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On adaptive experiments for nondeterministic finite state machines

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Abstract Adaptive experiments are well defined in the context of finite state machine (FSM) based analysis, in particular, in FSM based testing where homing and distinguishing experiments with FSMs are used for test derivation. In this paper, we define and propose algorithms for deriving adaptive homing and distinguishing experiments for non-initialized nondeterministic finite state machines (NFSMs). For NFSMs, the construction of adaptive experiments is rather complex as the partition over produced outputs does not define a partition over the set of states but a collection of intersecting subsets, and thus, the refinement of such set system is more difficult than the refinement of a partition. Given a complete non-initialized possibly non-observable NFSM, we establish necessary and sufficient conditions for having adaptive homing and distinguishing experiments and evaluate the height of these experiments.

Keywords Nondeterministic finite state machine · Conformance testing · Adaptive homing and distinguishing experiments

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1 Introduction

Finite state machines (FSMs) are widely used in various application domains, such as communication protocols, embedded control systems, sequential circuits, and other reactive systems. Moreover, FSMs are the underlying models for formal description techniques, such as statecharts, SDL, and UML. An FSM is a state transition system which has a finite number of inputs, outputs, states and a finite number of transitions each labeled by an input/output pair. Many conformance test derivation methods have been developed for deriving tests when the system specification and implementation are represented as FSMs (see, for example, [1–7]). In FSM-based testing, we have a machine or a black-box implementation under test (IUT) about which we lack some information, and we want to deduce this information by conducting experiments on this FSM. An experiment consists of deriving and applying input sequences to the IUT, observing corresponding output responses and drawing a conclusion about the machine under test. An experiment is *preset* if input sequences are known before performing the experiment and an experiment is *adaptive* if at each step of the experiment the next input is selected based on previously observed outputs [3,8].

Well-known types of experiments used in FSM-based testing are distinguishing and homing experiments which have been constructed for different types/classes of FSMs. An FSM is *weakly initialized* if it has several initial states. An FSM is *complete* if at every state of the machine there exists an outgoing transition under each input. A complete FSM is *reduced* if at each two different states, the FSM does not have the same behavior. An FSM is *nondeterministic* if at some state the machine has several transitions under a given input. A complete nondeterministic FSM is *observable* if at each state the machine has at most one transition under a given input/output pair, otherwise, it is *non-observable*. Given an FSM, assuming that the initial state is unknown, a *distinguishing experiment* determines the initial state of the FSM before the experiment, and such an experiment is widely used in FSM-based conformance testing for checking the correspondence between transitions of an IUT and those of the specification FSM. A *homing experiment* identifies the final state reached at the end of the experiment and it is used, for example, when deriving a checking sequence for non-initialized FSMs [9,10]. An input sequence applied when performing a preset distinguishing experiment is called a *distinguishing sequence* (DS), or a *separating sequence*, and such a sequence is called a *homing sequence* (HS) when performing a preset homing experiment.

Ongoing research on preset and adaptive homing experiments for deterministic FSMs started since the seminal paper on "gedanken experiments" by Moore [8]. For information and surveys on FSM-based experiments and some related algorithms, a reader may refer to [3–5,11]; in particular, Gill [3] and Lee and Yannakakis [5] presented methods for deriving preset and adaptive distinguishing experiments for deterministic FSMs with corresponding evaluation of the complexity of these experiments. Preset homing experiments are considered in [3,4,12,13]. Derivation of minimal length preset homing sequences can be done using the homing tree method introduced by Gill [3] and reported in details in Kohavi [4]. Any deterministic complete reduced FSM with n states has been shown to have a homing sequence of length up to $n(n-1)/2$ and Hibbard [14] showed that deterministic machines can require adaptive homing experiments with the height of the same order as that of preset experiments. It is worth mentioning that there has been also some work on the derivation of a *synchronizing sequence* that guarantees that the FSM from any initial state reaches the same state by the end of the experiment independent of produced output sequences. As in this case outputs are not important, such sequences are usually derived for the corresponding automata where only input actions are considered. A survey on homing and synchronization sequences is given by Sandberg [15]. Parallel algorithms for related problems are surveyed by Ravikumar (see, for example, [16]).

Currently, and mainly since the end of the 1980s, analysis of nondeterministic systems is capturing a lot of attention. Nondeterminism occurs due to various reasons such as performance, flexibility, limited controllability, and/or abstraction [17].

Preset distinguishing and homing experiments for nondeterministic FSMs are considered in [18–24]. In particular, Spitsyna et al. [19] presented a method for deriving a sequence that separates two initialized nondeterministic FSMs. An input sequence is a *separating sequence* of two FSMs if the sets of output sequences produced by the nondeterministic FSMs to this input sequence do not intersect [25]. Kushik et al. [21] showed that differently from deterministic FSMs a homing sequence does not necessarily exist for a complete reduced nondeterministic FSM and proposed an algorithm for deriving a preset homing sequence for a given observable nondeterministic FSM when such a sequence exists. A tight upper bound on a shortest preset separating sequence is shown to be of the order 2^{n^2} where n is the number of states of a complete nondeterministic observable FSM [19]. Hwang et al. [23] considered the non-equivalence relation between two states of a complete FSM and showed that the tight upper bound on the length of a sequence distinguishing two states of a non-observable FSM with n states is 2^n-2. Kushik and Yevtushenko [24] showed that there exists a special class of FSMs with n states and $(n-1)$ inputs, for which a shortest homing sequence has the length $2^{n-1}-1$, i.e., its length is exponential with respect to the number of FSM states. Zhang and Cheung studied related problems when deriving transfer and distinguishing trees for observable nondeterministic FSMs with probabilistic and weighted transitions [26].

Adaptive experiments for nondeterministic FSMs are considered in [18,20–22,27–29]. In particular, Petrenko and Yevtushenko [27] introduced the notion of a test case for describing an adaptive experiment as an initialized observable FSM with an acyclic transition diagram such that at each non-deadlock state only one input is defined with all possible outputs. A representation of a test case using the same formal model is widely used for transition systems such as LTS, input/output automata, etc. (see, for example, [30]). Such definition of a test case allows, at least for observable complete FSMs, defining distinguishing/checking/homing test cases based on the properties of the intersection of a transition system under experiment and a given test case. In [27–29] it is shown how a distinguishing test case can be derived for two states of a nondeterministic observable FSM when such a distinguishing test case exists. In particular, Alur et al. [18] show that the length of a shortest adaptive distinguishing test case that distinguishes two states of an observable nondeterministic FSMs with n states is at most $n(n-1)/2$. Petrenko and Yevtushenko [29] consider a set of adaptive test cases which have three parts: a preamble for reaching an appropriate state, a traversal input/output sequence and a state identifier. In this case, the length of an identifier can be optimized when distinguishing not two but several states with the same distinguishing test case. In addition, a distinguishing/checking sequence derived for a non-initialized FSM [9,10,20] can also be adaptive. Gromov et al. [31] and El-Fakih et al. [32] presented adaptive experiments for timed nondeterministic observable FSMs and some work on adaptive experiments for extended and communicating FSMs is reported in [22,33,34].

In this paper, adaptive homing and distinguishing experiments for non-initialized, possibly non-observable, nondeterministic finite state machines are considered. As in many other papers, an adaptive experiment is represented by a test case that is an initialized observable FSM with an acyclic flow diagram where only one input is defined at each intermediate state. Lee and Yannakakis [5] proposed an approach for deriving an adaptive distinguishing sequence for a deterministic FSM that is based on refining a partition of the set of states based on different outputs. In this paper, we deal with nondeterministic FSMs and unlike [5], the output partition defines not a partition of the set of states but rather a set system, which is a collection of intersecting subsets, for which it is difficult to define a corresponding refinement. For this reason, in this paper, necessary and sufficient conditions for having adaptive homing/distinguishing test cases are established based on extending the notion of k-r-distinguishability of two states [35] to subsets of states and an algorithm for deriving a homing/distinguishing adaptive test case with minimal length for a complete observable FSM is proposed. The upper bounds on the height of shortest homing/distinguishing experiments for observable and non-observable FSMs are established, and for observable FSMs an example is provided to illustrate that the presented bound for distinguishing test cases is expected to be tight. A preliminary version of this paper appeared in [36]. Here, that work is extended to complete, possibly non-observable, FSMs. In this case, the direct sum of observable equivalents over all initial states is constructed and a corresponding distinguishing/homing experiment is derived for the direct sum which is a complete observable FSM. In addition to the theoretical contribution, one natural application area of the proposed work is in further development of test derivation methods for nondeterministic, possibly non-observable, finite state machines. Problems related to testing from nondeterministic FSMs are summarized in several papers, for example, in [1,10,19,20,27,29,35].

This paper is organized as follows: Sect. 2 includes preliminaries. Homing and distinguishing test cases with related properties are introduced in Sect. 3. Section 4 contains an approach for deriving a homing/distinguishing test case for an observable FSM and then the approach is altered to deal with non-observable FSMs. Section 5 includes statements about the complexity evaluation. Section 6 concludes the paper.

2 Preliminaries

In this paper, adaptive experiments with weakly initialized finite state machines are considered. A *weakly initialized finite state machine* (FSM) S is a 5-tuple $(S, I, O, h_{\mathsf{S}}, S')$, where S is a finite set of states with the set $S' \subseteq S$ of initial states; I and O are finite non-empty disjoint sets of inputs and outputs, respectively; $h_{\mathsf{S}} \subseteq S \times I \times O \times S$ is a *transition relation*, where a 4-tuple $(s, i, o, s') \in h_{\mathsf{S}}$ is a *transition*. If $|S'| = 1$ then the FSM S is an *initialized* FSM. An input $i \in I$ is a *defined* input at state s of S if there exists a transition $(s, i, o, s') \in h_{\mathsf{S}}$ for some $s' \in S$ and $o \in O$. Sometimes when $S' = S$ the machine is called a *non-initialized* machine. Given a weakly initialized machine $\mathsf{S} = (S, I, O, h_{\mathsf{S}}, S')$, the notation S/s is used for denoting the initialized submachine of S with the single initial state $s \in S'$.

An FSM $\mathsf{S} = (S, I, O, h_{\mathsf{S}}, S')$ is *complete* if for each pair $(s, i) \in S \times I$ there exists a pair $(o, s') \in O \times S$ such that $(s, i, o, s') \in h_{\mathsf{S}}$; otherwise, the machine is *partial*. FSM S is *nondeterministic* if for some pair $(s, i) \in S \times I$, there exist at least two transitions $(s, i, o_1, s_1), (s, i, o_2, s_2) \in h_{\mathsf{S}}$, such that $o_1 \neq o_2$ and/or $s_1 \neq s_2$. FSM S is *observable* if for each two transitions $(s, i, o, s_1), (s, i, o, s_2) \in h_{\mathsf{S}}$ it holds that $s_1 = s_2$. FSM S is *single-input* if at each state there is at most one defined input at the state, i.e., for each two transitions $(s, i_1, o_1, s_1), (s, i_2, o_2, s_2) \in h_{\mathsf{S}}$ it holds that $i_1 = i_2$, and FSM S is *output-complete* if for each pair $(s, i) \in S \times I$ such that the input i is defined at state s, there exists a transition from s under i for every output in O [29].

A *trace* of S at state s is a sequence of input/output pairs of consecutive transitions starting from state s. Given a trace $i_1 o_1 \ldots i_k o_k$ at state s, the input projection $i_1 \ldots i_k$ of the trace is a *defined* input sequence at state s. Let $Tr(\mathsf{S}/s)$ denote the set of all traces of S at state s including the empty trace and let $Tr(\mathsf{S}/S')$ denote the union of $Tr(\mathsf{S}/s)$ over all states $s \in S'$. As usual, for a state s and a trace $\gamma \in (IO)^*$, $next\_state_{\mathsf{S}}(s, \gamma)$ denotes the set of all states that are reached from s by γ. If γ is not a trace at state s then the set $next\_state_{\mathsf{S}}(s, \gamma)$ is empty; otherwise, the set $next\_state_{\mathsf{S}}(s, \gamma)$ is the γ-*successor* of state s. For an observable FSM S, $|next\_state_{\mathsf{S}}(s, \gamma)| \leq 1$ for any $\gamma \in (IO)^*$. Given a nonempty subset b of states of the FSM S and $\gamma \in (IO)^*$, the set $next\_state_{\mathsf{S}}(b, \gamma)$ is the union of the sets $next\_state_{\mathsf{S}}(s, \gamma)$ over all $s \in b$ and this set is the γ-*successor* of the set b. An FSM S is *acyclic* if the set $Tr(\mathsf{S}/S')$ is finite, i.e., the FSM transition diagram has no cycles. An FSM S is (*initially*) *connected* if each state is reachable from an initial state. To characterize the common behavior of two weakly initialized machines, the operation of the intersection of initialized FSMs is extended as follows.

Given two complete FSMs S and P with the sets of initial states S' and P', the *intersection* $\mathsf{S} \cap \mathsf{P}$ is the connected FSM Q such that states of Q are pairs (b, c) of subsets of states of FSMs S and P, the initial state of Q is (S', P'), and h_{Q} is the smallest set derived using the following rule:

Given state (b, c), $b \subseteq S$ and $c \subseteq P$, and an input/output pair i/o, the FSM Q has a transition $((b, c), i, o, (b', c'))$ if there exist states $s \in b$ and $p, \in c$ with an outgoing transition labeled by the pair i/o, and b' and c' are the io-successors of subsets b and c. By definition, the FSM $\mathsf{S} \cap \mathsf{P}$ is observable even for non-observable FSMs S and P.

As an example of the FSM intersection, consider FSMs P (Fig. 1) and S (Fig. 2). FSM P is an initialized FSM while S

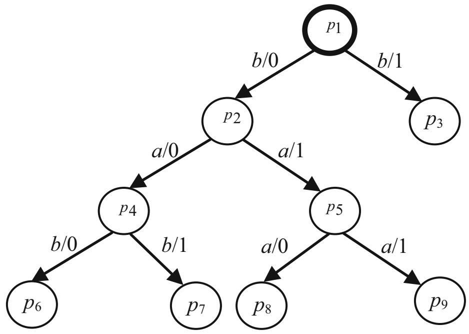

Fig. 1 A test case P over alphabets $I = \{a, b\}$ and $O = \{0, 1\}$

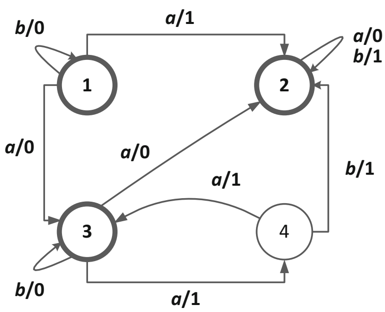

Fig. 2 FSM S with the three initial states shown in *bold*

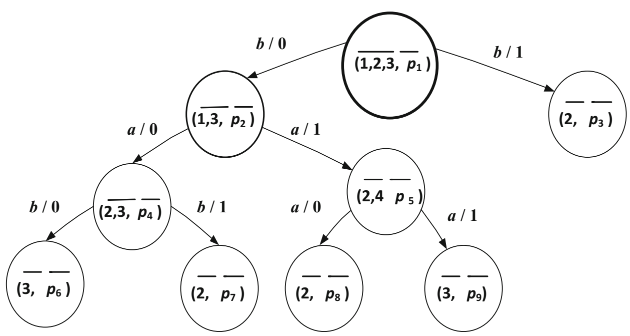

Fig. 3 The intersection $\mathsf{S} \cap \mathsf{P}$

has three initial states marked in bold. The intersection $\mathsf{S} \cap \mathsf{P}$ is shown in Fig. 3. As usual, the intersection of two weakly initialized FSMs describes the common behavior of component FSMs, and in addition, it also provides some information about the structure of their transition sets. For example, a state of the intersection provides information about which states of the corresponding machines are reachable from the initial states under a corresponding trace. In fact, the following proposition holds.

Proposition 1 *Given FSMs S and P with the sets S' and P' of initial states and state (b, c) of the intersection $\mathsf{S} \cap \mathsf{P}$ that is reachable from the initial state of the intersection under a trace γ, the set b (c) is the γ-successor of the set $S'(P')$.*

As in this paper adaptive experiments with nondeterministic FSMs are considered, in order to identify a state of a weakly initialized FSM before or after the experiment, a finite input sequence is applied to an FSM under experiment where the next input (except for the first one) of the sequence is determined based on the output of the FSM produced to the previous input. Formally, such an experiment can be described using a single-input output-complete observable FSM with an acyclic transition graph and similar to [27–29], such an FSM is called a *test case*.

Test case Given an input alphabet I and an output alphabet O, a *test case* $TC(I, O)$ is an initially connected single-input output-complete observable initialized FSM $\mathsf{P} = (P, I, O, h_{\mathsf{P}}, \{p_0\})$ with an acyclic transition graph.

If $|I| > 1$ then a test case is a partial FSM. A state $p \in P$ is a *deadlock* state of the FSM P if there are no defined inputs at this state. By definition, a state (b, c) of the intersection $\mathsf{S} \cap \mathsf{P}$, $b \subseteq S$ and $c \subseteq P$, is a deadlock state if for every $s \in b$, $p \in c$ and every input/output pair i/o labeling an outgoing transition from s, the io-successor of state p is empty.

A test case P over alphabets $I = \{a, b\}$ and $O = \{0, 1\}$ is shown in Fig. 1.

A test case $TC(I, O)$ over alphabets I and O defines an adaptive experiment with any complete FSM S over the same alphabets. As an example, consider the test case P in Fig. 1. An adaptive experiment with an FSM S over alphabets $I = \{a, b\}$ and $O = \{0, 1\}$ is conducted using P as follows. At the first step the input b is applied to S as this input is the only input defined at the initial state of P. If the output of the FSM S to this input is 1, then the experiment is over, since we reach the deadlock state p_3 of P. If the FSM S produces the output 0 to input b then the experiment is not over, since the test case P enters the intermediate state p_2 where the single input a is defined. As this input does not take the test case to a deadlock state, the next input which is also a is applied. If the output to a is 0 then the next input is b; otherwise, the next input is a. For this example, the length of a longest trace of the test case is three, i.e., at most three inputs are applied during this adaptive experiment.

In general, given a test case P, the *length* of the test case P is defined as the length of a longest trace from the initial state to a deadlock state of P and it specifies the length of the longest input sequence that can be applied to an FSM S during the experiment. The length of a test case is often called the *height* of the adaptive experiment and it is used for describing the complexity of the experiment. As usual, for testing, one is interested in deriving a test case (experiment) with minimal length (height).

3 Homing and distinguishing test cases

In this section, the notions of homing and distinguishing test cases that can be used in the context of adaptive testing of non-initialized nondeterministic observable FSMs are defined. A homing (distinguishing) experiment allows determining the unknown current (initial) state of a machine under experiment. Hereafter, if not stated otherwise, we consider an FSM under experiment to be a weakly initialized complete nondeterministic FSM and in the following sections we propose a method for deriving shortest homing and distinguishing test cases for an observable FSM (if such test cases exist). In Sect. 4.2, the work is extended to deal with complete non-observable FSMs.

3.1 Homing and distinguishing experiments for complete nondeterministic FSMs

Let S be a weakly initialized complete nondeterministic FSM over input and output alphabets I and O, and S' is the set of initial states of the FSM S.

Homing test case A test case P is a *homing* test case for the FSM S if for every trace γ of P from the initial state to a deadlock state, the γ-successor of S' has at most one state. If there exists a homing test case for the FSM S then the set S' is a *homing* set and the test case P is a *homing test case* for the set S' or we say that the test case P *homes* states of the set S'. Otherwise, the set S' is not homing.

Distinguishing test case A test case P is a *distinguishing test case* for the FSM S if every trace from the initial state to a deadlock state of P is a trace at most at one initial state of the set S'. If there exists a distinguishing test case for the FSM S then the set S' is a *distinguishing* set and the test case P is a *distinguishing test case* for the set S' or the test case P *distinguishes* states of the set S'. Otherwise, the set S' is not distinguishing.

A homing (distinguishing) test case is used for representing a homing (distinguishing) adaptive experiment with a nondeterministic FSM. An adaptive homing (distinguishing) experiment has two steps. At the first step, a finite input sequence is applied to an FSM under experiment where the next input (except for the first one) depends on the output of the FSM produced to the previous input. At the next step, after observing a produced output sequence, the conclusion is drawn about a state of the FSM after (before) the experiment. If all the traces of a test case from the initial state to a deadlock state have the same input projection, then the test case defines a preset input sequence and a corresponding adaptive experiment becomes a preset experiment. In the same way, when having a homing or a separating sequence for a given FSM a corresponding test case can be derived by augmenting this sequence with all possible output sequences. Therefore, given a weakly initialized FSM S there exists an adaptive homing (distinguishing) experiment for the FSM S if and only if the FSM S has a homing (distinguishing) test case.

Example 1 As an example of homing and distinguishing test cases, consider the weakly initialized FSM S in Fig. 2 with the set $\{1, 2, 3\}$ of initial states and the test case P in Fig. 1. By direct inspection, one can assure that for each trace γ of the test case P from the initial state to a deadlock state, γ is a trace at only one initial state of the FSM S. Thus, that the test case in Fig. 1 is a distinguishing test case for the FSM S, i.e., the set $\{1, 2, 3\}$ is a distinguishing set. In the same way, one can assure that the test case in Fig. 1 also is a homing test case.

3.2 Properties of homing and distinguishing experiments for complete observable FSMs

In this section, some properties of homing and distinguishing test cases for complete observable FSMs are established. When describing states and transitions of a test case, for simplicity of presentation, in order to reduce the number of brackets, we use the notations $\overline{s}$ and $\overline{s_1, \ldots, s_m}$ for representing a singleton $\{s\}$ and a subset $\{s_1, \ldots, s_m\}$ of several states of the FSM S.

The following two statements can be established based on Proposition 1 and the definitions of homing and distinguishing test cases.

Proposition 2 *Given a weakly initialized complete observable FSM S, a test case P is a homing test case for FSM S if and only if for each deadlock state $(b, \overline{p})$ of the intersection $\mathsf{S} \cap \mathsf{P}$, the set b is a singleton.*

Proposition 3 *Given a weakly initialized observable FSM S, a test case P is a distinguishing test case for FSM S if and only if* **(1)** *for each deadlock state $(b, \overline{p})$ of the intersection $\mathsf{S} \cap \mathsf{P}$, b is a singleton and* **(2)** *for each transition $((b, \overline{p}), i, o, (b', \overline{p'}))$ of the intersection $\mathsf{S} \cap \mathsf{P}$ the subset b does not have two different states which have the same io-successor, i.e.,*

$$\forall s_1, s_2 \in b\, ((s_1, i, o, s') \in h_{\mathsf{S}} \,\&\, (s_2, i, o, s') \in h_{\mathsf{S}} \Rightarrow s_1 = s_2).$$

For observable FSMs, each trace can take the FSM from a given state to not more than one state and thus, according to the above propositions, the following statement holds.

Proposition 4 *Given a complete observable FSM S, each distinguishing test case for S is also a homing test case. However, the converse is not necessarily true.*

Example 2 Consider again the weakly initialized FSM S in Fig. 2 and the test case P in Fig. 1. By direct inspection, one can assure that each deadlock state of the intersection $\mathsf{S} \cap \mathsf{P}$ (Fig. 3) is labeled by a pair of singletons and each two different states of any subset b such that $(b, \overline{p})$ labels an intermediate state of the intersection do not have the same *io*-successor. Thus, the set {1, 2, 3} is not only a homing but also a distinguishing set and the test case in Fig. 1 is a homing test case and a distinguishing test case for FSM S . For example, if the output 1 is produced to the input b at the initial state of the FSM S then the FSM reaches state 2 after the input b and we know that the initial state before applying the input b is 2.

Given a complete observable FSM S, under some conditions, there is a simple way for checking whether there exists a homing experiment.

Proposition 5 *Given a weakly initialized observable FSM S, there exists a homing test case for the FSM S if each subset $\{s_i,\ s_j\} \subseteq S$ of states of the FSM S is a homing set, i.e., each pair of two different states of S is homing.*

Proof Without loss of generality, it is assumed that $S = \{1, \ldots, n\}$ and $S' = \{1, \ldots, m\}$, $S' \subseteq S$. Let $\mathsf{P}_{i,j}$ be a homing test case for the set $\{i,\ j\} \subseteq S$. The statement is proven by construction. Given a homing test case $\mathsf{P}_{1,2}$ for the set {1, 2}, the set $\{1, 2, 3\} \subseteq S$ and a trace γ that takes the test case $\mathsf{P}_{1,2}$ from the initial state to a deadlock state, the γ-successor $next\_state(\{1, 2, 3\},\ \gamma)$ of the set $\{1, 2, 3\}$ has at most two states, since $\mathsf{P}_{1,2}$ is a homing test case for the set $\{1, 2\}$ and the FSM S is observable. If $next\_state(\{1, 2, 3\}, \gamma) = \{i,\ j\}$ for some $i,\ j \in S$, then append the test case $\mathsf{P}_{1,2}$ with the test case $\mathsf{P}_{i,j}$ at the corresponding deadlock state and obtain a homing test case $\mathsf{P}_{1,2,3}$.

Proceeding in the same way the test case $\mathsf{P}_{1,2,\ldots,m}$ can be derived. By construction, the test case $\mathsf{P}_{1,2,\ldots,m}$ is a homing test case for the FSMS. □

The constructive step of the proof of Proposition 5 is illustrated in Fig. 4 below. For the sake of simplicity, the input i and two outputs o_1 and o_2 are used for the illustration.

As a test case is an output complete FSM, in the illustration, the test case $\mathsf{P}_{1,2}$ has two traces io_1 and io_2. Let trace io_2 be a trace at state 1 and/or 2. The test case $\mathsf{P}_{1,2}$ is a homing test case for the pair $\{1, 2\}$, and thus, the io_2-successor of the set $\{1, 2\}$ is a singleton $\{d\}$. In Fig. 4, the corresponding deadlock of $\mathsf{P}_{1,2}$ is denoted by this singleton $\overline{d}$. The trace io_1 is not a trace at state 1 or 2; such trace takes the test case $\mathsf{P}_{1,2}$ to a deadlock state labeled with the empty set symbol Ø.

An FSM $\mathsf{P}_{1,2,3}$ is derived from the FSM $\mathsf{P}_{1,2}$ by preserving all the transitions of $\mathsf{P}_{1,2}$ and appending $\mathsf{P}_{1,2}$ with additional test cases at some deadlock states. If the io_1-successor f of state 3 in the FSM S is not empty then the io_1-successor of the set $\{1, 2, 3\}$ is a singleton $\{f\}$ and a correspondingly deadlock state is labeled by $\overline{f}$ when deriving $\mathsf{P}_{1,2,3}$ from the test case $\mathsf{P}_{1,2}$.

If another trace io_2 takes the FSM S from state 3 to state t then the corresponding io_2-successor of the set $\{1, 2, 3\}$ becomes $\{d, t\}$, and the corresponding deadlock state $\overline{d}$ of

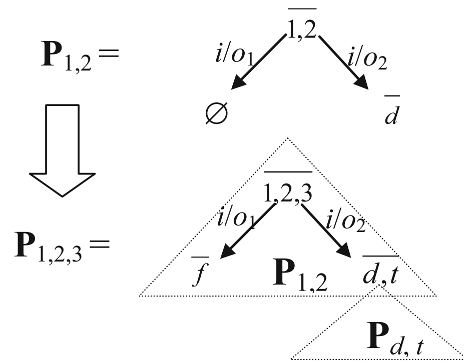

Fig. 4 Illustrating the constructive step of the proof of Proposition 5

$\mathsf{P}_{1,2}$ is labeled $\overline{d, t}$. In order to home the pair $\{d, t\}$, the test case $\mathsf{P}_{1,2}$ is appended with a homing test case $\mathsf{P}_{d,t}$ that has the deadlock state $\overline{d, t}$ as the initial state. By construction, the length of the test case $\mathsf{P}_{1,2,3}$ is the sum of length of test cases $\mathsf{P}_{1,2}$ and $\mathsf{P}_{d,t}$.

The FSM $\mathsf{P}_{1,2,3}$ is afterwards used for deriving a test case $\mathsf{P}_{1,2,3,4}$, etc. The procedure terminates when the test case $\mathsf{P}_{1,2,\ldots,m}$, $|S'| = m$, is derived.

Corollary *Given a non-initialized complete observable FSM S with the set $S' = S$ of initial states there exists a homing test case for FSM S if and only if there exists a homing test case for each subset $\{s_i, s_j\}$ of states of S.*

In other words, if each pair of states of an observable FSM S with n states can be adaptively homed then there exists a homing test case for any subset of FSM states. The proof of Proposition 5 shows a way for deriving such a homing test case when the conditions of the proposition hold.

Example 3 As an example of applying the construction stated in Proposition 5, consider the complete and observable FSM S with the set of initial states S' = {1, 2, 3} in Fig. 5. A homing test case $\mathsf{P}_{i,j}$ is derived for each subset $\{i,\ j\}$ of states of the FSM S and then a test case for the set S' is constructed. The set of transitions of $\mathsf{P}_{1,2}$ equals $\{(\overline{1,2}, i_1, o_1, \overline{1}),\ (\overline{1,2}, i_1,\ o_2, \overline{2})\}$; the set of transitions of $\mathsf{P}_{1,3}$ equals $\{(\overline{1,3}, i_3, o_1, \overline{1}), (\overline{1,3}, i_3, o_2, \overline{3})\}$; and the set of $\mathsf{P}_{2,3}$ transitions equals $\{(\overline{2,3}, i_2, o_1, \overline{2}), (\overline{2,3}, i_2, o_2, \overline{3})\}$. At the next step, state 3 is added to the set $\{1, 2\}$, transitions of $\mathsf{P}_{1,2}$ are included into $\mathsf{P}_{1,2,3}$ and the test case $\mathsf{P}_{1,2}$ is appended with the test case $\mathsf{P}_{1,2}$ at the deadlock state $\overline{1,2}$. More precisely, this is done as follows. First, consider input i_1 at state 3 for which a single output o_1 can be obtained, thus, the corresponding transitions at state $\overline{1,2,3}$ of $\mathsf{P}_{1,2,3}$ are $\{(\overline{1,2,3}, i_1, o_1, \overline{1,2}), (\overline{1,2,3}, i_1, o_2, \overline{2})\}$. At the deadlock state $\overline{1,2}$ of $\mathsf{P}_{1,2}$, corresponding transitions of the test case $\mathsf{P}_{1,2}$ are added and the following set of transitions for $\mathsf{P}_{1,2,3}$ is obtained: $\{(\overline{1,2,3}, i_1, o_1, \overline{1,2}), (\overline{1,2,3}, i_1, o_2, \overline{2}), (\overline{1,2}, i_1, o_1, \overline{1}), (\overline{1,2}, i_1, o_2, \overline{2})\}$. The length of such adaptive

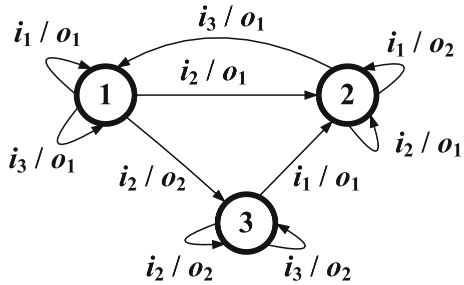

Fig. 5 An FSM S with three initial states

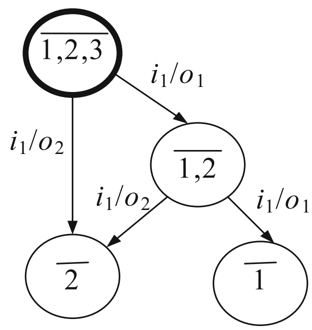

Fig. 6 A homing test case for the FSM in Fig. 5

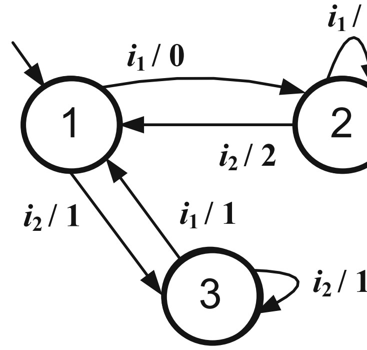

Fig. 7 An FSM that has no distinguishing test case though each two different states are distinguishable

homing test case (Fig. 6) equals two. However, if input i_3 is selected when deriving the set of transitions of $\mathsf{P}_{1,2}$ then the length of the corresponding adaptive homing test case for the set $S' = \{1, 2, 3\}$ equals one. Thus, when deriving an adaptive test case using the above approach the length of a returned test case significantly depends on the enumeration of states of the set S' as well as on selected inputs when deriving a test case and hence, such an approach does not guarantee the derivation of a homing test case of minimal length. Moreover, the approach does not guarantee the derivation of a homing test when there exists a subset with two states of an FSM under experiment that is not homing. Accordingly, in the following section, an algorithm for deriving a shortest homing test case for a complete observable FSM is proposed.

The constructive procedure used in the proof of Proposition 5 allows the evaluation of the maximal length of a shortest homing test case for a weakly initialized complete observable FSM S where each pair of two different states is homing.

Proposition 6 *Given a non-initialized complete observable FSM S with n states and m initial states, $m \le n$, if there exists a homing test case for each subset $\{s_i, s_j\}$ of S, then there exists a homing test case for the FSM S with the length $O(mn^2)$.*

Proof Each subset with two states of an observable FSM can be homed by a sequence of inputs of length at most C_n^2. The number of steps in the proof of Proposition 5, i.e., the number of steps for deriving the test case $\mathsf{P}_{1,2,\ldots,m}$ from $\mathsf{P}_{1,2}$ equals $(m-2)$. Thus, the maximal length of the trace in the test case $\mathsf{P}_{1,2,\ldots,m}$ is at most $O(mn^2)$. □

Proposition 6 only establishes that the maximal length of a shortest homing test case for a proper class of weakly initialized complete observable FSMs is polynomial with respect to the number of states of an FSM under experiment. Additional research is needed to evaluate the complexity of a homing test case for FSMs where some state pairs are not homing.

It should be noted that a proposition similar to Proposition 5 does not hold for a distinguishing test case. That is, similar to deterministic FSMs [3,9], given an observable FSM where each pair of two different states has a distinguishing test case, there is no guarantee that there exists a distinguishing test case for the FSM.

Example 4 Consider an FSM in Fig. 7. Each pair of states of the FSM has a distinguishing test case, for example, states 1 and 3, and also states 2 and 3 can be distinguished by the input i_1 while states 1 and 2 can be distinguished by the input i_2. Nevertheless, there is no distinguishing test case for the set $\{1, 2, 3\}$ since for each input there are two different states which have the same *io*-successor.

Given a complete observable FSM $\mathsf{S} = (S, I, O, h_S, S')$, in order to derive a homing/distinguishing test case with minimal length, the notions of *k-homing/k-distinguishing* sets not only for pairs but for arbitrary subsets of states are introduced.

A subset $g \subseteq S$ is 0-*homing*/0-*distinguishing* if g is a singleton. Let all $(k-1)$-homing/$(k-1)$-distinguishing sets, $k > 0$, be already defined.

The subset $g \subseteq S$ is a *k-homing set* if **(1)** g is a $(k-1)$-*homing*, or **(2)** there exists an input $i \in I$, such that for each $o \in O$, the *io*-successor of g is either empty or is a $(k-1)$-homing set.

A subset $g \subseteq S$ is a k-distinguishing set if **(1)** g is $(k-1)$-distinguishing, or **(2)** there exists an input $i \in I$, such that for each $o \in O$, the *io*-successor of $g$is either empty or is a

$(k-1)$-distinguishing set, and in addition, the *io*-successors of two different states of g do not coincide.

Proposition 7 *Given a weakly initialized observable complete FSM* S *with the set* S' *of initial states, the set* S' *is k-homing/k-distinguishing,* $k > 0$, *if and only if there exists a homing/distinguishing adaptive experiment of height k for the FSM* S. *If* S' *is k-homing/k-distinguishing,* $k > 0$, *but is not* $(k-1)$*-homing/*$(k-1)$*-distinguishing then k is the minimal height of a corresponding adaptive experiment.*

Proof The proposition is proven only for the case of a homing experiment as for the distinguishing case the proof is almost the same.

$\Rightarrow$ The statement is proven using the induction on k. Let $k = 1$. By definition, there exists an input $i \in I$, such that for each $o \in O$, the *io*-successor of S' is either empty or is a 0-homing set (a singleton), i.e., there exists an adaptive experiment of length 1 for the FSM $\mathsf{S} = (S, I, O, h_S, S')$.

Let the statement hold for all $k \leq K$ and S' is not K-homing set but S' is a $(K+1)$-homing set. In this case, by definition, there exists an input $i \in I$, such that for each $o \in O$, the *io*-successor of S' is either empty or is a K-homing set and according to the induction assumption, each *io*-successor can be homed by an adaptive experiment of length at most K. Thus, S' can be homed by an adaptive experiment of length at most $(K+1)$.

$\Leftarrow$ Suppose that there exists a test case P of length $k > 1$. By definition, the states of P that have transitions to deadlock states are 1-homing. Thus, the states of P that are connected to these states are 2-homing, etc. The states of P that are connected to the deadlock states via a sequence of length k are k-homing. □

Based on Propositions 2, 3 and 7, the following proposition holds.

Proposition 8 *Given a set* S', $|S'| > 1$, *of states of an observable FSM* S, *the set* S' *is homing/distinguishing if and only if* S' *is k-homing/k-distinguishing for some* $k > 0$.

4 Deriving homing and distinguishing test cases

In this section, a method is proposed (Procedures 1 and 2) for deriving homing/distinguishing test cases for a weakly initialized complete observable FSM S when such a test case exists. Otherwise, the method indicates that the states of the set S' of initial states cannot be homed/distinguished by an adaptive experiment. The same method can be used when deriving a homing test case for a complete, possibly non-observable, FSM. However, when deriving a distinguishing test case for non-observable FSMs the method has to be slightly modified as will be illustrated afterwards.

4.1 Deriving homing and distinguishing test cases for complete observable FSMs

Based on the notion of k-homing sets, Procedures 1 and 2 given below are proposed for deriving a homing/distinguishing test case for a homing/distinguishing set S' of states of a given observable FSM S. Procedure 1 is used for determining if all pairs of states of the FSM are homing/distinguishing and if so, it returns the empty set signaling this fact. Otherwise, Procedure 1 returns a non-empty set containing all pairs of states of the FSM that are not homing/distinguishing and

Procedure 1 : **Deriving the set of all state pairs of a given FSM which are not homing**
Input: A weakly initialized complete observable FSM $\mathsf{S} = (S, I, O, h_S, S')$, $|S'| > 1$
Output: The set N of all state pairs which are not homing
// The set N has each state pair of the FSM S that is not homing or $N = \varnothing$ when all pairs of states are homing

Step-1: $j := 0$; $N := \varnothing$;
  Derive the set Q_j of all singletons $\{s\}$ of the set S of FSM S;
Step-2:
// The set Q_{j+1} contains each subset $\{s_t, s_k\} \subseteq S$ that is $(j+1)$-homing but is not j-homing
Include into the set Q_{j+1} each subset $\{s_t, s_k\}$ of the set S for which there exists an input i such that for each input/output pair i/o, the *io*-successor of $\{s_t, s_k\}$ is a subset of some item of the set Q_j;
  If Q_{j+1} is not empty
  Then $Q_{j+1} := Q_{j+1} \cup Q_j$;
    $j := j + 1$;
    Go to Step-2
  Else
    Include into the set N each subset $\{s_t, s_k\}$ of the set S such that $\{s_t, s_k\} \notin Q_j$;
Return N

Procedure 2 : **Deriving a homing test case for a subset S' of states of an FSM**
Input: A weakly initialized complete observable FSM $\mathsf{S} = (S, I, O, h_S, S')$, $|S'| > 1$
Output: A homing test case P with minimal length for the FSM S or the message "there is no adaptive homing experiment for the FSM S"

Step-1: $j := 0$;
Derive the set $Q_j = H_j$ of all singletons $\{s\}$ of the set S of FSM S;
Call Procedure 1 that returns the set N ;
// The set N is either empty or has each state pair of the FSM S that is not homing
If N has a pair of initial states
Then Return the message "there is no adaptive homing experiment for the FSM S";
Step-2:
// The set Q_{j+1} contains each (j+1)-homing set of states of the FSM S that is not j-homing
$Q_{j+1} := \varnothing$;
For each nonempty subset A, $2 \le |A| \le |S'|$, of states of FSM S where A is not a subset of some item of the set $H_j \cup Q_{j+1}$ and no item of N is a subset of A
If there exists an input $i \in I$, such that for each $o \in O$, the io-successor of A is either empty or is a subset of some item of the set H_j
Then include the set A into Q_{j+1};
Derive a set Tr_A that contains each 4-tuple (A, i, o, A') where the io-successor of the set A is not empty, $A' \in H_j$ and A' contains the io-successor of the set A;
End For
If the set Q_{j+1} is empty
Then Return the message "there is no adaptive homing experiment for the FSM S".
If Q_{j+1} is not empty and does not contain the set S' as an item
Then
$H_{j+1} := H_j \cup Q_{j+1}$;
$j := j + 1$ and **Go-to Step-2**
Step-3: Derive a homing test case P with the set P of states as follows:
// States of P are subsets of states of the FSM S
The initial state of P is the set S'; i.e., include S' into P;
Mark the initial state of P labeled with the set S' as a "*non-visited*" state in P;
While there is a '*non-visited*' state A in P
For each 4-tuple (A, i, o, A') in Tr_A
Add to the test case P the transition (A, i, o, A');
If the final state A' of the transition is not in the set P
Then add A' to P and if the added state is not a singleton, mark the added state as a '*non-visited*' state;
EndFor
EndWhile
If P is not output-complete
Then
For each intermediate state p of P where a single input i is defined
If there is no transition (p, i, o, p') for some $o \in O$
Then add a transition (p, i, o, p') where p' is any state of P;
EndFor
Return P

this set allows reducing the computation efforts in the main procedure (Procedure-2) since if the set S' has a pair of states that is not homing/distinguishing then the states of the set S' cannot be homed/distinguished by an adaptive experiment and Procedure 2 ends signaling this fact. If all pairs of states of the FSM are homing/distinguishing, i.e., Procedure 1 returns the empty set, Procedure 2 proceeds to iteratively derive the set Q_j, $j = 1, 2, \ldots$, of state subsets of the cardinality at most $|S'|$ that are not homed/distinguished by the adaptive application of an input sequence of the length less than j while these states are homed/distinguished by the adaptive application of an input sequence of the length j. If for some j the set Q_j is empty and the $S' \notin \bigcup_{k=1}^{j-1} Q_k$ then a fixed point is reached and a homing experiment for the FSM S does not exist. In addition, in Procedure 2, a test case is derived when S' is homing/distinguishing; the states of the test case are labeled by subsets of states of the FSM S. We only include procedures for deriving a homing test case since procedures for deriving a distinguishing test case are almost the same. We note that illustrative comments included in the following procedures are preceded by the symbols //.

According to Procedure 1, each homing subset $\{s_t, s_k\} \subseteq S$ is an item of Q_j for appropriate integer $j > 0$. Correspondingly, the following statement holds.

Proposition 9 *Given a weakly initialized complete observable FSM S and a non-empty set N returned by Procedure 1, a subset $\{s_t, s_k\}$ of S is homing if and only if $\{s_t, s_k\} \notin N$.*

Corollary 1 *If the set N returned by Procedure 1 is empty then the FSM S is homing.*

Corollary 2 *If the set N contains a pair $\{s_t, s_k\} \subseteq S'$, then the FSM S is not homing.*

Remark Procedure 1 can be used for checking both homing and distinguishing properties of state pairs. However, Corollary 1 holds only for homing test cases. In addition, when checking the distinguishing property, in Step-2, the states s_t and s_k have to have different *io*-successors.

Theorem 1 *FSM P returned by Procedure 2 is a homing test case with minimal length for a complete observable FSM S with the set S' of initial states if and only if the set S' is a homing set.*

Proof $\Leftarrow$ The set S' is homing if this set is k-homing for some k. For this reason, when deriving at Step 2 $(j+1)$-homing sets an input $i \in I$ with the desired features always exists. By construction, FSM P returned by Procedure 2 is acyclic and at each intermediate state only one input is defined with all possible outputs, i.e., P is a test case. At Step 3, each trace $\gamma \in Tr_{\mathsf{S}}(s)$, $s \in S'$, that takes P to a deadlock state $\overline{s_k}$, takes the FSM S from any state of the set $S'$where this trace can be executed, to state s_k of S. Thus, the test case P is a homing test case for FSM S .

$\Rightarrow$ Let FSM P returned by Procedure 2 be a test case for FSM S of the height l. By definition, in this case the set S' is an l-homing set, i.e., S' is a homing set.

According to Procedure 2, if S' is an l-homing but it is not $(l-1)$-homing, then the procedure returns a test case (Step-3) of length l that is a test case of minimal length (Proposition 7). □

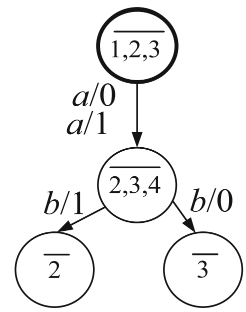

Fig. 8 The homing test case derived using Procedure 2 for the FSM in Fig. 2

By construction, in the above procedures, in order not to consider all subsets of states of the set S', we start with the subsets that have the same cardinality as S' and if some such set is homing, then we do not consider its subsets in any further investigation. On the other hand, if a non-homing pair is a part of a subset under investigation, then we know that the subset is not homing and thus, there is no need to consider such subsets in the investigation.

Example 5 Consider the FSM S in Fig. 2 with $S' = \{1, 2, 3\}$. Procedure 1 returns the empty set N since all pairs of different states of the set $S = \{1, 2, 3, 4\}$ are homing. At Step-1 of Procedure 2, $Q_0 = H_0 = \{\overline{1}, \overline{2}, \overline{3}, \overline{4}\}$. Then, at Step-2, by direct inspection, one can assure that the sets $\{2, 3, 4\}$ and $\{1, 4\}$ are 1-homing, i.e., $Q_1 = \{\overline{2, 3, 4}, \overline{1, 4}\}$. The corresponding sets of transitions are $Tr_{\{2,3,4\}} = \{(\overline{2, 3, 4}, b, 1, \overline{2}), (\overline{2, 3, 4}, b, 0, \overline{3})\}$ and $Tr_{\{1,4\}} = \{(\overline{1, 4}, b, 0, \overline{1}), (\overline{1, 4}, b, 1, \overline{2})\}$, respectively. Since the set $\{2, 3, 4\}$ is 1-homing, its subsets $\{2, 3\}$, $\{3, 4\}$, $\{2, 4\}$ are not considered at Step-2 of Procedure 2. As the set Q_1 does not contain the set $S' = \{1, 2, 3\}$, the set $H_1 = H_0 \cup Q_1 = \{\overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{2, 3, 4}, \overline{1, 4}\}$ is derived, j is incremented, and the procedure returns back to Step-2. The set $\{1, 2, 3\}$ is then observed to be 2-homing with the corresponding set of transitions $Tr_{\{1,2,3\}} = \{(\overline{1, 2, 3}, a, 1, \overline{2, 3, 4}), (\overline{1, 2, 3}, a, 0, \overline{2, 3, 4})\}$. The corresponding (shortest) homing test case derived by Procedure 2 for the set of states of the FSM S is presented in Fig. 8.

It is worth mentioning that the test case in Fig. 1 which has length 3 is not a shortest homing test case for the FSM S in Fig. 2 as there exists a homing test case of length 2.

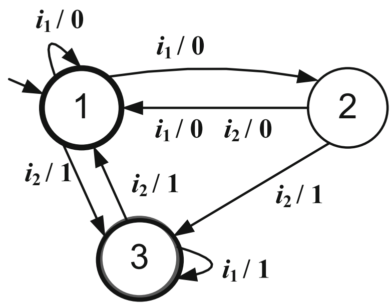

Fig. 9 An FSM S

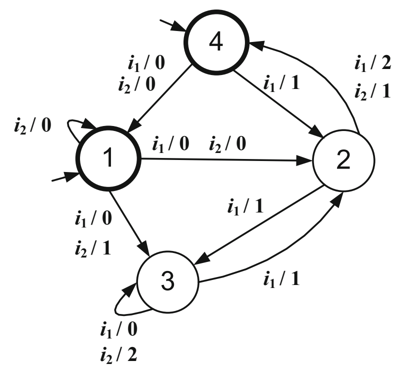

Fig. 10 A non-observable FSM S with the initial states 1 and 4

4.2 Deriving homing and distinguishing test cases for non-observable FSMs

For complete non-observable FSMs, Propositions 5 and 6 do not necessary hold since the *io*-successor of a set of two states can have more than two states. However, Proposition 2 is still valid for non-observable FSMs, and thus, Procedure 2 can be used for deriving a homing test case of minimal length for complete possibly non-observable FSM.

Unfortunately, for distinguishing test cases, Proposition 3 does not specify necessary and sufficient conditions. Consider an FSM in Fig. 9 with initial states 1 and 3, i.e., $S' = \{1, 3\}$. Then FSM P that contains only two outgoing transitions from the initial state p_0 labeled with the input/output pairs $i_1/0$ and $i_1/1$ is a distinguishing test case for S' despite of the fact that the intersection of the FSM in Fig. 9 and P has the deadlock state $(b, \overline{p_1})$ where $b = \overline{1, 3}$. However, given FSM S with the set S' of initial states, an observable FSM equivalent to S/s can be derived for each $s \in S'$. Given sets of states $z, z' \subseteq S$ of the FSM S $/s, i \in I, o \in O$, the 4-tuple $(z, i, o, z') \in h_{S/s}$ if and only if z' is the *io*-successor of the subset z.

In order to check whether the set of FSM states is distinguishing, the direct sum of FSMs S/s, $s \in S'$, will be used.

Given FSMs $\mathsf{S} = (S, I, O, h_\mathsf{S}, S')$ and $\mathsf{P} = (P, I, O, h_\mathsf{P}, P')$, the *direct sum* $\mathsf{S} \oplus \mathsf{P}$ of these FSMs is the FSM $(S \cup P, I, O, h_\mathsf{S} \cup h_\mathsf{P}, S' \cup P')$. In a usual way, the notion of a direct sum is extended to three or more FSMs. By definition, the direct sum of observable FSMs is an observable FSM.

The following statement holds.

Theorem 2 *Given FSM* $\mathsf{S} = (S, I, O, h_\mathsf{S}, S')$, $S' = \{s_1, \ldots, s_m\}$, *consider the observable FSMs* $\mathsf{S}_1, \ldots, \mathsf{S}_m$ *where each of these machines is equivalent to the corresponding FSM* $\mathsf{S}/s_1, \ldots, \mathsf{S}/s_m$. *The set* S' *of the FSM* S *is distinguishing if and only if the set of initial states of the direct sum* $\mathsf{S}_1 \oplus \cdots \oplus \mathsf{S}_m$ *is distinguishing.*

Proof For each $k = 1, \ldots, m$, $\mathsf{S}_k = (S_k, I, O, h_{S_k}, \overline{s_k})$ is an observable FSM, where S_k is the set of non-empty subsets of S and $\overline{s_k}$ is the initial state. The direct sum $\mathsf{S}_1 \oplus \cdots \oplus \mathsf{S}_m$ is the FSM $(\bigcup_{k=1}^m S_k, I, O, \bigcup_{k=1}^m h_{S_k}, \{\overline{s_1}, \ldots, \overline{s_m}\})$. By definition, the set S' of the FSM S is distinguishing if and only if there exists a distinguishing test case P for FSM S, such that every trace from the initial state to a deadlock state of P is a trace at only one initial state of the set S'. Given a trace $\gamma \in Tr(\mathsf{P}/p_0)$, there exists a single state s_k such that $\gamma \in Tr(\mathsf{S}/s_k)$. By definition of the direct sum, $\gamma \in Tr(\mathsf{S}/s_k)$ if and only if $\gamma \in Tr((\mathsf{S}_1 \oplus \cdots \oplus \mathsf{S}_m)/\overline{s_k})$ and $\overline{s_k}$ is a single state of the FSM $\mathsf{S}_1 \oplus \cdots \oplus \mathsf{S}_m$ where the trace γ is executed. □

The direct sum of observable FSMs is observable; thus, Procedure 2 can be applied for deriving a distinguishing test case for the direct sum. According to Theorem 2, this test case (if it exists) is a distinguishing test case for the initial non-observable FSM.

Example 6 As an example, consider the non-observable FSM S in Fig. 10 with the set $\{1, 4\}$ of initial states. Determinize the underlying automaton for the FSMs S/1 and S/4 and obtain the corresponding observable non-deterministic FSMs with the initial states $\overline{1}$ and $\overline{4}$, respectively. The direct sum of observable FSMs equivalent to S/1 and S/4 is the FSM shown in Fig. 11 with the set $\{\overline{1}, \overline{4}\}$ of initial states.

By direct inspection, one can assure that the test case in Fig. 12 is a distinguishing test case for the non-observable FSM in Fig. 10.

5 Evaluating the length of distinguishing and homing test cases

The length of a shortest adaptive homing/distinguishing experiment depends on the number of states and the number of initial states of an observable FSM under experiment.

Theorem 3 *Given a complete homing/distinguishing observable FSM* S *with* n *states and* m *initial states,* $m \le n$, *the length of a shortest homing/distinguishing test case is at most* $\sum_{i=2}^{m} C_n^i$.

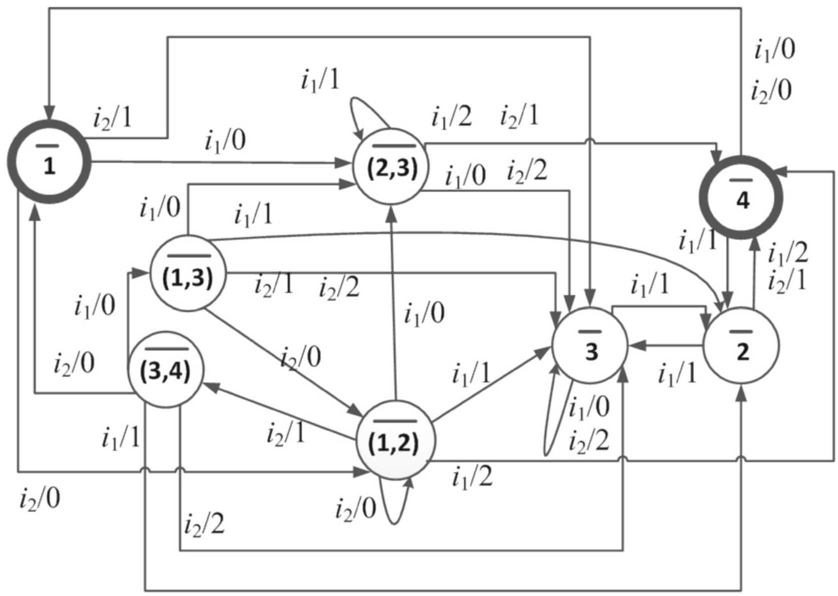

Fig. 11 The direct sum of observable FSMs equivalent to S/1 and S/4 for the FSM S in Fig. 10

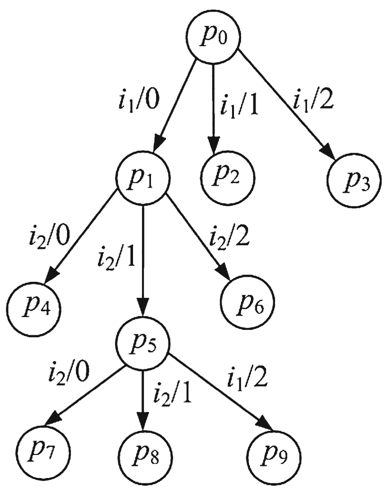

Fig. 12 A distinguishing test case for the FSM in Fig. 10

Proof For observable FSMs, a corresponding statement for a shortest adaptive homing test case is proven in [21]. Since the length of a shortest adaptive distinguishing test case is bounded by the same number of different subsets of states of the cardinality $(m-1), (m-2), \ldots, 2$, the same statement holds for a distinguishing test case. □

Corollary *Given a complete possibly non-observable FSM S with n states where each state can be an initial state, a shortest homing/distinguishing test case has the length at most* $2^{2^n-1} - 2^n - 2$.

Proof In fact, the maximal number of states in the corresponding direct sum equals $m = 2^n - 1$ and this bound is reachable [25]. Therefore, for $m = 2^n - 1$ the sum $\sum_{i=2}^{m} C_n^i = 2^{2^n-1} - 2^n - 2$. □

Additional research is needed to check whether the above bounds on the length of shortest adaptive homing/distinguishing test cases are reachable. For distinguishing test cases, an example of an observable FSM with four states is presented below, for which a shortest distinguishing test case has the length $\sum_{i=2}^{4} C_4^i = 6 + 4 + 1 = 11$.

Consider the FSM S with the flow table in Table 1 and with the set {1, 2, 3, 4} of initial states. The FSM S has 11 inputs and six outputs. The inputs are denoted $x_{123}, x_{124}, x_{134}, \ldots, x_{12}$ to demonstrate which subset is traversed when applying the input. For example, an input x_{123} defines $x_{123}0$-successor of the subset {1, 2, 3, 4} in the distinguishing test case, this successor coincides with the index of the input, i.e., it is the set {1, 2, 3}. The two last state subsets that are traversed by a longest trace in the distinguishing test case are {2, 3} and {1, 2}. The set {1, 2} is an $x_{12}0$-successor of the subset {2, 3} in the test case. The last input x_{sep} is a separating sequence for states 1 and 2 of the FSM

Table 1 Flow table of the FSM S

$x \backslash s$	1	2	3	4
x_{123}	1/0, 2, 3	2/0, 2, 4	3/0, 3, 4	4/2, 3, 4
x_{124}	1/0, 1	2/0, 1	4/0	3/0; 4/1
x_{134}	1/0, 1	3/0	2/0; 3/1	4/0, 1
x_{234}	2/0; 1/2	1/0; 2/1	3/0, 2	4/0, 1
x_{34}	1/1, 2, 4; 2/5	3/2; 2/4, 5	3/0, 1, 4	4/0, 1, 5; 2/2
x_{24}	1/0, 1, 2	3/0; 2/1	2/0; 3/2	4/0; 3/1
x_{14}	2/0; 1/1	1/0	3/0, 1	4/0, 1
x_{13}	1/0, 1	2/0, 1, 2	4/1; 3/0	3/0, 1; 4/2
x_{23}	2/0, 3; 1/1	1/0; 2/1, 2	3/0, 2 3	3/1; 4/2, 3; 1/0
x_{12}	3/0; 1/1	2/0, 1	1/0; 3/2	4/0, 1, 2
x_{sep}	1/0, 2	2/1, 3	2/1; 1/2	1/0; 2/1

S and takes the test case to different 0-distinguishing sets. A distinguishing test case can be derived for the FSM S using the last step of Procedure 2.

By definition, all singletons $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$ are 0-distinguishing sets. By direct inspection, one can assure that there is a single 1-distinguishing set for the FSM S which is the set $\{1, 2\}$ and this set is included into the set Q_1. The set Q_2 contains a single set $\{2, 3\}$ and the set $\{1, 3\}$ is a single 3-distinguishing set that is included into the set Q_3, etc. Thus, one can iteratively derive all the sets $Q_1, Q_2, Q_3, \ldots$, till reaching the set Q_{11} that contains the set $\{1, 2, 3, 4\}$ of initial states. The longest trace of the experiment traverses the sets $\{1, 2, 3, 4\}$, $\{1, 2, 3\}$, $\{1, 2, 4\}$, $\{1, 3, 4\}$, $\{2, 3, 4\}$, $\{3, 4\}$, $\{2, 4\}$, $\{1, 4\}$, $\{1, 3\}$, $\{2, 3\}$, and $\{1, 2\}$, respectively, and has length 11. All other traces do not allow to uniquely determine a state of the FSM before the experiment.

Since in the above example, a test case of minimal length traverses all subsets of some set, it is expected that an exponential upper bound can be reached for adaptive distinguishing experiments with observable nondeterministic FSMs.

6 Conclusion

Given a non-initialized complete FSM, a method for deriving adaptive homing/distinguishing experiments is proposed. Adaptive experiments are represented as special nondeterministic observable machines, called test cases. Necessary and sufficient conditions for having adaptive homing/distinguishing test cases with minimal length for observable and non-observable nondeterministic FSMs are established. The length of shortest homing/distinguishing test cases is evaluated. Possible extensions to the proposed work include the study of adaptive experiments for partial and/or timed nondeterministic FSMs. It would be also interesting to determine the tight upper bounds on the length of homing/distinguishing test cases for observable and non-observable FSMs. Another direction for future work is to study the complexity and evaluate the efficiency of sequential and/or parallel implementations of the algorithms proposed in this paper.

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