A Simple Econophysics Model of the Stock Market as a Nonequilibrium Open System

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Abstract. Mathematical modeling of a stock market functioning is one of the actual and at the same time complex task of the modern theoretical economics. From our point of view, building such mathematical models "ab initio", by using analogy between the stock market and a certain physical system (in our work, laser), is the most promising approach. This paper proposes a simple econophysical model of stock market as an open nonequilibrium system in form of Lorenz–Haken equation. In this system, variation of ask price, variation of bid price, and instantaneous difference between numbers of agents in active and passive state are intensity of external information flow is a control parameter. This model explains the impossibility of existence of an equilibrium state of the market and shows the presence of deterministic chaos in a stock market. AQ1

Keywords: Deterministic chaos · Lorenz–Haken equation · Stock market Ask price · Bid price

1 Introduction

From the second half of the XX century the general trend of science development is the penetration of the ideas and methods of physics into other natural and humanitarian disciplines. Methods of physical modeling are often used in sciences such as demography, sociology and linguistics.

An interdisciplinary research field, known as econophysics, was formed in the middle 1990s as an approach to solve various problems in economics, such as uncertainty or stochastic processes and nonlinear dynamics, by applying theories and methods originally developed by physicists. H. Eugene Stanley coined the term "econophysics" in order to describe the large number of papers written by physicists in the problems of stock markets (for econophysics reviews see refs. [1–4]).

Current state of theoretical economics allows one to effectively use advanced methods of physico-mathematical modeling for economical system. A remarkable example is applying nonlinear dynamics to analysis of financial time series [5, 6]. Moreover, in 1963 Mandelbrot [7] during his research of cotton prices found out that the prices follows a scaled distribution in time. That discovery originated a new approach in market research called fractal market analysis [8]. A systematic research of deterministic chaos in financial markets started from works of Savit [9].

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By the end of 20th century there were formed two lines of research of deterministic chaos in financial markets. The first one is related to discovery and analysis of deterministic chaos in the structure of financial markets. Studies of that kind are usually based on qualitative characteristic and quantitative measures of chaos [10], and their results show conclusively that deterministic chaos exists in financial markets [11–19]. The second line connected to retrieval of explicit form of such dynamical systems. Definitely, construction of such models is far more complex problem. That is why the number of relevant publications on this topic is relatively small. The most comprehensive survey of mathematical models of financial markets can be found in the book of Elliott and Kopp [20]. Although the book and other relevant publications contain numerous conceptual models, we have not found any "ab initio" econophysical model of a stock market that can explain its fundamental functioning mechanisms. Thus, the purpose of this work is building of econophysical model of a stock market using parallels between market functioning and physical principles of laser operation.

2 Construction of the Model

2.1 Model Assumptions

1. Stock market is a macroscopic system. Assume that the stock market is a dynamical system that consists of numerous market agents (investors) ($N \gg 1$). Modeling of such systems does not require detailed analysis of interactions between the agents on the micro-level. For macrosystem description we use macroscopic parameters and dynamic variables of the system. For macroscopic dynamic variables we have chosen aggregated flows of ask and bid price changes and dynamical difference of market agents in specific states.
2. Stock market is a point autonomous dynamical system.

$$\dot{\mathbf{X}} = \mathbf{F}(\mathbf{X}, \boldsymbol{\beta}). \tag{1}$$

This statement goes without saying, as it depends only on chosen modeling approach and objectives. However, the choice of (1) as the base model is made for reason. It is based on tests that the constructed mathematical model agrees with empirical (observed) data, for which we used available financial time series of ask/bid stock prices.

3. Every market agent can be in one of two possible states: active ($|\boldsymbol{a}\rangle$-state) or passive ($|\boldsymbol{p}\rangle$-state). A particular market agent being in $|a\rangle$-state has maximum amount of valuable information about financial asset ($I_{|a\rangle}$) and has minimum information ($I_{|p\rangle}$) otherwise. In $|a\rangle$-state the agent can generate local demand on deal with the asset and send an "ask-quantum" to other agents. If the agent is in $|p\rangle$-state, then the agent's rational decision is do not generate demand on deal ("bid-quantum"). Moreover, for the agent in $|p\rangle$-state generating of a deal offer depends on the agent's reaction on received "ask-quantum" (Fig. 1a) or can be his or her own

decision (Fig. 1b). General pattern in stock markets is that local "ask" waves ("quanta") induce local "bid" waves ("quanta").

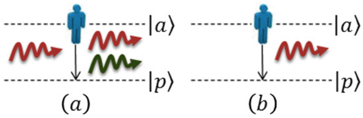

Fig. 1. Agents generate «ask-quanta» (red) and «bid-quanta» (green). (a) Forced generation of an «bid-quantum». (b) Spontaneous generation of an «ask-quantum». AQ2

4. Stock market is a nonequilibrium open system. Indeed, stock market is an open system that continuously interacts with the external world. Sources of external information include corporate financial reports, financial news feeds, stock-ticker data and others. This information flow, in some sense, "pump up" the stock market, making inverse population of market agents: $N_{|a\rangle} \gg N_{|p\rangle}$, where $N_{|a\rangle}$ is the number of agents being in $|a\rangle$-state, $N_{|p\rangle}$ is the number of agents being in $|p\rangle$-state.

With acceptable accuracy, the distribution of number of agents by their states can be represented as follows:

$$N_{|a\rangle} = N_{|p\rangle} \exp\left(-\left(I_{|a\rangle} - I_{|p\rangle}\right)/\theta\right), \quad (2)$$

where θ is average intensity of stochastic interactions between market agents Simple analysis of Eq. (2) allows to identify two macroscopic states of the market: stable equilibrium state and nonequilibrium state. If $I_{|a\rangle} - I_{|p\rangle} \gg \theta$, then $N_{|p\rangle} \gg N_{|a\rangle}$. In this case, the system is in stable equilibrium state. Otherwise, if $I_{|p\rangle} - I_{|a\rangle} \gg \theta$, then $N_{|a\rangle} \gg N_{|p\rangle}$. This case corresponds to nonequilibrium state of the system.

Taking into account continuous information pumping, stock market is always functioning in nonequilibrium state, making "avalanches" of ask and bid "quanta". Due to information pumping, the equilibrium state is almost unreachable. It is crucially important, that existence of chaotic states is a fundamental property of nonequilibrium open systems [24, 25].

2.2 Dynamic Variables of the Model and the Relationship Between Them

Let us define dynamic variables in Eq. (1) for constructing nonlinear dynamical model of stock market: $x_1(t) \equiv X_{ask}(t) - X_{ask}^{eq}$ is the variation of "ask" price ($X_{ask}(t)$) relative to equilibrium value (X_{ask}^{eq}) is the "ask" price in equilibrium state); $x_2(t) \equiv X_{bid}(t) - X_{bid}^{eq}$ is the variation of "bid" price ($X_{bid}(t)$) relative to equilibrium value (X_{bid}^{eq}) is the "bid" price in equilibrium state); $x_3(t) \equiv N_{|a\rangle}(t) - N_{|p\rangle}(t)$ instantaneous difference between numbers of agents in $|a\rangle$-state and $|p\rangle$-state.

The choice of these dynamic variables responds to possibility to test whether the constructed dynamical system agrees with empirical data, for which we have chosen available time series of ask and bid prices from real stock markets. However, it is impossible to compare the third dynamic variable with actual data, due to available datasets does not contain these values. Thus, the fit test can be performed only with two dynamic variables.

Let us establish connections between dynamic variables and their change rates. Variation rate of the ask price is defined by concurrency of two factors: rate decrease due to market relaxation ($-\alpha x_1(t)$) and rate increase due to growing bid price variation ($+\beta x_2(t)$):

$$\dot{x}_1(t) = -\alpha x_1(t) + \beta x_2(t) \tag{3}$$

Term $-\alpha x_1(t)$ in (3) is necessary due to relaxation of nonequilibrium system. According to Le Chatelier's principle [26], when the system at equilibrium is subjected to change by external force, then the system readjusts itself to counteract (partially) the effect of the applied change. Indeed, without term $+\beta x_2(t)$ the Eq. (3) has the following form:

$$\dot{x}_1(t) = -\alpha x_1(t). \tag{4}$$

A solution of differential Eq. (4) is a function of form $x_1(t) = A\exp(-\alpha t)$. Therefore, $X_{ask}(t) \to X_{bid}^{eq}$ when $t \to \infty$ (stock market tend to stable equilibrium). In Eq. (4) α – relaxation parameter, related to relaxation time (τ_1) according to: $\alpha = 1/\tau_1$. Term $+\beta x_2(t)$ in (3) refers to the fact, that increase of bid price variation leads to increase of variation rate of ask prices.

Variation rate of the bid price is defined by concurrency of two factors: rate decrease due to market relaxation ($-\gamma x_2(t)$) and rate increase due to $+cx_1(t)x_3(t)$.

$$\dot{x}_2(t) = -\gamma x_2(t) + cx_1(t)x_3(t) \tag{5}$$

Presence of the first term in (5) is explained by Le Chatelier' s principle. Term $+cx_1(t)x_3(t)$ is explained as follows: "bid quantum", on which every market agent reacts considering "ask quanta" flow, is proportional to ask price variation and depends on the agent's current state ($|a\rangle$-state or $|p\rangle$-state).

Dynamics of difference between numbers of market agents in $|\boldsymbol{a}\rangle$-state and $|\boldsymbol{p}\rangle$-state is defined as follows.

$$\dot{x}_3(t) = \varepsilon(I_0 - x_3(t)) + kx_1(t)x_2(t) \tag{6}$$

Again, term $-\varepsilon x_3(t)$ is in Eq. (6) due to Le Chatelier's principle. Parameter I_0 refers to intensity of external information pumping, so instantaneous difference between numbers of agents in $|a\rangle$-state and $|p\rangle$-state grows with increase of I_0.Term $+kx_1(t)x_2(t)$ represents the power that the aggregated ask price variation spends on creation of the aggregated bid price variation.

2.3 Modeling Results and Their Interpretation

The system of differential Eqs. (3), (5) and (6) represents the well-known Lorenz–Haken equation [27]:

$$\dot{x}_1 = -\alpha x_1 + \beta x_2, \dot{x}_2 = -\gamma x_2 + c x_1 x_3, \dot{x}_3 = \varepsilon(I_0 - x_3) + k x_1 x_2 \quad (7)$$

System (7) is one of the most studied 3-dimensional dynamical systems. General properties of (7) are presented in works [28, 29]. Let us consider (7) as a system with one control parameter I_0. From changing control parameter's value, we can make two important conclusions about system (7).

If $\beta c/\alpha\gamma \cong 1$ and $0 < I_0 < 1$, then $X_{ask}(t) \to X_{ask}^{eq}$, $X_{bid}(t) \to X_{bid}^{eq}$ and $N_{|a\rangle}(t) \to N_{|p\rangle}(t)$ as $t \to \infty$. In case of relatively small intensity of external information pumping, the stock market tends to stable equilibrium. However, practically this stable equilibrium state cannot be reached, since the market is an open system with permanent eternal information pumping. If $\beta c/\alpha\gamma \cong 1$ and $I_0 \cong 28$, then the stock market functions as an open nonequilibrium system with deterministic chaos. It is worth to mention that such behavior is typical for a financial market with considerably intense external information.

3 Conclusion

The constructed simple econophysical model of a stock market as an open nonequilibrium system allows explaining the unrealizability of equilibrium state of the market as well as appearance of deterministic chaos in the market. These phenomena are explained only by quantitative characteristics of external information pumping as a control parameter of the system.

However, this simple model cannot explain several other important phenomena, such as heavy-tailed distribution of financial time series, financial bubbles and economic downfalls. Actually, time series of ask and bid price variations has trimodal centered distribution with "cropped tails". Local maxima of the PDF represent three stable equilibrium points of the dynamical system (7). Formally, by increasing the number of stable equilibrium points one can fit theoretical PDF to empirical PDF. That will lead to significant growth of dynamical system dimension. Although such modification is possible, it also removes low-dimensional (deterministic) chaos, which is a characteristic feature of stock markets. We suppose, that explanation of heavy-tailed distribution of financial time series requires modification of dynamical system by introducing the noise of specific kind. Particularly, the form of time series with heavy-tailed distribution can be achieved by including a power-law multiplicative noise [30], what is the subject of our further research.

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