

References

1. Date, E., Kashiwara, M., Jimbo, M., Miwa, T.: Transformation groups for soliton equation. In: Jimbo, M., Miwa, T. (eds.). Proceedings of RIMS Symposium on Non-Linear Integrable Systems, pp. 39–119. World Science, Singapore (1983)
2. Dubrovin, B.A.: Theta functions and non-linear equations. Russ. Math. Surv. **36**(2), 11–92 (1981)
3. Dubrovin, B.A.: Riemann Surfaces and Nonlinear Equations [in Russian]. RHD, Moscow–Izhevsk (2001)
4. Dubrovin, B.A., Natanzon S.M.: Real theta-function solutions of the Kadomtsev–Petviashvili equation. Math. USSR Izv. **32**(2), 269–288 (1989)
5. Fricke, R., Klein, F.: Vorlesungen über die Theorie der automorphen Funktionen. Bd. 2. Teubner, Leipzig (1897, 1912)
6. Griffiths, P., Harris, J.: Principles of Algebraic Geometry. Wiley, New York (1978)
7. Its, A.R., Matveev, V.B.: Schrödinger operators with finite-gap spectrum and N -soliton solutions of the Korteweg–de Vries equation. Theor. Math. Phys. **23**(1), 343–355 (1975)
8. Klimov, Yu., Korzh, A., Natanzon, S.: From 2D Toda hierarchy to conformal maps for domains of the Riemann sphere. Am. Math. Soc. Transl. (2) **212**, 207–218 (2004)
9. Kostov, I., Krichever, I., Mineev-Weinstein, M., Wiegmann, P.B., Zabrodin, A.: The τ -function for analytic curves. In: Random Matrix Models and Their Applications. Mathematical Sciences Research Institute Publications, vol. 40, pp. 285–299. Cambridge University Press, Cambridge (2001)
10. Krichever, I.M.: Algebraic-geometric construction of the Zakharov–Shabat equations and their periodic solutions. Sov. Math. Dokl. **17**, 394–397 (1976)
11. Krichever, I.M.: Methods of algebraic geometry in the theory of nonlinear equations. Russian Math. Surveys **32**(6), 185–213 (1977)
12. Mineev-Weinstein, M., Wiegmann, P.B., Zabrodin, A.: Integrable structure of interface dynamics. Phys. Rev. Lett. **84**(22), 5106–5109 (2000)
13. Natanzon, S.M.: Invariant lines of Fuchsian groups. Russ. Math. Surv. **27**(4), 161–177 (1972)
14. Natanzon, S.M.: Moduli spaces of real curves. Tr. Mosk. Mat. Obs. **37**, 219–253 (1978)
15. Natanzon, S.M.: Formulas for A_n - and B_n -solutions of WDVV equations. J. Geom. Phys. **39**(4), 323–336 (2001)
16. Natanzon, S.M.: Witten solution for the Gelfand–Dikii hierarchy. Funct. Anal. Appl. **37**(1), 21–31 (2003)
17. Natanzon, S.M.: Moduli of Riemann Surfaces, Real Algebraic Curves, and Their Superanalogs. American Mathematical Society, Providence (2004)

18. Natanzon, S.: Towards an effectivisation of the Riemann theorem. *Ann. Global Anal. Geom.* **28**(3), 233–255 (2005)
19. Natanzon, S.M.: A Brief Course in Mathematical Analysis. MCCME, Moscow (2008)
20. Natanzon, S.M.: Dispersionless 2D Toda hierarchy, Hurwitz numbers and Riemann theorem. *J. Phys. Conf. Ser.* **670**, 1–6 (2016)
21. Natanzon, S., Zabrodin, A.: Symmetric solutions to dispersionless 2D Toda hierarchy, Hurwitz numbers and conformal dynamics. *Int. Math. Res. Not.* **2015**(8), 2082–2110 (2015)
22. Natanzon, S.M., Zabrodin, A.V.: Formal solutions to the KP hierarchy. *J. Phys. A* **49**(14), 145206 (2016)
23. Novikov, S.P.: The periodic problem for the Korteweg–de Vries equation. *Funct. Anal. Appl.* **8**(3), 236–246 (1974)
24. Shabat, B.V.: Introduction to Complex Analysis [in Russian]. Nauka, Moscow (1969)
25. Shiota, T.: Characterization of Jacobian varieties in terms of soliton equations. *Invent. Math.* **83**(2), 333–382 (1986)
26. Springer, G.: Introduction to Riemann Surfaces. Addison-Wesley, Reading (1957)
27. Teichmuller, O.: Extremale quasikonforme Abbildungen und quadratische Differentiale. *Abh. Preuss. Akad. Wiss. Math. Naturw. Kl.* **22**, 3–197 (1940)
28. Varchenko, A.N., Etingof, P.I.: Why the Boundary of a Round Drop Becomes a Curve of Order Four. American Mathematical Society, Providence (1991)
29. Wiegmann, P.B., Zabrodin, A.: Conformal maps and integrable hierarchies. *Commun. Math. Phys.* **213**(3), 523–538 (2000)

Index

- Abel–Jacobi map, 96
- Abel theorem, 96
- Abelian function, 94
- Abelian torus, 93
- Algebraic curve, 72
- Almost tau function, 107
- Analytic function, 46
- Antiderivative
 - along a curve, 10
 - on a domain, 10
- Argument, 26
- Argument principle, 26
- Atlas, 45
- Attracting fixed point, 50
- Automorphism
 - of a complex domain, 34
 - elliptic, 49
 - hyperbolic, 50
 - parabolic, 50
- Baker–Akhiezer function, 111
 - normalized, 115
- Biholomorphic equivalence, 33
- Biholomorphic map, 33
- Boussinesq equation, 111
- Canonical basis of cycles, 78
- Canonical class, 84
- Canonical element, 32
- Carathéodory theorem, 35
- Cauchy inequality, 20
- Cauchy integral formula, 12
- Cauchy–Riemann equations, 2
- Cauchy theorem, 9
- Chart, 45
- Compact family, 30
- Complex derivative, 1
- Complex structure, 45
- Complex torus, 69
- Conformal map, 5
- Continuous functional, 30
- Contour, 6
- Critical point, 70
- Critical value, 70
- Curve, 6
 - algebraic, 72
 - nonsingular, 73
- Degree
 - of a divisor, 83
 - of a map, 70
- Differential
 - holomorphic, 72
 - meromorphic, 71
- Dirichlet problem, 42
- Disk with a hole, 51
- Divisor, 83
 - effective, 84
 - linear equivalence, 84
 - of a meromorphic differential, 83
 - of a meromorphic function, 83
 - nonspecial, 101
 - positive, 83
 - principal, 84
 - special, 101
- Domain, 1

- Effective divisor, 84
- Elliptic automorphism, 49
- Entire function, 22
- Equicontinuous family, 29
- Equivalent atlases, 45
- Essential singularity, 21, 22
- Family of functions
 - compact, 30
 - equicontinuous, 29
 - uniformly bounded, 29
- Fixed point
 - attracting, 50
 - repelling, 50
- Formal exponential, 103
- Fuchsian group, 48, 53
 - of given type, 54
- Function
 - Abelian, 94
 - analytic, 46
 - entire, 22
 - harmonic, 37
 - holomorphic, 5
 - meromorphic, 22, 71
 - quasi-periodic, 118
 - univalent, 31
- Functional, 30
- Fundamental sequence of functions, 29
- Fundamental theorem of algebra, 27
- Gap, 90
- Gelfand–Dikii hierarchy, 110
- Green’s function, 41
- Green’s theorem, 39
- Group
 - Fuchsian, 48, 53
 - modular, 48
- Harmonic function, 37
- Hole, 51
- Holomorphic differential, 72
- Holomorphic function, 5
- Holomorphic map, 45, 46
- Hurwitz theorem, 31, 89
- Hyperbolic automorphism, 50
- Hyperbolic metric, 49
- Hyperbolic Riemann surface, 49
- Hyperelliptic surface, 69
- Implicit function theorem, 72
- Integral, 6
- Invariant line, 50
- Irreducible polynomial, 75
- Isolated singularity, 21, 22
- Isomorphism, 46
- Jacobian of a Riemann surface, 95
- Jacobi inversion problem, 97
- Kadomtsev–Petviashvili equation, 110
- KdV equation, 111
- KdV hierarchy, 110
- Laplace equation, 37
- Laplace operator, 37
- Lattice generated by a Riemann matrix, 93
- Laurent series, 20
- Linearly equivalent divisors, 84
- Liouville theorem, 21
- Map
 - biholomorphic, 33
 - conformal, 5
 - holomorphic, 46
- Mapping class group, 67
- Maximum modulus principle, 28
- Mean value theorem, 11
- Meromorphic differential, 71
- Meromorphic function, 22, 71
- Modular group, 48
- Moduli space, 49
 - of Riemann surfaces of given type, 51
- Montel theorem, 29
- Morera theorem, 16
- n*-KdV hierarchy, 110
- Nonsingular curve, 73
- Nonspecial divisor, 101
- Normalized Baker–Akhiezer function, 115
- Open mapping theorem, 27
- Order
 - of a pole, 22, 72
 - of a zero, 22, 72
- Parabolic automorphism, 50
- Path, 6, 32
- Period matrix, 79
- Period of a meromorphic differential, 79

- Poincaré model, 49
Poisson integral formula, 43, 44
Pole, 21, 22, 71
 simple, 71
Positive divisor, 83
Principal divisor, 84
Principal value integral, 24
Puncture, 51
Punctured disk, 51
- Quasi-periodic function, 118
- Ramification degree, 70
Ramification point, 70
Reducible polynomial, 75
Removable singularity, 21, 22
Repelling fixed point, 50
Residue
 of a differential, 80
 of a function, 23, 24
Richardson's harmonic moments, 119
Riemann bilinear relations, 82
Riemann–Hurwitz formula, 70
Riemann mapping theorem, 33
Riemann matrix, 90
Riemann–Roch theorem, 84
Riemann sphere, 22, 69
 with two punctures, 51
Riemann surface, 45
 of an analytic function, 46
 hyperbolic, 49
 of a polynomial, 74
 type, 51
Riemann vanishing theorem, 99
Rouché theorem, 26
- Schottky problem, 96
Schwarz integral formula, 44
- Schwarz lemma, 28
Semiperiod, 94
 even, 94
 odd, 94
Sequential set, 51
 of given type, 52, 54
Shift parameter, 50
Simple pole, 71
Singularity
 essential, 21, 22
 isolated, 21, 22
 removable, 21, 22
Sokhotski–Casorati–Weierstrass theorem, 22
Special divisor, 101
Special point, 111
Standard set of generators, 55
- Tau function, 107
Theta function, 91
 with characteristics, 93
Toda lattice, 121
- Uniformization theorem, 47
Uniformly bounded family, 29
Univalent function, 31
- Vector of Riemann constants, 99
- Weierstrass point, 89
Weierstrass theorem, 16
Weight of a Weierstrass point, 89
- Young diagram, 125
- Zero, 71