We take the initial velocity distribution density proportional to its square. In particular, Maxwell distribution doesn't satisfy this condition. We add the force field and wait for attaining equilibrium. Then we remove the force field and again wait for attaining equilibrium. Modelling many times these cycles, we obtain the different intermediate types of the density and the asymptotic behavior.

Criteria for foliations with transverse linear connection to be pseudo-Riemannian and Riemannian

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At present Riemannian foliations form the most deeply studied class of foliations with transverse geometric structures. Works of B. Reinhart, A. Haefliger, E. Ghys, Y. Carriere, E. Salem, V. Sergiesku and many others and also monographs of P. Molino, P. Tondeuer and V. Y. Rovenskii represent a significant contribution to the study of Riemannian foliations.

R. A. Wolak in [2] put the following question:

"When a G-foliation is a Riemannian one ?"

R. A. Wolak proved that every compact G-foliation of finite type is a Riemannian one. An analogous statement was proved by R. A. Wolak for foliations admitting transverse systems of differential equations of an arbitrary order. A number of other conditions for a compact foliation (M, F) to be Riemannian are well known [4].

For conformal foliations of a codimension $q > 2$ a criterion of Riemannianness was proved in [3]. For foliations (M, F) with trasverse parabolic geometry of rank one a criterion of Riemannianness is known from [5]. According to this criterion (M, F) is a Riemannian foliation if and only if all its holonomy groups are relatively compact.

Let (M, F) be a a foliation with transverse linear connection. We consider a general case when (M, F) is a foliation of a codimension q on n-dimensional manifold, $0 < q < n$.

We prove the following criterion for foliations with transverse linear connection to be pseudo-Riemannian.

Theorem 1 *Let (M, F) be a foliation of a codimension q with transverse linear connection given by (N, ∇)-cocycle. Let Q be the associated connection on the foliated bundle of transverse frames $\mathcal{R}(M, H)$, where $H = GL(q, \mathbb{R})$.*

Then (M, F) is a pseudo-Riemannian foliation given by (N, g)-cocycle, where g is a pseudo-Riemannian metric of a signature $(k, q-k)$, $0 \leq k \leq q$, parallel respectively ∇, if and only if there exists a point $v \in \mathcal{R}$ such that the holonomy group $\Phi(v)$ of the connection Q at v belongs to the pseudo-orthogonal subgroup $O(k, q-k)$ of the group H.

In the case when the dimension of (M, F) is zero, Theorem 1 implies the Schmidt's criterion for a torsion free linear connection to be the Levi-Civita connection for a pseudo-Riemannian metric [1].

As a corollary of Theorem 1 we get a necessary and sufficient conditions for a foliation (M, F) with transverse linear connection to be Lorentzian.

We prove also the following statement.

Theorem 2 *Let (M, F) be a foliation of a codimension q with transverse linear connection and let $H = GL(q, \mathbb{R})$.*

Then (M, F) is a Riemannian foliation if and only if the holonomy group $\Phi(u)$ at some point $u \in \mathcal{R}$ of the the foliated bundle of transverse frames $\mathcal{R}(M, H)$ is a relatively compact subgroup of the Lie group H.

The research was supported by RFBR (grant 16-01-00312-a) and the Basic Research Program at the HSE (project 90) in 2017.

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Lyapunov's exponents and multiplicative ergodic theorem for log-summable cocycles

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