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Leon A. Petrosyan • Vladimir V. Mazalov •  
Nikolay A. Zenkevich  
Editors

# Frontiers of Dynamic Games

Game Theory and Management,  
St. Petersburg, 2018

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# Preface

The content of this volume is mainly based on selected talks that were given at the 12th international conference “Game Theory and Management 2018,” held at the Saint Petersburg State University in Saint Petersburg, Russia, from June 27th to 29th, 2018.

Every year starting from 2007, an international conference “Game Theory and Management” (GTM) has taken place at the Saint Petersburg State University. Among the plenary speakers of this conference series were the Nobel Prize winners Robert Aumann, John Nash, Reinhard Selten, Roger Myerson, Finn Kidland, Eric Maskin, and many other famous game theorists. The underlying theme of the conferences is the promotion of advanced methods for modeling the behavior that each agent (also called player) has to adopt in order to maximize his or her reward once the reward does not only depend on the individual choices of a player (or a group of players), but also on the decisions of all agents that are involved in the conflict (game). In particular, the emphasis of the GTM 2018 conference was on the following topics:

- dynamic and differential games;
- cooperative solutions;
- dynamic game modeling in management and environmental issues;
- energy and recourse allocation;
- games in finance and marketing;
- stochastic and sequential games.

In this volume, two sorts of contributions prevail: chapters that are mainly concerned with the application of game theoretic methods and chapters where the theoretical background is developed.

In the chapter by Michael Flad, the interaction between a driver and a driving assistance system is described for the first time by means of a differential game. The system calculates the optimal control output by solving the game. It is interesting to note that this driving assistance system was implemented on a real-time system, integrated in a driving simulator and validated in a driving study.

In the chapter by Ekaterina Gromova and Yulia Lakhina on a special linear-quadratic differential game model for pollution control, an attempt is made to specify one solution from the set of solutions of the corresponding Hamilton–Jacoby–Bellman equations with the help of an economic criterion as well as a classical approach.

The chapter by Alexander Sidorov studies how the concentration of industries affects social welfare, measured as the consumer’s indirect utility. Based on the presented model, the author tries to explain that a lower concentration is not always harmful for consumers.

In the chapter by Benjamin Vallejo-Jimenez and Mario A. Gracia-Meza, it is demonstrated how the introduction of a time-inhomogeneous Markov-modulated diffusion process into asset portfolio decision problems yields higher returns for the rational decision maker.

The chapter by Nikolay A. Zenkevich, Irina Berezinetz, Natalia Nikolchenko, and Alina Ruchyova investigates the target sales rebate and buyback contracts in supplier-retailer supply chain games. Thereby, the Stackelberg model is used for the supply chain, under the condition of a fixed retail price and stochastic demand.

The chapter by Nikolay A. Krasovskii and Alexander M. Tarasyev is devoted to the analysis of competition and cooperation in dynamic auction models. A market equilibrium is defined in the Pareto set, and it is then asked how to shift the system from a competitive Nash equilibrium to the market equilibrium in the Pareto set. A shifting algorithm is proposed, and the results of the algorithm are demonstrated for a model of fast growing economies.

The chapter by Zeng Lian and Jie Zheng is concerned with the infinitely repeated Bertrand competition game among firms with stochastic entry and stochastic demand. The symmetric subgame-perfect Nash equilibrium of the game is characterized for the case when a firm’s strategy consists of two components, namely a positioning strategy and a pricing strategy.

The chapter by Leon A. Petrosyan and David W.K. Yeung deals with two-level cooperation: cooperation among members within a coalition bloc and cooperation between coalition blocs. The gain of each coalition is defined as components of the Shapley value. For the definition of the gains within a coalition, the proportional solution is used. The IDP is constructed for ensuring the time consistency of the two-level solution.

In their chapter, Vladimir Matveenکو, Maria Garmash, and Alexander Korolev investigate the equilibrium in a game theoretic model of production and externalities for a network with two types of agents having different productivities. Thereby, each player invests a part of her endowment in the first stage, and the consumption in the second stage depends on her investment and productivity as well as on investments of her neighbors in the network.

The chapter by Anna Khmelnitsakaya, Elena Parilina, and Artem Sedakov provides a comparative analysis of several procedures for the endogenous dynamic formation of the cooperation structure in TU games. The authors propose two approaches to endogenous graph formation, based on sequential link announcement and revision.

The chapter by Denis Kuzyutin, Yaroslavna Pankratova, and Roman Svetlov considers multistage multicriteria games in extensive form. The authors employ the so-called A-subgame concept for examining the dynamical properties of some non-cooperative and cooperative solutions.

The chapter by Ovanes Petrosian and Ildus Kuchkarov includes a complete description of the looking-forward approach for cooperative TU differential games. This approach can be used when the information about the game is updated—in each moment; the players receive the updated information about the motion equations and payoffs. Also, an example of the recourse extraction game is presented.

The chapter by Dmitrii Lozovanu investigates the problem of the existence of a Nash equilibrium in the class of stationary strategies for so-called average stochastic games.

In the chapter by Sergey N. Smirnov, a game theoretic approach is used for modeling the cheapest coverage of the contingent claim on an American option under all admissible scenarios.

The chapter by Dmitry B. Rokhlin and Gennady A. Ougolnitsky covers the dynamic incentive problem in the case of several followers playing a Markov game. The leader's strategy is determined by solving a stochastic control problem.

Denis Fedyanin's chapter is concerned with normal form games with an unknown parameter. The author investigates solutions based on the beliefs concerning this parameter.

The GTM 2018 program committee thanks all the authors for their active cooperation and participation during the preparation of this volume. Also, the organizers of the conference gratefully acknowledge the financial support given by the Saint Petersburg State University. Last but not least, we thank the reviewers for their outstanding contribution and the science editor Tobias Schwaibold.

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