

PAPER • OPEN ACCESS

## “Escargot Effect” and the Chandler Wobble Excitation

To cite this article: Leonid Zotov and Christian Bizouard 2018 *J. Phys.: Conf. Ser.* **955** 012033

View the [article online](#) for updates and enhancements.

# “Escargot Effect” and the Chandler Wobble Excitation

Leonid Zotov<sup>1</sup>, Christian Bizouard<sup>2</sup>

<sup>1</sup> National Research University Higher School of Economics, Moscow Institute of Electronics and Mathematics, Russia

<sup>2</sup> SYRTE, Observatoire de Paris, PSL Research University, CNRS, Sorbonne Universites, UPMC Univ. Paris 06

E-mail: wolftempus@gmail.com

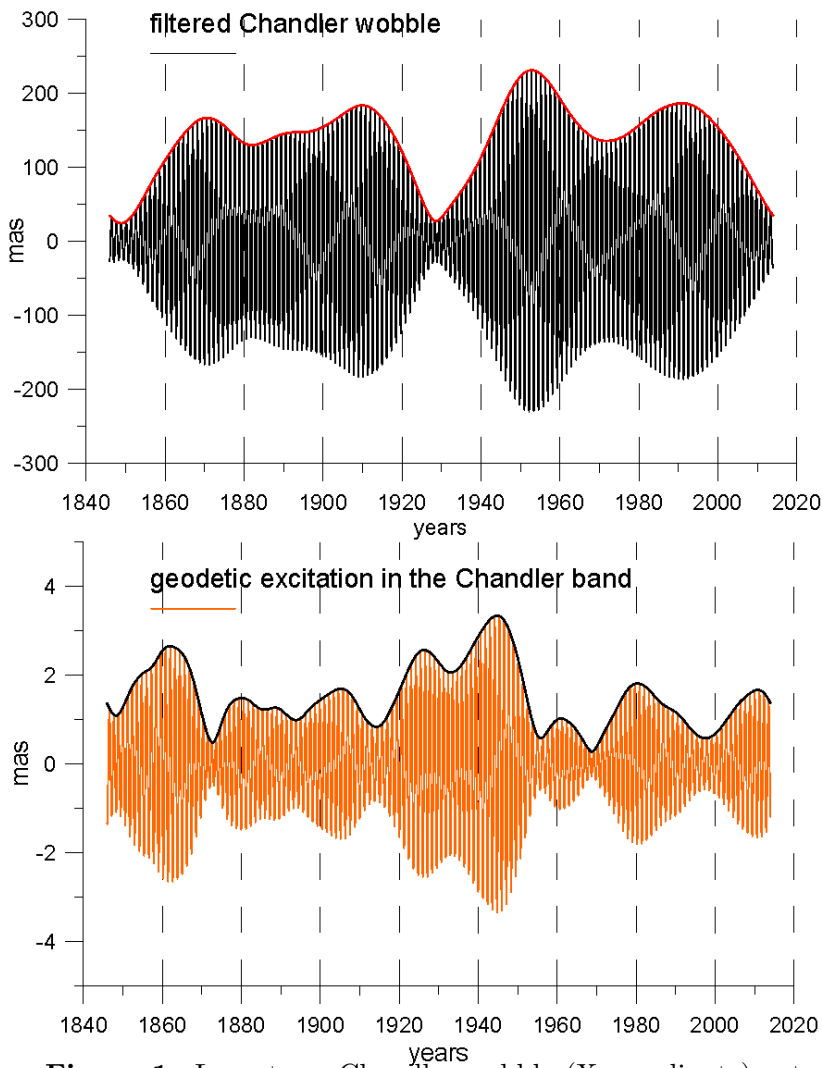
**Abstract.** We study the Chandler wobble (CW) of the pole from 1846 to 2017 extracted by the Panteleev filtering. The CW has period of 433 days, average amplitude of 0.13 milliarcseconds (mas) which is changing, and phase jump by  $\pi$  in 1930-th. The CW amplitude strongly (almost to zero) decreases in 1930-th and 2010-th with the phase jump in 1930th. The envelope model contains 83- and 42-years quasi-periodicities. We think the first one can be represented by the 166-years changes of the envelope, crossing zero in 1930th. We reconstruct Chandler input excitation based on the Euler-Liouville equation. Its amplitude has  $\sim 20$ -years variations. We explain this based on simple model and prove, that they appear in consequence of 42-years modulation of CW. The excitation amplifies the amplitude of CW for  $\sim 20$  years then damps it for another  $\sim 20$  years.

The analysis of the modulated CW signal in a sliding window demonstrates the specific effect, we called the “escargot effect”, when instantaneous “virtual” retrograde component appears in the purely prograde (at long-time interval) signal. Chandler excitation envelope shape is similar to this instantaneous retrograde component, which reflects the changes of ellipticity of the approximation ellipse.

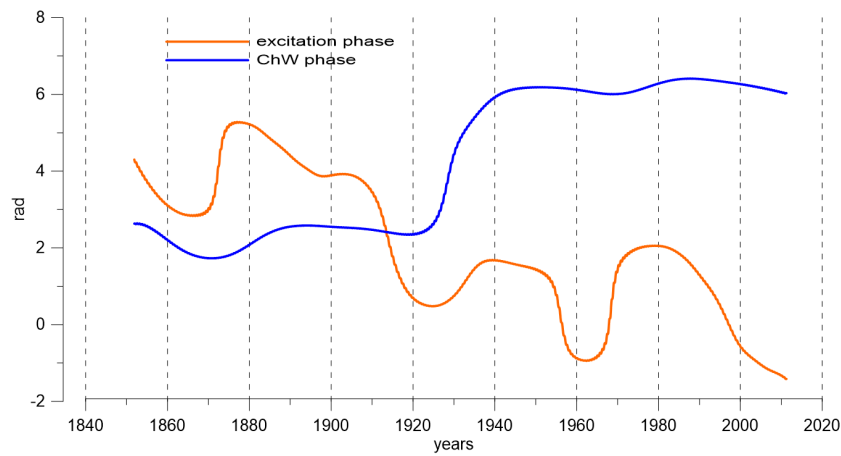
## 1. Introduction

Chandler wobble is one of the most enigmatic component of the polar motion (PM) of the Earth [6, 9]. Discovered at the end of XIX century it is observed for almost 160 years, but the origin of its amplitude changes remains unexplained. Being the resonant oscillation with the damping time of about  $\sim 50$ -years, the Chandler wobble maintenance requires small excitation, which is supposed to be provided by the ocean and atmosphere [1, 2, 4, 7]. Under this assumption the random phase jumps and amplitude changes of the CW are caused by the integration of noise, coming from the atmosphere and ocean (winds, currents, and pressure changes) [3]. In our previous paper [13] we developed several methods of Chandler wobble filtering and its excitation reconstruction based on singular spectrum analysis and/or the Panteleev filtering [12]. They gave similar results. Figure 1 shows CW and its excitation, whose amplitude variations have some regularities. We believe, they are causal.





**Figure 1.** Long-term Chandler wobble (X-coordinate) extracted by the Pantelev filter (top) and reconstructed excitation for it (bottom).



**Figure 2.** Phase changes of the Chandler wobble and its excitation.

**Table 1.** Components (for cos) of the Chandler wobble amplitude model.

| Period   | Amplitude | Phase (1880)        |
|----------|-----------|---------------------|
| 83.4 yrs | 42.6 mas  | 40.8 <sup>0</sup>   |
| 42.0 yrs | 54.6 mas  | -101.5 <sup>0</sup> |
| mean     | 134.8 mas |                     |

## 2. Chandler wobble modelling

Non-linear least squares (LS) fit of the CW amplitude reveals (Table 1) 42-years and 83-years quasi-periodicities, which can be used for the CW amplitude prediction [11]. The phase changes represented in Fig. 2 are much more difficult to predict. If the strong decrease of the CW amplitude in 2010th has the same origin as its decrease in 1930th [10, 8] we can expect the phase jump by  $\pi$  in CW phase now (in 2010th).

If the CW envelope in recent century can be modelled through simple combination of harmonics, what about its excitation? From Fig. 1 it is seen, that the excitation envelope has quasi-20 years periodicity. To explain its origin let's generate the signal with the Chandler carrying frequency  $f_c = 0.843$  cpy (cycles per year), and the envelope of 42-years period  $f_m = 1/42$  cpy (83-year component is skipped)

$$m(t) = [a_0 + a_1 \cos(2\pi f_m t)] \exp(i2\pi f_c t + \phi_\pi). \quad (1)$$

We used the amplitudes  $a_0 = 135$  mas,  $a_1 = 55$  mas, and zero-epoch for  $t$  argument  $t_0 = 1869.3$  yr. Through  $\phi_\pi$  we introduced the phase jump in 1930th, constant value  $\phi_\pi = 0$  before 1930 is abruptly changed to  $\phi_\pi = \pi$  in 1930.0. We introduced constant phase on both intervals of time and in calculations we assume that its jump-like input into derivative can be ignored.

The excitation for the model (1) was reconstructed by the same method of Panteleev corrective filtering in frequency domain [13]. As seen from Fig. 3, the result is very similar to the real excitation, obtained from the real CW, except for the border effects we can skip. The pikes behaviour are quite in phase thanks to  $\phi_\pi$  introduction and  $t_0$  choice.

Let's look at the Euler-Liouville equation, governing Earth rotation [9]:

$$i \frac{dm(t)}{dt} + \sigma_c m(t) = \sigma_c \Psi(t). \quad (2)$$

Here  $m(t) = m_1 + im_2$  is the 2D pole trajectory,  $\Psi(t)$  is the excitation; the complex frequency  $\sigma_c = \omega_c(1 + i/2Q)$  depends on the real Chandler frequency  $\omega_c = 2\pi f_c$  with  $f_c = 0.843$  cpy and the quality factor  $Q$  (we used  $Q = 100$ );  $i = \sqrt{-1}$ . It is easy to obtain the derivative of the model (1)

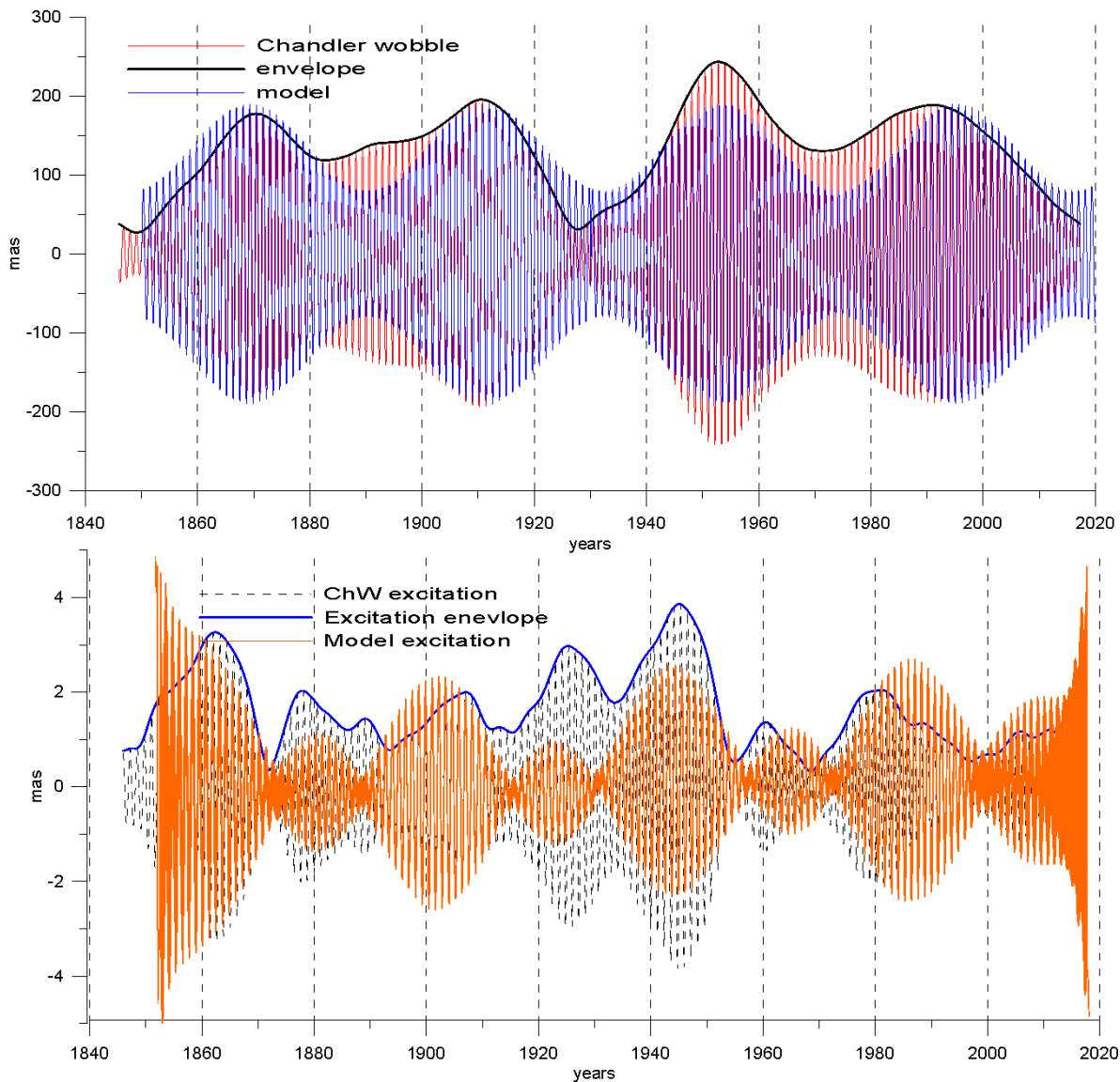
$$\frac{dm(t)}{dt} = i\omega_c m(t) + \frac{d|m(t)|}{dt} \exp(i\omega_c t + \phi_\pi).$$

Substitution of model (1) into (3) gives

$$\sigma_c \Psi(t) = -\omega_c m(t) + \sigma_c m(t) + i \frac{d|m(t)|}{dt} \exp(i\omega_c t + \phi_\pi). \quad (3)$$

The terms  $-\omega_c m$  and  $+\sigma_c m$  almost neutralize each other, but the term multiplied by  $1/2Q$  factor remains

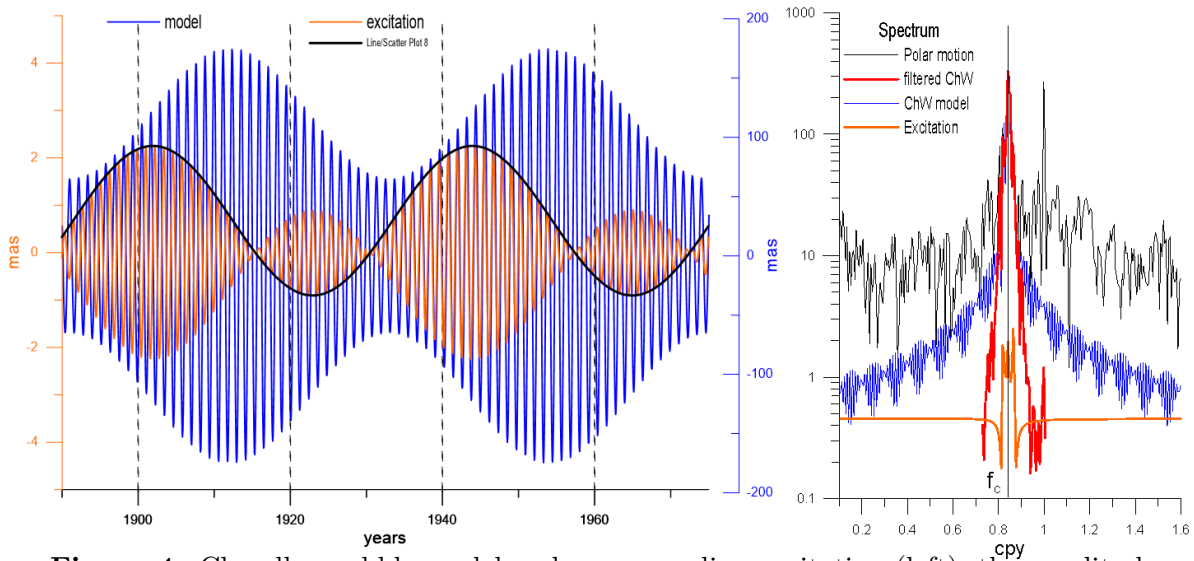
$$\Psi = \frac{i}{\sigma_c} \left( \frac{\omega_c}{2Q} [a_0 + a_1 \cos(\omega_m t)] - \omega_m a_1 \sin(\omega_m t) \right) \exp(i\omega_c t + \phi_\pi). \quad (4)$$



**Figure 3.** Chandler wobble and its model (top) and the corresponding excitations (bottom).

The sum in the large brackets defines the amplitude modulation of  $|\Psi|$ . Though  $\omega_c = 2\pi f_c > \omega_m = 2\pi f_m$ , the first term would dominate the second, related to the CW envelope, if not divided by  $2Q$ . The ratio  $\omega_c/2Q$  relative to the modulation frequency  $\omega_m$  plays important role, defining the phase and amplitude of  $\sin(\cdot)$  and  $\cos(\cdot)$  terms combination. The resulting excitation and its envelope are zoomed in Fig. 4, left, over the background of the CW model (1). The asymmetry of this envelope with respect to zero level is responsible for different amplitudes of excitation in maxima. If  $a_0$  is zero, the amplifying and damping counterparts are equal in size and last exactly 20-years each. Other combinations of  $a_0$ ,  $a_1$ ,  $f_m$  and  $Q$  (for example for 82-years modulation) would result in different amplitude ratio, but in reality we clearly observe  $\sim 20$ -years wave packages in excitation, corresponding to the epochs of CW amplification and deceleration.

Thus, in simple words, the  $\sim 20$ -years modulation of excitation is required to get  $\sim 40$ -years



**Figure 4.** Chandler wobble model and corresponding excitation (left), the amplitude spectra of CW, its model and excitation (right).

amplitude changes of CW. At the beginning the excitation amplifies the wobble for  $\sim 20$ -years, firstly slowly, then quicker ( $d|m|/dt = \max$ ), then slowly again ( $d|m|/dt = 0$ ). Then for  $\sim 20$ -years the excitation damps CW in the same manner, but with the opposite sign of derivative. Due to opposite sign of the envelope, the phase delay between excitation and CW is changing by  $\pi$  every  $\sim 20$  years (see Figs. 2 and 4). The excitation pikes are ahead of CW for  $\sim 20$  years, then the phase changes by  $\pi$  and the excitation pikes go behind the CW pikes. Now, let's pay attention to the  $\phi_\pi = \pi$  jump, occurred in 1930th. It reverses the pikes, and, according to the conclusions made above, it can be represented by long-term  $83 \cdot 2 = 166$  years CW envelope, crossing zero-level in 1930th. Under this model we should expect the phase jump in 2010th.

### 3. “Escargot effect”

The Pantelev filter designed to extract Chandler wobble has purely prograde transfer function [13]. Thus, the CW component studied here has purely prograde spectral pike, Fig. 4. The CW pike is split, having the side lobes representing the amplitude modulations. But when treating the exact frequency  $f_c$  at every moment, we should use instantaneous amplitude and phase. Being purely prograde at the 150-years time interval, the extracted CW being approximated at the short time interval shows ellipticity. This happens because the CW pole trajectory slightly twists and untwists every  $\sim 20$ -years, which produces the changes of the approximation ellipse semiaxes ratio and brings the “virtual” retrograde component to life. We called it “escargot effect” after the nice dish of French cousin (the snails with filling). In result, the LS-estimation of the prograde and retrograde complex amplitudes  $C_m^+$ ,  $C_m^-$  of the filtered CW from Fig. 1 in the 12-years sliding window has shown the presence of non-zero  $C_m^-$ , with module  $|C_m^-|$  oscillating with  $\sim 20$ -years period. Our attention was attracted to this effect, because the change of  $|C_m^-|$  reminds the shape of the CW excitation envelope. Further studies are needed to explain how “escargot effect” and the changes of ellipticity of the approximation ellipse are related to the difference between  $\sin(\cdot)$  and  $\cos(\cdot)$  in eq. (4). Such study can be done through the consideration of the sliding least-squares filter’s transfer function [5].

#### 4. Conclusion

In this short paper we explain why, if the Chandler wobble envelope has 40-years modulations, 20-years amplitude changes appear in its excitation. We do not discuss physical causes here, our goal is to prove this effect mathematically and to confirm it through simple modelling. We think that the Chandler wobble amplitude transition from positive to negative amplitudes can represent the 1930-th phase jump, which could repeat in 2010th epoch of CW amplitude minima.

The mathematical “escargot effect” observed while studying the CW amplitude in the sliding window is reported. The purely prograde long-term signal, if modulated, produce non-zero retrograde amplitude at instantaneous consideration. In case of CW, the behaviour of this “virtual” retrograde amplitude repeats the behaviour of the excitation envelope.

#### Acknowledgments

This work is supported by the Chinese “111 Plan”, NRU HSE support, RFBR grant No 16-05-00753, and monthly position at Paris observatory for the first author.

#### References

- [1] Bizouard C., F. Remus, S.B. Lambert, L. Seoane and D. Gambis, 2011, *The Earth's variable Chandler wobble*, (Astronomy & Astrophysics, vol. 526, A106 )
- [2] Brzezinski A., Dobslaw H., Dill R. et al., 2012, *Geophysical Excitation of the Chandler Wobble Revisited*, (Geod. for Planet Earth Int. Ass. of Geodesy Symposia, 136(3), 499-505)
- [3] Chao B.F. and Wei-Yung Chung, 2012, *Amplitude and phase variations of Earth's Chandler wobble under continual excitation*, ( J. of Geodynamics, Vol. 62, p. 35-39)
- [4] Gross R., 2000, *The excitation of the Chandler wobble*, (Geophys. Res. Lett., 27(15), 2329-2332).
- [5] Gubanov V.S., 2009, *Dynamics of the Earth core from VLBI observations*, (Astronomy letters. Vol. 35, Iss. 4, pp 270277).
- [6] Lambeck K., 1980, *The Earth's Variable Rotation; Geophysical Causes and Consequences*, Cambridge University Press
- [7] Liao D.C., Zhou Y.H., Liao X.H., 2007, *Comparison of Wind Contributions to Chandler Wobble Excitation*, (Chinese Astronomy and Astrophysics, 31, p. 57-65)
- [8] Wang G, Liu L., Su X., Liang X., Yan H., Tu Y., Li Z., Li W, 2016, *Variable Chandler and Annual Wobbles in Earths Polar Motion During 1900 2015*, Surveys in Geophysics, Vol 37, DOI: 10.1007/s10712-016-9384-0
- [9] Munk W., G. MacDonald, 1960, *The rotation of the Earth*, (Cambridge Univ. Press)
- [10] Miller N.O., 2011, *Chandler Wobble in Variations of the Pulkovo Latitude for 170 Years* (Solar System Research, Vol. 45, No. 4, p. 342-353)
- [11] Nastula J., A. Korsun, B. Kolaczek, W Kosek, W Hozakowski, 1993, *Variations of the Chandler and annual wobbles of polar motion in 1846-1988 and their prediction* (Manuscripta geodaetica, No 18 p. 131-135)
- [12] Panteleev V.L., Chesnokova T.S., 2011, *Problem deconvolution in inertial gravimetry*, (Moscow University Physics Bulletin, Vol. 66, N 1)
- [13] Zotov L., C. Bizouard, 2012, *On modulations of the Chandler wobble excitation* (Journal of Geodynamics, special issue “Earth rotation”, Vol. 62, p. 30-34), DOI: 10.1016/j.jog.2012.03.010