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Interference of Nonstationary Oblique Shock Waves

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Abstract—Special features of calculation of the flow parameters behind a nonstationary oblique shock wave moving in a stream of absolutely nonviscous gas are considered. The wave intensity at which the stream behind the shock wave may exhibit singularities is determined. The problem of calculating a nonstationary shock wave configuration formed during the interaction of a supersonic jet with an obstacle is solved. *© 2002 MAIK "Nauka/Interperiodica".*

Many gasdynamic problems [1–3] encounter the task of determining the parameters of flow f_2 behind a shock wave propagating in a gas stream, given the parameters of flow f_1 (velocity u_1, static pressure p_1, density ρ_1, temperature T_1, etc.) in front of the wave, the velocity w of the shock wave along a straight trajectory making an angle α with the gas stream direction, and the angle σ of the wave slope relative to the trajectory (Fig. 1a).

The main parameter of the problem is the shock wave intensity

$$J = \frac{p_2}{p_1} = (1+\varepsilon)\left(\frac{u_{1n}-D}{a_1}\right)^2 - \varepsilon, \quad \varepsilon = \frac{\gamma-1}{\gamma+1}, \quad (1)$$

which depends on the components of velocities $u_{1n} = u\sin(\sigma-\alpha)$ and $D = w\sin\sigma$ normal to the wave front and on the thermodynamic variables of the initial stream (sound velocity a_1 and adiabatic index γ). A physically justified condition $J \geq 1$ poses restrictions on the determining parameters, which will be considered below.

Using the conditions for dynamic consistency on the shock wave front [1–3], we can readily determine the Rankine–Hugoniot adiabate of the shock wave; establish relationships between specific enthalpies, temperatures, sound velocities, and the shock wave intensity,

$$E = \frac{\rho_2}{\rho_1} = \frac{J+\varepsilon}{1+\varepsilon J}, \quad \frac{h_2}{h_1} = \frac{T_2}{T_1} = \frac{a_2^2}{a_1^2} = \frac{J}{E} = \frac{J(1+\varepsilon J)}{J+\varepsilon}; \quad (2)$$

and derive expressions for the normal velocity components

$$D = u_{1n} + \chi I a_1, \quad u_{2n} = u_{1n} + \chi(1-\varepsilon)a_1\left(I - \frac{1}{I}\right), \quad (3)$$
$$I = \sqrt{\frac{J+\varepsilon}{1+\varepsilon}},$$

where $\chi = \pm 1$ is the index of wave propagation direction.

Expressing the velocity components through the angles $\sigma_e = \sigma - \alpha$ and β and using the condition $u_{1\tau} = u_{2\tau}$, we determine the stream velocity u_2 behind the shock wave:

$$u_2 = u_1 \frac{\cos\sigma_e}{\cos(\sigma_e - \beta)}. \quad (4)$$

Formulas (3) and (4) yield a relationship between the angles β and σ_e for the oblique shock wave:

$$\tan(\sigma_e - \beta) = \tan\sigma_e + \chi\frac{(1-\varepsilon)(I-1/I)}{M_1\cos\sigma_e}. \quad (5)$$

Combining formulas (4) and (2), we obtain a relati-

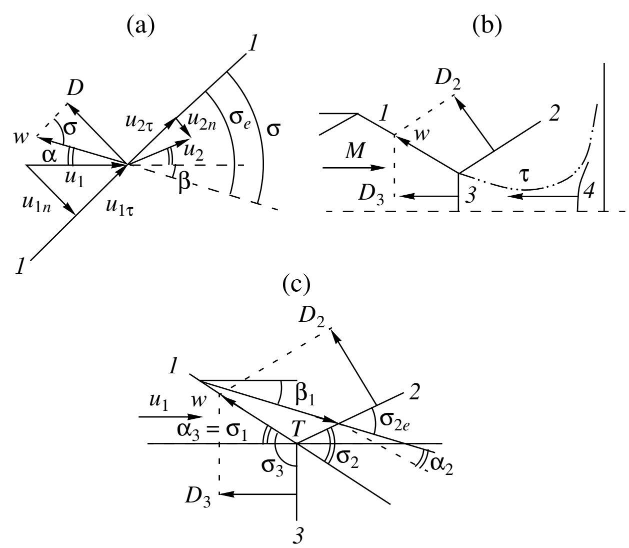

Fig. 1. The flow geometry for (a) an oblique shock wave moving downstream at a velocity w in a stream of velocity u, (b) overexpanded supersonic stream incident onto an infinite flat obstacle, and (c) triple shock wave configuration moving down a supersonic stream.

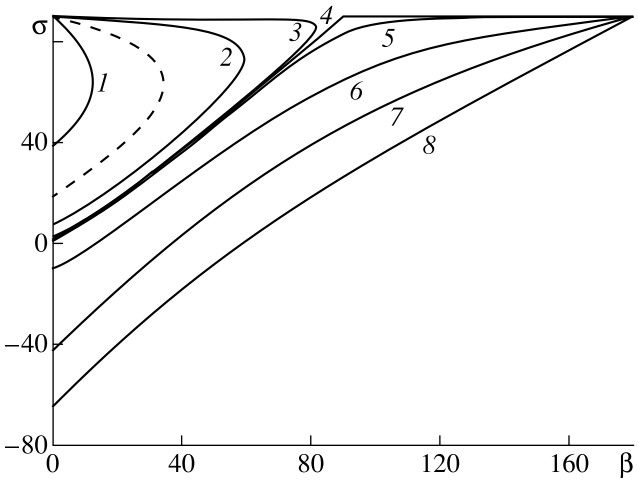

Fig. 2. Plots of the angle β of the stream rotation behind the shock wave versus the sloping angle σ of the wave.

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$$M_2 = M_1 \frac{\cos\sigma_e}{\cos(\sigma_e - \beta)} \sqrt{\frac{J + \varepsilon}{J(1 + \varepsilon J)}} \qquad (6)$$

between the Mach number $M_2 = u_2/a_2$ behind the shock wave and that ($M_1 = u_1/a_1$) in the initial stream.

Setting $J = 1$ in Eq. (1), we obtain a formula

$$\sigma_s = \arcsin\frac{M_D - \chi}{M_1}, \qquad (7)$$

which relates the Mach numbers $M_D = D/a_1$ and $M_1 > 0$ to the angle $\sigma_e = \sigma_s$ between the direction of the oncoming stream and the weak discontinuity front. Note that, in contrast to the case of a weak stationary shock wave ($M_D = 0$) for which $\sigma_s = \arcsin(1/M_1)$ is fixed, the σ_s value in the nonstationary case may vary within the interval $[-\pi/2, \pi/2]$. In the nonstationary case, the interval $\sigma_s \in (0, \pi/2]$ corresponds to a weak opposite shock wave, while the interval $\sigma_s \in [-\pi/2, 0)$ corresponds to a weak cocurrent shock wave. The special value $\sigma_s = 0$ refers to the case of a weak shock wave propagating perpendicularly to the oncoming stream, whereby the stream does not influence the shock wave (which propagates as if the stream were absent).

The value of $J = 1$ corresponds to the minimum possible intensity of the shock wave. A maximum intensity is attained for the J values corresponding to a forward shock wave ($\sigma = \pi/2$). Let us fix the Mach number $M_1 > 0$ and analyze the behavior of the main gasdynamic parameters for various M_D and $J \geq 1$, restricting the consideration to $\chi = -1$.

As can be seen from formula (7), the range of M_D ($\chi = -1$) is limited from above by $M_D = M_1 - 1$. For this M_D value, the stream can feature only a weak discontinuity with $J = 1$ and $\sigma = \pi/2$, which either propagates upstream (for $M < 1$) or is carried downstream (for $M > 1$). As the M_D value decreases, the lower boundary σ_s given by formula (7) goes down to reach zero for $M_D = -1$ and the value $\sigma_s = -\pi/2$ for $M_D = -M_1 - 1$. For $M_D < -M_1 - 1$, an oblique shock wave with an intensity $J > 1$ exists for any $\sigma \in [-\pi/2, \pi/2]$.

Figure 2 shows the $\sigma(\beta)$ curves constructed for various $M_D < M_1 - 1$, where $M_1 = 3$. As can be seen, the character of these curves significantly depends on the M_D value. For $M_D \in (M_*, M_1 - 1)$, the shape of $\sigma(\beta)$ (curves *1*–*3*) is qualitatively the same as in the case of a stationary shock wave ($M_D = 0$, dashed curve). For $M_D < M_*$, there always exists a shock wave which causes the stream to deviate by any preset angle $\beta \in [0°, 180°]$ (curves *5*–*8*).

The special Mach number M_* corresponds to the case whereby the M_2 value becomes zero behind the forward shock wave (curve *4*). Using formulas (1) and (3), we can readily obtain a relationship between M_* and the Mach number M_1 of the oncoming stream:

$$M_* = \frac{(1 - 2\varepsilon)M_1 - \sqrt{M_1^2 + 4(1 - \varepsilon)^2}}{2(1 - \varepsilon)}. \qquad (8)$$

For $M_D < M_*$, the stream behind the forward shock wave is rotated by 180°; for $\sigma < \pi/2$, the angle of rotation varies within the interval $\beta \in [0°, 180°]$. For $M_D < -M_1 - 1$, all the $\sigma(\beta)$ curves originate from the point (0°, –90°) and come to the point (180°, 90°).

In practical problems involving nonstationary shock waves, the analysis is usually performed using reversal of the wave motion [3]. The passage from a laboratory coordinate system to that related to the moving shock wave simplifies the problem by reducing the analysis to a simpler case of the jump in compression [2]. However, this transformation is not always possible in complicated cases involving the interference of oblique shock waves. For example, consider a shock wave configuration encountered in the study of nonstationary processes in jet streams [4]. When a homogeneous flat stream is outgoing from a profiled overexpanding nozzle ($n_a = p_n/p_a < 1$), there appears a straight oblique compression jump *1* (Fig. 1b) at the nozzle edge. In the case of an efficiently small n_a value, this shock wave exhibits irregular reflection from the stream axis. As a result of this reflection, there appear the Mach jump *3*, the reflected oblique jump *2*, and the tangential discontinuity τ separating the stream behind jumps *2* and *3*. For the sake of simplicity, let us assume that reflection of the oblique jump gives rise to a stationary Mach configuration of the shock waves in which the branching jump *3* is orthogonal to the current lines of the oncoming stream.

Let such a stream come onto an infinite flat obstacle (Fig. 1b). As was demonstrated previously (see, e.g., [5]), the dynamic and total pressures in the supersonic stream past the jumps *1* and *2* may significantly exceed the corresponding values behind the straight jump *3* in a subsonic flow. A large difference between these value may result in that the tangential discontinuity τ would instantaneously "stick" to the obstacle surface and block the subsonic flow. As a result, the "sticking" point will give rise to a curvilinear shock wave *4* interacting with the Mach jump *3* and propagating upstream. This will lead to a shift of the shock wave *2* containing the special point T with the branching jump *3* (Fig. 1c). Thus, we arrive at the problem of describing the stream behind the shock wave *2*.

In order to solve this problem, note that the shock wave *2* propagates along a trajectory coinciding with the line of jump *1* at a velocity of $w = D_2/\sin\sigma_2 = D_3/\sin\sigma_3$, where D_2 and D_3 are the velocities of shock waves *2* and *3*, respectively; σ_2 and $\sigma_3 = \pi/2 + \sigma_1$ are the angles between vector w and the surfaces of discontinuities *2* and *3*, respectively; and σ_1 is the sloping angle of jump *1* (Fig. 1c). The shock wave *3* moving upstream at a relative velocity of $M_{D3} < 0$ leads to an increase in the wave intensity,

$$J_3 = (1+\varepsilon)\left[\frac{u}{a}\sin(\sigma_3-\alpha_3) - \frac{w}{a}\sin\sigma_3\right]^2 - \varepsilon$$

$$= (1+\varepsilon)(M - M_{D3})^2 - \varepsilon,$$

where $\alpha_3 = \sigma_1$ is the angle between vector w and the oncoming stream velocity u. As a result, the intensity $J_2 = J_3/J_1$ of the shock wave *2* increases as well, this leading to a change in the angle σ_2 related to J_2 by the formula (1)

$$J_2 = (1+\varepsilon)(M_1\sin\sigma_{2e} - M_{D2})^2 - \varepsilon,$$
$$\sigma_{2e} = \sigma_2 - \alpha_2. \quad (9)$$

Here, M_1 is the Mach number behind jump *1* and $\alpha_2 = \sigma_1 - \beta_1$ is the angle between vector w and the stream velocity u_1 behind jump *1*. Upon numerically solving Eq. (9), we can calculate σ_2 and, hence, determine β_2, the angle of stream rotation (5) by shock wave *2*.

In the general case, the angle β_2 differs from β_{20}, the angle of stream rotation behind the immobile jump *2* in the stationary Mach configuration. As a result, the conditions of dynamic consistency at the tangential discontinuity τ separating the streams are broken. To restore these conditions, we have to admit that the initial straight jump *3* acquires curvature in the vicinity of the triple point T in the course of the shock wave motion. The curvature must provide both for the condition of equal pressures and for the collinearity of current lines above and below the tangential discontinuity τ (Fig. 1c). Assuming that the velocity w at which the shock wave *3* moves along jump *1* is still related to the velocity D_3 of the shock wave in the vicinity of the symmetry axis as $D_3 = w\cos\sigma_1$, we arrive at a system of equations for determining the gasdynamic parameters of flow at the branching point T (Fig. 1c):

$$J_3 = J_1J_2, \quad \beta_3 = \beta_2 - \beta_1. \quad (10)$$

Here, the intensities J_2 and J_3 are calculated by formulas (9), while J_3 is determined as

$$J_3 = (1+\varepsilon)(M\sin\sigma_{3e} - M_w\sin\sigma_3)^2 - \varepsilon,$$
$$\sigma_{3e} = \sigma_3 - \alpha_3, \quad M_w = w/a,$$

and the angles β_2 and β_3 are determined using formula (5). The results of calculations showed that, as the velocity D_3 of the Mach jump *3* grows, the intensities J_2, J_3 and the angles β_2, β_3 monotonically increase. A more complicated behavior is observed for the angle σ_3 describing the slope of jump *3* at the point T: the function $\sigma_3(M_{D3})$ initially (at small $|M_{D3}|$) decreases from $\sigma_{30} = 90° + \sigma_1$, then exhibits a minimum, and eventually increases again tending to σ_{30} as $M_{D3} \longrightarrow M_*$, where M_* is given by formula (8). At the point $M_{D3} = M_*$, the derivative of the function $\sigma_3(M_{D3})$ exhibits discontinuity; for $M_{D3} < M_*$, the σ_3 value monotonically decreases with increasing $|M_{D3}|$. It should be noted that, for $|M_{D3}| > |M_*|$, the stream behind jump *3* changes direction to the opposite and the gasdynamic parameters in the vicinity of point T have to be calculated using, instead of Eqs. (10), the system of equations

$$J_3 = J_1J_2, \quad \pi - \beta_3 = \beta_2 - \beta_1.$$

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REFERENCES

1. V. N. Uskov, *Traveling One-Dimensional Waves* (Balt. Gos. Tekh. Univ., St. Petersburg, 2000).
2. K. P. Stanyukovich, *Nonstationary Motions of Continuous Medium* (Gostekhizdat, Moscow, 1955).
3. G. Emanuel and T. H. Yi, Shock Waves, No. 10, 113 (2000).
4. B. G. Semiletenko, B. N. Sobkolov, and V. N. Uskov, Izv. Sib. Otd. Akad. Nauk SSSR, Tekh. Ser., No. 13 (1975).
5. A. V. Omel'chenko and V. N. Uskov, Izv. Akad. Nauk, Mekh. Zhidk. Gaza, No. 6, 118 (1995).

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