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PLANAR DYNAMIC MODEL OF PUBLIC INTERACTION BETWEEN OFFICIAL AND HIDDEN ECONOMY

Irina Yurievna Fedorova, Mikhail Nikolaevich Prokofiev, Victor Vladimirovich Moroz, Aleksey Sergeevich Sibiryaev, Natalia Sergeevna Sergienko, Lyubov Munirovna Khanova

Financial University under the Government of the Russian Federation (Financial University)
Leningradskiy Ave., 49, Moscow, 125468, Russia

Elena Vladimirovna Sinitsyna

Market Economy Institute of the Russian Academy of Sciences
Nakhimovsky Ave., 47, Moscow, 117418, Russia

ABSTRACT

The article considers the interaction mechanism between the official (legal) and illegal (shadow) economies described by a planar dynamic model, namely a system of two nonlinear ordinary differential equations. The authors obtained the analytical solution to these model equations, as well as formulated the conditions of coexistence of legal and hidden economies.

Key words: economic mathematical modeling, legal economy, hidden economy; coexistence trajectories; planar dynamic model

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1. INTRODUCTION

The phenomenon of the hidden economy (HE) is inherent in any economic system. There is not a single example in history in which, if a legal sector exists, there could be no hidden sector. This is also noted by F. Schnaider and D. Enste, the leading researchers of the HE issues, who note that the informal economy is present in each country [1]. The HE is particularly developed during the period of transformations, when the regulation of the legal sector by the state is less effective.

The HE parasites on the body of the official or legal economy (LE), having negative impact on the state budget. In this regard, the analysis of the interaction between the LE and HE is an urgent problem of our times.

The HE is a consequence of the inefficient functioning of the state management system that in turn can occur for many reasons, including the presence of a systemic crisis in the economy, active processes of socio-economic development transformation, and the like.

In general, the notion the HE refers to any unaccounted for or partially accounted for economic activity aimed at generating income, which is not shown to public authorities and, accordingly, is not reflected in the country's GDP. The HE is a multifaceted phenomenon, and thus there are many different approaches to its definition.

The works of fundamental nature [2-4] directly refer to the research subject. They are quite complex, because building the presented mathematical models (MM) requires a significant amount of comprehensive economic information, which is not always available or reliable. Given the role of the accuracy of the source data for the construction of an adequate MM [5, 6], it is worth dwelling on the aggregated dynamic model of smaller dimension than those presented in the noted works.

The purpose of the present study is to consider the mutual functioning mechanism of the LE and the HE, and to describe the mathematical dynamics of this process.

The objectives of the study include economic mathematical modeling of the interaction dynamics of the LE and HE in order to obtain an analytical solution to this model, which will describe the functional interdependence between the LE and the HE.

2. THE NATURE AND CHARACTERIZATION OF THE HIDDEN ECONOMY

The hidden sector of the economy can act as both a buffer to mitigate the destructive impact of economic crises, and a constraint that "mutes" or makes impossible the constructive impact of economic reforms.

The inability to identify and assess the real size of the hidden sector, its motives and driving forces, sets wrong goals, as well as vectors of economic reforms, thereby minimizing their usefulness.

Despite the existence of many methods to analyze illegal economic activity, no single definition of the concept of the "hidden economic activity" has yet been formed. Thus, Yu. Latov defines the HE as a combination of illegal business actions that are contrary to applicable law [7, p. 15-16].

Shestakov A.V. characterizes the HE as a complex socio-economic phenomenon represented by a set of uncontrolled and unregulated illegal, as well as legitimate, but immoral economic relations arising among economic entities in order to obtain abnormal profit from the concealment of income and tax evasion [8],

Tarasov Yu.V. has formulated the definition of the HE as the economy, which is not taken into account and not controlled by the official state bodies, as well as activity aimed at generating income through violation of the current legislation.

The HE can be interpreted as a complex socio-economic phenomenon, represented by a set of uncontrolled and unregulated both illegal and legal, but immoral economic relations among economic entities to obtain excess profits by concealing income and tax evasion [9].

Schneider F. believes that it is advisable to include the share of gross national product, not reflected in the official reporting, to the HE [10].

According to E. de Soto, "The hidden economy is a "shelter" for those for whom the costs of compliance with the relevant legislation in the implementation of economic activities exceed the benefits of achieving their goal" [11].

Three structural elements of the HE can be distinguished: informal, hidden, and underground (criminal) ones.

The informal economy is a segment of the HE, which is manifested in the permitted economic activity, though not formally registered, and therefore not included in the statistical reporting. This part of the HE plays the role of a buffer, because it creates jobs and produces goods that are available for consumption by the poorest people. This is exactly that positive development that, in an unstable political, economic, and social environment provides an opportunity for the survival of the marginal population.

The HE is a legal economic activity, which is not recorded in the statistical reports partially or completely, and consequently, evades taxation through the concealment of income. This part of the HE, although immoral in nature, does not pose a threat to society. It is the forced economy, caused by the necessity of survival and development of one's own businesses.

The underground (criminal) economy is a part of the HE, which is carried out intentionally, aimed at enrichment, and has an anti-social character. To a greater extent, it is the subject of legal studies.

The business entity takes the decision on the opportunistic behavior based on the ratio of "net cost/benefit". Therefore, the hidden economic activity, having lower transaction costs, makes it possible to obtain greater benefits.

The increase in transaction costs in the hidden sector of the economy may lead to the probability of being punished for deviant behavior. This approach makes it possible to conceptually solve the problem of the formation of the institutional environment in transformational systems by unshadowing the economy, which would create more favorable conditions for open (legal) labor, maximizing profits without deviant behavior [12].

The first researchers of the HE, in particular K. Hart, proceeded from the fact that actually the cause of the shadowing of society was marginalization, but, as it turned out later, the HE also existed and developed in the leading countries of the world.

This afforded ground to conclude that the condition for the spread of shadowing was not so much the marginalization of society as the imperfection of market relations that was manifested in their contradictory and spontaneous nature, as well as the ability of the HE to adapt to new conditions, its mobility, and low cost.

The spread and scale of the HE is determined by the lack of established rules and regulations of economic activity and noncompliance with the requirements of the time, when it becomes more profitable to work in the "shadow".

This situation leads to the institutionalization of the HE, which can be represented by the consolidation of the behavior of the hidden sector's entities into certain organizationally sustainable forms, recognized by all participants of this activity. Therefore, the government needs to pursue a balanced socio-economic policy, which should include measures to unshadow the economy.

In the context of the information society, a notion such as a communicative shadow already exists. It is understood as economic activity caused by the latest technological advances, which not only does not fall under the current legislation and accounting for official statistics, but also, like the factors of the "third wave" (E. Toffler), cannot be assessed by any of already known pricing mechanism, and therefore, it is difficult to determine tax revenues from such activities. It is associated with the Internet network, where everything is bought and sold, while it is very difficult to account for and control this activity.

In the studies of domestic economists, devoted to the problems of economy shadowization, the main attention is paid to the methodology, the justification of measures to

counteract the shadowization of economic activity, and the specifics of the implementation of these measures in the Russian realities. That is, we can state the existing shift in emphasis of domestic economic thought towards the practical aspects of the deshadowization processes, and at the same time, ignoring the obvious need for mathematic simulation of this phenomenon.

3. DYNAMIC MODEL OF INTERACTION BETWEEN OFFICIAL AND HIDDEN ECONOMY

At their essence, the sectors of LE and HE are competitive or antagonistic in terms of their purpose. Therefore, it is worth using a "victim-predator" model, well-known in science [7], where the role of the victim will be played by the LE, while of the predator – by the HE.

Let variable x1 = x1(t) describes the size of the LE society, while variable x2 = x2(t) describes the size of the HE.

In the absence of an illegal sector of the economy, the state of the official economy is traditionally described by the classical logistic equation $\dot{x}_1 = x_1(a_1 - b_1 x_1)$,

where $\dot{x}_1 = \frac{dx_1}{dt}$ is the derivative of variable x1,

The scalar coefficients a1 and b1 reflect, respectively, the growth factors and the depletion of available opportunities.

The presence of the HE contributes to taking into account not only the opportunities for domestic LE, but also requires taking into account the external effects of the depressive nature of the official economy.

It is assumed that the above is proportional to the bilinear product b12 x1 x2 with the coefficient of proportionality b12.

Obviously, such considerations are valid with regard to the effect of the LE on the HE. As a result, we obtain the following nonlinear system of ordinary differential equations:

$$\begin{cases} \dot{x}_1 = a_1 x_1 + b_{12} x_2 x_1 + c_1 x_1^2 \\ \dot{x}_2 = a_2 x_2 + b_{21} x_1 x_2 + c_2 x_2^2 \end{cases} \quad (1)$$

This is the so-called planar point mathematical model of economic dynamics (kind of the Lotka-Volterra model), but given the phenomenon of competition [2]. In fact, it is another example of the use of the competition concept in the real economy. It is the competition, being a sign of a market economy that possesses various types of interaction among market participants (in our case, it is a symbiosis, predation, etc.), which is taken into account through coefficients of MM (1).

The stability of the dynamic model (1) is determined by the coexistence between the products of intraspecific reduction factors c1, c2, as well as interspecific factors b12, b21 of suppressive nature [4].

If the inequality b12 x b21 < c1 x c2 is satisfied, that is, internal factors dominate (play greater role) or are more significant than the competition factors, then based on MM (1) dynamics the following is stated: coexistence of the LE and the HE is possible with a certain constant ratio between the coefficients of the model.

For convenience, we rewrite the dynamic model (1) as a series:

$$\begin{cases} \dot{x}_1 = \alpha x_1 + aqx_2x_1 + apx_1^2 \\ \dot{x}_2 = \beta x_2 + bpx_1x_2 + bqx_2^2 \end{cases} \quad (1a)$$

assuming:

$a_1 = \alpha; b_{12} = aq; c_1 = ap; a_2 = \beta; b_{21} = bp; c_2 = bq$

The first equation MM (1a) is subjected to the following transformations:

$$\frac{d}{dt}(\ln x_1) = \left[a(px_1 + qx_2) + \alpha\right]$$

$$\ln x_1(t)\Big|_0^t = \int_0^t \left[a(px_1 + qx_2) + \alpha\right]dt \Leftrightarrow \quad (2)$$

$$\Leftrightarrow \ln\left(\frac{x_1(t)}{x_1(0)}\right) = \int_0^t \left[a(px_1 + qx_2) + \alpha\right]dt$$

Hence, we obtain the expression:

$$x_1(t) = x_1(0) \times \exp\left\{\int_0^t \left[a(px_1 + qx_2) + \alpha\right]dt\right\} \quad (3)$$

Similarly, the same expression is obtained for the variable x2:

$$x_2(t) = x_2(0) \times \exp\left\{\int_0^t \left[b(px_1 + qx_2) + \beta\right]dt\right\} \quad (4)$$

The following equalities take place:

$$\int_0^t \left[a(px_1 + qx_2) + \alpha\right]dt = a\int_0^t \left[(px_1 + qx_2)\right]dt + \alpha t$$

$$\int_0^t \left[b(px_1 + qx_2) + \beta\right]dt = b\int_0^t \left[(px_1 + qx_2)\right]dt + \beta t$$

Since the equalities $e^{aZ} = (e^Z)^a$ and $\exp\{(a+b)\} = e^a \times e^b$ are valid, it is fair to write the following:

$$e^{a\int_0^t (px_1 + qx_2)dt} = \left(e^{\int_0^t (px_1 + qx_2)dt}\right)^a$$

$$\exp\left\{b\int_0^t \left[(px_1 + qx_2)\right]dt\right\} = \left(e^{\int_0^t \left[(px_1 + qx_2)\right]dt}\right)^b$$

Then expressions (3) and (4), respectively, take the form:

$$x_1(t) = x_1(0) \times \exp\left\{a\int_0^t (px_1 + qx_2)dt\right\} \times e^{\alpha t} \equiv$$

$$\equiv x_1(0) \times \left(\exp\left\{\int_0^t (px_1 + qx_2)dt\right\}\right)^a \times e^{\alpha t} \quad (3a)$$

$$(x_1(t))^{1/a} = (x_1(0))^{1/a} \times \exp\left\{\int_0^t (px_1 + qx_2)dt\right\} \times e^{\frac{\alpha}{a}t}$$

Similarly:

$$(x_2(t))^{1/b} = (x_2(0))^{1/b} \times \exp\left\{\int_0^t (px_1 + qx_2)dt\right\} \times e^{\frac{\beta}{b}t} \quad (4a)$$

The ratio of the last two expressions is written as:

$$\frac{(x_1(t))^{1/a}}{(x_2(t))^{1/b}} = \frac{(x_1(0))^{1/a}}{(x_2(0))^{1/b}} \times \exp\left\{\int_0^t \left(\frac{\alpha}{a} - \frac{\beta}{b}\right)t\right\} \Leftrightarrow$$

$$\Leftrightarrow \frac{x_1^{1/a}}{x_2^{1/b}} = C \times \exp\left\{\frac{(b\alpha - a\beta)t}{ab}\right\}$$

where C is the integration constant.

We raise the last expression to the power of ab and obtain the following expression

$$\frac{x_1^{b}}{x_2^{a}} = C^{ab} \times \exp\{(b\alpha - a\beta)t\}$$

which is the general integral of dynamic model (1a).

Hence, the functional dependence of the size of the LE on the size of the HE can be written as:

$$x_1 = x_2^{a/b} \times C^a \times \exp\left\{\left(\alpha - \frac{a}{b}\beta\right)t\right\} \quad (5)$$

the inverse dependence of the size of the HE on the size of the LE is as follows:

$$x_2 = x_1^{b/a} \times C^{-b} \times \exp\left\{\left(\beta - \frac{b}{a}\alpha\right)t\right\} \quad (6)$$

The resulting formulas show the following:

when inequality $\alpha < \frac{a}{b}\beta$ is true, there is a decline in production (economic recession) over time;

if inequality $\beta < \frac{b}{a}\alpha \Leftrightarrow \frac{\beta}{\alpha} < \frac{b}{a}$ is true, we have a decrease in the size of the HE in the society;

3) the above inequalities allow predicting the size of the LE and the HE for an arbitrary point in time t, selecting in advance the numerical values of the dynamic model coefficients.

Economic mathematical modeling of the dynamic regime is greatly simplified [4] when transiting to the so-called dimensionless model.

In our case, by replacing the variables $t = \tau/\alpha$, $x_1 = \frac{\alpha}{ap}u_1$, $x_2 = \frac{\beta}{bq}u_2$, we obtain a dynamic model in dimensionless form:

$$\dot{u}_1 = u_1(1 - u_1 - \varepsilon_1 u_2)$$
$$\dot{u}_2 = \gamma u_2(1 - u_2 - \varepsilon_2 u_1) \quad (7)$$

where values $\varepsilon_1 = \frac{\beta}{\alpha} \times \frac{a}{b}$, $\gamma = \frac{\beta}{\alpha}$, $\varepsilon_2 = \frac{\alpha}{\beta} \times \frac{b}{a}$ are scalar coefficients.

Finally, the values ε1 и ε2 are written as $\varepsilon_1 = \gamma\frac{a}{b}$; $\varepsilon_2 = \frac{b}{\gamma a}$;

Thus, an equality ε1 x ε2 = 1 is valid.

Note that unlike the initial mathematical (1a) model, the dimensionless dynamic model (7) includes only three scalars.

We rewrite formula (5) in dimensionless coordinates using the above replacement of variables. After the obvious transformation, it takes the form:

$$u_1 = \frac{ap}{\alpha} \times \left(\frac{\beta}{bq} u_2\right)^{a/b} \times C^a \times \exp\{(1-\varepsilon_1)\tau\} \qquad (8)$$

Consequently, the size of the official economy will grow provided ε1 < 1.

Similarly, the formula (6), which estimates the size of the HE, has the form:

$$u_2 = \frac{bq}{\beta} \times \left(\frac{\alpha}{ap} u_1\right)^{b/a} \times C^{-b} \times \exp\left\{\left(\frac{\beta}{\alpha} - \frac{b}{a}\right)\tau\right\} \qquad (9)$$

The size of the HE will decrease when the following inequality is valid:

$$\left(\frac{\beta}{\alpha} - \frac{b}{a}\right) < 0 \Leftrightarrow a\beta < \alpha b \qquad (10)$$

4. CONCLUSIONS

The growth of the economy shadowization is causing significant damage to society. In addition, an urgent social problem emerges: the public consciousness is developing doubts about the ability of state institutions, especially law enforcement agencies, to guarantee public safety, and to protect the identity and property of citizens.

In the article, the interaction mechanism between the official and illegal economies is reproduced using one of the point MM of nonlinear dynamics. For a planar dynamic model, an analytical solution is obtained, which allows predicting interrelated sizes of the LE and the HE at any point in time.

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