowledge limitations of the present work and consider opportunities for future research. First, while the results were consistent across studies, the effect sizes were relatively modest. That being said, the correlations we observed between measures of math determination and behavioral persistence (rs = .24-.34) were substantially larger than the mean correlation that Credé et al. (2017) found between grit and academic performance across 39 studies (r_{obs} = .15). In addition, a

wealth of research has shown that "small effects may have enormous implications in a practical

context" because "in ongoing processes [they] may accumulate over time to become large

effects" (Prentice and Miller, 1992, p. 163; see also Cohen & Sherman, 2014; Yeager & Walton,

2011). Future longitudinal research should explore this possibility.

Second, although we found an indirect effect of grit on behavioral persistence through

math determination in all three studies, grit and math determination were assessed at the same

time point. Thus, as with all correlational data, the causal links suggested by these intervening

variable analyses should be considered tentative. For instance, although we have argued that high

levels of grit may lead people to become more determined in the math domain, it is certainly

possible that students' domain-specific beliefs also contribute to their general conception of

themselves as gritty and perseverant in a bottom-up manner.[13] Future research should more

carefully investigate the causal directional of the associations observed in the present studies by

using experimental or longitudinal designs.

Relatedly, it is also important to consider potential unmeasured third variables. As

discussed, a broad personality factor like grit is likely to exert its influence on domain-specific

outcomes like math persistence through several indirect paths involving distinct intervening

variables. While the present studies provide consistent evidence supporting one such path—

through math determination—there are likely other important intervening variable to consider.

This point is supported by the results of Study 1, where the direct association between grit and

persistence went from non-significant to significantly negative when math determination was

accounted for. As an illustration of how a predictor variable may influence an outcome variable

in opposite directions via two separate indirect pathways, consider McFatter's (1979) discussion

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[13] Relevant to this, we found a significant *negative* indirect effect of math determination on behavioral persistence through grit in Study 1, but not Studies 2 and 3 (see the supplementary materials).

of the numerous potential relations between intelligence and performance. Whereas more

intelligent people are generally assumed to have higher levels of certain cognitive abilities and

are therefore expected to perform better on intellectual tasks, the same people may also exhibit a

lower tolerance for boredom, which could ultimately lead to disengagement and hurt

performance. As a result, the indirect effect of intelligence on performance through cognitive

ability would be positive, while its indirect effect through boredom would be negative.

Combined, these two effects may cancel each other out in some cases, resulting in a total effect

of intelligence on performance that is equal to zero. And there are likely many others cross-

cancelling paths by which intelligence might positively and negatively affect performance at the

same time. This may also be true of grit: while its indirect effect on math persistence through

math determination may be positive (as the present studies suggest), grit may also have indirect

effects on persistence that are negative. For example, students with higher levels of grit have

been shown to be more strategic in how they use their academic time (Wolters & Hussain, 2015),

which may make them spend less time (i.e., persist less) on academic tasks that are not

associated with their actual courses (i.e., lab-based math problems). Future research should

therefore continue to test for intervening variables between grit and important academic

outcomes.

Furthermore, our samples were primarily female. As prior research has found that women

report weaker math self-concepts than men (Else-Quest, Hyde, & Linn, 2010), future research

should seek to replicate the present results with more gender-balanced samples.

Finally, as previously discussed, the three studies reported here were originally conducted

as part of two separate projects. While the overlap in methods made it appropriate to examine

them together, the fact that these were initially separate projects with separate aims means that

most of the analyses conducted for the present paper were post hoc and exploratory in nature.

Indeed, we only formalized the conceptual relations between grit and math determination after beginning to interpret the results of the three studies. But, again, the results were very consistent across all three studies, which varied in terms of both culture and our assessment of persistence. It is therefore our hope that the present findings can serve as a starting place for future hypothesis-driven studies of the relations between grit, math determination, and math problem-solving persistence.

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Table 1

Zero-order correlations between primary Session 1 and 2 measures, Study 1.

	1	2	3
Session 1 Measures			
1. Grit	--		
2. Math determination	.24**	--	
Session 2 Measure			
3. Time-on-task (behavioral persistence)[a]	-.10	.25**	--
Mean	3.88	3.33	18.65[a]
Standard Deviation	.60	.72	6.09
Internal Consistency (α)	.80	.88	--

$N = 137$, $^*p < .05$, $^{**}p < .01$, [a] Behavioral persistence reflects the square root of the mean time (in seconds) participants spent on the two problems.

Table 2

Zero-order correlations between primary Session 1 and 2 measures in (a) Study 2a[1] and (b) Study 2b.[2]

(a)	1	2	3	4[a]	5
Session 1 Measures					
1. Grit	--				
2. Math determination	.35**	--			
Session 2 Measures					
3. Math task score (percent)	-.17*	.09	--		
4. Task choice (behavioral persistence 1)[a]	-.04	.20*	.24**	--	
5. Task preference (behavioral persistence 2)	.01	.24**	.24**	.85**	--
Mean	3.89	4.12	26.5%	56.6%	4.19
Standard Deviation	.66	.75	11.4%	49.7%	1.75
Internal Consistency (α)	.85	.87	--	--	--

$N = 156$, *$p < .05$, **$p < .01$. [a] Because task choice is a dichotomous variable, all correlations with other variables in this table are point-biserial. For Study 2a correlations involving this variable, $N = 152$.

(b)	1	2	3	4[b]	5
Session 1 Measures					
1. Grit	--				
2. Math determination	.29**	--			
Session 2 Measures					
3. Math task score (percent)	-.10	-.04	--		
4. Task choice (behavioral persistence 1)[b]	-.01	.12	.05	--	
5. Task preference (behavioral persistence 2)	.06	.25*	.08	.88**	--
Mean	3.51	3.44	23.8%	30.4%	3.46
Standard Deviation	.62	.63	11.2%	46.3%	1.68
Internal Consistency (α)	.73	.77	--	--	--

$N = 91$, *$p < .05$, **$p < .01$. Because not all participants completed the grit and task choice measures, correlations reflect differing degrees of freedom, with a maximum of 89. [b] Because task choice is a dichotomous variable, all correlations with other variables in this table are point-biserial.
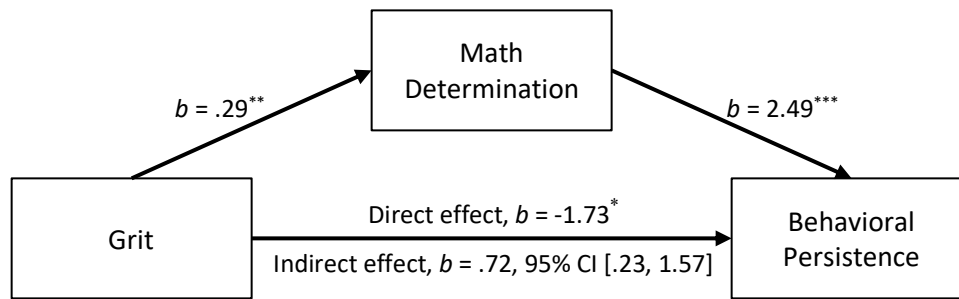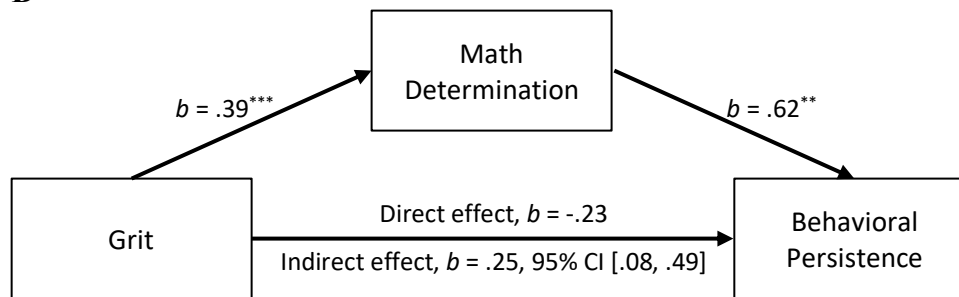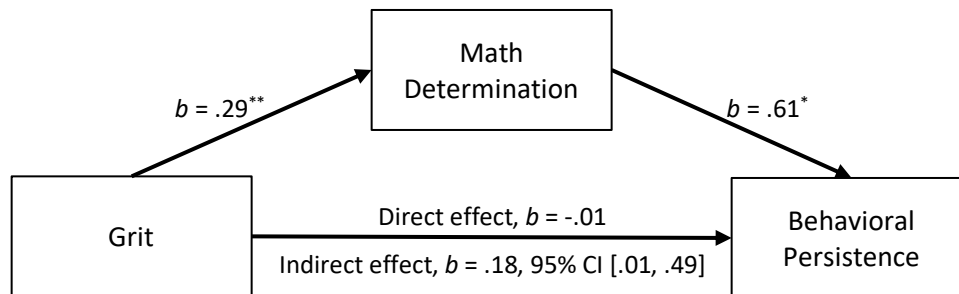
A

B

C

Figure 1. Intervening variable models illustrating the indirect effects of grit on math problem-solving persistence through math determination in Study 1 ($N = 137$) (A), Study 2a ($N = 156$) (B), and Study 2b ($N = 89$) (C). The unstandardized regression coefficient (b) for each indirect path was taken from a model that included all predictors preceding the path. $^{*}p < .05$. $^{**}p < .01$. $^{***}p < .001$.

Appendix

Behavioral Persistence Task (Study 1)

<u>Solvable problem</u>

A car travels downhill at 72 mph (miles per hour), on the level at 63 mph, and uphill at only 56 mph. The car takes 4 hours to travel from town A to town B. The return trip takes 4 hours and 40 minutes.

Find the distance between the two towns.

Answer: 273 miles

<u>Unsolvable problem</u>

Make two three-digit numbers using the following digits -- 1, 3, 4, 5, 6, 9 -- so that one of these three-digit numbers is exactly five times greater than the other (each digit should be used only once).

Math Determination (Studies 1, 2a, and 2b)

(formatting, including underlining, varied between administrations of the scale)

Please read each item carefully and <u>rate the extent to which the item describes you</u> using the following scale [the scale that appears below it]. There are no right or wrong answers. We are interested in your opinions.

1. If I am struggling to solve a problem, I try to think of alternative ways to solve it.

2. When I don't understand a problem, I keep working until I find the answer.

3. If I cannot do a math problem in a few minutes, I probably can't do it at all. (R)

4. Math problems that take a long time don't bother me.

5. If I can't solve a math problem quickly, I quit trying. (R)

6. I feel I can do math problems that take a long time to complete.

7. I find I can do hard math problems if I just hang in there.

8. I'm <u>not</u> very good at solving math problems that take a while to figure out. (R)

Supplementary Materials

Table of Contents

Order in which the Studies were Conducted, and Development of the Framing

As mentioned in the main text, these studies were originally designed as part of separate (but related) projects. Study 1 was conducted first, followed by Study 2b and then Study 2a. We analyzed the data for Study 1 with the general aim of identifying motivational and personality predictors of behavioral problem-solving persistence in the math domain, and identified a dissociation between the effects of grit and what we initially thought of as a grit-like, self-report measure of math persistence. At this point, we re-conceptualized our self-report measure of math persistence in terms of math-specific grit (rather than math determination). Later we examined the data from Study 2b and found the same dissociation between the effects of domain-general

grit and math-specific grit on behavioral persistence. Finally, after we finished collecting the data for Study 2a, we found the dissociation for a third time. Only after we had analyzed data from all three studies did we begin to explore the possibility that math-specific grit (math determination) might act as an intervening variable in the relation between general grit and math problem-solving persistence, which is why we described this aim of the investigation as post hoc.

We ultimately decided to re-conceptualize math-specific grit as math determination because we realized (based on feedback on a prior version of the manuscript) that our measure might be better characterized as an aspect of math self-concept (given its strong correlation with math confidence), rather than as a domain-specific version of a general personality trait. We also re-conceptualized what we had been calling math self-efficacy as math confidence, because our measure did not assess students' efficacy for specific tasks and because it seemed to be more in line with the confidence aspects of self-concept. In a previous version of the manuscript, we suggested math determination and math confidence could both be considered aspects of math self-concept – but noted that this post hoc realization was in part due to the strong correlations we observed between the two measures across Studies 1 and 2a. In the current version of the manuscript, we decided to focus on the results that were most consistent across studies and, therefore, moved our description and analysis of math confidence to the supplementary materials. For more information about the evolution of our theoretical framing of the manuscript, please contact the first author.

In this supplementary document, we report analyses that are relevant to understanding the results reported in the main text (e.g., the factor structure of the grit and math determination scales). We also document many (but not all) of the analyses we conducted while working on prior versions of this paper that are not central to its current focus. Note that, for each of the three studies, some preliminary analyses may have been conducted (but not carefully documented) that

differed from our final analyses and the analyses in this document in terms of which participants were included/excluded. However, in a section below, titled "Additional Details About Methods," we do point out some changes in participant exclusions that we made throughout the process of working on the current version of this paper. We also use this section to describe additional sample characteristics, procedures, and measures not reported in the main text.

Distinguishing Between Grit and Math Determination

To examine whether the items from the grit and math determination scales loaded into two distinct latent factors, we conducted a series of exploratory factor analyses (EFAs) in SPSS, followed by a series of confirmatory factor analyses (CFAs) using the lavaan package in R. The EFAs used principal axis factoring, an initial eigenvalue threshold of 1 (i.e., the Kaiser rule), and direct oblimin rotation (because the resulting factors were correlated).

In the Study 1 EFA ($N = 137$), two factors were extracted (initial eigenvalues: 4.83, 2.64), which were responsible for the common variance constituting 46.2% of the total variance. The pattern matrix (see Table S1) indicated that all items loaded cleanly onto the appropriate factors (loadings $\geq$ .3), with no cross-loadings. The results of Study 2a EFA ($N = 156$) were very similar to the results of the Study 1 factor analysis, except that a third factor which was below the eigen threshold in Study 1 (.96) was just above threshold (1.04). After looking at the pattern matrix for the three-factor solution (with only two items loading onto the third factor, one of which cross-loaded onto the math-determination factor), we decided to re-run the analyses and extract only two factors (initial eigenvalues: 5.23, 2.60), which were responsible for the common variance constituting 49.4% of the total variance. The pattern matrix (see Table S1) again indicated that all items loaded cleanly onto the appropriate factors, with no cross-loadings. The results of the Study 2b EFA ($N = 89$) diverged a bit from the results of the previous two studies (perhaps because the items in this study were presented in Russian). Initially, four factors surpassed the

eigen threshold (initial eigenvalues: 3.84, 1.99, 1.23, and 1.07); however, because extraction of

four factors failed to converge on a stable solution, we again decided to re-run the analysis and

extract only two factors, which were responsible for the common variance constituting 36.0% of

the total variance. The pattern matrix (see Table S1) indicated that most items tended to load

cleanly onto the appropriate factors. However, one of the grit items cross-loaded and one of the

math determination items did not load onto either factor.

We followed up the EFAs by testing a two-factor confirmatory model for each study

using maximum likelihood estimation. In this model, the latent factors were allowed to covary.

Across the three studies, we found that this model fit the data significantly better than a model

with only one factor or a two-factor model in which the factors were *not* allowed to covary, $\chi^2 >$

124.72, $ps < .001$. For Study 1, the model showed acceptable fit, $\chi^2(76) = 116.31$, CFI = .94,

RMSEA = .06, SRMR = .05. Three items from the grit scale and one item from the math

determination had relatively weak loadings (.38 to .49); all other loadings were greater than .5.

The correlation between the latent factors was .28, $p = .002$. For Study 2a, the model did not fit

the data as well as in Study 1, $\chi^2(76) = 174.60$, CFI = .90, RMSEA = .09, SRMR = .08.

Removing the two weakest loading items (one from each scale; .42 and .38), resulted in

acceptable fit, $\chi^2(53) = 99.99$, CFI = .94, RMSEA = .08, SRMR = .06. The correlation between

the latent factors in the original model was .35, $p < .001$. Finally, we found that the model for

Study 2b had the poorest fit, $\chi^2(64) = 114.88$, CFI = .82, RMSEA = .09, SRMR = .09. In this

model, one item from the math determination scale had a particularly small loading (.13).

Removing this item, along with two additional items (one from each scale; both .37) did not

improve the fit of the model $\chi^2(34) = 77.37$, CFI = .83, RMSEA = .12, SRMR = .09. The

correlation between the latent factors in the original model was .42, $p < .001$. The relatively poor

fit of this model can perhaps be partly attributed to the relatively small sample size ($N = 89$) (Iacobucci, 2010; Jackson, 2001; cf. Fan, Thompson, & Wang, 1999; Tanguma, 2001). In fact, the results of the CFAs from all three studies should be interpreted with caution due to the sample sizes being near or below the minimum sample size suggested by some authors for this type of model (e.g., Wolf, Harrington, Clark, & Miller, 2013).

Overall, the EFAs and CFAs suggest that the items from the grit and math determination scales loaded onto two separate latent factors that were moderately correlated with each other. Although there were some items with relatively weak loadings, only one item did not appear to measure the construct it was intended to capture, and this was only in the case of Study 2b. Furthermore, including this item as part of the math determination scale did not substantially decrease the scale's internal consistency ($\alpha = .78$ without the item versus .77 with the item). Therefore, we decided that it was fine to retain all of the items when computing mean scores for each scale (as we had done in our original analyses).

The Factor Structure of Math Determination

Our initial EFAs (see the section on math confidence below) examined the overlap between our math determination and math confidence scales, when math confidence was still a focal measure in the main manuscript. Because we did not include a measure of math confidence in Session 1 of Study 2b, we also conducted a factor analysis with only the math determination items ($N = 91$). This analysis served as initial examination of the scale's dimensionality (in a Russian sample). We found that, in Study 2b, the items actually loaded onto two factors (eigenvalues: 3.17, 1.18), with the first factor only accounting for the common variance constituting 32.6% of the total variance (prior to rotation) and the second factor accounting for the common variance constituting 7.4% of the total variance. In contrast, a comparable analysis

for Study 1 found that the items loaded onto one factor (eigenvalue: 4.37), which was responsible for the common variance constituting 48.3% of the total variance. Although the analysis for Study 2a found that the items loaded onto two factors (like in Study 2b), the pattern was actually more similar to Study 1 (eigenvalues: 4.25, 1.02), with the first factor responsible for the common variance constituting 47.5% of the total variance, and the second factor responsible for the common variance constituting 7.1% of the total variance. In addition, the communalities for the items in Study 2a were more similar to the communalities in Study 1 than those in Study 2b, which is not surprising considering that Studies 1 and 2a sampled from similar populations. With respect to the loadings in Study 2b (from the pattern matrix after oblimin rotation), six out of the eight items loaded onto the first factor, two items loaded on the second factor (including one item that also loaded onto the first factor), and one item did not load onto either factor.[1]

Math Confidence

  Assessing math confidence. As mentioned in Footnote 3 of the main text, math confidence—a construct traditionally associated with math self-concept in the educational psychology literature (see Marsh, Byrne, & Shavelson, 1988; Pajares & Miller, 1994; West, Fish, & Stevens, 1980)—was also assessed in Studies 1 and 2a (in addition to grit and math determination). Specifically, it was assessed during Session 1 using nine or eight (respectively) of the twelve items from the widely-used Confidence in Learning Mathematics Scale (Fennema & Sherman, 1976). Items included "I have a lot of self-confidence when it comes to math" and "I

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[1] This item also exhibited a relatively low item-total correlation in our reliability analyses for Study 2b. However, it did not substantially decrease the alpha for the full scale ($\alpha = .78$ without the item versus .77 with the item). Also note that the EFA for Study 2b was initially conducted with $N = 96$, but we decided to report the EFA conducted *without* participants who gave conflicting responses on the persistence measure, to be consistent with our reporting of the analyses from Study 2a.

don't think I could do advanced mathematics" (reverse-scored), and participants responded using a 1 ("Strongly disagree") to 6 ("Strongly agree") scale (see Table S2 for all items). We computed a math confidence index by reverse-coding the negative items and then averaging participants' ratings across all nine or eight items.

Results of analyses including math confidence.

Study 1. Although the two measures of math self-concept (confidence and determination) were strongly correlated with each other, only math determination was significantly correlated with grit (see Table S3). Furthermore, replicating prior research (e.g., Pajares & Graham, 1999; Skinner, Wellborn, & Connell, 1990), there was a statistically significant positive correlation between math confidence and behavioral persistence.

To examine the simultaneous effects of math confidence and the primary Session 1 variables on behavioral persistence, we conducted a hierarchical regression analysis with grit and math determination as predictors in Step 1. We then added math confidence as a predictor in Step 2. The variance inflation factor (VIF) for all variables in all steps was within an acceptable range. As shown in Table S4, math determination remained a significant predictor of behavioral persistence when controlling for grit. Interestingly, a significant negative effect of grit emerged in this model. When math confidence was added in Step 2, the effects of grit and math determination dropped to marginal significance, while the effect of math confidence was non-significant. For a description of additional regression analyses, see the relevant section below.

Furthermore, in contrast to the bootstrapping analyses discussed in the main text, when math confidence was included as the intervening variable between grit and math persistence, revealed no significant indirect effect, $b = .20$, 95% CI [-.20, .74]. In addition, the direct effect of grit was not significant in this analysis, $b = -1.21$, $p = .15$.

Study 2a. Replicating the results of Study 1, we found that the two measures of math self-concept (confidence and determination) were strongly correlated with each other (see Table S5). In addition, we found that grit was significantly correlated with math determination. However, in contrast Study 1, we also found a significant correlation between grit and math confidence. That is, the stronger students' trait-level tendency to persevere, the more they viewed themselves as being able to persevere on challenging math problems and the more confident they were in their math abilities in general.

To examine the simultaneous effects of the Session 1 variables on behavioral persistence, we conducted a hierarchical regression analysis with participants' continuous ratings of task preference as the dependent variable (given that it represents a more sensitive measure of persistence than task choice) and performance on the math-framed reasoning test, grit, and math determination as predictors in Step 1. We included math task score (percent correct) because it was significantly correlated with grit, math confidence, and both measures of persistence. We then added math confidence as a predictor in Step 2. The variance inflation factor for all variables in all steps was within an acceptable range.

As shown in Table S6, the results of the analysis showed that in Step 1, both math task score and math determination were significant predictors of task persistence, whereas grit was not. However, when math confidence was added as a predictor in Step 2, it accounted for nearly all of the shared variance between math determination and task persistence and explained an additional 4% of the variance in task persistence.

Furthermore, in contrast to Study 1, an indirect effect analysis with bootstrapping that included math confidence as the intervening variable revealed a significant indirect effect of grit on persistence via confidence, $b = .22$, 95% CI [.06, .46], but no direct effect of grit on persistence, $b = -.20$, $p = .33$. Given that math confidence appeared to account for the association

between math determination and behavioral persistence in our regression analysis, we also conducted a number of additional indirect effect analyses to explore whether or not math determination and math confidence represented distinct pathways in the relation between grit and behavioral persistence. The results of these analyses, which are reported in a subsequent section of this document, revealed a significant sequential pathway from grit to math determination to math confidence and ultimately behavioral persistence. That is, the indirect effect of grit through determination appeared to be itself accounted for by an indirect path through math confidence. The same sequential path was not significant in Study 1.

Note that the causal links suggested by this sequential analysis should be considered tentative. Although it is possible that there was an effect of math determination on behavioral persistence that was mediated by math confidence, it could instead be the case that confidence was a common cause of both determination and persistence (such that these two variables were not causally related). Interestingly, we found evidence of a reciprocal relation between math determination and math confidence when testing indirect effects models that included these two variables as either the mediator or outcome and grit as the predictor (see below).

Distinguishing between math determination and math confidence. Due to the fact that math determination and math confidence were strongly correlated in Studies 1 and 2a, we were interested in the potential overlap between these measures, as well as the extent to which they independently predicted differences in math persistence. The analyses we conducted to explore these relations are reported below.

In order to identify items from the two scales that uniquely loaded onto two different factors we conducted exploratory factor analyses for Studies 1 and 2a, using principal axis factoring, an eigenvalue threshold of 1, and direct oblimin rotation (because the resulting factors

were correlated).[2] Both analyses resulted in two-factor solutions, which were responsible for the common variance constituting 61% and 62% of the total variance, respectively. However, the pattern matrix for each analysis indicated that 2 items from the math determination scale cross-loaded on the math confidence factor (only one of these items was the same across analyses) and 0-1 items from the math confidence scale cross-loaded on the math determination factor. In addition, in Study 2a, 1 item from the determination scale loaded only on the confidence factor (loadings ≥ .3; see Table S2). Note that in a previous analysis for Study 2a (which included 2 additional participants that were later determined to have given conflicting responses on the persistence measures [see relevant section below]), 4 items from the math determination scale cross-loaded on the math confidence factor.

To produce a measure of math determination that was distinct from math confidence, we averaged across the four items from our earlier analyses that did not cross-load in either study (α = .78 in both studies). This reduced math determination measure was still significantly correlated with math confidence, $r(135) = .52$, $p < .001$ (Study 1), $r(154) = .57$, $p < .001$ (Study 2a), and with math problem-solving persistence, $r(135) = .26$, $p = .002$ (Study 1), $r(154) = .17$, $p = .04$ (Study 2a). To see if these associations were independent of each other, we conducted additional hierarchical regression analyses, with grit and math determination in Step 1 and math confidence added in Step 2. The results of the analysis for Study 1 were similar to the results for the hierarchical regression analysis reported in the previous section (see Tables S4 and S7). That is, math determination was a significant predictor of problem-solving persistence in Step 1.

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[2] We initially conducted a series of analyses using varimax rotation for Study 1, but decided to switch to direct oblimin based on the recommended practice of using an oblique rotation when the factors are correlated (Beavers et al., 2013; Fabrigar, Wegener, MacCallum, & Strahan, 1999; Russell, 2002; Worthington & Whittaker, 2006), particularly if the correlation is .32 or above (Tabachnick & Fidell, 2013, p. 651). Our original analyses based on varimax rotation yielded more cross-loading items, such that only 3 items loaded uniquely onto the math determination factor and 5 items loaded uniquely onto the confidence factor. For correlational analyses based on the results of this EFA (along the lines of what is described in the next paragraph), contact the corresponding author.

However, as opposed to being reduced to marginal significance in Step 2, math determination was still a significant predictor of persistence when controlling for math confidence. In contrast, math confidence was not a significant predictor when controlling for math determination.[3] The results of the analysis for Study 2a were also similar to the results of the analysis in the previous section (see Tables S6 and S8).[4] That is, math determination was a significant predictor of problem-solving persistence in Step 1, but not Step 2. In addition, math confidence was still a significant predictor when controlling for math determination.

Because Study 2b did not include a measure of math confidence in Session 1, we were not able to conduct an EFA for this study that distinguished between math determination and math confidence. However, we did create a subscale score for math determination that was based on the four items identified by the EFAs from the other studies ($\alpha = .62$). The correlation between this subscale score and math problem-solving persistence was not significant, $r(89) = .11, p = .29$. This association was also non-significant when controlling for grit. One possible explanation for the lack of correlation in this study compared to the other studies is that our math determination scale may not have had the same factor structure when translated into Russian as it did in English. If so, then the four cross-loading items (which increased the significance of the correlation with persistence) may have been tapping primarily into the math determination factor.[5]

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[3] For Study 1, an initial regression analysis was conducted with a version of the math determination scale that was based on the six items that did not cross-load onto the math confidence factor. The analysis also did not include grit. In this analysis, math determination was only a marginally significant predictor of persistence when controlling for math confidence.

[4] Although, for Study 2a, the analysis did not include math task score, as in the previous section.

[5] For Study 2b, an initial correlation analysis was conducted with a version of the math determination scale that was based on the six items that did not cross-load onto the math confidence factor in Study 1. The analysis revealed a marginally significant correlation with persistence.

The interactive effect of math determination and math confidence on persistence.

Before we explored indirect effects involving grit, math determination, math confidence, and

math problem-solving persistence, we initially explored the possibility that the association

between math determination and persistence was moderated by math confidence. In particular,

for Study 1, we performed a hierarchical regression analysis with math determination and math

confidence in the first step and the corresponding interaction term in the second step. The results

of the analysis revealed that, although the main effects of the two predictors were not significant

when controlling for each other, $\beta s < .16$, $t s < 1.43$, $p s > .15$, there was a significant interaction, β

$= -.22$, $t(133) = 2.62$, $p = .01$.[6]

To explore the nature of this interaction, we conducted follow-up analyses with the math

confidence variable centered at 1 *SD* above or below the mean (see Aiken & West, 1991). Math

determination was a significant predictor of time on task for participants with relatively low math

confidence, $\beta = .28$, $t(133) = 2.36$, $p = .02$, but not for participants with high math confidence, β

$= -.13$, $t(133) = .81$, $p = .42$.[7]

Next, we examined whether the relation between math determination and task persistence

was moderated in Study 2b. Note, however, that unlike Study 1, Study 2b did not include a

measure of math confidence that was administered *prior* to the persistence task. Rather, math

confidence in the current study was assessed after participants had already made a choice of

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[6] An analysis in which problem type, problem order, and challenge manipulation were entered as additional factors (along with all two-, three-, and four-way interactions, except those involving manipulation and order together) showed that the math determination x math confidence interaction was marginally moderated by order, but not by any of the other variables. A subsequent analysis in which we added the manipulation x order two-way interaction, which was found to be significant in other analyses, again revealed a marginal order x math determination x math confidence interaction.

[7] Note, for this study and for Study 2b, we conducted additional region of significance analyses to explore the interactions (see Preacher, Curran, & Bauer, 2006). Details and figures pertaining to these analyses are available upon request. Also note that we initially conducted follow-up analyses at 1.5 *SD* above and below the mean, in keeping with some our previous research.

whether or not to persist on the math-framed reasoning task. Yet, the current study did provide us

with a measure of task performance, which allowed us to explore whether the association

between math determination and task persistence varied depending on participants' good versus

poor performance on the math-framed reasoning task.[8] The hierarchical regression analysis that

we conducted included standardized versions of the math determination and test score variables

in the first step and their interaction term in the second step. The results revealed a main effect of

math determination, $\beta = .25$, $t(88) = 2.46$, $p = .02$, but no effect of test score, $\beta = .09$, $t(88) = .91$,

$p = .37$. The effect of math determination was qualified by a marginal interaction, $\beta = -.18$, $t(87)$

$= 1.76$, $p = .08$. Parallel to Study 2a, we conducted follow-up analyses with the task performance

variable centered at 1 *SD* above or below the mean (see Aiken & West, 1991). Math

determination was a significant predictor of time on task for participants with relatively low task

performance, $\beta = .44$, $t(87) = 2.98$, $p = .004$, but not for participants with high task performance,

$\beta = .02$, $t(87) = .13$, $p = .90$.[9]

Finally, for Study 2a (which was conducted after Studies 1 and 2b), we investigated both

interactions reported above. However, our separate analyses revealed that neither the math

determination × math confidence interaction, $\beta = .11$, $t(152) = 1.44$, $p = .15$, nor the math

determination × task performance interactions, $\beta = .06$, $t(152) = .75$, $p = .45$, were significant.

While we therefore cannot be as confident in this interaction as in our primary results, we

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[8] In fact, we explored the interaction between math determination and task performance in this study *before* we examined the interaction between math determination and math confidence in Study 1.

[9] In separate analyses conducted *after* we identified the interaction between task performance and math determination (as well as the interaction between math confidence and math determination in Study 1), we did check to see if the math confidence measures that were administered in Study 2b subsequent to participants completing the task and receiving feedback also interacted with math determination; however, neither of these interaction effects were significant. Although one might expect these math confidence measures to reflect differences in task performance and, thus, to also interact with math determination, neither measure was actually correlated with task performance. Also note that we conducted interaction analyses for Studies 1 and 2b using the 6-item version of the math determination scale that excluded items that cross-loaded onto the math confidence factor in the EFA from Study 1. These analyses are available upon request.

include these details with the hope that future research might continue to investigate the possibility that math determination may help compensate for a tendency to withdraw from challenging math tasks when one feels (or is) less capable of performing well in this domain than one's peers.

Discussion of the relation between math determination and math confidence. In Studies 1 and 2a, we found that math determination and math confidence were significant predictors of math problem solving persistence. Because these constructs can both be considered facets of students' math self-concept, it is worth considering whether they account for the same variance in students' behavioral persistence, or whether they serve as unique predictors. In Study 1, when we added math confidence to a regression model that already included math determination, neither determination or confidence had fully significant effects on persistence. However, in Study 2a, we found that math confidence did emerge as a significant predictor when it was added to the model, whereas the effect of math determination dropped to non-significance.

This difference in findings between the two studies is in line with the findings from some of the additional analyses of indirect effects that we conducted (see below). That is, in Study 2a, but not Study 1, we found that math confidence was an additional intervening variable in the indirect path connecting grit and math determination. One possible explanation for this difference is that, when assessing persistence in terms of students' choice or preference to continue working on a challenging task (i.e., in terms of motivation), students with high levels of grit and math determination may be particularly likely to persist if this determination leads them to become more confident about their math ability in general. But, when assessing persistence in terms of the amount of time students spend working on challenging math problems, math determination may have a more direct or independent influence on the extent to which students persist.

However, one question that remains in regard to Study 1 is why the effects of both math determination and math confidence became marginal or non-significant when controlling for each other. One possibility is that both variables had small direct effects on persistence, *as well as* small indirect effects through the other variable. The direct effects may have been too small to emerge as significant in our regression analyses, but strong enough to reduce the effects of the other variable to non-significance.

Another possibility is that math determination and math confidence, as operationalized in our studies, are overlapping constructs and are best assessed by an omnibus measure of math self-concept. Certainly, the two variables were strongly correlated in both studies (rs = .67-.72). In addition, a number of the items from the math determination scale were framed using confidence- or efficacy-oriented language (e.g., "I find I can do hard math problems if I just hang in there"). However, there are several reasons to believe that math determination and math confidence functioned as distinct constructs in our analyses. First, these constructs were differentially associated with grit. As one might expect, math determination was more strongly correlated with grit than was math confidence in both Study 1, $z = 2.56$, $p = .01$, and Study 2a, $z = 2.00$, $p = .046$ (using Lee & Preacher's [2013] test for dependent correlations). Second, only math determination was a reliable intervening variable in the relation between grit and math problem-solving persistence across studies. Third, in a series of exploratory factor analyses (see supplementary materials), we found that many of the items from the math determination and math confidence scales loaded onto two distinct factors, although there were several items from the determination scale and one item from the confidence scale that loaded on both factors.[10]

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[10] When we created new math determination scores based on the subsets of items that loaded onto a single factor, the correlation between math determination and math confidence was still strong (though substantially smaller than before). See supplementary materials for additional analyses involving these revised math determination scores.

In sum, the present studies are a first step in understanding the relation between students' math determination and math confidence. Although these variables can both be considered aspects of math self-concept and were strongly correlated, they functioned as distinct constructs in multiple analyses. However, because most of these analyses were exploratory in nature, there is a need for additional studies that are designed to carefully test the relation between math determination and math confidence before any strong conclusions can be drawn.

Additional Details About Methods

We use this section to describe additional sample characteristics, procedures, and measures not reported in the main text. We also point out some changes in participant exclusions that we made throughout the process of working on the current version of this paper – though it should be noted that some preliminary analyses may have been conducted (but not carefully documented) that differed from what is reported below (and in the main text) in terms of which participants were included/excluded.

Study 1. Although Session 2 was supposed to be completed 14 to 35 days after Session 1, one participant ended up completing Session 2 only 10 days after Session 1. We only noticed that this individual did not adhere to the stipulated time interval after having analyzed the data. In contrast to Study 2a (see below), we kept this participant in the dataset. It seems less likely that this individual's data would deviate from other participants' data due to differences in time, whereas the participants excluded in Study 2a either completed Session 2 soon after Session 1, or in a different semester. We did not note any substantial changes in the results after excluding this additional participant. The association between math determination and problem-solving persistence was still statistically significant with the individual excluded, $r(134) = .25$, $p = .003$.

Because a number of participants who completed the first session of Study 1 did not complete the second session, we examined whether these participants ($N = 37$) differed from the

participants in the final sample (N = 137) in terms of their grit, math determination, and

demographic characteristics. These analyses did not include participants who were excluded

from the final sample for other reasons (e.g., because they completed a preliminary version of

one of the surveys that did not contain all of the items from the final version). The results

showed that the two sets of participants did not vary in their mean levels of grit, $t(172)$ = .96, p =

.34, or math determination, $t(76.96)$ = .56, p = .58 (with a correction for unequal variances). The

two groups marginally differed in terms of gender composition, χ^2 = 2.78, p = .095, with 16.1%

of the final sample being male versus only 5.4% of the participants who did not complete

Session 2. However, the two groups did not differ significantly in terms of age, $t(172)$ = 1.54, p =

.12.

In addition to completing the measures of grit, math determination, and math confidence

in Session 1, participants also completed measures assessing participants' Big 5 personality

traits, their general and math-specific growth mindsets (which we discuss in another paper;

Authors, 2016), and their regulatory focus (promotion vs. prevention).

Participants' Big 5 traits were assessed using the Ten-Item Personality Inventory (TIPI;

Gosling, Rentfrow, & Swann, 2003), which is a brief measure of these personality dimensions.

Gosling and colleagues (2003) have established that this shortened inventory is adequate in terms

of its convergence with longer Big-Five measures, test-retest reliability, and predictive validity.

Each of the Big-Five dimensions in this inventory was measured with two statements, one

positively-keyed (e.g., "I see myself as dependable, self-disciplined") and one negatively-keyed

(e.g., "I see myself as disorganized, careless"), using a six-point scale (1 = "Strongly disagree" to

6 = "Strongly agree"). The statements were presented in a fixed order. Participants' scores on

each dimension was calculated by reverse-coding their responses to the negatively-keyed item

and averaging them with their responses to the positively-keyed item. We had originally included correlational analyses involving the Big 5 in the main manuscript (see Table S3).

The regulatory focus measure was included in the study because of prior research suggesting that different regulatory orientations may be associated with task persistence (see Molden & Rosenzweig, 2016); however, because it was not a focus of the present paper and because it did not correlate with any measures other than extraversion and agreeableness (which has already been established; Lanaj, Chang, & Johnson, 2012), we did not discuss it in the main manuscript.

In addition to the measures already described, we also administered several unpublished questionnaires that assessed participants' beliefs about the nature of memory and effort. The items within each of these questionnaires, which we constructed ourselves, were presented in a random order. These measures are available from the first author upon request. As mentioned in the main manuscript the order of the various measures was randomized across participants (with the exception that two of the memory beliefs questionnaires were always presented in the same order). At the end of Session 1, participants completed a demographic questionnaire.

Session 2 began with an experimental manipulation that involved having participants solve easy or difficult arithmetic problems for four minutes. This manipulation was included primarily to examine the effects of academic mindsets on task persistence (Authors, 2016). See the next section for a description of the analyses that demonstrated the lack of influence this manipulation had on the primary outcome measure and on the relation between each of the primary predictors and the outcome. After completing the experimental task, participants complete an initial pair of questions about participants' math ability: "How good are you at solving math problems?" (1 = "not at all good" to 7 = "very good") and "How confident are you in your math ability?" (1 = "not at all confident" to 7 = "very confident"). After answering these

questions (which were very similar to the items in our math confidence measure and which we label in terms of ability to avoid confusion), participants completed the behavioral persistence task described in the main manuscript. They then completed a second set of math ability questions. This math ability judgment measure was meant to capture any changes in participants' math confidence during Session 2 of the study. Two indexes of math ability judgments were created by averaging together the two items from each administration of the measure. The correlations between these indices and the primary variables discussed in the manuscript are presented in Table S3.

After completing the second set of math ability questions, participants answered questions about the effort and difficulty associated with each of the two tasks in Session 2 (i.e., the experimental task and the behavioral persistence task). They then answered a question about their familiarity with the math questions they completed, as well as some questions about how many college-level math questions they had taken and planned to take. Finally, they completed a suspicion check and a second demographic questionnaire (in addition to the one they completed at the end of Session 1), which repeated questions from the first demographic questionnaire, but also included questions about how long they had lived in the U.S. and how long they had spoken English.

Studies 2a and 2b. In terms of graduating class, 51.6% of participants in Study 2a were freshmen, 27.3% were sophomores, 13.7% were juniors, and 7.5% were seniors. In Study 2b, 76% of participants were in their first year of college, and 24% were in their fourth. With respect to age in Study 2b, we did not have a precise estimate of age because, although we collected the birthdate of participants, the exact date of participation was not readily accessible to us. Thus, we reported an estimate of age based on the average participation date (i.e., the average of the two days when Session 2 data was collected).

With respect to the exclusions for Study 2a, we had conducted analyses with a final sample of 163 participants (160 of who were determined *not* to have given conflicting responses on the two behavioral persistence measures), but then excluded two additional participants when we realized that one of them completed Session 2 two days after Session 1 and another one completed Session 2 in a separate semester from Session 1 (90 days apart). We conducted another set of analyses with a sample of 161 participants (158 who were determined *not* to have given conflicting responses) before realizing that two additional participants (5 in all) gave conflicting responses. Therefore, our primary analyses involving the preference measure of persistence were conducted with only 156 participants. We did not note any substantial changes in the results after excluding the four, additional participants. The association between math determination and problem-solving persistence was significant with them included, but the three initially-identified conflicting responses excluded, $r(158) = .24, p = .003$.

Because a number of participants who completed the first session of Studies 2a and 2b did not complete the second session, we examined whether these participants (Study 2a: $N = 33$, Study 2b: $N = 13$) differed from the participants in the final samples (Study 2a: $N = 156$, Study 2b: $N = 91$) in terms of their grit and math determination. These analyses did not include participants who were excluded from the final sample for other reasons (e.g., because they were exposed to key information about Session 2 of the study before completing it [Study 2a], or because they did not complete Session 1 [Study 2b]). The results showed that the two sets of participants did not vary in their mean levels of grit, $|t|s < 1.28, ps > .20$, or math determination, $|t|s < 1.45, ps > .15$ (with a correction for unequal variances when appropriate). We could not compare the two groups in terms of their demographic characteristics because this information was not collected until the second session.

In Session 1, before completing the math determination and grit measures, participants in both studies completed measures of participants' growth mindsets about intelligence and math ability, as well as their growth mindsets about their capacity for hard work (in general and for math problem solving; for analyses involving these mindset variables, contact the first author). And, as explained above, participants in Study 2a (but not Study 2b) completed the math confidence measure after the math determination and grit measures in Session 1. In addition, in Session 1 of Study 2a (but not Study 2b), participants also completed a measure of math value that we created based on expectancy-value theory (Wigfield, Tonks, & Kluada, 2016) and on previous expectancy-value questionnaires (e.g., Eccles & Wigfield, 1995; Luttrell et al., 2010). The measure included 12 items assessing four different types of value (3 items per type): interest (e.g. "I enjoy working on math problems"), utility value (e.g. "I see no real use in being good at math"), attainment value (e.g. "Being good at math is an important part of who I am"), and cost (e.g. "Solving math problem is too stressful"). For each item, participants indicated their level of agreement on a six-point scale (1 = "Strongly disagree" to 6 = "Strongly agree"). To create an index of participants' math value, we reverse coded the items that were negatively framed and then averaged participants' responses to all 12 items. We had originally included some analyses involving the math value in the main manuscript. Correlations involving math value can be found in Table S5 and regression analyses including this variable are reported later in this document and in Table S6.

After completing the math-framed reasoning task in Session 2 of Studies 2a and 2b, participants were asked to complete a brief demographic questionnaire while they waited for their scores on the task. The questionnaire included questions about their majors, their parents' highest level of education, their favorite subjects in high school and college, and their hopes and expectations about their future profession or career. However, as opposed to being provided with

their actual scores, participants received randomly assigned fake feedback. Half of the

participants were assigned to receive "positive" feedback, while the other half received

"negative" feedback. The specific feedback participants received was, "Your results on the math

reasoning task were [good/weak]. In fact, you did [better/worse] than [70/71/72%] of students at

your University who previously took this test." See the next section for a description of the

analyses that demonstrated the lack of influence this manipulation had on the primary outcome

measure and on the relation between each of the primary predictors and the outcome.

After receiving this feedback, participants were asked to complete the preference measure

of persistence. They were then asked to complete _task_-_specific_ measures of confidence and

value. The measures, which were administered together, consisted of two confidence items (e.g.,

"I would expect to perform well on another test that is like the math reasoning test I just took")

and three value items (e.g., "I would enjoy working on math-reasoning items like the one in the

test I just took") that participants responded to using a six-point agreement scale (1 = "Strongly

Disagree" to 6 = "Strongly Agree"). For Study 2b, participants then went on to complete the

measures of math confidence and value (participants in Study 2a completed these measures as

part of Session 1). For Study 2a, the participants went on to complete a manipulation check. For

Study 2a, the correlations between the two task-specific measures and other variables from the

study are presented in Table S5. In Study 2b, one participant (of the final 91) was missing

responses for the four confidence and value measures. The correlations between these four

variables and the primary variables discussed in the manuscript are presented in Table S9.

Effects of Problem Order and of the Feedback/Challenge Manipulations

Study 1. As mentioned in the main text, we conducted analyses to determine whether the

zero-order associations between behavioral persistence and the Session 1 variables were

moderated by problem type (solvable vs. unsolvable) or problem order (unsolvable first vs.

solvable first). Specifically, we submitted the square-root transformed persistence times to separate 2 (problem type: solvable vs. unsolvable) × 2 (problem order: unsolvable first vs. solvable first) mixed ANCOVAs, where problem type was a within-participants variable and problem order was a between-participants variable. Separate ANCOVAs were conducted with each of the eight motivational/personality variables listed in Table S3 as the continuous predictor/covariate (although we only reference three of these analyses in the main text).

We used mixed ANCOVA in order to examine the interaction between a repeated measures factor and a continuous individual difference variable (see Thomas, 2009; Thomas et al., 2009). Each motivational/personality variable was standardized before being entered into the corresponding analysis. We found no significant interactions involving the problem type or problem order variables. In fact, the only fully significant effects involving these variables in any of the analyses were the main effects of problem type (ps < .001), such that participants spent more time attempting to complete the unsolvable problem. The only marginal effects were a main effect of order in two of the eight analyses and an order × extraversion interaction.

To examine the effects of the *challenge manipulation* described above, we conducted an additional series of mixed ANCOVAs with time (square-root transformed) on the behavioral persistence task as the dependent measure, experimental condition as a dichotomous predictor, and one of the eight primary motivation/personality variables from Session 1 of the study as a continuous predictor. The other predictors were problem type (solvable vs. unsolvable), which was a repeated measures variable, and problem order (unsolvable first vs. solvable first). All two-, three-, and four-way interaction terms were included in the model. The results showed that there was a significant challenge manipulation x problem order interaction present in all analyses. The only other significant effect involving the challenge manipulation was a marginal (p = .08) challenge manipulation x problem order x emotional stability interaction, which we did

not attempt to interpret. Other fully significant effects ($ps < .05$) included the main effects of math determination, math confidence, and openness, as well as the order × extraversion interaction.

Study 2a. To examine whether the feedback manipulation described above affected the primary findings of the present study, we conducted four regression analyses with participants' continuous ratings of their task preference (i.e., behavioral persistence) as the dependent variable. In Step 1 of each analysis we entered experimental condition as a predictor, along with either grit, math determination, math confidence, or math value. We then entered the interaction term in Step 2. Across the analyses, grit was not a significant predictor of persistence ($p = .85$), while the main effects of the three other variables were significant ($ps < .004$). Neither the main effects of experimental condition ($ps > .14$) nor the interaction terms were significant in any of the analyses ($ps > .18$). That is, there was no moderation by experimental condition.

Study 2b. To examine whether the feedback manipulation described above affected the primary findings of the present study, we conducted two regression analyses with participants' continuous ratings of their task preference (i.e., behavioral persistence) as the dependent variable. In Step 1 of each analysis, we entered experimental condition as a predictor, along with either grit or math determination. We then entered the interaction term in Step 2. For the analysis involving grit, there were no significant effects ($ps > .56$). For the analysis involving math determination, the main effect of math determination was significant ($p = .02$), but all other effects were non-significant ($ps > .32$). That is, there were no main effects of experimental condition and no moderation by experimental condition.

Additional Correlational and Regression Analyses

Study 1. First, note that in an analysis that we initially conducted containing only math-determination and math confidence as predictors (but not grit), the effects of *both* variables were non-significant ($ps > .15$). This is in slight contrast to the results of the analyses described above and reported in Table S4, where math determination remained a marginal predictor when controlling for both grit and math confidence.

Some of our additional regression analyses for Study 1 included openness as a predictor. We added openness because the correlation between it and behavioral persistence was at the threshold of significance ($p = .05$). In one of these analyses, grit and math determination were entered as predictors in Step 1. We then added openness as a predictor in Step 2 and math confidence as a predictor in Step 3. The variance inflation factor for all variables in all steps was within an acceptable range (VIFs < 2.12). As reported above, math determination remained a significant predictor of behavioral persistence when controlling for grit, and grit was negatively associated with persistence when controlling for math determination (see Table S4). When openness was added in Step 2, both grit and math determination remained significant predictors, $\beta s > |.21|$, ps < .02, while openness emerged as a marginal positive predictor of persistence, $\beta = .16$, $t(133) = 1.70$, $p = .09$. However, when math confidence was added in Step 3, math determination was no longer a significant predictor, $\beta = .15$, $t(132) = 1.27$, $p = .21$, nor was math confidence, $\beta = .14$, $t(132) = 1.25$, $p = .21$. Openness remained a marginal predictor of behavioral persistence, $\beta = .17$, $t(132) = 1.83$, $p = .07$, and the effect of grit remained significant, $\beta = -.21$, $t(132) = 2.37$, $p = .02$. See Table S3 for correlations between openness (and the other Big 5 traits) and the other Study 1 measures.

Studies 2a and 2b. In Study 2a, a point-biserial correlation revealed that math value was positively correlated with participants' likelihood of choosing A (see Table S5). Including the five participants who gave contradictory responses did not change this pattern of results.

Furthermore, a Pearson correlations revealed that math value was positively correlated with a preference for "working on the items that [they] didn't finish or that [they] were unsure about from the previous [math reasoning] test." This pattern of results remained the same when excluding participants who were missing responses to the task choice item (in addition to the participants who gave conflicting responses; $N = 152$) and when including all participants (even those who gave conflicting responses; $N = 161$). Thus, despite the fact that grit was positively correlated with math determination, math confidence, and math value, $r(154) > .23$, $ps < .004$, it appears that only these domain-specific variables were associated with participants' preference for continuing to work on a challenging math task (i.e., their persistence). In addition, when math value was included in the hierarchical regression analysis discussed above, it was only a marginal predictor of task persistence (i.e., when controlling for the other independent variables; see Table S6). However, it did cause the effect of math confidence to become non-significant. This is perhaps because math value was very strongly correlated with math confidence as well as math determination.

In addition to the point-biserial correlations we reported for the dichotomous choice measure of persistence in Studies 2a and 2b, we conducted a series of logistic regression analyses, which yielded results that were highly similar to the correlations. Thus, for simplicity, we only reported the correlations. The results of logistic regression analyses are available upon request.

In addition to the multiple regression analyses we reported for the three studies, there were a number of additional analyses that we did not report. These analyses mostly differed in terms of the step at which certain variables were entered. For instance, in some analyses, we did not add math task score until the last step (if at all), such that models were computed that did not include this control variable. In other analyses, we did not include grit as a predictor. The basic

pattern of results for these models (with respect to the primary predictors of interest) did not differ substantially from the pattern we reported above. Ultimately, we selected analyses to report that entered variables into the model in a manner that we found to be maximally informative.

Finally, note that for Study 2a, we conducted a three-step hierarchical logistic regression analysis that was similar to the analyses reported in Tables S6, except that task choice (rather than task preference) was the outcome measure and math task score was not included as a predictor. The analysis yielded a pattern of results that was highly similar to the pattern reported in Tables S6, except that math confidence remained the sole marginal predictor in the third step.

Additional Tests of Indirect Effects

Study 1. In addition to our main tests of intervening variable models discussed in the main text, we conducted an additional analysis that included math determination as the independent variable, math confidence as the intervening variable, and task persistence as the outcome. The indirect effect of math determination on task persistence through math confidence was not significant, $b = .80$, 95% CI [-.44, 2.08], nor was the direct effect, $b = 1.34$, $p = .16$. In addition, the results of an analysis testing the reverse pathway showed that there was no indirect effect of math confidence on persistence through math determination, $b = .52$, 95% CI [-.21, 1.17], nor was the direct effect significant, $b = .70$, $p = .20$.

In addition, we tested a sequential model that included grit as the independent variables, math determination and math confidence as intervening variables, and task persistence as the outcome. Three intervening variable pathways were included in the model: (a) grit → math determination → persistence, (b) grit → math confidence→ persistence, and (c) grit → math determination → math confidence → persistence. Although the *total* indirect effect of the model

was significant, *b* = .61, 95% CI [.04, 1.49], the only significant pathway to emerge was the one containing math determination as the sole intervening variable, *b* = .52, 95% CI [.05, 1.46], such that the indirect effect of grit on task persistence went through math determination. The direct effect was marginally significant, *b* = -1.62, *p* = .06.

We also tested a parallel model that included only two pathways: (a) grit → math confidence→ persistence, and (b) grit → math determination → math confidence → persistence. Consistent with the results of the previous analysis, the only significant pathway to emerge was the one containing math determination as the intervening variable, *b* = .52, 95% CI [.03, 1.33]. The direct effect of grit was again marginal, but the total indirect effect was not significant.

Study 2a. As in Study 1, in addition to our main tests of intervening variable models discussed in the main text, we conducted an additional analysis that included math determination as the independent variable, math confidence as the intervening variable, and task persistence as the outcome. There was a significant indirect effect of math determination on task persistence, *b* = .59, 95% CI [.20, .91], but no direct effect, *b* = -.03, *p* = .90. We also tested the reverse pathway, in which math confidence was the predictor and math determination was the intervening variable. In this analysis, the indirect effect was not significant, *b* = -.01, 95% CI [-.21, .22], but the direct effect was significant, *b* = .51, *p* = .002.

As in Study 1, we also tested a sequential model that included grit as the independent variable, math determination and math confidence as intervening variables, and task persistence as the outcome. Three intervening variable pathways were included in the model: (a) grit → math determination → persistence, (b) grit → math confidence → persistence, and (c) grit → math determination → math confidence → persistence. We did not include math value in the model because it was not assessed in Session 1 of the other datasets and we wanted to compare

results across studies. Although the *total* indirect effect of the model was again significant, b = .23, 95% CI [.002, .51], unlike in Study 1, the only significant intervening variable pathway to emerge was the one containing both intervening variables, b = .23, 95% CI [.07, .46], such that the effect of grit on task persistence went through math determination, which in turn went through math confidence. The direct effect was not significant, b = -.21, p = .33.

We also tested a parallel model that included only two pathways: (a) grit → math confidence→ persistence, and (b) grit → math determination → math confidence → persistence. Consistent with the idea that math confidence was an intervening variable in the indirect path of grit on persistence through math determination, the only significant pathway to emerge was the one containing math confidence as the intervening variable, b = .21, 95% CI [.06, .48]. Again, the direct effect of grit was non-significant, but the total indirect effect was significant.

In another set of analyses, we found some evidence of a reciprocal relation between math determination and math confidence. Not only did math determination emerge as a significant intervening variable in the path from grit to math confidence, b = .45, 95% CI [.25, .68], but math confidence also emerged as a significant intervening variable between grit and math determination, b = .18, 95% CI [.05, .33]. This suggests that, in certain cases, math confidence may function as both a predictor and outcome of math determination. However, in Study 1, only the first of these intervening variable pathways was significant.

Studies 1, 2a, 2b. Finally, because grit and math determination were collected at the same point in time, we report analyses from each study in which math determination was the predictor, grit was the intervening, and persistence was the outcomes. If there are no indirect effects of determination through grit (and only indirect effects of grit through determination, as reported in the main text), this would provide additional evidence for the possibility of a causal

path from grit to math determination to math problem-solving persistence. Across all three studies, we found that there were no *positive* indirect effects of math determination through grit, while the positive direct effect of determination on persistence was statistically significant in all three studies (ps < .04). Interestingly, in Study 1, there was a significant *negative* indirect effect of math determination on persistence through grit. This seems to pertain to the negative direct of grit in the analysis displayed in Figure 1a of the main text. We are not entirely sure how to interpret these negative effects, but it may have something to do with gritty students sometimes over-persisting on challenging tasks, such that they spend a lot of time working on problems that they are unlikely to solve correctly at the expense of working on other, more easily-solvable problems (see Lucas, Gratch, Cheng, & Marsella, 2015).

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Table S1.

Factor Loadings from EFAs examining Grit and Math Determination.

	Study 1 Pattern Matrix		Study 2a Pattern Matrix		Study 2b Pattern Matrix	
	1	2	1	2	1	2
Grit Scale (Perseverance of Effort)						
1. I have overcome setbacks to conquer an important challenge.		.662		.617	.376	.400[a]
2. Setbacks don't discourage me.		.451		.451	N/A	
3. I am a hard worker		.797		.816		.676[b]
4. I finish whatever I begin.		.588		.723		.415
5. I have achieved a goal that took months of work.		.519		.750		.701
6. I am diligent.		.810		.836		.731[c]
Math Determination Scale						
1. If I am struggling to solve a problem, I try to think of alternative ways to solve it.	.538		.347			
2. When I don't understand a problem, I keep working until I find the answer.	.623		.627		.625	
3. If I cannot do a math problem in a few minutes, I probably can't do it at all. (R)	.779		.795		.538	
4. Math problems that take a long time don't bother me.	.640		.584		.565	
5. If I can't solve a math problem quickly, I quit trying. (R)	.713		.778		.723	
6. I feel I can do math problems that take a long time to complete.	.794		.802		.550	
7. I find I can do hard math problems if I just hang in there.	.710		.677		.376	
8. I'm not very good at solving math problems that take a while to figure out. (R)	.723		.690		.674	

Note: Blank cells indicate suppressed loadings (less than .3).

[a] In the Russian version of the grit scale (Tyumeneva, Kardanova, & Kuzmina, in press), this item says "I have usually continued working after failed attempts to solve the problem."
[b] In the Russian version, this item says "Without irony, I am a hard worker."
[c] In the Russian version, this item says "At work, I am diligent."

Table S2.

Factor Loadings from EFAs Examining Math Confidence and Math Determination.

	Study1 Pattern Matrix		Study 2a Pattern Matrix	
	1	2	1	2
Math Confidence Scale				
1. I am sure I could do advanced work in mathematics.	.777		N/A	
2. I think I could handle more difficult mathematics.	.754		.575	.357
3. I have a lot of self-confidence when it comes to math.	.797		.824	
4. I'm no good in math. (R)	.858		.867	
5. I don't think I could do advanced mathematics. (R)	.744		.713	
6. I am not the type to do well in math. (R)	.892		.914	
7. For some reason even though I study, math seems unusually hard for me. (R)	.818		.879	
8. Most subjects I can handle OK, but I have a knack for flubbing up math. (R)	.907		.948	
9. Math has been my worse subject. (R)	.899		.952	
Math Determination Scale				
1. If I am struggling to solve a problem, I try to think of alternative ways to solve it.		**.554**		**.631**
2. When I don't understand a problem, I keep working until I find the answer.		**.742**		**.731**
3. If I cannot do a math problem in a few minutes, I probably can't do it at all. (R)		**.677**		**.644**
4. Math problems that take a long time don't bother me.	.424	.352	.388	
5. If I can't solve a math problem quickly, I quit trying. (R)		**.774**		**.580**
6. I feel I can do math problems that take a long time to complete.	.329	.564		.573
7. I find I can do hard math problems if I just hang in there.		.656		.518
8. I'm not very good at solving math problems that take a while to figure out. (R)		.563	.427	.335

Note: Blank cells indicate suppressed loadings (less than .3). Items in **bold** were included in the reduced version of the math determination scale. (R) indicates the item was reverse-coded before being entered into the EFA.

Table S3

Zero-order correlations between primary Session 1 and 2 measures and additional Session 1 measures, Study 1.

	1	2	3	4	5	6	7	8	9	10	11
Part 1 Measures											
1. Grit	--										
2. Math determination	**.24****	--									
3. Math confidence	.08	.67**	--								
4. Conscientiousness	.58**	.20*	.16†	--							
5. Emotional stability	.32**	.32**	.27**	.29**	--						
6. Openness	.38**	.37**	.15†	.22**	.31**	--					
7. Extraversion	.15†	.05	.06	.09	.23**	.45**	--				
8. Agreeableness	.24**	.22*	.12	.28**	.25**	.37**	.16†	--			
Part 2 Measures											
9. Math ability judgments 1	-.004	.53**	.69**	.12	.17*	.12	.04	.15	--		
10. Time-on-task (behavioral persistence)[a]	**-.10**	**.25****	.25**	-.02	.03	.17†	.07	.13	.24**	--	
11. Math ability judgments 2	.07	.46**	.60**	.12	.23**	.13	.15†	.17*	.70**	.11	--
Mean	3.88	3.33	3.55	4.73	3.88	4.27	3.93	4.51	4.03	18.65[a]	3.03
Standard Deviation	.60	.72	1.25	1.00	1.09	.95	1.29	.92	1.40	6.09	1.46
Internal Consistency (α)	.80	.88	.95	.58[b]	.57[b]	.42[b]	.72[b]	.37[b]	.88[b]	--	.91[b]

N = 137, $^\dagger p < .10$, *$p < .05$, **$p < .01$, Primary measures are in bold. [a] Behavioral persistence reflects the square root of the mean time (in seconds) participants spent on the two problems. [b] Because the measures of Big 5 traits and math ability judgments only had two items each, internal consistency was computed as a Pearson correlation (*r*).

Table S4

Summary of hierarchical regression analysis for math confidence and primary
Session 1 variables predicting behavioral persistence for Study 1

	Model 1				Model 2			
Variable	*B*	*SE*	*β*	*VIF*	*B*	*SE*	*β*	*VIF*
Grit	-1.73	.86	-.17*	1.06	-1.62	.86	-.16†	1.08
Math determination	2.49	.71	.30**	1.06	1.80	.96	.21†	1.93
Math confidence					.58	.54	.12	1.83
R² Change		.09**				.01		
F for *R² Change*		6.82				1.13		

N = 137, †*p* < .10. **p* < .05. **p < .01.

Table S5

Zero-order correlations between primary and additional measures from both sessions, Study 2a.

	1	2	3	4	5	6[a]	7	8	9
Session 1 Measures									
1. Grit	--								
2. Math determination	.35**	--							
3. Math confidence	.23**	.72**	--						
4. Math value	.28**	.71**	.82**	--					
Session 2 Measures									
5. Math task score (percent)	-.17*	.09	.18*	.11	--				
6. Task choice (behavioral persistence 1)[a]	-.04	.20*	.30**	.26**	.24**	--			
7. Task preference (behavioral persistence 2)	.01	.24**	.34**	.34**	.24**	.85**	--		
8. Task-specific confidence	.13	.48**	.51**	.51**	.002	.28**	.37**	--	
9. Task-specific value	.06	.42**	.53**	.56**	.21**	.50**	.55**	.57**	--
Mean	3.89	4.12	3.85	3.69	26.5%	56.6%	4.19	3.69	3.51
Standard Deviation	.66	.75	1.19	.81	11.4%	49.7%	1.75	1.02	1.24
Internal Consistency (α)	.85	.87	.96	.89	--	--	--	.64[b]	.92

$N = 156$, *$p < .05$, **$p < .01$. [a] Because task choice is a dichotomous variable, all correlations with other variables in this table are point-biserial. For correlations involving this variable, $N = 152$. [b] Because the measure of task-specific confidence only had two items, internal consistency was computed as a Pearson correlation (*r*).

Table S6

Summary of hierarchical regression analysis for variables predicting behavioral persistence for Study 2a

	Model 1				Model 2				Model 3			
Variable	*B*	*SE*	*β*	*VIF*	*B*	*SE*	*β*	*VIF*	*B*	*SE*	*β*	*VIF*
Math task score	.03	.01	.21**	1.06	.03	.01	.18*	1.08	.03	.01	.18*	1.09
Grit	-.10	.22	-.04	1.19	-.11	.22	-.04	1.19	-.14	.22	-.05	1.20
Math determination	.54	.19	.23**	1.17	.03	.26	.01	2.22	-.09	.27	-.04	2.41
Math confidence					.45	.16	.31**	2.12	.23	.20	.16	3.49
Math value									.50	.29	.23†	3.34
R² Change		.11**				.04**				.02†		
F for *R² Change*		6.04				7.88				2.88		

N = 156, † *p < .10,* * *p < .05,* ** *p < .01.*

Table S7.

Summary of hierarchical regression analysis for variables predicting behavioral persistence for Study 1.

Variable	Model 1				Model 2			
	B	*SE*	*β*	*VIF*	*B*	*SE*	*β*	*VIF*
Grit	-1.61	.85	-.16†	1.04	-1.56	.85	-.16†	1.04
Math determination (4 items)	2.42	.70	.29**	1.04	1.76	.81	.21*	1.42
Math confidence					.74	.47	.15	1.37
R² Change		.09**				.02		
F for R² Change		6.75				2.50		

*N = 137, †p < .10. *p < .05. **p < .01.*

Table S8.

Summary of hierarchical regression analysis for variables predicting behavioral persistence for Study 2a.

Variable	Model 1				Model 2			
	B	*SE*	*β*	*VIF*	*B*	*SE*	*β*	*VIF*
Grit	-.15	.23	-.06	1.13	-.19	.22	-.07	1.13
Math determination (4 items)	.43	.20	.19*	1.13	-.04	.22	-.02	1.59
Math confidence					.54	.14	.37**	1.49
R² Change		.03†				.09**		
F for *R² Change*		2.40				15.77		

N = 156, $^{†}p < .10.$ $^{*}p < .05.$ $^{**}p < .01.$

Table S9.

Zero-order correlations between additional Session 2 measures and primary measures, Study 2b.

	1	2	3	4
Additional Session 2 Measures				
1. Math confidence	--			
2. Math value	.79**	--		
3. Task-specific confidence	.46**	.31**	--	
4. Task-specific value	.43**	.46**	.51**	--
Primary Session 1 Measures				
5. Grit	.02	-.09	.18	.11
6. Math determination	.62**	.52**	.37**	.38**
Primary Session 2 Measures				
7. Math Task Score (Percent)	.15	.12	.15	.03
8. Task Choice (Behavioral Persistence 1)[a]	.12	.09	.26*	.43**
9. Task Preference (Behavioral Persistence 2)	.28**	.23*	.37**	.52**
Mean	3.57	3.81	3.75	3.87
Standard Deviation	1.02	.76	.97	1.06
Internal Consistency (α)	.90	.88	.64[b]	.82

$N = 90$, $^{*}p < .05$, $^{**}p < .01$. For correlations between primary measures, see Table 2b in the main manuscript. Because pairwise exclusion was used, correlations reflect differing degrees of freedom, with a maximum of 88. [a] Because task choice is a dichotomous variable, all correlations with other variables in this table are point-biserial. [b] Because the measure of task-specific confidence only had two items, internal consistency was computed as a Pearson correlation (*r*).