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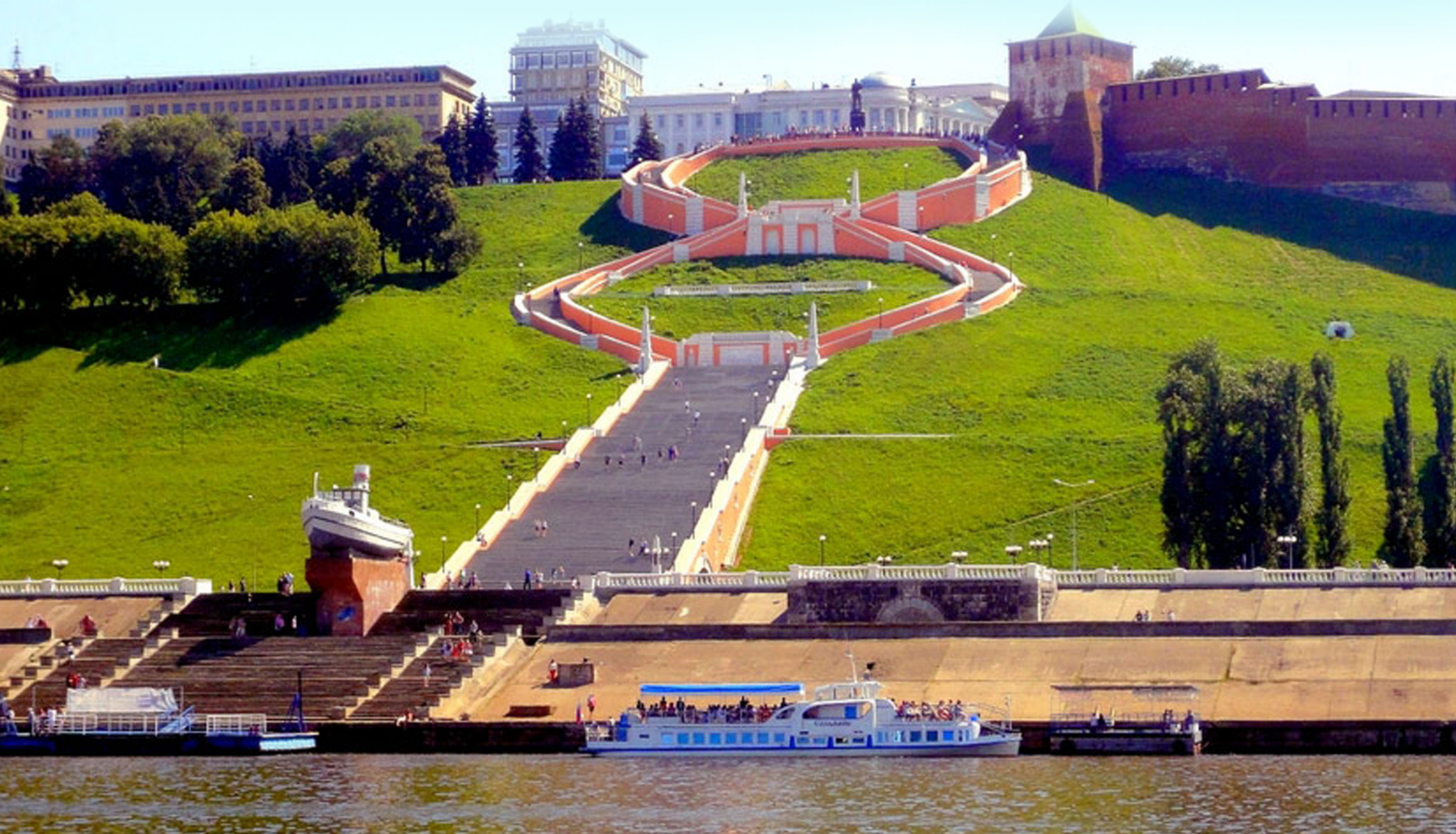
BOOK of ABSTRACTS



Steklov Mathematical Institute of the RAS

Institute of Information Technologies, Mathematics and Mechanics of the
National Research Lobachevsky State University of Nizhni Novgorod

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Metric characteristics of complexity in dynamical systems

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Lyapunov function method and estimation of state space structure of nonlinear maps

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Investigation of a macro-structure of the state space of nonlinear maps is connected with investigation of stability "in the large", where the size of an attraction domain of a stable state is calculated or estimated. For study of the macro-structure of the state space it is convenient to use the direct Lyapunov method [1,2]. To analyze stability and obtain qualitative characteristics of a nonlinear map admitting linearization near a stable state one can use Lyapunov function of the quadratic form which is Lyapunov function for the linearized map. Possibility of choosing of the coefficients of Lyapunov function with using of the simple relations such as the inequality $\Delta V(x) < 0$ for the linearized map is discussed in [3,4].

In the current work these Lyapunov functions are used to permit analytic evaluation $V(x) < V_{0y}$ of the attraction domain of the stable state of the nonlinear map (with help of methods described in [5]). This evaluation, in particular, contains the value δ such that $\max \Delta V(x)/V(x) = \delta V_{0y}$ over the level surface $V(x) = V_{0y}$ for the Lyapunov function.

Theorem. Let nonlinear map be given in the form

$$\bar{x}_i = a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n + \Omega_i(x_1, x_2, \ldots, x_n) \quad (i = 1, 2, \ldots, n), \qquad (1)$$

where

$$|\Omega_i(x_1, x_2, \ldots, x_n)| \leq N(x_1^2 + x_2^2 + ... + x_n^2) \quad (i = 1, 2, \ldots, n),$$

and $x_1 = x_2 = ... = x_n = 0$ is its stable state. And let the positively defined quadratic form

$$V(x_1, x_2, \dots, x_n) = \sum_{i=1}^{n}\sum_{j=1}^{n} K_{ij}x_ix_j \quad (K_{ij} = K_{ji}).$$

be the Lyapunov function for the linearized map which satisfies the condition

$$\max_{V=V_0}(\Delta V/V) = -\delta, \ \ (\max_i\{|z_i|^2 - 1\} \leq -\delta \leq 0).$$

Then the domain $V(x_1, x_2, \dots, x_n) < V_{0y}$ for

$$V_{0y} = ((-\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n}|a_{il}||K_{ij}|C_lM + ((\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n}|a_{il}||K_{ij}|C_lM)^2 +$$

$$+\delta\sum_{i=1}^{n}\sum_{j=1}^{n}|K_{ij}|M^2)^{1/2})(\sum_{i=1}^{n}\sum_{j=1}^{n}|K_{ij}|M^2)^{-1})^2,$$

is the evaluation of the attraction domain for the stable state of the map (1), where

$$M = N(C_1^2 + C_2^2 + ... + C_n^2), \quad C_l = \sqrt{\frac{A_{ii}}{detK}} \quad (l = 1, 2, \dots, n),$$

A_{ii} is the cofactor of the entry K_{ii} in matrix $K = (K_{ij})$.

Nevertheless, the value V_{0y} that was obtained may be essentially undervalued in comparison with the value V_0 for the section $V(x) = V_0$ which is actually inscribed in the attraction domain. In order to make the evaluation more exact we propose the qualitative numeric procedure similar to that used in [6. pp. 355-381] in synthesis of automated control systems.

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On the quadratic Morse-Smale endomorphisms

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We consider the one-parameter family of quadratic maps

$$F_\mu(x,y) = (xy, (x-\mu)^2) \tag{1}$$

where $(x;y)$ is a point of the plane R^2, $\mu \in (0,1]$.

The following theorem is proved here.

Theorem. *Let F_μ be a map of type (1). Then for every $\mu \in (0,1)$ the map F_μ is the Morse-Smale endomorphism such that its nonwandering set $\Omega(F_\mu)$ consists of three fixed points (sink $A_1(0;\mu^2)$, source $A_2(\mu+1;1)$, saddle point $A_3(\mu-1;1)$) and periodic orbit B of period two, which is formed by two sources $B_1\left(\frac{\mu^2+1-\sqrt{\mu^2+1}}{\mu}; \frac{\mu^2}{(1-\sqrt{\mu^2+1})^2}\right)$, $B_2\left(\frac{\mu^2+1+\sqrt{\mu^2+1}}{\mu}; \frac{\mu^2}{(1+\sqrt{\mu^2+1})^2}\right)$.*

The map F_1 is the singular Morse-Smale endomorphism such that its nonwandering set $\Omega(F_1)$ consists of two fixed points (nonhyperbolic point $A_1(0;1)$, source $A_2(2;1)$) and periodic orbit B of period two, which is formed by two sources $B_1\left(2-\sqrt{2}; \frac{1}{3-2\sqrt{2}}\right)$, $B_2\left(2+\sqrt{2}; \frac{1}{3+2\sqrt{2}}\right)$.

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Piecewise smooth maps with strange and wild attractors

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Classification method of mixed chaotic/stochastic data

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The understanding of the dynamic behavior in real physical or industrial system is of almost importance, for analysis, synthesis, prediction, etc.

It's sensible to consider that the behavior of many physical systems like phytoplankton, solar activity, oscillation of waves is a combination between chaotic or stochastic processes, which can be successfully used for prediction of health applications, meteorological phenomena etc.

Many physical/chemical or sometimes financial phenomena are considered as being only chaotic ((ex. Belousov - Zhabotinsky reaction or purely stochastic (stock model price, integral Ito, Black-Scholes model), but in fact they are both deterministic and stochastic.

So it is of utmost interest to find new models taking into account both behaviors, stochstic and chaotic, to understand and predict better the real physical phenomena, but also to model data for biomedical applications like (ECG, IRM, . . .).

The original idea in this paper is to juxtapose methods from stochastic signal analysis (nonstationary Gaussian processes, statistics from limit theorems by Nordin, Hurst exponent), and nonlinear (chaotic) dynamical system analysis (phase portrait, phase delayed plot, Lyapunov exponents), to develop a common methodology to analyze complex time series. Assuming that these two behaviors are inherently correlated, we are analyzing if there exists a correlation exists between the stochastic quantifiers (Hurst exponent, Garch method, ARMA) and chaotic quantifiers (Lyapunov exponents). To do that, different kind of stochastic-chaotic mixed processes shall be modeled and analyzed from different points of view to be developed.

Proposed methodology. As classical approach, we assume a priory stochastic nature of time series model and construct a mathematical model as a random process. Hurst exponent is defined like the estimate $\hat{\mathrm{H}}$ of approximated fractional Brownian motion for these time series. On the other hand, for some deterministic systems, where the state is the solution of nonlinear differential or difference equation like $x_n = X(t_n)$, the behavior can be highly irregular and extremely complex. In some cases the behavior is estimated like chaotic. In the first approximation, we can determine the chaocity by the property of the system to construct i trajectories in a bounded domain of the phase space. Properties of dynamical systems which generate chaotic solutions, has been widely discussed (results and references in the monographs The simplest example is an one-dimensional dynamical system

$$x_{n+1} = f(x_n, m)$$

which generates chaotic solution for some functions f and values of parameter m. In particular, for logistic function f

$$x_{n+1} = mx_n(1 - x_n), 0 < x_n < 1, m > 0$$

the plot of solution looks like white noise with some values $m > 3,6$.

So, the problem statement the nature of time series analysis nature is do the observed data have stochastic nature, or deterministic. A lot of papers have been devoted to this problem by 90s. The essence of these results is as follows. Let's construct some statistics of observed time series, the values of which will be different from random or deterministic chaotic sequences. There are a lot of criteria of difference between chaotic and stochastic nature of time series developed in the last years. One of the main characteristics of the a priori deterministic series is the Lyapunov exponent I. It's using a presence of dynamical system, which is generating research data by estimation of Lyapunov exponent, so it doesn't work for the algorithm of random process for calculation I. The criterion of chaotic for a deterministic time series is a positive Lyapunov exponent. It's equal $I = \ln 2$ for logistic sequence $x_{n+1} = 4x_n(1 - x_n)$.

Note that the above results have been proved only for a certain class of dynamical systems which generated deterministic chaos. As usual, the situation of mixture "chaotic-randomness" is a normal for the natural observed data (one of the main task is to determine their correlation in the time series). It's normal to expect that the quality of the approximation of this mixture depends on the specified ratio in the proposed model

fbm	$H = 0.2$	$H = 0.5$	$H = 0.6$
Tent map	0.5766	0.0078	1.0569
Mixture ($\alpha = 0.2$)	0.9591	0.8721	2.6039
Lorenz	1.8544	1.6360	1.9381
Mixture ($\alpha = 0.5$)	0.9244	1.0678	2.8903

(approximation of a random process fbm and the quality is defined by the specified statistics A_n, B_n, D_n).

Key words: fractional Brownian motion, Lyapunov exponent, Lorenz system.

Influence of Chaotic Behavior in Complex Networks by Changing Network Topology

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In our society, there are various type networks. We have lived our life by using networks. Examples of networks are transportation network, flight network and so on. Recently, various networks around our life have became more complex and large scale. Complex network have attracted grate deal of attention from various fields. Some researchers discover small-world network [1] and scale-free network [2]. These network models have various types of feature quantities. Examples of feature quantities are path length, degree distribution, clustering coefficient and so on. Moreover, in the complex network, there are various network with propagation. The pandemic outbreak of viral infection and the traffic jam of the transportation network are mentioned as an example of propagation in the real network. However, there are not many studies of large- scale network of continuous-time real physical systems such as electrical circuits Therefore, it is important to investigate the chaos propagation and the spread of chaotic behavior under some difficult situations for the circuits.

As previous studies, the chaos propagation and the spread of chaotic behavior have been investigated only in simple networks such as ladder and ring topology [3]. In this simple network, the periodic attractors change to the chaotic attractors by increasing the coupling strength.

The chaotic circuit is shown in Fig. 1. This circuit consists of a negative resistor, two inductors, a capacitor and dual- directional diodes. This chaotic circuit is called Nishio-Inaba circuit. We propose different topology complex networks with coupled chaotic circuit. Figure 2 shows the proposed two types networks. Proposed network models consist of many nodes and edges. We set chaotic circuit in node, and resistor R in edge. Each node is coupled by one edge. We use 25 coupled chaotic circuits in Fig. 2(A) and 49 coupled chaotic circuits in Fig. 2(B). Furthermore, one circuit is set to generate chaotic attractor and the other circuits are set to generate three-periodic attractors.

In this study, we investigate the influence of chaotic behavior in complex networks by changing network topology. First, we investigate ratio of spreading chaotic behavior by changing network topology in small network. Second, we verify chaotic behavior in large-scale complex network. Finally, we observe how to spread of chaotic behavior by increasing the coupling strength.

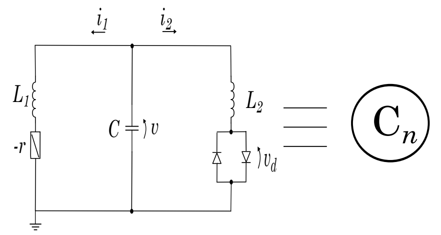

Figure 1: Chaotic circuit.

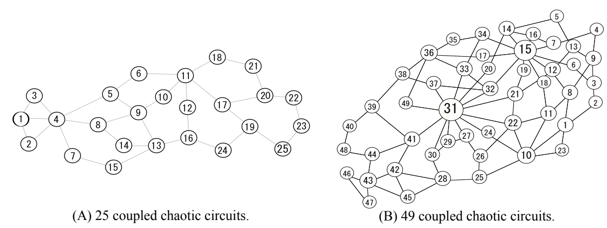

Figure 2: Attractors of chaotic circuit.

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The Concept of Integrability for Multifunctions and Dynamics of the Trace Map

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The concept of integrability of a continuous map in the plane introduced in [1] (see also [2]), is generalized for an upper semicontinuous two-valued map defined in a convex unbounded domain of the plane.

Criterion is proved for integrability of above multivalued maps. This criterion is based on the reduction of a considered two-valued map to an upper semicontinuous two-valued skew product of maps of an interval defined on an unbounded (with respect to second variable) rectangle of the plane.

Obtained results are applied to the investigation of the upper semicontinuous two-valued map connected with the trace map

$$F(x,\, y) = (xy,\, (x-2)^2).$$

This trace map arises in quasicrystal physics.

Considerations of this work are based on use of geometric results obtained in [1] for the above trace map.

This is the joint work with S.S. Belmesova.

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On the Totally Transitive Skew Products on n-Dimensional Cell

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This work is the continuation of research [1] where the class of skew products $T^3_{fb}(I^n)$ (defined below) is explored. Here $I^n = \prod\limits_{j=1}^{n} I_j$ is n-dimensional cell, $I_j = [a_j, b_j]$ for every $1 \leq j \leq n$, $n \geq 2$. Sufficient conditions of topological transitivity but not total topological transitivity for maps from class $T^3_{fb}(I^n)$ are proved in [1].

In this work we obtained sufficient conditions of total topological transitivity (it means that any iteration of a map is topologically transitive) for maps from $T^3_{fb}(I^n)$.

A map $F : I^n \to I^n$, where

$$F(x_1, x_2, ..., x_n) = \Big(f_1(x_1), f_{2,x_1}(x_2), ..., f_{n,x_1,x_2,...,x_{n-1}}(x_n)\Big)$$

is called to be a skew product in n-dimensional cell I^n.

For every natural number $n \geq 2$ we have

$$\hat{I}^{n-1} = \prod_{j=1}^{n-1} I_j, \quad \hat{f}_{n-1} = (f_1, ..., f_{n-1,x_1,x_2,...,x_{n-2}}),$$

$$\hat{x}_{n-1} = (x_1, ..., x_{n-1}), \ \ (x_1, ..., x_n) = (\hat{x}_{n-1}, x_n).$$

A map $\hat{f}_{n-1} : \hat{I}^{n-1} \to \hat{I}^{n-1}$ is called to be quotient map of a skew product $F : I^n \to I^n$. For every $\hat{x}_{n-1} \in \hat{I}^{n-1}$ a map $f_{n,\hat{x}_{n-1}}(x_n) : I_n \to I_n$ is called to be fiber map over a point $\hat{x}_{n-1}$.

For every $\hat{x}_n \in \hat{I}^n$ and $k \geq 2$ the following equality is valid

$$F^k(\hat{x}_{n-1}, x_n) = \left(\hat{f}^k_{n-1}(\hat{x}_{n-1}), f_{n,\hat{x}_{n-1},k}(x_n)\right),$$

where

$$f_{n,\hat{x}_{n-1},k}(x_n) = f_{n,\hat{f}^{k-1}_{n-1}(\hat{x}_{n-1})} \circ f_{n,\hat{f}^{k-2}_{n-1}(\hat{x}_{n-1})} \circ ... \circ f_{n,\hat{x}_{n-1}}(x_n).$$

Denote by $T^3_{fb}(I^n)$ a class of C^3-smooth skew products satisfying the following conditions:

$(C.1)$ *for any* $2 \le i \le n$ *and* $\hat{x}_{i-1} \in \hat{I}^{i-1}$ *Schwarzian of a map* $f_{i,\hat{x}_{i-1}}(x_i)$*:*

$$S\left(f_{i,\hat{x}_{i-1}}(x_i)\right) = \frac{\frac{\partial^3}{\partial x_i^3} f_{i,\hat{x}_{i-1}}(x_i)}{\frac{\partial}{\partial x_i} f_{i,\hat{x}_{i-1}}(x_i)} - \frac{3}{2}\left(\frac{\frac{\partial^2}{\partial x_i^2} f_{i,\hat{x}_{i-1}}(x_i)}{\frac{\partial}{\partial x_i} f_{i,\hat{x}_{i-1}}(x_i)}\right)^2$$

is negative for every $x_i \in I_i$ *such that* $\frac{\partial}{\partial x_i} f_{i,\hat{x}_{i-1}}(x_i) \neq 0$*;*

$(C.2)$ *for every* $2 \le i \le n$ *and* $\hat{x}_{i-1} \in \hat{I}^{i-1}$ *a map* $f_{i,\hat{x}_{i-1}}(x_i) : I_i \to I_i$ *as a function of variable* x_i*, has at most one critical point in interval* (a_i, b_i) *and this point is nonsingular*[1]

$(C.3)$ $a_i < f_{i,\hat{x}_{i-1}}(a_i) \le b_i$ *and* $f_{i,\hat{x}_{i-1}}(b_i) = a_i$ *for every* $2 \le i \le n$, $\hat{x}_{i-1} \in \hat{I}^{i-1}$.

Let a skew product $F \in T^3_{fb}(I^n)$ has surjective fiber maps $f_{i,\hat{x}_{i-1}}$ such that

$$f_{i,\hat{x}_{i-1}}(a_i) = z_{i,1},$$

where $z_{i,1}$ is a periodic repeller of a map $f_{i,\hat{x}_{i-1}}$ with an odd period $s_i > 1$. Moreover, $z_{i,1}$ does not depend on $\hat{x}_{i-1} \in \hat{I}^{i-1}$.

Let us state the main result of the paper.

Theorem 1. *Let a skew product* $F \in T^3_{fb}(I^n)$ *satisfy the following conditions:*

$(Y.1)$ *the map* f_1 *is totally topologically transitive;*

$(Y.2)$ *for every* $2 \le i \le n$ *and* $\hat{x}_{i-1} \in \hat{I}^{i-1}$ *a map* $f_{i,\hat{x}_{i-1}}$ *is surjective and the equality* $f_{i,\hat{x}_{i-1}}(a_i) = z_{i,1}$ *is valid.*

Then a skew product F *is totally transitive and has a dense set of periodic points on the phase space* I^n*.*

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[1]Critical point c of map $f_{i,\hat{x}_{i-1}}(x_i)$ is called to be *nonsingular* if $\left.\frac{\partial^2 f_{i,\hat{x}_{i-1}}}{\partial x_i^2}(x_i)\right|_{x_i=c} \neq 0$

Almost- and near-solutions of equations in unitary matrices.

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The questions under discussion is, roughly formulated, as follows: "when an almost-solution to an equation is a near-solution to the equation". Let us illustrate it with an example of commutation. I say that unitary matrices U, V are δ-almost-commuting if $\|UVU^{-1}V^{-1} - 1\| \leq \delta$. (Here 1 denotes the unit matrix.) I say that U, V are ϵ-near-commuting if there exist commuting unitary U', V' such that $\|U - U'\| < \epsilon$ and $\|V - V'\| < \epsilon$. Is the following statement valid:

*) For any $\epsilon > 0$ there exists $\delta > 0$ such that any δ-almost-commuting unitary matrices are ϵ-near-commuting.

Remark. Note that the dimension of the matrices is not fixed. So, the $\delta(\epsilon)$ is independent of dimensions of the matrices. This makes the problem difficult. Particular, the answer to the question depends on the norm $\|\cdot\|$. The statement *) fails if $\|\cdot\|$ in the definitions is the operator norm. But it is true for $\|\cdot\| = \|\cdot\|_2$, the normalized Hilbert-Schmidt norm:

$$\|A\|_2 = \sqrt{\frac{1}{n} Trace(AA^*)}.$$

Where n is the dimension of A.

In the talk I am going to discuss the similar questions for systems of equations, not only for the commutator. In what follows I will suppose that $\|\cdot\|$ is the operator norm. Let $F = \langle x_1, x_2, \ldots, x_j \ldots \rangle$ be the free group on $x_1, x_2, \ldots$. Let $W = \{w_1(\bar{x}), w_2(\bar{x}), \ldots, w_k(\bar{x})\} \subset F$. I will consider W as a system of equations (or, precisely, as right hand sides of the equations).

- A tuple $\bar{U} = U_1, U_2 \ldots U_k$ of unitary matrices is a δ-almost-solution of W if $\|w(\bar{U}) - 1\| \leq \delta$ for every $w \in W$;
- A tuple $\bar{U}$ is an ϵ-near-solution if there a solution $\bar{V}$ (i.e. $w(\bar{V}) = 1$ for every $w \in W$) such that $\|U_i - V_i\| < \epsilon$ for $i = 1, \ldots, k$;
- A system of equations W is said to be unitary testable if for any $\epsilon > 0$ there exists $\delta > 0$ such that any unitary δ-almost-solution to W is an ϵ-near-solution to W.

A non difficult but important fact is that unitary testability is a group property, that is, the following statement is true:

Let W and W' be systems of equations such that the group $F/N(W)$ is isomorphic to $F/N(W')$. (Here $N(X)$ is the normal subgroup of F generated by X.) Then W is unitary testable if and only if W' is.

So, in the view of the above statement one may speak about unitary testable finitely presented groups.

Adopting the techniques of D.Kazhdan and A.Zuk we give a sufficient condition on the group to be unitary testable. Examples:

- $\mathbb{Z}^2$ is not unitary testable (equation: $XYX^{-1}Y^{-1} = 1$).
- $Z_k \times Z$ is unitary testable (equations: $XYX^{-1}Y^{-1} = 1$ and $X^k = 1$).

We hope that our conditions allows to find more examples of unitary testable groups.

Three types of dynamical chaos

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When we speak of dynamical chaos, we usually mean one of two quite different types of dynamics. In Hamiltonian systems, we have conservative chaos – something like a "chaotic sea" with elliptic islands inside. Chaos in dissipative systems is quite different and is associated with strange attractors.

Our goal in this talk is to attract attention to one more type of chaos, the third one, which is called "mixed dynamics". This type of behavior is characterized by *inseparability* of attractors, repellers, and conservative elements (elliptic orbits, KAM-tori, etc.) in the phase space.

Characteristics of networks with hyperbolic geometry and symbolic dynamics

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A graph as a mathematical idealization of a network is completely different than a Riemannian manifold. However, the recent development of the so-called coarse geometry under the leadership of Mikhael Gromov has given the two mathematical structures-graphs and manifolds-the unifying framework of geodesic spaces.

The fundamental mathematical idea behind this unification is to realize that the traditional Riemannian curvature, which relies on the differentiable structure of the manifold, can be rephrased in terms of the more primitive concept of distance.

Ideas related to hyperbolicity have been applied in numerous networks applications: secure transmission of information on the internet; spread of viruses through the network; distance estimation; sensor networks; traffic flow and congestion; large-scale data visualization.

There are models commonly used for hyperbolic geometry: the Klein model, the Poincaré disc, the Poincaré half plane, and the Lorentz model. It is possible, using the symbolic dynamics, to study the variation in the length of the geodesic, allowing the determination of thresholds for these lengths that are closely related to the shortest path length. This study has implications on the characteristics of networks particularly in the navigability.

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On topology of manifolds admitting Morse-Smale systems without intersections of codimension one separatrices

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We show that if a closed manifold M^n ($n \geq 3$) admits a Morse-Smale system F without intersections of codimension one separatrices, and if, in addition in the case when F is a flow, F has no periodic trajectories, then M^n is either an n-sphere S^n or the connected sum of copies of $S^{n-1} \otimes S^1$'s and of special manifolds N_i^n which admit polar Morse-Smale systems. Moreover, if some N_i^n contains a unique saddle, then N_i^n is projective-like (in particular, $n \in \{4, 8, 16\}$, and N_i^n is a simply-connected orientable manifold).

We present a formula which provides interrelation between the topology of the manifold M^n and the number of sinks, sources and saddle periodic points. As a consequence, we get conditions of existence of heteroclinic intersections for Morse-Smale diffeomorphisms and of existence of periodic trajectories for Morse-Smale flows.

The above results have been obtained in collaboration with E.V. Zhuzhoma and V.S. Medvedev.

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On Embedding of Morse–Smale Diffeomorphisms in Topological Flows

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The report is devoted to discussion of the results of the papers [2]-[3] obtained in collaboration with V. Grines and O. Pochinka.

One of the important indicators of an adequacy of the numerical solution of an autonomous system of differential equations is the topological conjugacy of the obtained discrete model to the time-one shift map of the initial flow. In this connection the question on necessary and sufficient conditions for the embedding of a cascade in a flow naturally arises.

In [1] there were stated necessary conditions of the embedding of a Morse–Smale diffeomorphism (structurally stable diffeomorphism with finite non-wandering set) $f\colon M^n \to M^n$ in a topological flow. These conditions are: 1) the non-wandering set Ω_f coincides with the set of fixed points; 2) the restriction of the diffeomorphism f on every invariant manifold of any fixed point $p \in \Omega_f$ preserves its orientation; 3) if for any different saddle points $p, q \in \Omega_f$ the intersection $W^{\mathrm{s}}_p \cap W^{\mathrm{u}}_q$ is not empty then it does not contain any compact connected components.

Palis has proved, that in the case $n = 2$ this conditions are not only necessary but also sufficient. In [2] the case $n = 3$ was considered. It was shown that there is an additional obstruction to the embedding of such diffeomorphisms in topological flows, which is connected with a possibility of a non-trivial embedding of separatrices of saddle points in the ambient manifold, and the necessary and sufficient conditions of the embedding of a 3-dimensional Morse–Smale diffeomorphism in a topological flow were obtained.

In [3] it was announced that for the class $G(S^n)$ of Morse–Smale diffeomorphisms without heteroclinic intersections, defined on the sphere S^n of the dimension $n \geqslant 4$ and satisfying to Palis conditions there are no other obstructions and the following theorem is true.

Theorem.*Any diffeomorphism $f \in G(S^n)$, $n \geqslant 4$, is embedded to a topological flow.*

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Superposition principle for the continuity equation in a bounded domain

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The connection between the ordinary differential equation

$$\dot\gamma(t) = b(t, \gamma(t)) \tag{1}$$

and the continuity (or Liouville) equation for measures

$$\partial_t \mu_t + \operatorname{div}(b\mu_t) = 0 \tag{2}$$

is an important tool in the theory of Dynamical Systems. For smooth vector fields $b\colon \mathbb{R} \times \mathbb{R}^d \to \mathbb{R}^d$ this connection is described by the classical method of characteristics.

For non-smooth vector fields the connection between (1) and (2) is more involved and has been studied by Ambrosio, Bernard, Colombo, Figalli, Gigli, Savare and many others [1, 2, 3]. In particular the following *Superposition Principle* is known: any non-negative finite measure-valued solution of the Cauchy problem for the continuity equation in $\mathbb{R}^d$ with a bounded Borel vector field can be represented as a superposition of measures concentrated along characteristics.

More precisely, suppose that $b\colon [0,T] \times \mathbb{R}^d \to \mathbb{R}^d$ is a bounded Borel vector field, $T > 0$. Let $\Gamma := C([0,T];\mathbb{R}^d)$ and for any $t \in [0,T]$ let $e_t\colon \Gamma \to \mathbb{R}^d$ denote the *evaluation map* defined by $e_t(\gamma) := \gamma(t)$. Let $\{\mu_t\}_{t\in[0,T]}$ be a Borel family of non-negative measures on $\mathbb{R}^d$ such that $\operatorname{esssup}_{t\in[0,T]} \mu_t(\mathbb{R}^d) < \infty$. If the family $\{\mu_t\}_{t\in[0,T]}$ solves (2) in sense of distributions then there exists a finite non-negative measure η on Γ such that

- μ_t is the image (pushforward) of η under the map e_t for a.e. $t \in [0,T]$
- η is concentrated on the set of integral curves of b, i.e. for η-a.e. $\gamma \in \Gamma$

$$\gamma(t) = \gamma(0) + \int_0^t b(s, \gamma(s))\, ds, \quad \forall t \in [0,T].$$

In this work we propose an analog of the Superposition Principle for the continuity equation in a bounded domain $\Omega \subset \mathbb{R}^d$. Our results are based on the structure of normal

1-currents (or charges), described by Smirnov [4] and later by Paolini and Stepanov [5, 6]. We also discuss corollaries of this framework for the case of the whole space $\mathbb{R}^d$.

The presentation is based on a joint work with P. Bonicatto.

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On basic sets of Smale-Vietoris A-diffeomorphisms

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We introduce Smale-Vietoris diffeomorphisms that include the classical DE-mappings with Smale solenoids. We describe the correspondence between basic sets of Smale-Vietoris A-diffeomorphisms and basic sets of nonsingular A-endomorphisms. For Smale-Vietoris diffeomorphisms of 3-manifolds one proves the uniqueness of nontrivial solenoidal basic set. We construct bifurcations between different types of solenoidal basic sets, and one of them can be considered as a destruction (or birth) of Smale solenoid.

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Networks of pulse delay coupled oscillators: reduction to discrete maps

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Interaction via pulse signals is typical for oscillators of various nature, as well as the presence of coupling delays. Neural networks are a prototypical example of such systems, where the interaction between neurons takes form of the exchange by short spikes that propagate along axons with finite speed. Other examples are networks of chemical oscillators, ensembles of mode-locked lasers, telecommunication systems, etc. We present a technique that allows to reduce networks of pulse delay coupled oscillators to multidimensional discrete map. The obtained map governs the dynamics of the network and allows easier analytical or numerical investigation. We also present some recent results obtained in the framework of the suggested approach for various types of networks.

Some properties of singular hyperbolic and Lorenz-type attractors

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Singular hyperbolic attractors were introduced in [1] as a generalization of the geometric Lorenz attractors, including all C_1 robustly transitive sets with singularities on 3-manifolds up to flow reversing. An attractor is singular-hyperbolic if it has singularities (all hyperbolic) and is partially hyperbolic with volume expanding direction.

We discuss a problem on constructing models of singular-hyperbolic attractors. Let us consider the models of attractors in form of the inverse limit of semiflows on branched manifolds that are suspensions over a discontinuous expanding map of a closed line interval with several, rather than one, discontinuity points. These models are generalization of geometrical model of the attractor in the Lorenz system. Let $t : [0, 1] \to [0, 1]$ be a discontinuous map with several discontinuity points. Assume that t is locally eventually onto. We make a branched manifold L carrying a semiflow ϕ_t, $t > 0$, such that the corresponding Poincare map is a map t [2]. Consider the inverse limit

$$\widehat{L} = \lim_{\leftarrow}(L, \phi_t, t > 0).$$

The elements $\widehat{z} \in \widehat{L}$ are the branches of the negative semitrajectories of the semiflow ϕ_t. A flow $\widehat{\phi}_t$ is naturally defined on the space $\widehat{L}$. The pair $(\widehat{L}, \widehat{\phi}_t)$ generalizes well known geometrical Lorenz attractor by Williams.

It appears that all models $(\widehat{L}, \widehat{\phi}_t)$ can be realized as real attractors of singular-hyperbolic flows with handlebody basins. A handlebody of genus $g \geq 1$ is a compact three-dimensional manifold H which can be represented as a closed regular neighborhood in R^3 of an embedded bouquet of g circles (a cube with handles). In [3] C. A. Morales proved that every orientable handlebody except for the solid torus and 3-ball can be realized as the basin of attraction of a singular-hyperbolic attractor.

A problem on representation of a general topological transitive singular hyperbolic attractor as the inverse limit of a semiflow on a branched manifold is considered.

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An investigation of phenomena observed in scale-free coupled circle maps

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In this study, phenomena observed in scale-free coupled circle maps are investigated. The circle map is a one-dimensional discrete-time map and exhibits various kinds of behavior as the parameters of the function change. As the topology of the coupled circle map, star-, ring-, or all-to-all coupling, and a coupled map lattice have been studied. However, the scale-free coupled circle map has not been well investigated so far. In the scale-free coupled circled map, the synchronization of each map and the expansion of chaotic behavior could be controlled by varying the parameters of the circle map of the hub nodes. In this study, the set of the parameter values of the circle maps at the hub nodes which leads to the synchronization and the expansion of the chaotic behavior of the scale-free coupled circle maps would be elucidated.

Besicovich cascads and Hölder condition

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Let $T_\rho\colon \mathbb{T} \to \mathbb{T}$ be an irrational circle rotation: $T_\rho x = x + \rho \pmod 1$, $f\colon \mathbb{T} \to \mathbb{R}$ be a continuous function with zero mean. We consider a cylindrical cascade $T_{\rho,f}\colon \mathbb{T}\times\mathbb{R} \to \mathbb{T}\times\mathbb{R}$ with a cocycle f

$$T_{\rho,f}(x,y) = (T_\rho x, y + f(x)).$$

It is known that $T_{\rho,f}$ is topologically transitive if and only if f is not a coboundary over T_ρ and has zero mean.

A.S. Besicovitch [1] showed that for any irrational circle rotation T_ρ, there exists a continuous f such that $T_{\rho,f}$ is topologically transitive and has discrete orbits (we call it the Besicovitch transformation).

If f has bounded variation T_f is not Besicovitch because the sequence $T^{q_n}_{\rho,f}(x,y)$ is bounded for denominators q_n of ρ, as $||f^{q_n}||_{C(\mathbb{T})} \leqslant Var(f)$. On the other hand, there exist the Besicovitch transformations with Hölder cocycles [2, 3].

The «Besicovitch set» of points in the circle $\mathbb{T}\times\{0\}$ having discrete orbits, has the null Lebesgue measure, but may have the positive Hausdorff dimension. The examples show some relationship between the Hölder exponent γ of f and the Hausdorff dimension of the Besicovitch set B. For f continuous [5] it may be $\dim_H(B) = 1$. On the other hand, there exists an example [4] with a γ-Hölder cocycle f such that

$$\dim_H(B) \geqslant 1 - \gamma.$$

We will try to find an upper estimate for the Hausdorff dimension of the Besicovitch set for cylindrical cascade with a γ-Hölder function, and construct «more smooth» Besicovitch transformation.

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Nonlinear maps of noncommutative algebras

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Let $\mathcal{A}$ be a noncommutative associative normed algebra. For nonlinear maps of the algebra it is easy to give a usual definition of the derivative, but even simplest nonlinear maps, for example, power maps do not have such derivatives. However there exists analogue of the differential, containing only first power of the "argument increment". Namely,

$$(x + \Delta x)^2 - x^2 = x \cdot \Delta x + \Delta x \cdot x + (\Delta x)^2,$$

$$(x + \Delta x)^3 - x^3 = x^2 \cdot \Delta x + x \cdot \Delta x \cdot x + \Delta x \cdot x^2 + (\Delta x)^3$$

This was the reason for the author to introduce [1] and investigate the maps as $f_1(\Delta x) = x \cdot \Delta x + \Delta x \cdot x$ and $f_2(\Delta x) = x^2 \cdot \Delta x + x \cdot \Delta x \cdot x + \Delta x \cdot x^2$.

The map $f : \mathcal{A} \to \mathcal{A}$ is called a mapping of degree 1 (1-degree map for short) if $f(x) = \sum_{i=1}^{n} a_i \cdot x \cdot b_i$ for some $n \in N$ and $a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n \in \mathcal{A}$. Algebraic and topological properties of 1-degree maps were researched in the author's papers [2,3].

Analogously n-degree maps and polynomial maps are defined. The expression $a_1 \cdot x_1 \cdot a_2 \ldots a_n \cdot x_n \cdot a_{n+1}$ is said to be a n-degree monomial over $\mathcal{A}$ $(a_1, a_2, \ldots, a_n, a_{n+1} \in$

$\mathcal{A}$, $x_1, x_2, \dots x_n$ are variables, some of them may be equal). A finite sum of n-degree monomials is called a homogeneous n-degree polynomial and a finite sum of homogeneous polynomials is called a polynomial over $\mathcal{A}$. The map is called an analytical map whenever it is convergent series of the homogeneous polynomials. Analytical and polynomial maps are the maps which have a 1-degree "differential".

A certain application of 1-degree maps is described lower.

In probability theory a discrete Markov chain is described by the transition matrix consisting of elements from [0,1]. Markov's theorem proclaims that if some power of this matrix does not contains zeroes, then limit transitions probabilities exist. We assume that kth power of the transition matrix does not contains zeroes and so the question is arising: how the elements of transition matrix that can be changed for kth power of matrix remain without zeroes.

Assume that the transition matrix is an $n \times n$-matrix and consider the set of the real $n \times n$-matrices as a normed involutive algebra with transposition as the involution and the norm $\|A\| = n \cdot \max_{1 \le i,j \le n} |a_{i,j}|$ (equivalent to the norm $\|A\| = \sup_{x \ne 0} \|Ax\|/\|x\|$ but more suitable for calculation). The inequality $\|AB\| \le \|A\| \cdot \|B\|$ for the normed algebra is fulfilled.

Let $P = (p_{ij})_{n \times n}$ be a transition matrix, $P^k = (p_{ij}^{(k)})_{n \times n}$ be the kth power of P and $f(P) = P^k$. Using the equality $f(P+\Delta P) - f(P) = (P+\Delta P)^k - P^k = df + o(\Delta P)$ we approximate the difference at the left-hand side by the "differential" df and instead of the inequality $\|\Delta f\| = \|f(P+\Delta P) - f(P)\| \le \varepsilon$ we solve the inequality $\|df\| \le \varepsilon$. Here $df = P^{k-1} \cdot \Delta P + P^{k-2} \cdot \Delta P \cdot P + \ldots + P \cdot \Delta P \cdot P^{k-2} + \Delta P \cdot P^{k-1}$ is a 1-degree map of ΔP.

Taking into account the estimate $\|df\| \le \|P^{k-1}\| \cdot \|\Delta P\| + \|P^{k-2}\| \cdot \|\Delta P\| \cdot \|P\| + \ldots + \|P\| \cdot \|\Delta P\| \cdot \|P^{k-2}\| + \ldots + \|\Delta P\| \cdot \|P^{k-1}\| \le k \cdot \|P\|^{k-1} \cdot \|\Delta P\|$, it is sufficient to solve the inequality $k \cdot \|P\|^{k-1} \cdot \|\Delta P\| < \varepsilon$. Thus we get $\|\Delta P\| < \delta_1(\varepsilon) = \frac{\varepsilon}{k \cdot \|P\|^{k-1}}$, therefore for all $i, j \in \{1, 2, \ldots, n\}$ $|\Delta p_{i,j}| < \frac{\varepsilon}{n \cdot k \cdot \|P\|^{k-1}}$. If we will not reject $o(\Delta P)$, than obtain more exact estimate:

$$\|\Delta P\| < \delta_2(\varepsilon) = \frac{\sqrt{(k\|P\|^{k-1})^2 + 2\varepsilon k(k-1)(\|P\|+n)^{k-2}} - k\|P\|^{k-1}}{nk(k-1)(\|P\|+n)^{k-2}}$$

and $|\Delta p_{ij}| < \frac{\delta_2(\varepsilon)}{n}$. This is sufficient condition for $\|\Delta f\| < \varepsilon$.

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Topological classification of Ω-stable flows on surfaces

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A traditional method of qualitative studying of the flows dynamics with a finite number of special trajectories on surfaces consists of a splitting of the ambient manifold by regions with predictable trajectories behavior known as *cells*. In the classical works of A.A. Andronov, L.S. Pontryagin, E.A. Leontovich, A.G. Mayer, M. Peixoto there is a topological classification of some classes of flows on the surfaces, resulting from a canonical description of dynamics in such cells and their contiguity. And specifically, the scheme of Leontovich-Mayer is a full topological invariant for flows with finite number of specific trajectories in limited part of plane, and the graph of Peixoto makes full description of topological equivalence class of structurally stable flow on arbitrary surface. We consider the class of Ω-stable flows that includes rough flows and generalizes flows of Leontovich-Mayer in the sense of removing restrictions of lifting surface. The Ω-stable flows has trajectories joining saddle points which is make such flows not structural stable. However, we prove that the topological classification of such flows is also reduced to a combinatorial problem.

We divide the surface into regions depending on location of limit cycles and build the directed multigraph with vertices corresponding to the regions and edges corresponding to bounds of the regions. Next we equip some vertices with some additional information about dynamics in regions corresponding to that vertices: four-colour multigraph or weight depending on a type of a region. We prove that an isomorphicity class of this equipped multigraph is the complete topological invariant for our class of flows and, besides, we give a standard flow for every isomorphicity class of the equipped multigraphs.

On existence of one-dimensional Cantor type repeller on two-torus

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F. Przytycki generalized an axiom A for endomorphisms [2] that was previously introduced by S. Smale for diffeomorphisms [3]. According to M. Shub [4] there is an A-endomorphism of the circle whose non-wandering set contains a basic set homeomorphic to the Cantor set. It was shown by R. V. Plykin [1] that any one-dimensional basic set of an A-diffeomorphism of surfaces is locally homeomorphic to product of the Cantor set and the interval. There exists an A-endomorphism of the two-torus which is not a diffeomorphism whose non-wandering set consists of one-dimensional attractor and repeller homeomorphic to the circle as well as there exists an A-endomorphism of the two-sphere whose non-wandering set contains repeller homeomorphic to some fractal set (such endomorphisms naturally appear in holomorphic dynamics).

We construct an A-endomorphism of two-torus whose nonwandering set contains one-dimensional repeller locally homeomorphic to product of the Cantor set and the interval. The key idea of construction consists in applying the surgery introduced by S. Smale [3] to an algebraic endomorphism of the two-torus. We present the results of computational experiment that demonstrate correctness of our construction.

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Functional Iteration Models for Random Markets

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Some economic models for random conservative markets where agents trade by pairs are addressed. These are gas-like economic models where the time evolution of the wealth distribution is given by nonlinear functional mappings. In these models, an operator governs the discrete time evolution of the wealth distribution of an out-of-equilibrium economic gas-like system. These operators are nonlinear maps in the space of wealth distributions, which are shown to conserve the total and mean wealth of the economic system, and even an H-Theorem can be verified for some cases. Different asymptotic results for several models are presented. The decay to the exponential distribution is found in some of them and a transition to power-like distributions is sketched when a naive bank system is suggested. Simulations and implementations of these systems in different topologies are also investigated.

Keywords: Functional iterations, Gas-like economic models, Quantitative methods, Economic modeling, Random markets.

Mastering high quality randomness via chaos theory

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By randomness we designate several topics including generation of random numbers, chaos based cryptography, new algorithms for hash functions, etc.
The last few decades have seen the tremendous development of new IT technologies that incessantly increase the need for new and more secure cryptosystems. Random numbers are useful for many purposes, such as cryptography, modeling in ecology, games and gambling, and for selecting random samples from larger data sets. Instead of wasting time in tossing coin, which is largely insufficient for generating billions of random numbers, computers are nowadays used routinely for such generation. Generally one distinguishes between true random numbers, pseudo-random numbers and chaotic numbers.
True random numbers are only generated by physical devices of phenomena. They are non-deterministic, however they are difficult to produce in large amount and at fast pace, due to the necessarily interface between physical device and computer. They are not reproducible, limiting their use in algorithms. Pseudo-random numbers (PNR) which are deterministic are widely used. However the most common formulas producing them, based on arithmetic functions are not very flexible. Chaotic numbers when used in raw form are easily recognizable. Nevertheless it is possible to combine chaotic numbers in such a way that they produce pseudo-random numbers of very high quality, passing every NIST test and more sophisticated ones. They can be used for generating a new kind of hash function.
Contrarily to most (PNR) generator algorithms that are used nowadays and based on a limited number of arithmetic or algebraic methods (like elliptic curves), networks of coupled chaotic maps offer quasi-infinite possibilities to generate parallel streams of pseudo-random numbers at a rapid pace when they are executed on modern multicore processors. We explore several topologies of network of 1-D coupled chaotic mapping (mainly tent map and logistic map) in order to obtain good Chaotic Pseudo Random Number Generators (CPRNG). Higher dimensional systems make it possible to achieve better randomness and uniform point distribution, because more perturbations and non-

linear mixing are involved. Therefore we focus on a particular network from dimension 2 to dimension 5. All NIST tests for dimensions 3 to 5 for every variable of this special network are successful, showing that these realizations in 3-D up to 5-D are good CPRNGs. In addition to those tests, we study this multidimensional mapping more thoroughly, far beyond the NIST tests which are limited to a few million iterates and which seem not robust enough for industrial mathematics, although they are routinely used worldwide. Very long computations on modern multicore machines are used: they generate up to one hundred trillion iterates in order to assess such network.

In order to check the portability of the computations on multicore architectures, we have implemented all our numerical experiments on several different multicore machines. The pace of generation of random bits can be incredibly high (up to 200 billion random bits per second).

The TQ-bifurcations and generation of the `T8N`, `T8P` symbols

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Let us introduce a discrete dynamical system:

$$\mathbf{s}_{k+1} = \mathbf{f}\left(\mathbf{s}_k,\, \mathbf{p}\right), \quad \boldsymbol{\phi}_{\mathbf{p}} : \mathrm{S} \times \mathrm{K} \to \mathrm{S}, \quad \boldsymbol{\phi}_{\mathbf{p}}(\mathbf{s},\, k) \equiv \mathbf{f}^k\left(\mathbf{s},\, \mathbf{p}\right), \tag{1}$$

with the properties:

$$\mathbf{s} \in \mathrm{S} \subset \mathbb{R}^N,\ \mathbf{p} \in \mathrm{P} \subset \mathbb{R}^L,\ \mathbf{f} \in \mathrm{C}^0(\mathrm{S} \times \mathrm{P}),\ k \in \mathrm{K} \subseteq \mathbb{Z},\ n \in \overline{1,\, N},\ l \in \overline{1,\, L},$$

where $\mathbf{s}$ is the state variable of map, $\mathbf{p}$ is the parameters vector of map, k is discrete time. Let us relate system (1) with the evolution trajectory of its state $\mathbf{s}$: $\left\{\mathbf{f}^k(\mathbf{s},\, \mathbf{p})\right\}_{k \in \mathrm{K}}$, where $\mathbf{f}^k$ is the composition of functions.

In this article, we define a new class of bifurcations and propose methods to diagnose them and analyze their properties. We introduce and study TQ-bifurcations that are realized in recurrent relations and manifest as qualitative change of map trajectory shapes in the extended space of states $\mathrm{S} \times \mathrm{K}$ [1]. The method is based on the formalism of symbolic CTQ-analysis, proposed earlier by the author (see [2] and references therein).

Define the main map that encodes the shape of the n-th component of sequence $\{\mathbf{s}_k\}$ in the space $\mathrm{S} \times \mathrm{K}$ in terms of the finite T-alphabet:

$$\left\{\mathbf{s}_{k-1}^{(n)},\, \mathbf{s}_{k}^{(n)},\, \mathbf{s}_{k+1}^{(n)}\right\} \Rightarrow T_k^{\alpha\varphi}|_n, \quad T_k^{\alpha\varphi} = [T_k^{\alpha\varphi}|_1,\, \ldots,\, T_k^{\alpha\varphi}|_N]\,. \tag{2}$$

In addition to $T_k^{\alpha\varphi}|_n$ symbols, introduce symbols $Q_k^{\alpha\varphi}|_n$:

$$Q_k^{\alpha\varphi}|_n \equiv T_k^{\alpha\varphi}|_n \to T_{k+1}^{\alpha\varphi}|_n,\ Q_k^{\alpha\varphi} = [Q_k^{\alpha\varphi}|_1,\ \ldots,\ Q_k^{\alpha\varphi}|_n,\ \ldots,\ Q_k^{\alpha\varphi}|_N]\,. \tag{3}$$

Let us assume that the graphs Γ_a^{TQ} and Γ_b^{TQ} are symbolic TQ-images of the dynamical system $\big(\mathrm{S},\ \mathrm{K},\ \boldsymbol{\phi}_{\mathbf{p}}\big)$ [2], when values of the vector of parameters are $\mathbf{p}_a$ and $\mathbf{p}_b$ respectively. Let us introduce another definition.

Definition 1. *TQ-bifurcation in the discrete dynamical system $\big(\mathrm{S},\ \mathrm{K},\ \boldsymbol{\phi}_{\mathbf{p}}\big)$ is the change of the symbolic TQ-image of the dynamical system that satisfies the condition:*

$$\Gamma_a^{\mathrm{TQ}} \xrightarrow[\mathbf{p}=\mathbf{p}_b]{\text{TQ-bif}} \Gamma_b^{\mathrm{TQ}}, \quad \Gamma_a^{\mathrm{TQ}} \neq \Gamma_a^{\mathrm{TQ}}, \quad \mathbf{p}_a \neq \mathbf{p}_b.$$

Where Γ_a^{TQ} and Γ_b^{TQ} are symbolic TQ-images of the dynamical system $\big(\mathrm{S},\ \mathrm{K},\ \boldsymbol{\phi}_{\mathbf{p}}\big)$ before and after bifurcation respectively, and $\mathbf{p}_b$ is the bifurcation value of the vector of parameters.

The nature of the TQ-bifurcations suggest that they define homogenous dynamics areas of the system $\big(\mathrm{S},\ \mathrm{K},\ \boldsymbol{\phi}_{\mathbf{p}}\big)$ from the standpoint of the symbolic CTQ-analysis, i.e., with regard to the trajectory shape of the dynamical system in the extended space of states $\mathrm{S} \times \mathrm{K}$.

Note the characteristic role of the `T8`$\circ$ symbols, $\circ =$ `N`, `P`. These symbols facilitate the transition from the equilibrium state (stationary point, `T0` symbol) to non-trivial evolution of the dynamic system [2].

The next theorem is true.

Theorem. *The necessary conditions for the existence of* `T8`$\circ$ *symbols in class* (1) *maps:* (i) $N \geqslant 2$; (ii) $\exists\, n,\ \mathbf{s}_k : \mathbf{f}^{(n)}(\mathbf{s}_k) = \mathbf{f}^{(n)}(\mathbf{s}_{k+r}),\ \mathbf{s}_{k'} \neq \mathbf{s}_{k''}$, *where* $k' \neq k'',\ k',\ k'' = \overline{k,\ k+r}$, $r = \overline{1,\ R}$, $\mathbf{s}_k,\ \mathbf{s}_{k+r} \in \mathrm{S}$.

Corollary 1. *The following condition is true for class* (1) *maps:* $\forall\, \mathbf{s} \in \mathrm{S},\ \forall\, \mathbf{p} \in \mathrm{P}$, $\nexists \big\{\mathrm{T}_k^{\alpha\varphi}\big\}_{k\in\mathrm{K}}|_{\mathbf{s},\mathbf{p}} \ni \mathtt{T}\, i_1 \cdots i_N : \forall\, i_{1\ldots N} = 8\circ,\ \circ =$ `N`, `P`.

Corollary 2. *The existence of* `T8`$\circ$ *symbols is impossible in the trajectories of homeomorphisms.*

Corollary 3. *If the trajectories of a class* (1) *dynamical system contain* `T4`$\circ$ *symbols and the system does not satisfy the conditions of the theorem, then, correct time inversion is impossible for this system.*

The following examples serve to illustrate the propositions of the theorem.

Example 1. *Define the map:* $x_{k+1} = \sin\big(\tau\, z_k + \theta\big),\ z_{k+1} = z_k + 1,\ k \in \mathbb{Z},\ x,\ z \in \mathbb{R}$, *where* $\tau, \theta \in [0,\ 2\,\pi]$ *are control parameters. The system dynamics (on the x variable) is studied on the* $\mathbb{T}^2 = \mathbb{S}^1 \times \mathbb{S}^1$ *torus.*

Example 2. *Define the map:* $x_{k+1} = w_{xx}\, x_k + w_{xy}\, y_k + b_x,\ y_{k+1} = \max\big(0,\ w_{yx}\, x_k + w_{yy}\, y_k + b_y\big)$, $x,\ y \in \mathbb{R}$, *where* $w_\circ,\ b_\circ \in \mathbb{R}$ *are control parameters.*

Also the examples possess inherent value (it have a applied interest for specialists): 1 – the model of discretization of a continuous signal and the model of stroboscopic

Poincare map; 2 – the model, from a certain standpoint, of the most basic recurrent neural network.

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The structure of dendrites and continuous maps on them

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By continuum we mean a compact connected metric space. Dendrite is a locally connected continuum without subsets homeomorphic to a circle.

Let X be a dendrite. The next properties are well known (see, e.g., [1] — [2]):

1) a dendrite is one-dimensional continuum;

2) for any points $x, y \in X$ there is a unique arc containing these points;

3) the set of branch points of X is at most countable;

4) the number of connected components of the set $X \setminus \{p\}$ is at most countable for any point $p \in X$.

In spite of last two properties there are dendrites with complicated structure. For example, Gehman dendrite has uncountable set of end points or there are dendrites with everywhere dense set of branch points (see, e.g., [2]).

Let $f : X \to X$ be a continuous map of a dendrite X. There are many examples of continuous maps on dendrites showing that dynamics of such maps depends on the structure of dendrites (see, e.g., [3] — [6]).

In the report the correlation between the structure of dendrites and dynamics of continuous maps on them is investigated.

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On kneading constructions for invariant measures of discontinuous one-dimensional maps with zero entropy

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We consider one-dimensional piecewise monotone discontinuous maps with zero topological entropy and apply the technique of kneading invariants and kneading series. The kneading technique was introduced first by J. Milnor and W. Thurston for continuous piecewise monotone one-dimensional maps and was applied before to maps with positive topological entropy. In [1], [2], by using the kneading technique we studied properties of one-dimensional maps with positive entropy and also for their multidimensional perturbations.

In the present talk we consider more complicated case in a neighborhood of a map with zero topological entropy. We show how to use the kneading techique for Lorenz maps with zero entropy and for generalized interval exchange transformations, i.e., at the border of the convergence disk for kneading series in the complex plane, in order to construct the invariant measures and thus, to construct semiconjugacy (being actually a conjugacy in the transitive case) with minimal model maps of unit slope, i.e., for rigid interval exchange transformations.

Concerning Lorenz maps in a neighghborhood of a map with zero entropy, note that the entropy function may have jumps in C^0-topology. Nevertheless, if one considers Lorenz maps with zero one-sided derivatives at the discontinuity point and with respect to C^1-topology, it still depends continuously on the map. More precisely, the result is as follows.

Theorem The function $f \to h_{top}(f)$ in the class of Lorenz maps with C^0-topology is continuous at f_0, except for the case when $h_{top}(f_0) = 0$ and the kneading invariants $K^+_{f_0}, K^-_{f_0}$ of f_0 are periodic with the same period; in the latter case, the jump of topological entropy is precisely $\frac{1}{p}\log 2$, where p is the common period of the kneading invariants. Moreover, for the class of Lorenz maps having zero one-sided derivatives at the discontinuity point and with C^1-topology, such an exceptional case is impossible, and thus, the topological entropy depends continuously on the map.

We also discuss multidimensiodal perturbations of Lorenz maps with zero entropy.

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Transient sequences in a hypernetwork generated by an adaptive network of spiking neurons

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We propose a paradigmatic model of an adaptive network of spiking neurons that gives rise to a hypernetwork of its dynamic states at the upper level of description. Despite the simplicity of the coupling structure, the neuron model, and the evolutionary operator we show the basic idea: how dynamics of the adaptive oscillatory network of spiking neurons leads to the emergence of various transient behaviours in the hypernetwork. Left to itself, the network exhibits a sequence of transient clustering which relates to a traffic in the hypernetwork in the form of a random walk. Receiving inputs the system is able to generate reproducible sequences corresponding to stimulus-specific paths in the hypernetwork. We illustrate these basic notions by a simple network of map-based spiking neurons together with its FPGA realization and analyze their properties.

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Maps in a laser harmonically mode-locked by optoelectronic feedback

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We analyze the dynamics of a harmonically [1, 2] mode-locked solid state laser controlled with the combination of two inertial optoelectronic feedback loops. The case of $k > 1$ short pulses circulating in the laser cavity can be obtained not only by external modulation, but also by use of single inertial optoelectronic feedback [3]. Optoelectronic control is characterized by two time constants, the first one is feedback response time corresponding to the charging process of intracavity modulator capacity C by short pulse. The second constant defines the inertiality of feedback loop and equals RC, where R is the discharge resistance of modulator control circuit. Necessary conditions for self mode-locking are proper feedback delay and fast feedback response, the latter means that the response time is much less than a cavity round trip time T_r. In [4] it was concluded that, for harmonic self-mode-locking, single feedback delay should be equal to $(1 - \frac{1}{k} + M)T_r$, where $M = 0, 1, 2....$ Using the cyclicality of equations that describe the laser radiation dynamics from round-trip to the next one, we compactify the system of maps into a single map with discrete time count T_r/k. In general case, short laser pulses interact via feedback inertiality and the map corresponding to single-feedback control is

$$x_{n+1} = rx_{n-(k-1)} \left(1 - \sum_{i=0}^{i_{max}} x_{n-(k-1)-k\cdot M-i} \cdot \gamma^i\right), \quad n = 0, 1, 2..., \qquad (1)$$

where n is the number of laser pulse, x_n is the pulse energy, r is the total gain, $0 \leq \gamma < 1$ describes feedback inertiality, i_{max} is the number of preceding pulses that add in control signal, i.e. the memory depth of feedback. Pulse interaction can be controlled by parameters γ and i_{max}. Inertial feedback control corresponds to $i_{max} \to \infty$, $\gamma = e^{-RC\cdot k/T_r} > 0$, whereas single negative memory-free ($i_{max} = 0$) feedback control is achieved at $\gamma = 0$. Other values of i_{max} can be realized by means of previously proposed dual-feedback memory erasing technique [5]: negative inertial feedback is complemented by $(i_{max} + 1)T_r$-delayed positive inertial feedback with relative sensitivity chosen to cancel the residual action of negative feedback. This control method allowed to obtain high-frequency regimes in the form of regular bursting with period less than the laser cavity round-trip time. For example, bursting with period $3T_r/7$ was observed in map (1) at $i_{max} = 1$, $k = 7$, $M = 0$, $\gamma = 0.367$:

$$x_{n+1} = rx_{n-6}(1 - x_{n-6} - 0.367x_{n-7}). \qquad (2)$$

The results of map simulation agree with the numeric simulation taking into account the evolution of fine time structure of laser radiation. Our findings hold up a hope for harmonically mode-locked dual-feedback-controlled solid-state laser to be a promising object of nonlinear science.

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Complex maps of exp(iz) kind: solitary and lattice coupled by linear relation

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As it known, the topology of Mandelbrot and Julia sets of hyperbolic maps like as $ae^z + be^{-z},\ a, b, z \in \mathbb{Z}$, is greatly different from classical case as square-law $z^2 + c, c \in \mathbb{Z}$. This topology shows the elements of Cantor bouquet, which is similar to Cantor set in the case of two dimensions. In [1], we have exhibited some additional properties of $\lambda e^{iz}, \lambda \in \mathbb{C}$ map that is easily expressed as hyperbolic one. Despite from the accepted term «coupled map lattice» [2], its dynamics is poorly studied on the complex plane, especially with cellular automata (CA) formalization and for this map. We carry out our research with mostly using numerical methods of MATLAB and original software SoftCAM for CA design aim. The purpose of our study is to compare the dynamics of solitary map with group dynamics of its several copies.

The studied object is defined by CA structure on the hexagonal field and on the grid with $N = 2, 4$ edge with cyclic boundary conditions and local transition function with the coupling factor μ_0 :

$$z^{t+1} = \lambda(z_1^t, z_2^t, ..., z_n^t, \mu)\exp(iz^t), \quad t \in 0, 1, 2, ...,\ z^t \in \mathbb{C},\ \mu \in \mathbb{C}$$
$$z^* : z^* = \mu \exp(iz^*), \quad \lambda(t) = \mu + \mu_0 \left| \frac{1}{n} \sum_{k=1}^{n} z_k^t - z^*(\mu) \right|, \quad \mu \doteq 1 \tag{1}$$

Index n takes all values of the neighboring cells for current cell, which is not indexed. The task of CA homogeneous equilibrium searching is leads to the necessary additional study of linear-exponential map (2):

$$z^{t+1} = f(\mu_0, z^t) = (1 + \mu_0|z^t - z^*|)\exp(iz^t),$$
$$z^* \equiv 0.57641 + 0.37470i \tag{2}$$

It has been shown that when μ_0 has small values, z^* is stable, but when the imaginary part of μ_0 is increasing from $|\mu_0^{double}| = 2.1682$, equilibrium doubles and is drifting to z^* at an 146^0 angle taking its equilibrium. Julia sets (fig. 1-2) have been studied in the cases of map (2) and $\mu_0 = 0.25, 0.25i, -0.9i, 2, 5i$. The hypothesis that stable heterogeneous equilibriums do not exist is proving. On the basis of the introduced definitions of point stable and «in-direction» stable equilibrium it is possible to construct an analogue of the Julia sets for the group dynamics of CA. In the latter case, there are some differences, such its partial destruction (transition from a solid to a fractal structure). Note computational instability in the computer construction of these sets originating from the expression (1) structure.

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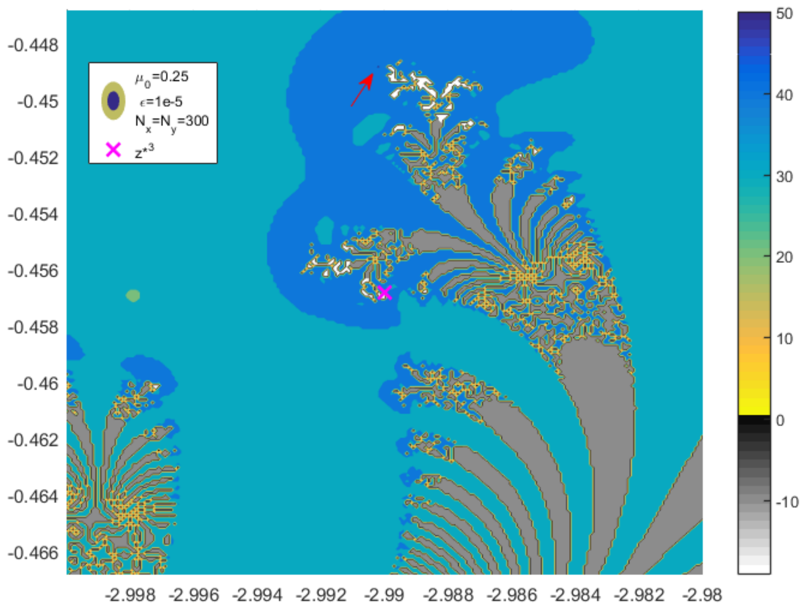

Figure 1: Julia and Fatou sets in the neighborhood of an unstable equilibrium of a solitary map (2). Cantor bouquets are distinguishable even on a small scale, as well as the points of the local increase convergence rate within Fatou set. The colors

mark the area of convergence to z^*, gray tones mark Julia set. The darker color, the faster escape to infinity.

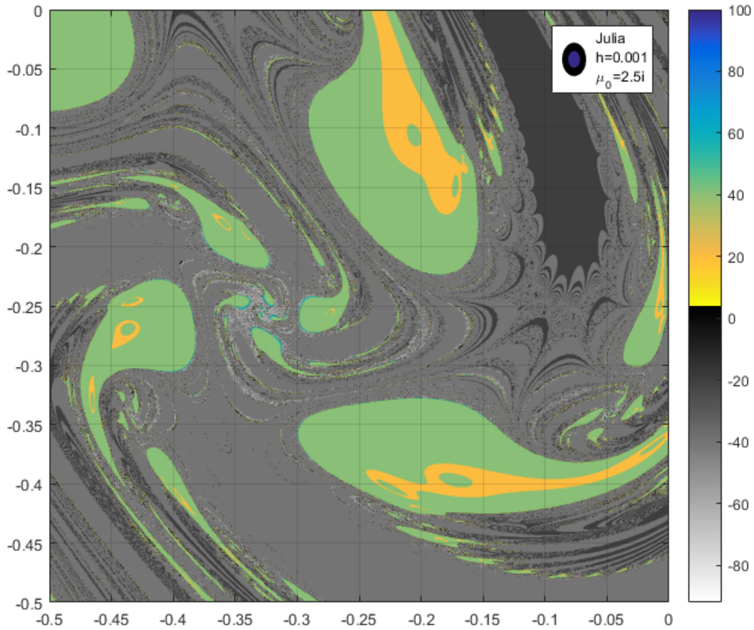

Figure 2: The common structure of Julia and Fatou sets for a solitary map with $\mu_0 = 2.5i$. As in Figure 1, the axes represent the real and imaginary parts of the initial condition. Color indicates the number of iterations to convergence, gray tones indicate the number of iterations before leaving orbit to infinity.

Synchronization Phenomena in Rings of Coupled Three van der Pol Oscillators

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Synchronization phenomena have been studied in various fields since a long time ago, such as in electrical systems, in mechanical systems, in biological systems and basically everywhere. Among them, synchronization phenomena of van der Pol oscillator are similar to natural phenomena by changing frequency. The coupled system of van der Pol oscillators is simple and easy to handle. Many researchers have proposed various coupled oscillatory networks of van der Pol oscillators [1] - [2]. We focus on the coupling strength of coupled oscillatory networks consisted of two kinds of oscillators including van der Pol oscillator.

In this study, we propose a novel coupled oscillatory system. Figure 1 shows circuits of van der Pol oscillators. Figure 2 shows the circuit model. We use two ring circuits with six oscillators. Three VDP of the first ring are connected by resistors, three NC of the second ring are connected by inductors and resistors. The first and the second ring are connected by resistors (R_1, R_2, R_3). We investigate how to change synchronization phenomena of adjacent oscillators by changing the value of R_1, R_2 and R_3 by computer simulations and circuit experiments. Furthermore, when we change only the value of R_2 from 0 to 0.03 at intervals of 0.001, we investigate relationship between coupling strength and phase difference. This research obtained an interesting results which synchronization phenomena are observed by magnitude correlation between R_1, R_2 and R_3.

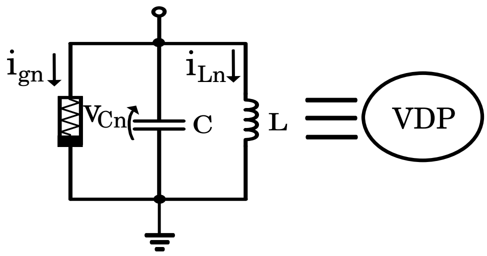

Figure 1: Circuit of van der Pol oscillators.

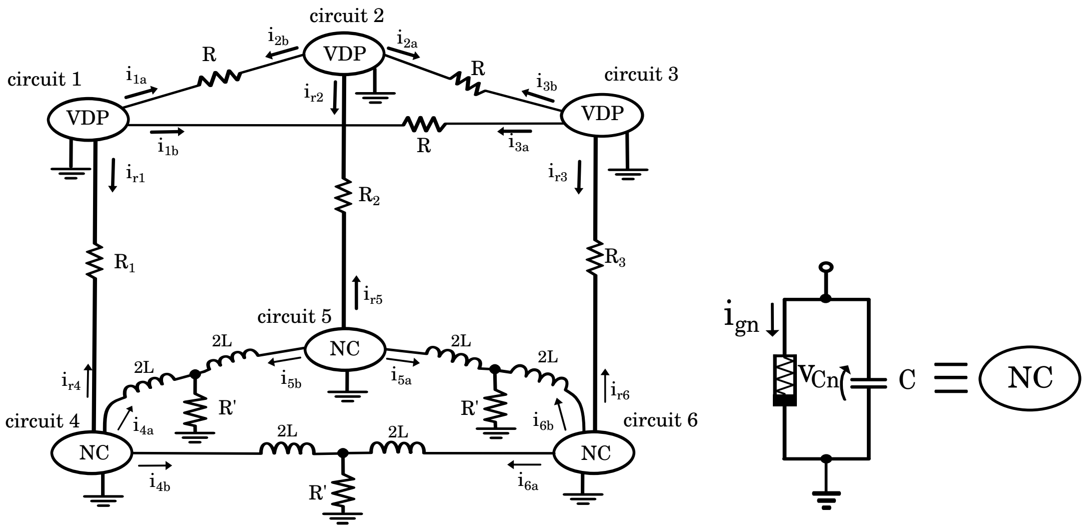

Figure 2: Circuit model.

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On the shadowing property and odometers

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When we investigate the space of invariant measures from ergodic theory point of view, we are usually not that much interested in the topological structure of underlying space. By famous Jewett-Krieger theorem, we can view invariant measures as supported on minimal systems and numerous further generalizations allow to add even more topological (dynamical) properties to the underlying system. On the other hand, there are examples of systems with quite rich dynamical structure (e.g. topologically mixing) but not that much interesting invariant measures (e.g. only trivial measure, only atomic measures, etc.). In other words, connections between topology and ergodic theory (on compact metric spaces) is not that tight.

In this talk we will provide some characterizations of invariant measures in the case when a dynamical system (X,T) has the shadowing property. We will show that often invariant measures can be approximated by a special class of minimal dynamical systems. We will also comment on possibilities of approximation of entropy.

Adiabatic cycles and geometric phases in maps.

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The behaviour of discrete dynamical systems under adiabatic cyclic variations of their parameters, is investigated in discrete versions of adiabatically-rotated rotators. We generalize the concept of geometric phase and apply this generalization to discrete dynamical systems and look for the conditions where such phases exists in these rotators. In the case of rotated sine circle maps -a strongly dissipative system- we found an analytical relationship between the geometric phase and the rotation number of the map. On the other hand, the discrete version of the Hamiltonian rotated rotator considered by Berry turns out to be a rotated generalization of the standard map. For this one, we further explore the connection of the geometric phase with the rotation number as well as the role of the geometric phase at the onset of chaos. Further into the chaotic regime, we find that the geometric phase is also related to the diffusive behaviour of the dynamical variables as discover a surprising connection with the scaling of the Lyapunov exponent as the nonlinearity strength increases.

In continuous time dynamics, the study of adiabatic cyclic variations of parameters is related to the concepts of anholonomy and geometric phase [1], that is the failure of certain variables to return to their original values after a closed circuit in the parameters. Physical manifestations of anholonomies are the rotation of the plane of oscillation of a Foucault pendulum, the swimming of microorganisms at low Reynolds numbers, mixing in the stomach, the way a falling cat manage to reorientate itself in order to land on its feet [2]. The geometric phase originally encountered in quantum mechanics [3] was then generalized to classical integrable systems then to nonintegrable Hamiltonian systems, and finally to dissipative systems [4]. Berry and Morgan, investigated the geometric phase of a continuous-time Hamiltonian rotated rotator [3] as a convenient example to understand its principles. Geometric phases however, have not been considered hitherto in discrete time dynamical systems and the general question of how a mapping-defined dynamics behaves under adiabatic parametric cyclic perturbation has not been addressed until now. Here, we then introduce a discrete analogue of the geometric phase and show that it is linked to important aspects of the dynamics of maps. For this purpose, we follow the lines off the rotated rotator considering first adiabatic cyclic perturbations of the sine circle map to find that he geometric phase is intimately related to the behaviour of the rotation number as a function of the bare frequency parameter. We then study the rotated Hamiltonian standard map, in which we discover surprising relationships between the geometric phase, not only with the rotation number as in the former case, but also with the Lyapunov exponent and the diffusive behaviour of both action and phase variables [5].

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Bifurcations structure in an embedding of Dim1 generic growth maps into a Dim2 diffeomorphism

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The main purpose of this talk is to present dynamics and bifurcations properties of an embedding of Hénon's map type: a "continuous" embedding of one-dimensional (Dim1) generic growth functions $f : [0,1] \to [0,1]$ into a two-dimensional (Dim2) real continuous map $T_c : \mathbf{R}^2 \to \mathbf{R}^2$. The 1D generic growth functions are a 3-parameter family of unimodal maps, defined as follows:

$$f(x; r, \beta, \gamma) = r\ x^{1+\beta(1-\gamma)} \left(1 - x^{\beta}\right)^{\gamma}, \tag{1}$$

where the variable $x \in [0,1]$ and the real parameters $r, \beta, \gamma > 0$, with $\gamma < 1 + \frac{1}{\beta}$. The chaotic dynamics and the bifurcations structure of Dim1 generic growth functions have been studied in [2].

The Dim2 real continuous map $T_c : \mathbf{R}^2 \to \mathbf{R}^2$ is defined in the form of recurrence relationship as follows:

$$T_c \equiv \begin{cases} x_{n+1} = f(x_n; \Sigma) + y_n \\ y_{n+1} = c x_n \end{cases} \Leftrightarrow T_c \equiv \begin{cases} f_1(x_n, y_n; \Sigma) = r x_n^{1+\beta(1-\gamma)} \left(1 - x_n^{\beta}\right)^{\gamma} + y_n \\ f_2(x_n, y_n; \Sigma) = c x_n \end{cases}, \tag{2}$$

where $0 \leq c \leq 1$ is the embedding parameter, $(x_n, y_n) \in [0,1] \times [0,1]$ and $n \in \mathbf{N}_0$. The parameters $(\beta, \gamma, r) \equiv \Sigma$ have the same meaning as in Eq.(1). Note that the Dim2 map T_c is defined in the $(\Sigma, c)-$parameters space and is a generalization of the embedding studied at [3]. When the parameter c of the map T_c is equal to the critical value $c = 0$, from the initial condition (x_0, y_0), with $y_0 \neq 0$, the sequence of iterated points (x_n, y_n), with $n \in \mathbf{N}$, are located on the abscises axis $y = 0$. Thus, after one iteration the degenerated Dim2 map $T_{c=0}$ turns into the Dim1 generic function T_0, with initial condition x_1, given by $x_{n+1} = r x_n^{1+\beta(1-\gamma)} \left(1 - x_n^{\beta}\right)^{\gamma}$, with $n \in \mathbf{N}$. In fact it is verified a "degenerated" behavior, i.e, $T_{c=0} \equiv T_0$, for $n \geq 1$. For this reason it is said that the Dim1 generic growth function $T_0 \equiv f$ is embedded into the Dim2 invertible map T_c. This topic of "germinal dynamics" has been extensively studied by Mira and co-authors in the books [4] and [5], see also, for example, [1] and references therein.

The bifurcations structure of $T_0 \equiv f$ ($f \equiv f_1$, for $c = 0$), Eq.(1), in the Σ parameters space is a "germ" for what occurs on the parameters space (Σ, c) of the map T_c. The evolution of the bifurcation structures of T_c in the parameters plane will be studied by varying the embedding parameter c. More particularly properties that characterize the big bang bifurcations are considered in relation with this coupling of two population size functions. These bifurcation cascades converge to curves of singular points, as sets of codimension-2 bifurcation points. In this embedding or coupling problem, the knowledge of the big bang bifurcation structures in Dim1 situation gives us a germinal state for understanding the Dim2 case. We provide and discuss suficient conditions for the stability of the fixed points of T_c, related to their eigenvalues and the embedding parameter c. Analytical results will be illustrated with the help of numerical simulation results and appropriate bifurcation diagrams.

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Iterations of independent random flows generated by the differential equations with random parameters

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In the paper [1] the sufficient and necessary condition of the law of large number for the sequence of compositions of independent random semigruops of bounded linear operators had been obtained. The extention of law of large number on the compositions of independent random semigruops of bounded nonlinear maps will be investigated. As the result the method of probabilistic approximation of differential equation with singularities is contructed.

Let E be the Hilbert space and $H:\ E\to E$ is the Borel vector field. If Cauchy problem for the nonlinear differential equation

$$x'_t(t)=H(x(t)),\ t>0; x(0)=x_0\in E, \qquad (1)$$

for the unknown function $x:\ [0,+\infty)\to E$ has the unique solution on the semiaxe $[0,+\infty)$ for arbitrary $x_0\in E$ then it generates the one-parametric semigroup (Flow) of nonlinear maps $X_t:\ E\to E$, $t\geq 0$.

Let λ be the measure on the measurable space (E,Λ). We investigate the conditions on the field H and measure λ under the which the flow X_t, $t\geq 0$, induces the semigroups of linear maps

$$T_t:\ T_tu(x)=u(X_t^{-1}(x)),\ x\in E,\ t\geq 0,\ u\in\mathcal{H} \qquad (2)$$

in the space $\mathcal{H} \equiv L_2(E, \Lambda, \lambda)$. We obtain the condition on the function $u \in \mathcal{H}$ under the which the function $v(t, x) \equiv u(X_t^{-1}(x))$ is the solution of linear differential equation

$$v_t(t, x) + (\mathbf{H}(x), \nabla v(t, x)) = 0; \; t > 0, \, x \in E. \tag{3}$$

If $\dim(H) < \infty$, the function $u \in \mathcal{H}$ is smooth and $H : E \to E$ is the smooth nondegenered vector field then any solution of the equation (3) is the first integral of the equation (1) and any phase trajectory of the equation (1) can be defined by the full set of independent sulutions of the equation (3).

The linear differential equation (3) can be approximated in the frame of the method of elliptic regularization and the nonlinear differential equation (1) can be considered with some directed set of its random regularizations (see [2, 3]).

If $\{\mathbf{H}_\omega, \, \omega \in \Omega\}$ is the measurable map of the probability space $(\Omega, \mathcal{A}, \mu)$ into the space of vector fields $E \to E$ such that for any $\omega \in \Omega$ Cauchy problem (1) has the unique solution $X_{\omega,t}(x_0)$, $t \in R$, for any $x_0 \in E$ then the one-paramrter semigroups $X_{\omega,t}(\cdot)$, $t \in R$, of the maps of the space E are defined for any $\omega \in \Omega$.

If for any $\omega \in \Omega$ the maps $X_{\omega,t}$, $t \geq 0$, are measurable maps of the space (E, Λ, λ), then the one-paramrter semigroups $T_\omega(t)$, $t \in R$, of linear operators in the space $\mathcal{H}$ are defined by the equalities (2) for any $\omega \in \Omega$.

If the maps $X_\cdot : \; \Omega \to C(R_+, C(E, E))$ and $T_\cdot : \; \Omega \to C(R_+, B(\mathcal{H}))$ are the measurable maps of the measurable space into the topological space with the minimal algebra of subsets containing the topology then the one-parameter semigroups $X_{\cdot,t}$, $t \in R$, and $T_\cdot(t)$, $t \in R$, are the random semigroups ([2]) of maps of the space E and $\mathcal{H}$ respectively.

We investigate the limit behavior of the sequence of averaged composition of independent random semigroups $(X(t/n, \cdot))^n = X_{\omega_n}(\frac{t}{n}, \cdot) \circ ... \circ X_{\omega_1}(\frac{t}{n}, \cdot)$ in the space E and $(T(t/n))^n = T_{\omega_n}(\frac{t}{n}) \circ ... \circ T_{\omega_1}(\frac{t}{n})$ in the space $\mathcal{H}$.

We show that the validity of the law of large numbers for the sequence of averaged composition of independent random semigroups of linear maps in the space $\mathcal{H}$ provides the validity of law of large numbers for the sequence of averaged composition of independent random semigroups of nonlinear maps in the space E (see [1]). Some application to the different form of Cauchy problems (1) (see [3]) will be considered.

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Non-equilibrium thermodynamics in the Poincaré cycles

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Henri Poincaré considers a one-dimensional ideal gas uniformly filling an interval. The ideal gas is considered as a system of noninteracting particles. In particular, they cannot collide with each other. Each particle of this medium moves inertially, independently of the other particles, reflecting elastically from the boundaries of this interval. Poincaré's basic observation was that, independently of the initial distribution, gas eventually tends to uniform filling of interval. Thus, the ideal gas shows the irreversible behavior. Every particle of the gas approaches arbitrary close to the initial position infinitely many times. However, such individual returnability is not uniform, which results in a diffusion in a reversible and conservative system. Thus, the compatibility of the reversibility and retainability properties with irreversible behavior of a dynamical system was shown.

Let's consider an equilibrium of a one-dimensional ideal gas. A gravitating body approaches the interval from infinity, the gas is allowed to attain a new equilibrium, after which the body recedes back to infinity. After that, the gas tends to fill the interval uniformly. So, the colisionless gas has performed a closed cycle. This cycle is defined by V. V. Kozlov as a Poincaré cycle (similarly to the Carnot cycle). But in contrast to the Carnot cycle, the Poincaré cycle is non-equilibrium and irreversible.

We take the initial velocity distribution density proportional to its square. In particular, Maxwell distribution doesn't satisfy this condition. We add the force field and wait for attaining equilibrium. Then we remove the force field and again wait for attaining equilibrium. Modelling many times these cycles, we obtain the different intermediate types of the density and the asymptotic behavior.

Criteria for foliations with transverse linear connection to be pseudo-Riemannian and Riemannian

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At present Riemannian foliations form the most deeply studied class of foliations with transverse geometric structures. Works of B. Reinhart, A. Haefliger, E. Ghys, Y. Carriere, E. Salem, V. Sergiesku and many others and also monographs of P. Molino, P. Tondeuer and V. Y. Rovenskii represent a significant contribution to the study of Riemannian foliations.

R. A. Wolak in [2] put the following question:

"When a G-foliation is a Riemannian one ?"

R. A. Wolak proved that every compact G-foliation of finite type is a Riemannian one. An analogous statement was proved by R. A. Wolak for foliations admitting transverse systems of differential equations of an arbitrary order. A number of other conditions for a compact foliation (M, F) to be Riemannian are well known [4].

For conformal foliations of a codimension $q > 2$ a criterion of Riemannianness was proved in [3]. For foliations (M, F) with trasverse parabolic geometry of rank one a criterion of Riemannianness is known from [5]. According to this criterion (M, F) is a Riemannian foliation if and only if all its holonomy groups are relatively compact.

Let (M, F) be a a foliation with transverse linear connection. We consider a general case when (M, F) is a foliation of a codimension q on n-dimensional manifold, $0 < q < n$.

We prove the following criterion for foliations with transverse linear connection to be pseudo-Riemannian.

Theorem 1 *Let (M, F) be a foliation of a codimension q with transverse linear connection given by (N, ∇)-cocycle. Let Q be the associated connection on the foliated bundle of transverse frames $\mathcal{R}(M, H)$, where $H = GL(q, \mathbb{R})$.*

Then (M, F) is a pseudo-Riemannian foliation given by (N, g)-cocycle, where g is a pseudo-Riemannian metric of a signature $(k, q-k)$, $0 \le k \le q$, parallel respectively ∇, if and only if there exists a point $v \in \mathcal{R}$ such that the holonomy group $\Phi(v)$ of the connection Q at v belongs to the pseudo-orthogonal subgroup $O(k, q-k)$ of the group H.

In the case when the dimension of (M, F) is zero, Theorem 1 implies the Schmidt's criterion for a torsion free linear connection to be the Levi-Civita connection for a pseudo-Riemannian metric [1].

As a corollary of Theorem 1 we get a necessary and sufficient conditions for a foliation (M, F) with transverse linear connection to be Lorentzian.

We prove also the following statement.

Theorem 2 *Let (M, F) be a foliation of a codimension q with transverse linear connection and let $H = GL(q, \mathbb{R})$.*

Then (M, F) is a Riemannian foliation if and only if the holonomy group $\Phi(u)$ at some point $u \in \mathcal{R}$ of the the foliated bundle of transverse frames $\mathcal{R}(M, H)$ is a relatively compact subgroup of the Lie group H.

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Lyapunov's exponents and multiplicative ergodic theorem for log-summable cocycles

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Investigation of K-means Algorithm Using an Improved Firefly Algorithm

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Clustering is a popular data analysis technique used for data analysis, image analysis, data mining and the other fields of science and engineering. The goal of clustering is to find homogeneous groups of data points in a data set. Each group is called a cluster and is characterized by the fact that objects belonging to the same group are more similar than objects belonging to different groups. The K-means algorithm is one of the most famous clustering methods. It is used if the number of clusters is known and the clusters tend to be spherical. The goal of the method is to find K cluster centers and assign each object to the closest cluster center such that the sum of the squared distances between the objects and the corresponding cluster centers is minimal. This means that the K-means clustering problem is an optimization problem.

Senthilnath et al. [1] proposed an algorithm that used the firefly algorithm for K-means clustering (KMFA). Numerical experiments have indicated that this algorithm is more efficient algorithm than the standard algorithm or other optimization heuristics. The firefly algorithm (FA) has been proposed by Yang in 2007 and is based on the idealized behavior of the flashing characteristics of fireflies [2–4]. FA is an efficient optimization algorithm because it has a deterministic component and a random component. Almost all algorithms having only the deterministic component are local search algorithms, for which there is a risk of being trapped in a local optimum. However, the random component makes it possible to escape from such a local optimum.

In this paper, we propose a new clustering algorithm that combines K-means clustering and improved firefly algorithm (KMIFA). In our proposed algorithm, each firefly has its own value of $\alpha(t)$:

$$\alpha(t)_i = \lambda_i \left(\frac{10^{-4}}{0.9}\right)^{t/t_{max}}. \tag{1}$$

In the case of firefly i, if the assignment of all objects does not change, the value of λ_i decreases. We set all initial values of λ to 1.0 when initializing the population of fireflies and define the minimum value of λ is 0.

$$\lambda_i = \begin{cases} \lambda_i^{old} - V & \text{, the assignment does not change} \\ \lambda_i^{old} & \text{, otherwise} \end{cases} \tag{2}$$

The parameter V is a predefined value. At the beginning of the search, all fireflies move with a relatively strong random influence. Hence, they can more easily escape from local optima. As the number of iterations increases, the firefly tends to converge. We compare the conventional K-means algorithm, KMFA and our proposed algorithm KMIFA using several data models that have several spherical clusters. These experiments indicate that our algorithm is more efficient than the other algorithms.

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The preliminary version of this study is presented at NCSP2017.

On locally linearizable billiard systems

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On positive metric entropy conjecture

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We prove the folowing conjecture by Herman:

Arbitrarily close, in the C^∞-topology, to the identity map of a two-dimensional disc there exists an area-preserving diffeomorphism with positive metric entropy.

Investigation of Ring - Star Network of van der Pol Oscillators

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There are a lot of synchronization phenomena in this world. This is one of the nonlinear phenomena that we can often observe by natural animate beings which do collective actions. For example, firefly luminescence, cry of birds and frogs, applause of many people, and so on. Synchronization phenomena have a feature that the set of small power can produce very big power by synchronizing at a time. Therefore, study of synchronization phenomena has been widely reported not only in the engineering but also the physical and the biological fields. Investigation of coupled oscillators is focused on many researchers, because coupled oscillatory network produces interesting synchronization phenomena, such as the phase propagation wave, clustering, and complex patterns. In addition, it has the advantage of being able to manufacture for circuit on the board[1, 2, 3].

In this study, we investigate synchronization phenomena observed in the system model containing a ring and a star of van der Pol oscillators by circuit experiment and computer simulation. We observe several types of synchronization phenomena by increasing the coupling strength of the ring. Then, we observe the synchronization phenomena with computer simulation. van der Pol oscillator is shown in Fig. 1.

Figure 2 shows a system model constituted van der Pol oscillators (VDP-A and VDP-B). We couple each VDP-B via inductor L and ground by coupling resistor R. In addition, We couple VDP-A via resistor r. VDP-A is the only one central circuit which is connected to all VDP-B in this system by resistor r.

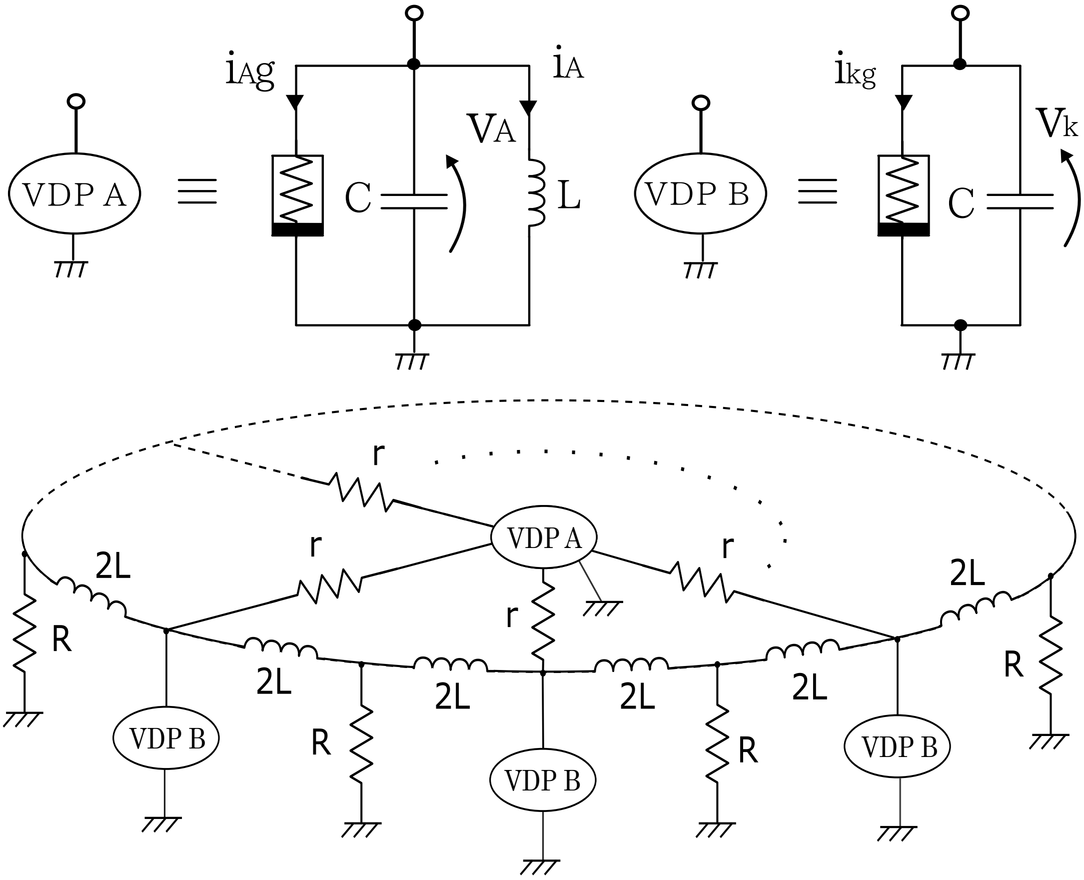

Figure 2: System model.

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Dynamics of monotone maps on a one-dimensional locally connected continuum

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By continuum we mean a compact connected metric space.

Let X be a one-dimensional locally connected continuum, $f : X \to X$ be a continuous map. A map f is called to be monotone, if for every connected subset $C \subset X$, $f^{-1}(C)$ is connected.

One-dimensional locally connected continua have a complicated topological structure. Therefore, even monotone maps on them have nontrivial dynamics (see, e.g., [1] - [5]).

In this report dynamics of monotone maps on a one-dimensional locally connected continuum is studied.

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Boltzmann extremals and ergodic theorem for group representations.

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The main problem of the ergodic theory is the problem of describing a limit, which a solution of Liouville equation (the equation for density or for particle distribution functions) goes to with time going to infinity. The problem of the justification of the method of Gibbs is a special case of this problem, reducing the question to clarifying the conditions for a Hamiltonian system, when the limit is the exponent on the energy. In the works of Boltzmann the concept of maximum of entropy with fixed linear conservation laws (Boltzmann extremal) was introduced. In studies of Poincare and Kozlov and Treshov it has been shown, what the law of growth of entropy for Liouville equations is: entropy of a temporary average greater than or equal to the entropy of the initial distribution, while along the solutions it persists. In the works of V.V.Vedenyapin it is shown that the time averages of Liouville equation coincides with the Boltzmann extremal. We prove this coincidence for representations of groups by introducing the entropy and studying its properties in representation theory. Then we find out what it gives to ergodic problem.

Let us call a convex function $S(x)$, $x \in V$ an entropy of the representation ρ of a group G, if $S(gx) \geq S(x)$ for all $g \in G$.

Such property, when any decreasing functionality is persistent, can be regarded as the property of reversibility of the dynamics. Here reversibility is just related to the group property of the dynamics.

The concept of average (similar to the temporary average) for the action of the group G is introduced:

$$[x] = \frac{1}{|G|}\sum_{g\in G}\rho(g)x = \frac{1}{|G|}\sum_{g\in G} gx \tag{1}$$

Here $|G|$ is the number of elements in the group.

We prove the existence of entropy and then the analogue of the H-theorem for representations of groups: $S([x]) \geq S(x)$.

In the proof we use the convexity of $S(x)$. This is the analogue of the theorem of Poincare-Kozlov-Treshov for Liouville equation.

The results, which are obtained link the reversibility and the irreversibility in the most clear form. This relationship, which worried the classics Boltzmann, Loschmidt, Zermelo, Poincare, may have been one of the motivations of ergodic theory and it continues to bother some modern researchers. The growth of entropy in theorem above is associated with averaging: an observer in a fast averaging sees precisely the average, like the spokes in a rotating wheel or a white color of a multicolored Maxwell rotating spinning top. This is entirely consistent with the works of Poincare and Treshov, where a group considered is real numbers (time analogue): there, too, during the evolution the entropy paradoxically is saved, and its limit is greater than or equal to (but in the examples is often strictly more) than this conserved quantity. Note that the analogy is not literal, because in the classical ergodic theorems of Birkhoff, Von Neumann, Riesz, and Bogolyubov semigroups are always concernd, as averaging occurs in the positive half-line. In the case of non-compact groups it is necessary to take care of convergence, but Von Neumann and Riesz obtained, in fact, an alternative formulation in the form of the projection method.

In one of the works of Boltzmann the H-theorem is proved and with the example of the discrete models of the Boltzmann equation the concept of extremal of an entropy with fixed linear conservation laws is explored - the extremal, to which the solution of the equation goes when time goes to infinity. In the work of Boltzmann it is found what is called Boltzmann statistics. Here the Boltzmann extremal is already used as a fundamental concept and as a working tool: a conditional maximum of entropy with Lagrange multipliers in integrals of the number of particles and kinetic energy is found, and Maxwell distribution is obtained. We define this concept in case of group representations in a similar manner as the conditional extremum of the entropy under the same invariants as the original vector space, where a representation is operating, using the decomposition of von Neumann-Riesz.

The obtained results can be generalized to the case of compact groups.

H-theorem by Boltzmann and Poincare.

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H-theorem was first discussed by Boltzmann in [1]. Boltzmann linked this theorem, proving the convergence of solutions of Boltzmann type equations to Maxwell's distribution, with the law of entropy increasing [2]. The proof of the H-theorem not only proves the second law of thermodynamics, but also makes the behavior of the solution of the equation is clear, as it allows to see where the solution of this equation converges with the time going to infinity.

We consider the generalization of equations of chemical kinetics, including classical and quantum chemical kinetics [3]. H-theorem for these generalizations of the equations of chemical kinetics: (2) and (3) in the case of continuous time were studied [3]. A generalized condition of detailed equilibrium (balance) and the generalized condition of dynamic equilibrium (or generalized condition by Stuckelberg Batisheva–Pirogov), under which the H-theorem is fair, were studied. In [4], [5] it is shown how does the law of growth of entropy for Liouville equations hold true: entropy of a temporary average greater than or equal to the entropy of the initial distribution, while along the solutions it persists. In [6], [7], it is shown that the time averages of Liouville equation coincides with the Boltzmann extremal in cases where the conditional maximum of entropy with fixed conservation laws is achieved. We prove the coincidence for the representations of groups by introducing the entropy and studying its properties in representation theory. Then we find out what it provides for the ergodic problem by getting the generalization and refinement of the ergodic theorems of Riesz, Birkhoff-Khinchin, von Neumann and Bogoliubov from one point of view.

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The Hamilton–Jacobi Method in the Non Hamiltonian Situation and Boltzmann extremals

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The hydrodynamic substitution, which is wellknown in the theory of the Vlasov equation [1]-[3], has recently been applied to the Liouville equation and Hamiltonian mechanics [4]-[8]. In [4]-[6], Kozlov outlined the simplest derivation of the Hamilton–Jacobi(HJ) equation, and the hydrodynamic substitution simply related this derivation to the Liouville equation [7]-[8]. The hydrodynamic substitution also solves the interesting geometric problem of how a surface of any dimension subject to an arbitrary system of nonlinearordinary differential equations moves in Euler coordi nates (in Lagrangian coordinates, the answer is obvious). This has created prerequisites for generalizing the HJ method to the non Hamiltonian situation. The H-theorem is proved for generalized equations of chemical kinetics, and important physical examples of such generalizations are considered: a discrete model of the quantum kinetic equations (the Uehling–Uhlenbeck equations) and a quantum Markov process (a quantum random walk). The time means are shown to coincide with the Boltzmann extremes for these equations and for the Liouville equation [9]. This give possibility to prove existence of analogues of action-angles variables in nonhamiltonian situation.

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On the dynamics of non-invertible branched coverings of surfaces

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Let $f : M \to M$ be a branched covering, i.e. an inner (open and isolated) map of a surface M. A map is open if the image of an open set is open. A map is isolated if the pre-image of a point consists of isolated points.

For the sets of diffeomorphisms and smooth flows there are notions of structural stability. An element of the set is structurally stable when it is equivalent to each element of some its neighborhood, where equivalence is the topological conjugacy in case of diffeomorphisms and trajectory equivalence in case of flows.

It is natural to introduce by analogy a notion of structural stability for a smooth inner map. The paper [1] introduced a set of new invariants of topological conjugacy of non-invertible inner mappings that are modeled from the invariant sets of dynamical systems generated by homeomorphisms. Using those results we show that the topological conjugacy is a bad choice for the neighborhood equivalence of inner maps and suggest another equivalence to be used to define structural stability of inner maps.

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Denoising Auto Encoder with Intermittency Chaos to Express Space Features

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Recently, deep learning is used as commercial services and it becomes hot topic. It is difficult to learn whole network, because deep learning has complicated network. It is known that giving good initial values in advance is effective for learning the whole network. Auto encoder is used to give initial values.

Auto encoder has three layers. They are input, hidden and output layers. The goal of the auto encoder is to obtain the output values to match the input values. The number of neurons in the hidden layer is smaller than those of input and output layers. When input data are sent to the hidden layer, they are moved to a dimensional space. This process is called dimensional reduction. So we can think of input layer as an encoder because it compresses data. Then, output layer as a decoder is try to reconstruct the original data by using relation between the hidden layer and the output layer. Also we use input data with the noise to obtain more robust value. This method is called denoising auto encoder and we obtain good values for deep learning [1] . Equation (1)

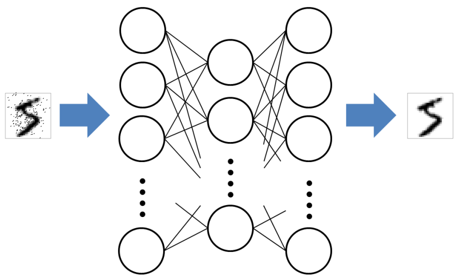


Figure 1: Denoising auto encoder.

shows encode and decode.

$$\begin{cases} y = s(Wx + b) \\ z = s(W'y + b') \end{cases} \quad (1)$$

x means input data. y means encoded information. z means reconstructing data from x. s means sigmoid function. W and W' mean weight. We define W_x equal W'_x by tied weight. We update the weight so that output z becomes closer to input x with calculating an error function. After updating the parameters W, b and b' the network calculates to minimize error function. The cross entropy (2) is used as error function.

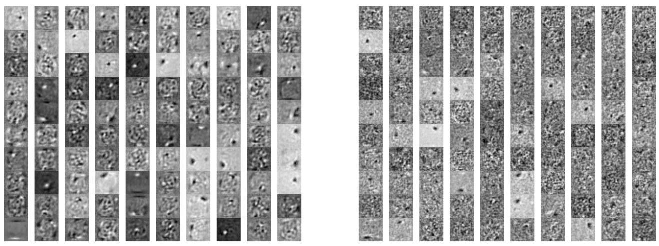
$$L_H(x, z) = -x \log z - (1 - x) \log(1 - z) \tag{2}$$

In this study, we use logistic map to make noise. Equation (3) shows logistic map.

$$f(x_{n+1}) \quad = \quad ax_n(1 - x_n) \tag{3}$$

Parameter a contrals the logistic map behavior. We set the parameter a as 3.828327 and use the intermittency chaos. We generate a random number by the logistic map and compare the number with the threshold. When it exceeds a threshold, it output 0. The output is multiplied by each pixel of the input data. The pixel which is multiplied 0 is painted black and becomes a noise. We use the logistic map for binarization. In this study, we forcus on weight in network due to difference of the number of neurons in hidden layer.

We set the number of neurons between 100 and 600 and visualize them in Fig. 2. As the number of neurons in hidden layer decrease, every neuron works and express features each other. Visualization of weight with 100 neurons has more space features than one with 600 neurons.



(a) (b)

Figure 2: Visualization of weight due to the number of neurons. (a) 100. (b) 600.

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On pseudo-Anosov homeomorphisms with non-orietable invariant foliations

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Let M be the closed oriented surface of genus g and $f : M \to M$ be generalized pseudo-Anosov homeomorphism with invariant foliations $\mathcal{W}^s$ (contracting) and $\mathcal{W}^u$ (expanding). It says that these foliations are orientable if for each loop made up of two arcs of leaves of them intersection index is the same in all intersection points. Let invariant foliations of f are non-orientable and singular type of f be $\{b_d; d \in \mathbb{N}\}$. This means that the number of singularities of valency d for $\mathcal{W}^s$ and $\mathcal{W}^u$ is b_d. In the talk the explicit construction of the surface $\widetilde{M}$, the map $p : \widetilde{M} \to M$ and homeomorphism $\widetilde{f} : \widetilde{M} \to \widetilde{M}$ with the properties listed below will be given.

1. $\widetilde{M}$ is closed orientable surface of genus $\widetilde{g} = 2g - 1 + \frac{1}{2}\sum_d b_d$.

2. $p : \widetilde{M} \to M$ is two-fold covering of M, if f has no singularities of odd valencies, and branched two-fold covering with branch points of multiplicity 2 in singularities of odd valencies in other case.

3. $\widetilde{f}$ covers f and is pseudo-Anosov homeomorphism whose invariant foliations $\widetilde{\mathcal{W}}^s$ and $\widetilde{\mathcal{W}}^u$ are orientable.

4. $\mathcal{W}^s = p(\widetilde{\mathcal{W}}^s)$, $\mathcal{W}^u = p(\widetilde{\mathcal{W}}^u)$.

5. Singular type of $\widetilde{f}$ is defined by $\widetilde{b}_d = 0$, if d is odd or $d = 2$, $\widetilde{b}_d = 2b_d + b_{d/2}$ if d is even and no multiply of 4, and $\widetilde{b}_d = 2b_d$ if d is multiply of 4.

Geometric structures on orbifolds and their automorphisms

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The automorphism group is associated with every object of a category. One of the central problems is the question, whether the group of all automorphisms of an object may be endowed with a finite-dimensional Lie group structure.

According to the results of Cartan, Myers and Steenrod, Nomizy, Kobayashi, Ehresmann and others, the groups of all automorphisms of different geometries on smooth manifolds are often Lie groups of transformations.

Recall that a G-structure on an n-dimensional orbifold $\mathcal{N}$ is a reduction of the $GL(n,\mathbb{R})$-bundle of frames over $\mathcal{N}$ to a Lie subgroup G of the Lie group $GL(n,\mathbb{R})$.

A Lie subalgebra $\mathfrak{g} \subset \mathfrak{gl}(n,\mathbb{R})$ is referred to as elliptic if $\mathfrak{g}$ contains no matrix of rank one. A G-structure is called elliptic if the Lie algebra $\mathfrak{g} \subset \mathfrak{gl}(n,\mathbb{R})$ corresponding to the Lie subgroup $G \subset GL(n,\mathbb{R})$ is elliptic.

An orbifold can be regard as a manifold with singularities. The topological space of an n-dimensional orbifold is locally homeomorphic to a quotient space of $\mathbb{R}^n$ by a finite group Γ of diffeomorphisms of $\mathbb{R}^n$. The group Γ is not fixed and can vary from one chart of an orbifold to another. Orbifolds were introduced by Satake, and they were named V-manifolds. Famous results of Thurston on the classification of closed 3-manifolds use the classification of 2-dimensional orbifolds. Orbifolds appear naturally in many branches of mathematics and mathematical physics: in the foliation theory; in the theory of deformation quantization on symplectic orbispaces, which include symplectic orbifolds; in the string theory as spaces of propagation of strings. Orbifolds were being used by physicists in the study of conformal field theory. An overview of the orbifold history can be found in [1].

The existence of a Lie group structure in the group of all automorphisms of G-structure of finite type was proved in [3] and [4].

As a tool we use the Ehresmann connection for a foliation in the sense of Blumenthal and Hebda.

For a given orbifold $\mathcal{N}$, a smooth foliation (M,F) admitting an Ehresmann connection is called associated with $\mathcal{N}$ if on the leaf space M/F there exists an orbifold structure such that the canonical projection $M \to M/F$ is submersion in the category of orbifolds and there exists an isomorphism $f : M/F \to \mathcal{N}$ in this category.

We present a new method of investigation of G-structures on orbifolds. This method is founded on the consideration of a G-structure on an n-dimensional orbifold as the corresponding transversal structure of an associated foliation.

For a given orbifold, there are different associated foliations. We construct and apply a compact associated foliation (M,F) on a compact manifold M for a compact orbifold. If an orbifold admits a G-structure, we construct and use a foliated G-bundle for the compact associated foliation.

Using our method we prove the following statement.

Theorem 1 *On a compact orbifold $\mathcal{N}$, the group of all automorphisms of an elliptic G-structure is a Lie group, this group is equipped with the compact-open topology, and its Lie group structure is defined uniquely.*

By the analogy with manifolds we define the notion of an almost complex structure on orbifolds and get the following statement.

Theorem 2 *The automorphism group of an almost complex structure on a compact orbifold is a Lie group, its topology is compact-open and its Lie group structure is defined uniquely.*

For manifolds, the statements of Theorems 1 – 2 are classical results [2]. Theorem 1 for manifolds was proved by Ochiai. In particular, in the case of flat elliptic G-structures on manifolds, Theorem 1 was proved by Guillemin and Sternberg and also by Ruh. Theorem 2 for manifolds was proved by Boothby, Kobayashi, Wang.

We also generalize the main result of [3] to orbifolds with rigid geometries. Emphasize that rigid geometries contain G-structure of finite type, Cartan geometries and rigid geometries of the sense of Gromov.

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An algorithm for the simulation of nonlinear oscillators

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The main property of the all simulations is the corresponding accuracy. It is too tempting to present some algorithm that gives for practically arbitrary nonlinear oscillator such accuracy that restricted by lattice size only. In order to show how it is possible let us consider the simplest nonlinear free van der Pol oscillator although the same reasoning holds for wide range of other cases. The 1-dimensional free van der Pol oscillator is described by the well-known equation

$$\ddot{x} - \lambda(1 - x^2)\dot{x} + x = 0. \tag{1}$$

Let us make analytical continuation for the x function into some subset of C. Also let us perform a variable change to y such that $x = -e^{iy}$. Putting the lattice with time step τ we have from (1)

$$e^{i(\tau^2 D^2 y + 2\tau Dy)} - (2 + \lambda\tau)e^{i\tau Dy} + \lambda\tau e^{i(2y+\tau Dy)} - \lambda\tau e^{i2y} + 1 + \lambda\tau + \tau^2 = 0, \tag{2}$$

where Dy is finite derivation of the y.

It is hinted from (2) that it might be possible to use discrete Fourier transformation to determine Dy and D^2y and therefore the values of the y and corresponding complex and then real x in different time positions t. The study which will be covered in the report discovers the possibility of using for the simulation purpose the quantum Fourier transformation instead of too slow FFT algorithm. Using the existing quantum computer IBM it is impossible to make any really significant computation but on the same resource one has emulation possibilities which could help to test the algorithm. The sample of the free van der Pol oscillator will be shown during the talk.

The main result of the study is the quantum algorithm for simulation of any oscillator by means of quantum Fourier transformation and SWAP-test for every register. The algorithm has polynomial computational complexity and could be executed on the any quantum computer with enough quantity of qubits (starting from 100 but optimal merely 1000).

On the birth of separators in magnetic fields

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We consider the model of magnetic field with point charges that model regions with an intensive magnetic flux. Using methods of Dynamical Systems Theory, one gets the conditions of the absence and existence of separators in a nice conducting fields (plasma). One gives a typical bifurcation of the birth of separators.

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