PHYSICAL REVIEW D **94,** 123501 (2016)

General properties and kinetics of spontaneous baryogenesis

E. V. Arbuzova,[1,2,*] A. D. Dolgov,[1,3,4,†] and V. A. Novikov[3,‡]

[1]*Novosibirsk State University, Novosibirsk 630090, Russia*
[2]*Department of Higher Mathematics, Dubna State University, 141980 Dubna, Russia*
[3]*ITEP, Bol. Cheremushkinsaya ul., 25, 117259 Moscow, Russia*
[4]*Dipartimento di Fisica e Scienze della Terra, Università degli Studi di Ferrara Polo Scientifico e Tecnologico—Edificio C, Via Saragat 1, 44122 Ferrara, Italy*

(Received 23 July 2016; published 1 December 2016)

General features of spontaneous baryogenesis are studied. The relation between the time derivative of the (pseudo) Goldstone field and the baryonic chemical potential is revisited. It is shown that this relation essentially depends upon the representation chosen for the fermionic fields with nonzero baryonic number (quarks). The calculations of the cosmological baryon asymmetry are based on the kinetic equation generalized to the case of nonstationary background. The effects of the finite interval of the integration over time are also taken into consideration. All these effects combined lead to a noticeable deviation of the magnitude of the baryon asymmetry from the canonical results.

DOI: 10.1103/PhysRevD.94.123501

I. INTRODUCTION

The usual approach to cosmological baryogenesis is based on three well-known Sakharov's conditions [1]: (a) nonconservation of baryonic number, (b) breaking of C and CP invariance, and (c) deviation from thermal equilibrium. There are however some interesting scenarios of baryogenesis for which one or several of the above conditions are not fulfilled. A very popular scenario is the so-called spontaneous baryogenesis (SBG) proposed in Refs. [2–4]; for reviews see, e.g., [5,6]. The term spontaneous is related to spontaneous breaking of underlying symmetry of the theory. It is assumed that in the unbroken phase the Lagrangian is invariant with respect to the global $U(1)$-symmetry, which ensures conservation of the total baryonic number including that of the Higgs-like field, Φ, and the matter fields (quarks). This symmetry is supposed to be spontaneously broken and in the broken phase the Lagrangian density acquires the term

$$\mathcal{L}_{SB} = (\partial_\mu \theta) J^\mu_B, \tag{1.1}$$

where θ is the Goldstone field, or in other words, the phase of the field Φ and J^μ_B is the baryonic current of matter fields (quarks). Depending upon the form of the interaction of Φ with the matter fields, the spontaneous symmetry breaking (SSB) may lead to nonconservation of the baryonic current of matter. If this is not so and J^μ_B is conserved, then integrating by parts Eq. (1.1) we obtain a vanishing expression and hence the interaction (1.1) is unobservable.

The next step in the implementation of the SBG scenario is the conjecture that the Hamiltonian density corresponding to $\mathcal{L}_{SB}$ is simply the Lagrangian density taken with the opposite sign,

$$\mathcal{H}_{SB} = -\mathcal{L}_{SB} = -(\partial_\mu \theta) J^\mu_B. \tag{1.2}$$

This could be true, however, if the Lagrangian depended only on the field variables but not on their derivatives, as it is argued below.

For the time being we neglect the complications related to the questionable identification (1.2) and proceed further in description of the SBG logic.

For the spatially homogeneous field $\theta = \theta(t)$ the Hamiltonian (1.2) is reduced to $\mathcal{H}_{SB} = -\dot\theta n_B$, where $n_B \equiv J^4_B$ is the baryonic number density of matter, so it is tempting to identify $\dot\theta$ with the chemical potential, μ, of the corresponding system; see, e.g., [7]. If this is the case, then in thermal equilibrium with respect to the baryon nonconserving interaction the baryon asymmetry evolves to

$$n_B = \frac{g_S B_Q}{6}\left(\mu T^2 + \frac{\mu^3}{\pi^2}\right) \to \frac{g_S B_Q}{6}\left(\dot\theta T^2 + \frac{\dot\theta^3}{\pi^2}\right), \tag{1.3}$$

where T is the cosmological plasma temperature, and g_S and B_Q are respectively the number of the spin states and the baryonic number of quarks, which are supposed to be the bearers of the baryonic number.

It is interesting that for successful SBG two of the Sakharov's conditions for the generation of the cosmological baryon asymmetry, namely, breaking of thermal equilibrium and a violation of C and CP symmetries, are unnecessary. This scenario is analogous to the baryogenesis in the absence of CPT (CPT is the product of charge conjugation (C), mirror reflection (P), and time reversal (T) transformations. According to CPT theorem the usual field

*arbuzova@uni-dubna.ru
†dolgov@fe.infn.it
‡novikov@itep.ru

 © 2016 American Physical Society

theories are invariant with respect to this transformation. In our case CPT-invariance is broken by external field $\theta(t)$) invariance, if the masses of particles and antiparticles are different. In the latter case the generation of the cosmological baryon asymmetry can also proceed in thermal equilibrium [8,9]. In the SBG scenario the external field $\theta(t)$ plays the role of the CPT "breaker".

However, in contrast with the usual saying, the identification $\dot{\theta} = \mu_B$ is incorrect. Indeed, if $\dot{\theta}(t)$ is constant or slowly varying, then according to Eq. (1.2) it shifts the energies of baryons with respect to antibaryons at the same spatial momentum, by $\dot{\theta}$. Thus there would be different number densities of baryons and antibaryons in the plasma even if the corresponding chemical potential vanishes. In this case the baryon asymmetry is determined by effective chemical potential $\mu_{\text{eff}} = \mu - \dot{\theta}$ to be substituted into Eq. (1.3) instead of μ. The detailed arguments are presented in Sec. IV. It is also shown there that the baryonic chemical potential tends to 0 when the system evolves to the thermal equilibrium state. So in equilibrium the baryon asymmetry is nonzero with vanishing chemical potential.

The picture becomes different if we use another representation for the quark fields. Redefining the quark fields by the phase transformation, $Q \to \exp(i\theta/3)Q$, we can eliminate the term (1.1) from the Lagrangian, but instead it appears in the interaction term that violates B-conservation; see Eq. (2.5). Clearly in this case $\dot{\theta}$ is not simply connected to the chemical potential. However, as is shown in the present paper, the baryonic chemical potential in this formulation of the theory tends in equilibrium to $c\dot{\theta}$ with a constant coefficient c. Anyway, as we see from the solution of the kinetic equation presented below, the physically meaningful expression of the baryon asymmetry, n_B, expressed through θ, is the same independently of the above-mentioned two different formulations of the theory, though the values of the chemical potentials are quite different. Seemingly this difference is related to nonaccurate transition from the Lagrangian $\mathcal{L}_{SB}$ to the Hamiltonian $\mathcal{H}_{SB}$, made according to Eq. (1.2). Such identification is true if the Lagrangian does not depend on the time derivative of the corresponding field, $\theta(t)$, in the case under scrutiny. The related criticism of spontaneous baryogenesis can be found in Ref. [10]; see also the review [6].

Recently the gravitational baryogenesis scenario was suggested [11]; see also [12]. In these works the original SSB model was modified by the substitution of curvature scalar R instead of the Goldstone field θ. With the advent of the $F(R)$-theories of modified gravity the gravitational baryogenesis was studied in their frameworks [13] as well.

In this paper the classical version of SBG is studied. We present an accurate derivation of the Hamiltonian for the Lagrangian that depends upon the field derivatives. For a constant $\dot{\theta}$ and sufficiently large interval of the integration over time the results are essentially the same as obtained in the previous considerations. With the account of the finite time effects, which effectively break the energy conservation, the outcome of SBG becomes significantly different. We have also considered the impact of a nonlinear time evolution of the Goldstone field,

$$\theta = \dot{\theta}_0 t + \ddot{\theta}_0 t^2/2, \tag{1.4}$$

and have found that there can be significant deviations from the standard scenario with $\dot{\theta} \approx \text{const}$.

A strong deviation from the standard results is also found for the pseudo-Goldstone field oscillating near the minimum of the potential $U(\theta)$.

The paper is organized as follows. In Sec. s-ssb the general features of the spontaneous breaking of baryonic $U(1)$-symmetry are described and the (pseudo) Goldstone mode, its equation of motion, as well as the equations of motion of the quarks are introduced. In Sec. III the construction of the Hamiltonian density from the known Lagrangian is considered. Next, in Sec. IV the standard kinetic equation in stationary background is presented. Section V is devoted to the generation of cosmological baryon asymmetry with out-of-equilibrium purely Goldstone field. The pseudo-Goldstone case is studied in Sec. VI. In Sec. VII we derive the kinetic equation in the time dependent external field and/or for the case when energy is not conserved because of finite limits of integration over time. Several examples, for which such a kinetic equation is relevant, are presented in Sec. VIII. Lastly in Sec. IX we conclude.

II. SPONTANEOUS SYMMETRY BREAKING AND THE GOLDSTONE MODE

We start with the theory of a complex scalar field Φ interacting with fermions Q and L with the Lagrangian,

$$\mathcal{L}(\Phi) = g^{\mu\nu}\partial_\mu\Phi^*\partial_\nu\Phi - V(\Phi^*\Phi) + \bar{Q}(i\gamma^\mu\partial_\mu - m_Q)Q + \bar{L}(i\gamma^\mu\partial_\mu - m_L)L + \mathcal{L}_{\text{int}}(\Phi, Q, L), \tag{2.1}$$

where it is assumed that Q and Φ have nonzero baryonic numbers, while L do not. Here $V(\Phi^*\Phi)$ is the self-interaction potential of Φ defined below in Eq. (2.4). The interaction Lagrangian $\mathcal{L}_{\text{int}}$ describes the coupling between Φ and fermionic fields. In the toy model studied below we take it in the form

$$\mathcal{L}_{\text{int}} = \frac{\sqrt{2}}{m_X^2}\frac{\Phi}{f}(\bar{L}\gamma_\mu Q)(\bar{Q}^c\gamma_\mu Q) + \text{H.c.}, \tag{2.2}$$

where Q^c is the charged conjugated quark spinor and m_X and f are parameters with dimension of mass. We prescribe to Φ and Q the baryonic numbers (-1) and $1/3$, respectively, so the interaction (2.2) conserves the baryonic number. The interaction of this type can appear, e.g., in

$SU(5)$ grand unified theory (GUT). For simplicity, in our toy model we do not take into account the quark colors.

Q and L can be any fermions, not necessarily quarks and leptons of the standard model. They can be, e.g., new heavy fermions possessing similar or the same quantum numbers as the quarks and leptons of the standard model. They should be coupled to the ordinary quarks and leptons in such a way that the baryon asymmetry in the Q-sector is transformed into the asymmetry of the observed baryons.

Other forms of $\mathcal{L}_{\rm int}$ can be considered leading, e.g., to transition $3L \leftrightarrow Q$ or $2Q \leftrightarrow 2\bar{Q}$. They are not permitted for the standard quarks. However, for the usual quarks the process $3q \leftrightarrow 3\bar{q}$ is permitted. Note that the kinetics of all these processes is similar. We denote by q the usual quarks or the fermionic field with the same quantum numbers.

The Lagrangian (2.1) is invariant under the following $U(1)$ transformations with constant α:

$$\Phi \to e^{i\alpha}\Phi, \qquad Q \to e^{-i\alpha/3}Q, \qquad L \to L. \tag{2.3}$$

In the unbroken symmetry phase this invariance leads to the conservation of the total baryonic number of Φ and of quarks. In the realistic model the interaction of left- and right-handed fermions may be different but we neglect this possible difference in what follows.

The global $U(1)$-symmetry is assumed to be spontaneously broken at the energy scale f via the potential of the form,

$$V(\Phi^*\Phi) = \lambda(\Phi^*\Phi - f^2/2)^2. \tag{2.4}$$

This potential reaches the minimum at the vacuum expectation value of Φ equal to $\langle\Phi\rangle = fe^{i\phi_0/f}/\sqrt{2}$ with an arbitrary constant phase ϕ_0.

Below scale f we can neglect the heavy radial mode of Φ with the mass $m_{\rm radial} = \lambda^{1/2}f$, since being very massive it is frozen out, but this simplification is not necessary and is not essential for the baryogenesis. The remaining light degree of freedom is the variable field ϕ, which is the Goldstone boson of the spontaneously broken $U(1)$. Up to a constant factor the field ϕ is the angle around the bottom of the Mexican hat potential given by Eq. (2.4). Correspondingly we introduce the dimensionless angular field $\theta \equiv \phi/f$ and thus $\Phi = \langle\Phi\rangle \exp(i\theta)$.

The low energy limit of the Lagrangian (2.1) in the broken phase, which effectively describes the dynamics of the θ field, has the form

$$\mathcal{L}_1(\theta) = \frac{f^2}{2}\partial_\mu\theta\partial^\mu\theta + \bar{Q}_1(i\gamma^\mu\partial_\mu - m_Q)Q_1 + \bar{L}(i\gamma^\mu\partial_\mu - m_L)L + \left(\frac{e^{i\theta}}{m_X^2}(\bar{L}\gamma_\mu Q_1)(\bar{Q}_1^c\gamma_\mu Q_1) + \text{H.c.}\right) - U(\theta). \tag{2.5}$$

Here we added the potential $U(\theta)$, which may be induced by an explicit symmetry breaking and can lead, in particular, to a nonzero mass of θ. We use the notation Q_1 for the quark field to distinguish it from the phase rotated field Q_2 introduced below in Eq. (2.7). In a realistic model the quark fields should be (anti) symmetrized with respect to color indices, omitted here for simplicity.

If $U(\theta) = 0$, the theory remains invariant with respect to the global $U(1)$-transformations (i.e. the transformations with a constant phase α),

$$Q \to e^{-i\alpha/3}Q, \qquad L \to L, \qquad \theta \to \theta + \alpha. \tag{2.6}$$

The phase transformation of the quark field with the coordinate dependent phase $\alpha = \theta(t, \mathbf{x})$ introduces the new field $Q_1 = e^{-i\theta/3}Q_2$. In terms of this field the Lagrangian (2.5) turns into

$$\mathcal{L}_2(\theta) = \frac{f^2}{2}\partial_\mu\theta\partial^\mu\theta + \bar{Q}_2(i\gamma^\mu\partial_\mu - m_Q)Q_2 + \bar{L}(i\gamma^\mu\partial_\mu - m_L)L + \left(\frac{1}{m_X^2}(\bar{Q}_2\gamma_\mu L)(\bar{Q}_2\gamma_\mu Q_2^c) + \text{H.c.}\right) + (\partial_\mu\theta)J^\mu - U(\theta), \tag{2.7}$$

where the quark baryonic current is $J_\mu = (1/3)\bar{Q}\gamma_\mu Q$. Note that the form of this current is the same in terms of Q_1 and Q_2.

The equation of motion for the quark field Q_1 that follows from Lagrangian (2.5) has the form

$$(i\gamma^\mu\partial_\mu - m_Q)Q_1 + \frac{e^{-i\theta}}{m_X^2}[\gamma_\mu L(\bar{Q}_1\gamma_\mu Q_1^c) + 2\gamma_\mu Q_1^c(\bar{Q}_1\gamma_\mu L)] = 0. \tag{2.8}$$

Analogously the equation of motion for the phase rotated field Q_2 derived from Lagrangian (2.7) is

$$\left(i\gamma^\mu\partial_\mu - m_Q + \frac{1}{3}\gamma^\mu\partial_\mu\theta\right)Q_2 + \frac{1}{m_X^2}[\gamma_\mu L(\bar{Q}_2\gamma_\mu Q_2^c) + 2\gamma_\mu Q_2^c(\bar{Q}_2\gamma_\mu L)] = 0. \tag{2.9}$$

Equations for the θ field derived from these two Lagrangians in flat space-time have respectively the forms

$$f^2(\partial_t^2 - \Delta)\theta + U'(\theta) + \left[\frac{ie^{-i\theta}}{m_X^2}(\bar{Q}_1\gamma_\mu L)(\bar{Q}_1\gamma_\mu Q_1^c) + \text{H.c.}\right] = 0 \tag{2.10}$$

and

$$f^2(\partial_t^2 - \Delta)\theta + U'(\theta) + \partial_\mu J_B^\mu = 0, \tag{2.11}$$

where $U'(\theta) = dU/d\theta$.

Using either the equation of motion (2.8) or (2.9) we can check that the baryonic current is not conserved. Indeed, its divergence is

$$\partial_\mu J^\mu_B = \frac{ie^{-i\theta}}{m_X^2}(\bar{Q}_1\gamma_\mu Q_1^c)(\bar{Q}_1\gamma^\mu L) + \text{H.c.} \tag{2.12}$$

The current divergence in terms of the rotated field Q_2 has the same form but without the factor $\exp(-i\theta)$. So the equations of motion for θ in both cases (2.10) and (2.11) coincide, as expected.

Equation (2.11) expresses the law of the total baryonic current conservation in the unbroken phase. When the symmetry is broken, the nonconservation of the physical baryons (in our case of quarks) becomes essential and may lead to the observed cosmological baryon asymmetry. Such a B-nonconserving interaction may have many different forms. The one presented above describes the transition of three quark-type fermions into the (anti) lepton. There may be transformation of two or three quarks into an equal number of antiquarks. Such an interaction describes neutron-antineutron oscillations, now actively looked for [14]. There even can be a quark transition into three leptons. Depending on the interaction type the relation between $\dot\theta$ and the effective chemical potential has different forms, i.e., different values of the proportionality coefficient c mentioned in the introduction.

If we consider realistic modes dealing only with the known quarks and leptons, the processes of nucleon decay into a lepton or leptons plus some other particles with vanishing total leptonic and baryonic numbers or the processes of three quarks transforming into three antiquarks are of interest. Among them is, first, the proton or neutron decay. These processes have been actively searched for and the existing bound on the proton lifetime is about $\tau_p \geq 10^{33}$ years [15]. It means that the mass m_X should be above 10^{14} GeV, which makes such processes not very promising for baryogenesis. The B-nonconserving neutron decay, see, e.g., [16], especially into invisible modes, e.g., into 3ν, is much more weakly restricted (approximately by 6 orders of magnitude). This leaves such processes more appropriate for baryogenesis. Quark-antiquark transformations leading to neutron-antineutron oscillations may also be of interest for baryogenesis [14].

In the spatially homogeneous case, when $\partial_\mu J^\mu_B = \dot n_B$ and $\theta = \theta(t)$, and if $U(\theta) = 0$, Eq. (2.11) can be easily integrated giving

$$f^2[\dot\theta(t) - \dot\theta(t_{\text{in}})] = -n_B(t) + n_B(t_{\text{in}}). \tag{2.13}$$

It is usually assumed that the initial baryon asymmetry vanishes; $n(t_{\text{in}}) = 0$.

The evolution of $n_B(t)$ is governed by the kinetic equation discussed in Sec. IV. This equation allows us to express n_B through $\theta(t)$ and to obtain the closed systems of, generally speaking, integrodifferential equations. In thermal equilibrium the relation between $\dot\theta$ and n_B may become an algebraic one, but this is true only in the case when the interval of the integration over time is sufficiently long and if $\dot\theta$ is a constant or slowly varying function of time.

In the cosmological Friedmann-Robertson-Walker (FRW) background the equation of motion of θ (2.11) becomes

$$f^2(\partial_t + 3H)\dot\theta - a^{-2}(t)\Delta\theta + U'(\theta) = -(\partial_t + 3H)n_B, \tag{2.14}$$

where $a(t)$ is the cosmological scale factor and $H = \dot a/a$ is the Hubble parameter. For the homogeneous theta field, $\theta = \theta(t)$, this equation turns into

$$f^2(\partial_t + 3H)\dot\theta + U'(\theta) = -(\partial_t + 3H)n_B. \tag{2.15}$$

We do not include the curvature effects in the Dirac equations because they are not essential for what follows. Still we have taken into account the impact of the cosmological expansion on the current divergence using the covariant derivative in the FRW space-time: $\mathcal{D}_\mu J^\mu = \dot n_B + 3Hn_B$.

III. HAMILTONIANS VERSUS LAGRANGIANS

Though, as we see in Secs. IV and VII, the baryon asymmetry originated in the frameworks of SBG is proportional to $\dot\theta$ in many interesting cases, as justly envisaged in Refs. [2,3]; the identification of $\dot\theta$ with baryonic chemical potential, $\dot\theta = \mu_B$, is questionable, as we argue below.

A. General consideration

In the canonical approach the Hamiltonian density, $\mathcal{H}$, is derived from the Lagrangian density, $\mathcal{L}$, in the following way. The Lagrangian density is supposed to depend upon some field variables, ϕ_a, and their first derivatives, $\partial_\mu\phi_a$. First, we need to define the canonical momentum conjugated to the coordinate ϕ_a,

$$\pi_a = \frac{\partial\mathcal{L}}{\partial\dot\phi_a}. \tag{3.1}$$

The Hamiltonian density is expressed through the canonical momenta and coordinates as

$$\mathcal{H} = \sum_a \pi_a\dot\phi_a - \mathcal{L}, \tag{3.2}$$

where the time derivatives, $\dot\phi_a$, should be written in terms of the canonical momenta, π_a.

The Hamilton equations of motion,

$$\dot{\phi} = \frac{\partial \mathcal{H}}{\partial \pi} \quad \text{and} \quad \dot{\pi} = -\frac{\partial \mathcal{H}}{\partial \phi}, \tag{3.3}$$

are normally equivalent to the Lagrange equations obtained by the least action principle from the Lagrangian.

For example for a real scalar field with the Lagrangian

$$\mathcal{L}(\chi) = (\partial\chi)^2/2 - m_\chi^2\chi^2/2 \tag{3.4}$$

the canonical momentum is $\pi_\chi = \dot{\chi}$ and the Hamiltonian density is:

$$\mathcal{H}(\chi) = (1/2)[\pi_\chi^2 + (\nabla\chi)^2 + m_\chi^2\chi^2], \tag{3.5}$$

while for a complex scalar field with

$$\mathcal{L}(\phi) = |\partial\phi|^2 - m_\phi^2|\phi|^2 \tag{3.6}$$

the canonical momenta are $\pi_\phi = \dot{\phi}^*$ and $\pi_{\phi^*} = \dot{\phi}$ and the Hamiltonian density is

$$\mathcal{H}(\phi) = \pi_\phi\pi_{\phi^*} + |\nabla\phi|^2 + m_\phi^2|\phi|^2. \tag{3.7}$$

The corresponding Hamilton equations lead, as expected, to the usual Klein-Gordon equations for ϕ or χ.

For the Dirac field with

$$\mathcal{L}(\psi) = \bar{\psi}(i\partial\!\!\!/ - m_\psi)\psi \tag{3.8}$$

the canonical momenta are $\pi_\psi = i\psi^\dagger$ and $\pi_{\psi^\dagger} = 0$, so we arrive to the well-known expression,

$$\mathcal{H}(\psi) = \psi^\dagger(i\gamma_4\gamma_k\partial_k + \gamma_4 m)\psi. \tag{3.9}$$

Let us now do the same exercise but with the symmetric Lagrangian, which differs from the canonical one by a total derivative,

$$\mathcal{L}_{\text{sym}}(\psi) = [\bar{\psi}(i\partial\!\!\!/ - 2m_\psi)\psi - i(\partial_\mu\bar{\psi})\gamma_\mu\psi]/2. \tag{3.10}$$

The corresponding canonical momenta are $\pi_\psi = i\psi^\dagger/2$ and $\pi_{\psi^\dagger} = -i\psi/2$ and the Hamiltonian density is

$$\mathcal{H}_{\text{sym}}(\psi) = m_\psi\psi^\dagger\gamma_4\psi + \frac{i}{2}(\psi^\dagger\gamma_4\gamma_k\partial_k\psi - \partial_k\psi^\dagger\gamma_4\gamma_k\psi), \tag{3.11}$$

which differs from the usual expression (3.9) by the space divergence, $(i/2)\partial_k(\psi^\dagger\gamma_4\gamma_k\psi)$. The total Hamiltonian, defined as

$$H = \int d^3x\mathcal{H}, \tag{3.12}$$

remains the same in both cases, (3.9) and (3.11), if the fields vanish at spatial infinity. Below the field θ depending only on time is considered, but one can assume that it weakly depends upon the space coordinates and vanishes at infinity. The local dynamics in this case remains undisturbed.

B. The case of SSB

Let us consider now a model with the coupling

$$\mathcal{L}_{SB}(\Theta) = (\partial_\mu\Theta)J_B^\mu, \tag{3.13}$$

where Θ is some scalar field and J_B^μ is a vector baryonic current. It has the form

$$J_B^\mu = B\bar{\psi}\gamma^\mu\psi, \tag{3.14}$$

where ψ is some fermionic baryon (e.g., quark) and B is its baryonic number. Such an interaction is postulated in spontaneous baryogenesis scenarios [2–5] or in gravitational baryogenesis [11,12]. In the former case $\Theta = \theta$ is a (preudo) Goldstone field, while in the latter $\Theta = R/m_R^2$ with R being the curvature scalar and m_R is a constant parameter with dimension of mass.

In what follows we confine ourselves to consideration of the Goldstone field θ and distinguish between the following two possibilities:

(A) θ is a dynamical field with the free Lagrangian of the form given by Eq. (3.4) where $\chi = f\theta$. This is exactly the situation that is realized in the case of spontaneous symmetry breaking.

(B) θ is an external "fixed" field. The term fixed is used here in the sense that the dependence of θ on coordinates is fixed by some dynamics which does not enter into the Lagrangians under scrutiny. This is the case which is studied both in the spontaneous baryogenesis and in the gravitational baryogenesis. It is considered in the next subsection.

In the canonical case A the Hamiltonian density is calculated in accordance with the specified above rules. Correspondingly, for the Lagrangian (2.5) we obtain

$$\begin{aligned}\mathcal{H}_1(\theta) = &\frac{f^2}{2}(\dot{\theta}^2 + (\nabla\theta)^2) + Q_1^\dagger\gamma_4(i\gamma_k\partial_k + m_Q)Q_1 \\ &+ L^\dagger\gamma_4(i\gamma_k\partial_k + m_L)L \\ &- \left(\frac{e^{-i\theta}}{m_X^2}(Q_1^\dagger\gamma_4\gamma_\mu L)(Q_1^\dagger\gamma_4\gamma_\mu Q_1^c) + \text{H.c.}\right) + U(\theta),\end{aligned} \tag{3.15}$$

where the θ-conjugated canonical momentum is $\pi_{1\theta} = f^2\dot{\theta}$.

Analogously for the Lagrangian (2.7) the Hamiltonian density is

$$\mathcal{H}_2(\theta) = \frac{f^2}{2}(\dot\theta^2 + (\nabla\theta)^2) + Q_2^\dagger\gamma_4(i\gamma_k\partial_k + m_Q)Q_2 + L^\dagger\gamma_4(i\gamma_k\partial_k + m_L)L - \left(\frac{1}{m_X^2}(Q_2^\dagger\gamma_4\gamma_\mu L)(Q_2^\dagger\gamma_4\gamma_\mu Q_2^c) + \text{H.c.}\right) + U(\theta) - (1/3)(\partial_k\theta)(Q_2^\dagger\gamma_4\gamma_k Q_2), \tag{3.16}$$

where the canonical momentum is $\pi_{2\theta} = f^2\dot\theta + n_B$. Correspondingly, $\dot\theta$ should be expressed through the canonical momentum $\pi_{2\theta}$ according to

$$\dot\theta = (\pi_{2\theta} - n_B)/f^2. \tag{3.17}$$

Taking into account that $Q_1 = e^{-i\theta/3}Q_2$ we can check that the Hamiltonians (3.15) and (3.16) interchange under this transformation. Thus we see that the calculation of Hamiltonians according to the specified rules is self-consistent.

Note that both Hamiltonians, as they are presented in Eqs. (3.15) and (3.16), do not contain chemical potential, $\dot\theta$, in the form $\dot\theta n_B$ and in this sense contradict the presumption (1.2). However, the case is somewhat more tricky. Written in terms of the canonical momentum the corresponding part of the Hamiltonian (3.16) (the first term) has the form $\delta\mathcal{H}_2(\theta) = (\pi_{2\theta} - n_B)^2/(2f^2)$. In the spatially independent case and in the absence of $U(\theta)$ the Hamiltonian equation of motion for $\mathcal{H}_2$ has the form $\dot\pi_{2\theta} = 0$, so its solution is $\pi_{2\theta} = \text{const}$. Evidently this equation is equivalent to the Lagrange equation of motion for the θ field (2.11) (where the cosmological expansion is neglected).

The presence of the $(-\pi_{2\theta}n_B/f^2)$ term in the Hamiltonian (3.16) implies that $\pi_{2\theta}/f^2$ can be understood as the baryonic chemical potential, μ_B. Since it is usually assumed that initially $n_B(t_{\text{in}}) = 0$, $\pi_{2\theta} = f^2\dot\theta(t_{\text{in}})$ and thus $\mu_B = \dot\theta(t_{\text{in}})$, but not $\mu_B = \dot\theta(t)$ taken at the running t for which thermal equilibrium is established.

C. External field θ

The assertion (1.2) might be, in principle, valid, if θ was an external fixed field with the dynamics determined "by hand," as it is noted in Sec. III B. In this case expression (1.2) could be formally true but, as we show here, such a theory possibly has some internal inconsistencies.

Let us study previously considered theories with Lagrangians (2.5) and (2.7), where the kinetic and potential terms for θ are omitted. We have two options for the construction of Hamiltonians: either to proceed along the usual lines specified above or to assume the validity of the prescription $\mathcal{H}_{\text{int}} = -\mathcal{L}_{\text{int}}$ for the interaction parts of Lagrangians. There is an unambiguous procedure for Lagrangian (2.5), since its interaction part does not contain derivatives. It is not so for Lagrangian (2.7), because of the term $(\partial_\mu\theta)J_B^\mu$ for which the conjecture $\mathcal{H}_{\text{int}} = -\mathcal{L}_{\text{int}}$ is not true. As we have seen in Sec. III B, the standard approach leads to the Hamiltonian (3.16), which does not contain the term $\dot\theta(t)n_B$. To arrive at the mechanism of spontaneous baryogenesis described in the literature we need to postulate $\mathcal{H}_{\text{int}} = -\mathcal{L}_{\text{int}}$ independently of the presence of the field derivatives. If this postulate is true, the Lagrangian (2.7) leads to a Hamiltonian containing the necessary term $\dot\theta n_B$. On the other hand, if we apply the standard procedure to calculate the Hamiltonian from the Lagrangian without the kinetic term, we find $\pi_\theta = n_B$ and arrive at the striking result

$$\mathcal{H}(\theta) = \pi_\theta\dot\theta - \mathcal{L} = 0, \tag{3.18}$$

which clearly demonstrates an inconsistency of a theory without the kinetic term.

Additional problems appear if we consider the theory with the Lagrangian

$$\mathcal{L}_{SB}^{(1)} = -\theta\partial_\mu J_B^\mu, \tag{3.19}$$

which differs from the original $\mathcal{L}_{SB}$ (1.1) by the total divergence and thus leads to the same Lagrangian equations of motion, so these Lagrangians are physically equivalent. However, it may be not so for the Hamiltonian densities. The Lagrangian (3.19) does not contain the time derivative of the theta field but contains time derivatives of the dynamical fermionic fields. So the Hamiltonian obtained from $\mathcal{L}_{SB}^{(1)}$ through the specified above standard rules, applied to fermions, has the form

$$\mathcal{H}_{SB}^{(1)} = (\partial_k\theta)J_k - \partial_k(\theta J_k) \to (\partial_k\theta)J_k, \tag{3.20}$$

where at the last step we omitted the spatial divergence. Evidently the Hamiltonian $\mathcal{H}_{SB}^{(1)}$ differs from $\mathcal{H}_{SB}$ (1.2), though they are obtained from the equivalent Lagrangians. It means that the Hamiltonian equations of motion corresponding to $\mathcal{H}_{SB}$ and $\mathcal{H}_{SB}^{(1)}$ are different. It can be checked that the equations derived from the Hamiltonian (3.20) disagrees with the Lagrangian ones. However, this is not a problem inherent to SBG but to the problem with the determination of the Hamiltonian density of the fermionic fields, related to the degeneracy between the coordinate ψ and the canonical momentum $\psi^\dagger$; see Sec. III A. These problems are considered elsewhere, while in this work we concentrate on the kinetics of the standard scenario of SBG, which in many cases leads essentially to the usual results presented in the literature. However, this is not always so.

IV. KINETIC EQUATION FOR TIME INDEPENDENT AMPLITUDE

A. Kinetic equilibrium

The study of the kinetics of fermions in the cosmological background is grossly simplified if the particles are in equilibrium with respect to elastic scattering, to their possible annihilation, e.g., into photons, and to other baryoconserving interactions. The equilibrium with respect to elastic scattering implies the following form of the phase space distribution functions:

$$f_{\rm eq} = [1 + \exp(E/T - \xi)]^{-1}, \tag{4.1}$$

where the dimensionless chemical potential $\xi = \mu/T$ has equal magnitude but opposite signs for particles and antiparticles. The baryonic number density for small ξ is usually given by the expression

$$n_B = g_S B_Q \xi_B T^3/6 \tag{4.2}$$

[compare to Eq. (1.3)]. Here ξ_B is the baryonic chemical potential. This equation which expresses baryonic number density through chemical potential is true only for the normal relation between the energy and three-momentum, $E = \sqrt{p^2 + m^2}$, with equal masses of particles and antiparticles.

Vanishing baryon asymmetry implies $\xi_B = 0$, as is usually the case. If the baryonic number of quarks is conserved, n_B remains constant in the comoving volume and it means in turn that $\xi = \text{const}$ for massless particles. If $n_B = 0$ initially, then ξ_B remains identically 0. If baryonic number is not conserved, then as we see below from the kinetic equation, equilibrium with respect to B-nonconserving processes leads to $\xi_B = c\dot{\theta}/T$, as is envisaged by SBG. The constant c depends upon the concrete type of reaction. Complete thermal equilibrium in the standard theory demands $n_B \to 0$, but a deviation from thermal equilibrium of B-nonconserving interaction leads to generation of nonzero ξ_B and correspondingly to nonzero n_B.

The situation changes if quarks and antiquarks satisfy the equation of motion (2.9), for which the following dispersion relation is valid,

$$E = \sqrt{p^2 + m^2} \mp \dot{\theta}/3, \tag{4.3}$$

where the signs $\mp$ refer to particles or antiparticles, respectively. So the energies of quarks and antiquarks with the same three-momentum are different. This is similar to mass difference that may be induced by CPT violation. It is noteworthy that the above dispersion relation is derived under the assumption of constant or slow varying $\dot{\theta}$. Otherwise the Fourier transformed Dirac equation cannot be reduced to the algebraic one and the particle energy is not well defined.

The baryon number density corresponding to the dispersion relation (4.3) is given by the expression

$$\begin{aligned} n_B &\equiv g_S B_Q \int \frac{d^3 p}{(2\pi)^3} [f(p) - \bar{f}(p)] \\ &= \frac{g_S B_Q}{6} \left(\xi_B + \frac{\dot{\theta}}{3T} \right) T^3, \end{aligned} \tag{4.4}$$

where $\bar{f}$ is the distribution function of antiparticles. If the baryon number is conserved and is 0 initially, the condition $\xi_B + \dot{\theta}/(3T) = 0$ is always fulfilled. If B is not conserved, then the equilibrium with respect to B-nonconserving processes demands $\xi_B = 0$, as it follows from the kinetic equation presented below. So evidently $\xi_B \neq \dot{\theta}$ but nevertheless the baryon asymmetry is proportional to $\dot{\theta}$ as follows from Eq. (4.4).

B. Relation between $n_B(t)$ and $\theta(t)$ in the pure Goldstone case

The equation of motion for the theta field in cosmological background (2.15) with $U(\theta) = 0$ can be easily integrated expressing baryon asymmetry, n_B, through $\dot{\theta}$. In the case when the relation (4.2) is fulfilled, we obtain

$$f^2 \left[\frac{\dot{\theta}(t)}{T^3(t)} - \frac{\dot{\theta}(t_{\rm in})}{T^3_{\rm in}} \right] = -\frac{g_S B_Q}{6} [\xi_B(t) - \xi_B(t_{\rm in})], \tag{4.5}$$

assuming that the temperature drops according to the law $\dot{T} = -HT$.

The initial value of the baryon asymmetry is usually taken to be 0, so according to Eq. (4.2) we should also take $\xi_B(t_{\rm in}) = 0$. Let us remind the reader that Eq. (4.2) is valid for the case of the normal dispersion relation, $E = p$ (in the massless case), both for quarks and antiquarks.

In the theory with the Lagrangian (2.7) and with the Dirac equation (2.9) the dispersion relation changes to (E-split) and the relation between n_B and ξ_B becomes (4.4). Now Eq. (2.15) is integrated as

$$\begin{aligned} &f^2 \left[\frac{\dot{\theta}(t)}{T^3(t)} - \frac{\dot{\theta}(t_{\rm in})}{T^3_{\rm in}} \right] \\ &\quad = -\frac{g_S B_Q}{6} \left[\xi_B(t) - \xi_B(t_{\rm in}) + \frac{\dot{\theta}(t)}{3T} - \frac{\dot{\theta}(t_{\rm in})}{3T_{\rm in}} \right]. \end{aligned} \tag{4.6}$$

If initially $n_B = 0$, then $\xi_B(t_{\rm in}) = -\dot{\theta}_{\rm in}/(3T_{\rm in})$.

In the pseudo-Goldstone case, when $U(\theta) \neq 0$, equations of motion (2.11) or (2.15) cannot be so easily integrated, but in thermal equilibrium the system of equations containing $\theta(t)$ and $\xi_B(t)$ can be reduced to ordinary differential equations that are easily solved numerically. Out of equilibrium one has to solve a much more complicated system of the ordinary differential equation of motion for $\theta(t)$ and the integrodifferential kinetic equation. It is discussed below in Sec. IV.

C. Kinetic equation in (quasi) stationary background

The probability of any reaction between particles in quantum field theory is determined by the amplitude of transition from an initial state $|\text{in}\rangle$ to a final state $|\text{fin}\rangle$. In t he lowest order of perturbation theory the transition amplitude is given by the integral of the matrix element of the Lagrangian density between these states, integrated over the four-dimensional space d^4x. Typically the quantum field operators are expanded in terms of creation-annihilation operators with the plane wave coefficients as

$$\psi(t,\mathbf{x}) = \int \frac{d^3q}{2E(2\pi)^3}[a(\mathbf{q})e^{-iqx} + b^\dagger(\mathbf{q})e^{iqx}], \tag{4.7}$$

where a, b, and their conjugate are the annihilation (creation) operators for spinor particles and antiparticles and $qx = Et - \mathbf{qx}$.

If the amplitude of the process is time independent, then the integration over dtd^3x of the product of the exponents of iqx in infinite integration limits leads to the energy-momentum conservation factors,

$$\int dtd^3xe^{-i(E_{\text{in}}-E_{\text{fin}})t+i(\mathbf{P}_{\text{in}}-\mathbf{P}_{\text{fin}})\mathbf{x}} = (2\pi)^4\delta(E_{\text{in}} - E_{\text{fin}})\delta((\mathbf{P}_{\text{in}} - \mathbf{P}_{\text{fin}}), \tag{4.8}$$

where E_{in}, E_{fin}, $\mathbf{P_{in}}$, and $\mathbf{P_{in}}$ are the total energies and three-momenta of the initial and final states, respectively. The amplitude squared contains a delta function of 0 that is interpreted as the total time duration, t_{max}, of the process and as the total space volume, V. The probability of the process given by the collision integral is normalized per unit time and volume, so it must be divided by V and t_{max}.

The temporal evolution of the distribution function of ith-type particle, $f_i(t,p)$, in an arbitrary process $i + Y \leftrightarrow Z$ in the FRW background, is governed by the equation

$$\frac{df_i}{dt} = (\partial_t - Hp_i\partial_{p_i})f_i = I_i^{\text{coll}}, \tag{4.9}$$

with the collision integral equal to

$$I_i^{\text{coll}} = \frac{(2\pi)^4}{2E_i}\sum_{Z,Y}\int d\nu_Z d\nu_Y \delta^4(p_i + p_Y - p_Z) \times \left[|A(Z \to i+Y)|^2 \prod_Z f \prod_{i+Y}(1 \pm f) - |A(i+Y \to Z)|^2 f_i \prod_Y f \prod_Z (1\pm f)\right], \tag{4.10}$$

where $A(a \to b)$ is the amplitude of the transition from state a to state b, Y and Z are arbitrary, generally multi-particle states, $(\prod_Y f)$ is the product of the phase space densities of particles forming the state Y, and

$$d\nu_Y = \prod_Y \bar{dp} \equiv \prod_Y \frac{d^3p}{(2\pi)^3 2E}. \tag{4.11}$$

The signs $+$ or $-$ in $\prod(1\pm f)$ are chosen for bosons and fermions, respectively. We neglect the effects of space-time curvature in the collision integral, which is generally a good approximation.

We are interested in the evolution of the baryon number density, which is the time component of the baryonic current J^μ: $n_B \equiv J^4$. Because of the quark-lepton transitions the current is nonconserved and its divergence is given by Eq. (2.12). The similar expression is evidently true in terms of Q_2 but without the factor $\exp(-i\theta)$. Let us first consider the latter case, with the interaction described by the Lagrangian (2.7), which contains the product of three quark and one lepton operator, and take as an example the process $q_1 + q_2 \leftrightarrow \bar{q} + l$.

Since the interaction in this representation does not depend on time, the energy is conserved and the collision integral has the usual form with conserved four-momentum. Quarks are supposed to be in kinetic equilibrium but probably not in equilibrium with respect to B-nonconserving interactions, so their distribution functions have the form

$$f_q = \exp\left(-\frac{E}{T} + \xi_B\right) \quad \text{and} \quad f_{\bar{q}} = \exp\left(-\frac{E}{T} - \xi_B\right). \tag{4.12}$$

Here and in what follows the Boltzmann statistics is used. According to Ref. [17], Fermi corrections are typically at the 10% level. Since the dispersion relation for quarks and antiquarks (4.3) depends upon $\dot{\theta}$, the baryon asymmetry in this case is given by Eq. (4.4) and the kinetic equation takes the form

$$\frac{g_S B_Q}{6}\frac{d}{dt}\left(\xi_B + \frac{\dot{\theta}}{3T}\right) = -c_1\Gamma\xi_B, \tag{4.13}$$

where c_1 is a numerical factor of order unity and Γ is the rate of baryononconserving reactions. If the amplitude of this reaction has the form determined by the Lagrangian (2.7), then $\Gamma \sim T^5/m_X^4$.

For constant or slow varying temperature the equilibrium solution to this equation is $\xi_B = 0$ and the baryon number density (4.4) is proportional to $\dot{\theta}$, $n_B = (g_S B_Q/18)\dot{\theta}T^2$, with $\dot{\theta}$ evolving according to Eq. (4.6) as

$$\dot{\theta} = \frac{f^2}{f^2 + g_S B_Q T^2/18}\left(\frac{T}{T_{\rm in}}\right)^3 \dot{\theta}(t_{\rm in}). \qquad (4.14)$$

We see that the equilibrium value of n_B drops down with decreasing temperature as T^5. However at small temperatures baryon-nonconserving processes switch off and n_B tends to a constant value in comoving volume.

Let us check now what happens if the dependence on θ is moved from the quark dispersion relation to the B-nonconserving interaction term (2.10). The collision integral (4.10) contains delta functions imposing conservation of energy and momentum if there is no external field that depends upon coordinates. In our case, when quarks "live" in the $\theta(t)$-field, the collision integral should be modified in the following way. We have now an additional factor under integral (4.8), namely, $\exp[\pm i\theta(t)]$. In the general case this integral cannot be taken analytically. However if $\theta(t)$ can be approximated as $\theta(t) \approx \dot{\theta}t$ with constant or slowly varying $\dot{\theta}$, the integral is reduced to delta-function of the energy difference between the initial and final states shifted by $\dot{\theta}$. For example, in the process of two quark transformation into an antiquark and lepton, $q_1 + q_2 \leftrightarrow \bar{q} + l$ the energy balance condition would be imposed by $\delta(E_{q_1} + E_{q_2} - E_{\bar{q}} - E_l - \dot{\theta})$. In other words the energy is nonconserved due to the action of the external field $\theta(t)$. The approximation of linear evolution of θ with time can be valid if the reactions are fast in comparison with the rate of the θ-evolution.

Note in passing that with a nonzero $\theta(t)$ the current nonconservation (2.12), in principle, may induce baryogenensis because it breaks not only baryonic number conservation, but also CP, due to the complexity of the coefficients. However, in this particular model no baryon asymmetry is generated. The model is quite similar to the model of the baryon asymmetry generation in heavy particle decays, such as, e.g., GUT baryogenesis. However, as it is argued, e.g., in Refs. [8,18], for the generation of the asymmetry at least three different channels of baryononconserved reactions are necessary. Thus one needs to add some extra fields into the model to activate this mechanism.

Returning to our case we can see that the collision integral taken over the three-momentum of the particle under scrutiny [i.e., particle i in Eq. (4.10)], e.g., for the process $q_1 + q_2 \to l + \bar{q}$, turns into

$$\dot{n}_B + 3Hn_B \sim \int d\tau_{l\bar{q}} d\tau_{q_1 q_2} |A|^2 \delta(E_{q_1} + E_{q_2} - E_l - E_{\bar{q}} - \dot{\theta}) \delta(\mathbf{P}_{\rm in} - \mathbf{P}_{\rm fin}) e^{-E_{\rm in}/T} (e^{\xi_L - \xi_B + \dot{\theta}/T} - e^{2\xi_B}), \qquad (4.15)$$

where $d\tau_{l,\bar{q}} = d^3 p_l d^3 p_{\bar{q}} / [4E_l E_{\bar{q}} (2\pi)^6]$. We assumed here that all participating particles are in kinetic equilibrium, i.e., their distribution functions have the form (4.12). In expression (4.15) ξ_B and ξ_L denote baryonic and leptonic chemical potentials respectively and the effects of quantum statistics are neglected but only for brevity of notations. The assumption of kinetic equilibrium is well justified because it is enforced by the very efficient elastic scattering. Another implicit assumption is the usual equilibrium relation between chemical potentials of particles and antiparticles, $\bar{\mu} = -\mu$, imposed, e.g., by the fast annihilation of quark-antiquark or lepton-antilepton pairs into two and three photons. Anyhow the assumption of kinetic equilibrium is one of the cornerstones of spontaneous baryogenesis.

The conservation of $(B + L)$ implies the following relation: $\xi_L = -\xi_B/3$. Keeping this in mind, we find

$$\dot{n}_B + 3Hn_B \approx -(1 - e^{\dot{\theta}/T - 3\xi_B + \xi_L}) I \approx \left(\frac{\dot{\theta}}{T} - \frac{10}{3}\xi_B\right) I, \qquad (4.16)$$

where we assumed that ξ_B and $\dot{\theta}/T$ are small. In relativistic plasma with temperature T the factor I, coming from the collision integral, can be estimated as $I = T^8/m^4$, where m is a numerical constant with dimension of mass. It differs from m_X, introduced in Eq. (2.5), by a numerical coefficient.

For a large factor I we expect the equilibrium solution

$$\xi_B = \frac{3}{10}\frac{\dot{\theta}}{T}, \qquad (4.17)$$

so $\dot{\theta}$ up to the numerical factor seems to be the baryonic chemical potential, as expected in the usually assumed SBG scenario. The value of the coefficient $c = 3/10$ in Eq. (4.17) may be different for other types of B-nonconserving reactions, e.g., for the reaction $3q \leftrightarrow 3\bar{q}$ one can find that $c = 1/6$. Let us remind the reader that for the dispersion relation (4.3) the baryonic chemical potential is not proportional to $\dot{\theta}(t)$, but is equal to 0; see Eq. (4.13) and comments below.

V. OUT-OF-EQUILIBRIUM GENERATION OF BARYON ASYMMETRY IN THE PURELY GOLDSTONE CASE

As we have seen in the previous section the equilibrium value of the baryon asymmetry in comoving volume drops down as T^2. So for an effective generation of the asymmetry the B-nonconserving reactions must drop out of equilibrium at sufficiently high temperatures. Below we estimate the asymptotic value of the baryon asymmetry.

Let us first study the case when the cosmological expansion is very slow and the temperature can be considered as constant or, better to say, adiabatically decreasing. The proper equations in this limit can be solved analytically and it allows better insight into the problem. With constant T the equilibrium is ultimately reached if time is sufficiently large and asymptotically the baryonic chemical potential is indeed proportional to $\dot\theta(t)$, but one should remember that this is true in the case when $\theta(t)$ enters the interaction term but not the quark dispersion relation. A similar situation is realized in cosmology with decreasing temperature of the cosmic plasma but it is interesting that the magnitude of the resulting baryon asymmetry is a nonmonotonic function of the strength of B-violation. With very strong and very weak interaction the asymmetry goes to 0 and the best conditions for baryogenesis are realized in the intermediate case.

Using Eqs. (4.2), (4.5), and (4.16) we find

$$\dot\xi_B = \gamma\left[\frac{\dot\theta_{\rm in}}{T} - \xi_B\left(\frac{10}{3} + \frac{C_B T^2}{f^2}\right)\right], \tag{5.1}$$

which is solved as

$$\xi_B(t) = \frac{\dot\theta_{\rm in}}{T\kappa}\left[1 - e^{-\kappa\gamma(t-t_{\rm in})}\right], \tag{5.2}$$

where $C_B = g_S B_Q/18$, $\gamma = T^5/(C_B m^4)$, $\kappa = 10/3 + C_B T^2/f^2$, $t_{\rm in}$ is the initial value of time, at which $\xi_B(t_{\rm in}) = 0$, and $\dot\theta_{\rm in} = \dot\theta(t_{\rm in})$.

The time derivative of the Goldstone field evolves as

$$\dot\theta(t) = \dot\theta_{\rm in}\left[1 - \frac{C_B T^2}{f^2\kappa}\left(1 - e^{-\kappa\gamma(t-t_{\rm in})}\right)\right]. \tag{5.3}$$

So $\dot\theta(t)$ drops down asymptotically at large time with respect to its initial value, and the baryonic chemical potential exponentially tends to $\xi_B \to \dot\theta_{\rm in}/(\kappa T)$, as it is expected in the SBG scenario.

As follows from Eq. (5.3), $\dot\theta$ tends to a constant value at large t; however at the beginning the second time derivative $\ddot\theta$ may be non-negligible,

$$\ddot\theta = -\frac{\dot\theta_{\rm in} C_B T^2 \gamma}{f^2} e^{-\kappa\gamma(t-t_{\rm in})}. \tag{5.4}$$

The variation of $\dot\theta$ with time is considered in Sec. VIII B.

Let us turn now to more realistic cosmology when the temperature drops down according to

$$\dot T = -HT \tag{5.5}$$

with the Hubble parameter equal to

$$H = \left(\frac{8\pi^3 g_*}{90}\right)^{1/2}\frac{T^2}{m_{\rm Pl}} \equiv G_*\frac{T^2}{m_{\rm Pl}}, \tag{5.6}$$

where $m_{\rm Pl} = 1.2\times 10^{19}$ GeV is the Planck mass and g_* is the number of species in the primeval relativistic plasma. In the interesting temperature range $g_* \sim 100$.

Now $\dot\theta(t)$ is expressed through $\xi_B(t)$ according to Eq. (4.5) and instead of Eq. (5.1) we obtain

$$\dot\xi_B = \gamma\left(\frac{\dot\theta_{\rm in}T^2}{T_{\rm in}^3} - \kappa\xi_B\right). \tag{5.7}$$

This equation can be more conveniently solved if we change the time variable as $dt = -dT/(HT)$ and introduce dimensionless inverse temperature according to $\eta = T_{\rm in}/T$. So the baryonic chemical potential evolves as a function of $\eta = T_{\rm in}/T$ as

$$\xi_B(\eta) = K\int_1^\eta \frac{d\eta'}{(\eta')^6}\exp\left[-N\int_{\eta'}^{\eta}\frac{d\eta''}{(\eta'')^4}\left(\frac{10}{3} + \frac{C_B T_{\rm in}^2}{f^2\eta''^2}\right)\right], \tag{5.8}$$

where $K = \dot\theta_{\rm in} m_{\rm Pl} T_{\rm in}^2/(C_B m^4 G_*)$, $N = m_{\rm Pl}T_{\rm in}^3/(C_B m^4 G_*)$. If $K \gg 1$, which corresponds to the equilibrium case, the integral can be evaluated up to the terms of the order of $1/K$ and we find

$$\xi_B^{\rm eq}(\eta) = \frac{(\dot\theta_{\rm in}/T_{\rm in})}{(10\eta^2/3) + (C_B T_{\rm in}^2)/f^2}. \tag{5.9}$$

This result coincides, as expected, with the equilibrium solution of Eq. (5.7): $\xi_B = \dot\theta_{\rm in}T^2/(T_{\rm in}^3\kappa)$. Note that in equilibrium both ξ_B and $\dot\theta/T$ fall down as T^2 with decreasing temperatures.

It is instructive to consider a different model of baryonic number nonconservation through quark-antiquark transformation $2Q \leftrightarrow 2\bar Q$. For realistic quarks such a process is forbidden, but the process $3q \leftrightarrow 3\bar q$ is allowed in, e.g., the $SO(10)$ model of grand unification. However, we consider the first one just for simplicity. The kinetic equation (4.16) in this case is transformed into

$$\dot n_B = \left(\frac{\dot\theta}{T} - 4\xi_B\right)\frac{T^8}{m^4}, \tag{5.10}$$

so in equilibrium with respect to the process $2Q \leftrightarrow 2\bar Q$ the baryonic chemical potential tends to $\xi_B \to \dot\theta/(4T)$.

Now we see what happens out of equilibrium. To this end we numerically take the integral in Eq. (5.8) for different values of K and $C_B T_{\rm in}^2/f^2$. The results for $\xi_B(\eta)$ and the ratio of ξ_B to the equilibrium value $(3/10)\dot\theta/T$ as functions of $\eta = T_{\rm in}/T$ are presented in Fig. 1, in left and right panels respectively. As is seen from the left panel, the baryon asymmetry is a nonmonotonic function of the rate of the baryononconserving processes. For a large rate (large K and N) baryon asymmetry is quickly generated and reaches

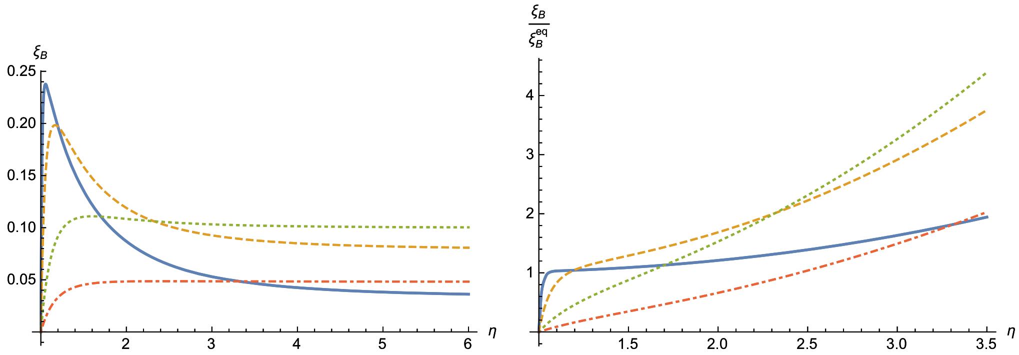

FIG. 1. Left: Evolution of $\xi_B(\eta)$ according to Eq. (5.8) where $C_B T_{\rm in}^2/(5f^2) = 0.1$ for K = N = 20 (thick), 5 (dashed), 1 (dotted), and 0.3 (dot dashed). Right: Ratio of $\xi_B(\eta)$ to its equilibrium value, $\xi_B^{\rm eq}(\eta)$, determined by (5.9) for the same values of the parameters.

high value, but it drops down as the equilibrium one, $\sim 1/\eta^2$, till lower temperatures. As a result the final baryon asymmetry is smaller for larger rates. On the other hand, if the rate is very small, the generation of the baryon asymmetry is not efficient from the very beginning and because of that the final value is also small. So there is an intermediate magnitude of the rate for which the baryon asymmetry is maximal.

The variation of $\dot\theta(\eta)$ calculated according to Eq. (4.5) with $\xi_B(\eta)$ determined from Eq. (5.8) is presented in Fig. 2. It is clearly seen that $\dot\theta$ is not constant, but quite strongly changes as a function of temperature or time, especially near the initial moment. It means that the basic assumption of the SBG scenario is violated.

VI. PSEUDO-GOLDSTONE CASE

If the potential $U(\theta)$ is nonzero, the equation of motion (2.15) cannot be so easily integrated. This case is more efficient for the generation of the cosmological baryon asymmetry because the field $\theta(t)$ naturally oscillates around the potential minimum, while the mechanism leading to nonzero $\dot\theta$, especially after inflation, is unclear. The potential is usually taken in the form

$$U(\theta) = -f^2 m_\theta^2 \cos\theta \to f^2 m_\theta^2 \theta^2/2, \qquad (6.1)$$

where the last equality corresponds to expansion of the cosine near the minimum of the potential.

To obtain a closed system of equations describing the evolution of $\theta(t)$ with an account of backreaction of the created baryons, one needs to average the quantum operator $(\dot n_B + 3Hn_B)$ over the medium. In Ref. [10] the averaging was performed over the vacuum state. It corresponds only to decay of $\theta(t)$ while the backreaction of the particles in cosmic plasma restoring the θ field is neglected. To include this backreaction we need to use kinetic equation (4.9), expressing $\dot n_B$ through the collision integral, which depends upon $\theta(t)$ and $\xi_B(t)$. As a result a system of the ordinary differential and integral equations is obtained that completely determines the evolution of $\theta(t)$ and $n_B(t)$. The problem becomes much simpler in thermal equilibrium when the collision integral is reduced to an algebraic relation between $\theta(t)$ and ξ_B. However, this is true only

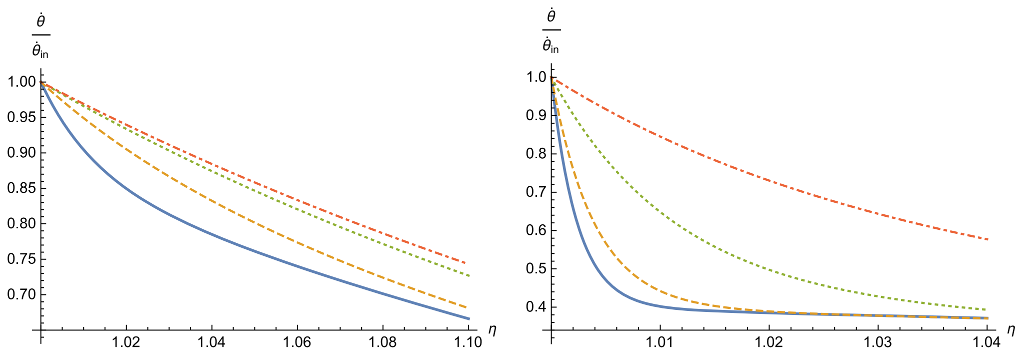

FIG. 2. Left: Evolution of $\dot\theta(\eta)$, normalized to its initial value, $\dot\theta_{\rm in}$, for $C_B T_{\rm in}^2/(5f^2) = 0.1$ and K = N = 20 (thick), 5 (dashed), 1 (dotted), and 0.3 (dot dashed). Right: The same with $C_B T_{\rm in}^2/(5f^2) = 1$ and K = N = 50 (thick), 30 (dashed), 10 (dotted), and 3 (dot dashed).

if θ is a slowly varying function of time and $\dot{\theta}$ is essentially constant. If this is so, we return to the situation considered in the previous section. The case when the variation of $\theta(t)$ is of importance demands modification of the kinetic equation for the time dependent background, discussed in the following section.

Note that if the B-nonconserving reactions are frozen, the baryon number density remains constant in the comoving volume, i.e., $\dot{n}_B + 3Hn_B = 0$, so the evolution of θ is governed by the free Klein-Gordon equation. Correspondingly $\theta(t)$ during the equilibrium period simply oscillates near the minimum of the potential with adiabatically decreasing amplitude induced by the cosmological expansion.

In Ref. [2,3] a different approach was taken. It was assumed that the backreaction of the particle production on the evolution of θ could be described by the "friction" term $\Gamma\dot{\theta}$ that was added to the equation of motion,

$$f^2(\partial_t + 3H)\dot{\theta} + f^2\Gamma\dot{\theta} + U'(\theta) = 0, \tag{6.2}$$

where Γ is the rate of the B-nonconserving processes. Comparing this equation with Eq. (2.15) the authors concluded that $\theta(t)$ oscillates with exponentially decreasing amplitude, $\sim\exp(-\Gamma t)$, and that

$$\dot{n}_B + 3Hn_B = f^2\Gamma\dot{\theta}. \tag{6.3}$$

However, this might be true only for the decays into the empty or overcooled state, as was mentioned in Ref. [3]. In this case thermal equilibrium is broken and the identification of $\dot{\theta}/T$ with ξ_B is questionable. Another problem is a possibility of description of particle production by $\Gamma\dot{\theta}$. As it is shown in the paper [19], such description can only be true, but not necessarily so, for harmonic potential of the field, which produces particles. In the case when the interaction is given by $(\dot{n}_B + 3Hn_B)$, one has to average this quantum operator over the medium with external field $\theta(t)$. As a result a nonlocal in time expression containing $\theta(t)$ emerges leading to a integrodifferential equation for θ, which is not reduced to Eq. (6.2). The problem is treated this way in Ref. [10], where the results are different from those obtained in the papers [2,3].

VII. KINETIC EQUATION FOR TIME-VARYING AMPLITUDE

The canonical kinetic equation (4.9) is usually presented for scattering or decay processes in the time independent or slowly varying background with the collision integral giving by Eq. (4.10).

In the case when the interaction proceeds in the time dependent background and/or the time duration of the process is finite, the energy conservation delta function does not emerge and the described approach becomes invalid, so one has to make the time integration with an account of time-varying background and integrate over the phase space without energy conservation.

In what follows we consider the two-body inelastic process with baryonic number nonconservation with the amplitude obtained from the last term in Lagrangian (2.5). At the moment we do not specify the concrete form of the reaction and only say that it is the two-body reaction

$$a + b \leftrightarrow c + d, \tag{7.1}$$

where a, b, c, and d are some quarks and leptons or their antiparticles. The expression for the evolution of the baryonic number density, n_B, follows from Eq. (4.9) after integration of both its sides over $d^3p_i/(2\pi)^3$. Thus we obtain

$$\dot{n}_B + 3Hn_B = -\frac{(2\pi)^3}{t_{\max}}\int d\nu_{\rm in}d\nu_{\rm fin}\delta(\mathbf{P}_{\rm in} - \mathbf{P}_{\rm fin})|A|^2(f_af_b - f_cf_d) \tag{7.2}$$

where, e.g., $d\nu_{\rm in} = d^3p_ad^3p_b/[4E_aE_b(2\pi)^6]$ and the amplitude of the process is defined as

$$A = \left(\int_0^{t_{\max}} dt e^{i[(E_c+E_d-E_a-E_b)t+\theta(t)]}\right)F(p_a, p_b, p_c, p_d), \tag{7.3}$$

and F is a function of four-momenta of the participating particles, determined by the concrete form of the interaction Lagrangian. In what follows we consider two possibilities, $F = \text{const}$ and $F = \psi^4 m_X^{-2}$, where in the last case ψ^4 symbolically denotes the product of the Dirac spinors of particles a, b, c, and d.

In the case of equilibrium with respect to baryon-conserving reactions the distribution functions have the canonical form $f_a = \exp(-E_a/T + \xi_a)$, where $\xi_a \equiv \mu_a/T$ is the dimensionless chemical potential. So for constant F the product $|A|^2(f_af_b - f_cf_d)$ depends upon the particle four-momenta only through $E_{\rm in}$ and $E_{\rm fin}$, where

$$E_{\rm in} = E_a + E_b, \quad \text{and} \quad E_{\rm fin} = E_c + E_d. \tag{7.4}$$

Now we can perform almost all (but one) integrations over the phase space in Eq. (7.2). To this end it is convenient to change the integration variables, according to

$$\frac{d^3p_a}{E_a}\frac{d^3p_b}{E_b} = d^4P_{\rm in}d^4R_{\rm in}\delta(P_{\rm in}^2 + R_{\rm in}^2)\delta(P_{\rm in}R_{\rm in}), \tag{7.5}$$

where $P_{\rm in} = p_a + p_b$ and $R_{\rm in} = p_a - p_b$ and masses of the particles are taken to be 0. Analogous expressions are valid for the final state particles. Evidently the time components of the 4-vectors P are the sum of energies of the incoming and outgoing particles, $P_{\rm in}^{(4)} = E_{\rm in}$ and $P_{\rm fin}^{(4)} = E_{\rm fin}$.

First we integrate over the initial momenta $d^4P_{\rm in}d^4R_{\rm in}$ through the following steps (to avoid an overload of the equations we skip below the subindex "in" where it is not necessary):

(1) Integration over $d^3P_{\rm in}$ (or $d^3P_{\rm fin}$) with $\delta(\mathbf{P}_{\rm in} - \mathbf{P}_{\rm fin})$ gives simply 1.

(2) Taking the integral over $d^4R = 2\pi dR_4\mathbf{R}^2 d|\mathbf{R}|d\zeta$ we first integrate over the polar angle using

$$\delta(PR) = \delta(P_4R_4 - |\mathbf{P}||\mathbf{R}|\zeta), \tag{7.6}$$

so $\zeta = Q_4R_4/(|\mathbf{R}||\mathbf{Q}|)$ and using the delta function $\delta(Q_4^2 - \mathbf{Q}^2 + R_4^2 - \mathbf{R}^2)$ we find that R_4 is bounded by $R_4^2 < \mathbf{Q}^2$, because $|\zeta| < 1$. The integral over $\mathbf{R}^2/|\mathbf{Q}|$ is taken with the above-written delta function and there remains only the integration over dR_4 in the limits $(-|\mathbf{Q}|)$ and $(+|\mathbf{Q}|)$. So the integration over the initial momenta is reduced finally to $2\pi dQ_4$.

(3) Proceeding along the same lines with the integration over the phase volume of the final particles, but without $\delta(\mathbf{P}_{\rm in} - \mathbf{P}_{\rm fin})$, we obtain

$$(2\pi)^3 \int d\nu_{\rm in} d\nu_{\rm fin} \delta(\mathbf{P}_{\rm in} - \mathbf{P}_{\rm fin}) = \frac{1}{2^9\pi^6}\int dE_{\rm in}dE_{\rm fin}d|\mathbf{Q}_{\rm fin}||\mathbf{Q}_{\rm fin}^2|. \tag{7.7}$$

Naively we should expect that the integration over $|\mathbf{Q}_{\rm fin}|$ lies in the limits from 0 to $E_{\rm fin}$ because

$$\mathbf{Q}_{\rm fin}^2 = E_c^2 + E_d^2 + 2E_cE_d\zeta < (E_c + E_d)^2 = E_{\rm fin}^2, \tag{7.8}$$

but there is a constraint $\mathbf{Q}_{\rm fin} = \mathbf{Q}_{\rm in}$, so the upper limit on $|\mathbf{Q}_{\rm fin}|$ is the smaller out of $E_{\rm fin}$ and $E_{\rm in}$. Let us introduce new notations: $E_+ = E_{\rm in} + E_{\rm fin}$ and $E_- = E_{\rm in} - E_{\rm fin}$. It is easy to check that $E_{\rm fin} > E_{\rm in}$ for $E_- < 0$ and $E_{\rm fin} < E_{\rm in}$ for $E_- > 0$. Thus for $E_- < 0$ the integration over $d|\mathbf{Q}_{\rm fin}|$ in Eq. (7.7) gives $E_{\rm in}^3/3$, while for $E_- > 0$ the result is $E_{\rm fin}^3/3$.

(4) So we are left with the integral over $dE_{\rm in}dE_{\rm fin}$, which is convenient to rewrite as

$$\int dE_{\rm in}dE_{\rm fin} = dE_+dE_-/2. \tag{7.9}$$

Note that the amplitude A (7.3) depends only on E_- but not on E_+, while the products of the particle densities in the phase space are

$$f_af_b = \exp\left(-\frac{E_+ + E_-}{2T} + \xi_a + \xi_b\right) \quad \text{and}$$
$$f_cf_d = \exp\left(-\frac{E_+ - E_-}{2T} + \xi_c + \xi_d\right). \tag{7.10}$$

(5) The integral over dE_+ can be taken explicitly but first we need to establish the integration limits. The original integration over $dE_{\rm in}dE_{\rm fin}$ is taken from 0 to ∞, so the integral over dE_+ runs from $|E_-|$ to ∞ and the integral over dE_- runs from $(-\infty)$ to $(+\infty)$. It is convenient to separate the integration over dE_+ into two parts for positive and negative E_-. For positive E_- we find

$$\int_{E_-}^{\infty} dE_+\left(\frac{E_+ - E_-}{2}\right)^3 \exp\left(-\frac{E_+ + E_-}{2T}\right) = 12T^4e^{-y},$$
$$\int_{E_-}^{\infty} dE_+\left(\frac{E_+ - E_-}{2}\right)^3 \exp\left(-\frac{E_+ - E_-}{2T}\right) = 12T^4, \tag{7.11}$$

where $y = E_-/T$. For negative E_- we obtain the same results with an interchange of the initial and final states, i.e., $f_af_b \leftrightarrow f_cf_d$ and with $y \to |y|$. Effectively it corresponds to the change of sign of $\theta(t)$ in Eq. (7.3).

Thus, collecting all the factors (7.10), we finally obtain

$$\dot n_B + 3Hn_B = -\frac{T^5}{2^5\pi^6 t_{\rm max}}\int_0^\infty dy[e^{\xi_a+\xi_b}(|A_+|^2 + |A_-|^2e^{-y}) - e^{\xi_c+\xi_d}(|A_-|^2 + |A_+|^2e^{-y})], \tag{7.12}$$

where A_+ is the amplitude taken at positive E_-, while A_- is taken at negative E_-. With the substitution $E_- \to |E_-|$ the only difference between A_+ and A_- is that $A_-(\theta) = A_+(-\theta)$.

The equilibrium is achieved when the integral in Eq. (7.12) vanishes. This point determines the equilibrium values of the chemical potentials in the external $\dot\theta$ field. Clearly it takes place at

$$\xi_a + \xi_b - \xi_c - \xi_d = \frac{\langle |A_+|^2e^{-y} + |A_-|^2\rangle}{\langle |A_+|^2 + |A_-|^2e^{-y}\rangle} - 1, \tag{7.13}$$

where the angular brackets mean integration over dy as indicated in Eq. (7.12).

The results above are obtained for the amplitude that does not depend upon the participating particle momenta. The calculations would be somewhat more complicated if this restriction were not true. For example if the baryon nonconservation takes place in four-fermion interactions, then the amplitude squared can contain the terms of the form $(p_ap_b)^2/m_X^4$ or $(p_ap_c)^2/m_X^4$, etc. The effect of such terms results in a change of the numerical coefficient in Eq. (4.16) but the latter is unknown anyhow, and what is more important is that the temperature coefficient in front of the integral in this equation would change from T^5 to T^9/m_X^4.

VIII. EXAMPLES OF TIME-VARYING θ

A. Constant $\dot\theta$

This is the case usually considered in the literature and the simplest one. The integral (7.3) is taken analytically resulting in

$$|A|^2 \sim \frac{2 - 2\cos[(\dot\theta - E_-)t_{\max}]}{(\dot\theta - E_-)^2}. \tag{8.1}$$

Here E_- is running over the positive semiaxis, see Eq. (7.11), and comments around it.

For large $t_{\max}$ this expression tends to $\delta(E_- - \dot\theta)$, so $|A_+|^2 = 2\pi\delta(E_- - \dot\theta)t_{\max}$ and $|A_-|^2 = 2\pi\delta(E_- + \dot\theta)t_{\max} = 0$, if $\dot\theta > 0$ and vice versa otherwise. Hence the equilibrium solution is

$$\xi_a + \xi_b - \xi_c - \xi_d - \dot\theta/T = 0, \tag{8.2}$$

coinciding with the standard result.

The limit of $\dot\theta = \text{const}$ corresponds to the energy nonconservation by the rise (or drop) of the energy of the final state in reaction (7.1) exactly by $\dot\theta$. However if $t_{\max}$ is not sufficiently large, the nonconservation of energy is not equal to $\dot\theta$ but somewhat spread out and the equilibrium solution is different. There is no simple analytical expression in this case, so we have to take the integrals over y in Eq. (7.13) numerically.

The results of the calculations are presented in Fig. 3. In the left panel the values of the rhs of Eq. (7.13) are presented as a function of $\dot\theta/T$ for the cutoff of the time integration in Eq. (8.1) equal to $\tau \equiv t_{\max}T = 30, 10, 3$. The larger the integration time is, the closer the lines are to $\dot\theta/T$, which is also depicted.

In the right panel the relative differences between the rhs of Eq. (7.13) and $\dot\theta/T$, normalized to $\dot\theta/T$, as a function of $\dot\theta/T$ for a different maximum time of the integration, are presented. We see that for $\tau = 30$ the deviations are less than 10%, while for $\tau = 3$ the deviations are about 30%. If we take τ close to unity, the deviations are about 100%. The value of $\dot\theta/T$ is bounded from above by approximately 0.3 because at large $\dot\theta/T$ the linear expansion, used in our estimates, is invalid.

The realistic values of τ depend upon the model parameters. There is one evident limit related to the cosmological expansion, which implies $\tau < t_{\text{cosm}}T \sim T/H \sim m_{\text{Pl}}/T$. Here m_{Pl} is the Planck mass, H is the Hubble parameter, and $T_{\text{cosm}} \sim 1/H$, so the effects of the expansion may be significant only near the Planck temperature. Another upper bound on τ is presented by the kinetic equations, which demand the characteristic time variation to be close (at least initially) to the inverse reaction rate $\gamma \sim T^5/m_X^4$. The discussed effects would have an essential impact on the approach to equilibrium for $T \sim m_X$, which might be realistic.

B. Second order Taylor expansion of $\theta(t)$

As we have seen in the previous subsection the approximation $\dot\theta = \text{const}$ is noticeably violated. Here we assume that $\theta(t)$ can be approximated as

$$\theta(t) = \dot\theta t + \ddot\theta t^2/2, \tag{8.3}$$

where $\dot\theta$ and $\ddot\theta$ are supposed to be constant or slowly varying. In this case the integral over time (7.3) can also be taken analytically but the result is rather complicated. We need to take the integral

$$\int_0^{t_{\max}} dt \exp[i\theta(t)]. \tag{8.4}$$

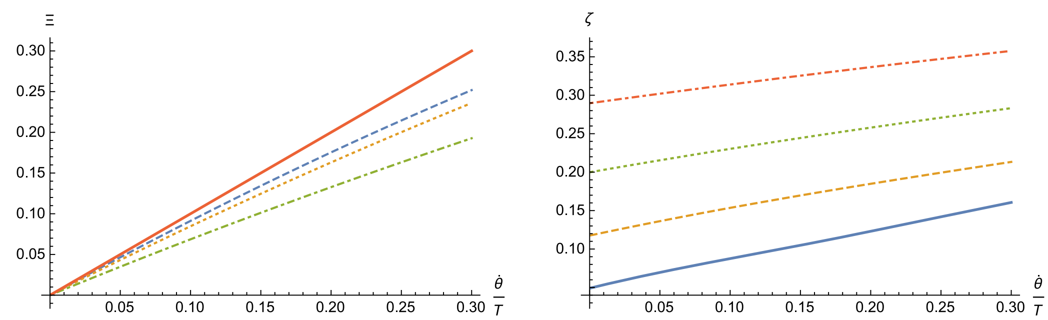

FIG. 3. Left: The rhs of Eq. (7.13), denoted by Ξ, as a function of $\dot\theta/T$ for the cutoff of the time integration in Eq. (8.1): $\tau \equiv t_{\max}T = 30$ (dashed), 10 (dotted), 3 (dot dashed), and $\dot\theta/T$ (thick). Right: The relative difference: $\zeta = \Xi/(\dot\theta/T) - 1$, as a function of $\dot\theta/T$ for $\tau = 30$ (thick), 10 (dashed), 5 (dotted), and 3 (dot dashed).

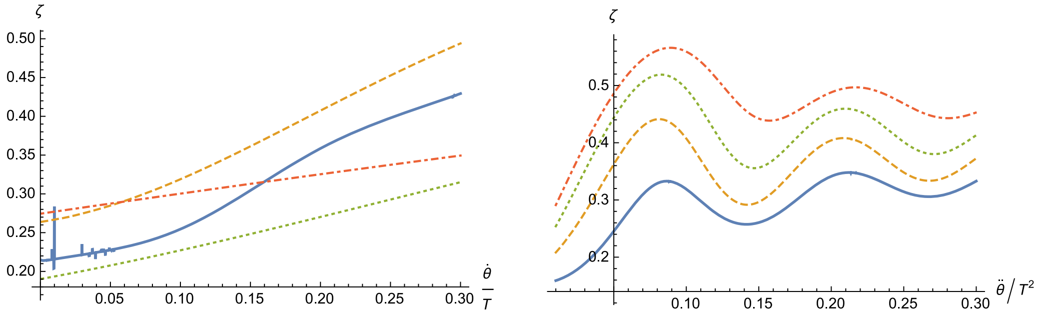

FIG. 4. Left: The relative difference, $\zeta = \Xi/(\dot{\theta}/T) - 1$, as a function of $\dot{\theta}/T$ for the cutoff of the time integration $\tau = 30$ (thick), 10 (dashed), 5 (dotted), and 3 (dot dashed) for fixed $\ddot{\theta}/T^2 = 0.1$. Right: The same difference as a function of $\ddot{\theta}/T^2$ for fixed time of integration $\tau = 10$ and different $\dot{\theta}/T = 0.1$ (thick), 0.2 (dashed), 0.3 (dotted), and 0.4 (dot dashed).

Its real and imaginary parts are easily expressed though the Fresnel functions. So the amplitude squared is given by the functions tabulated in *Mathematica* and the position of the equilibrium point can be calculated, as in the previous case, by numerical calculation of the one-dimensional integral.

The rhs of Eq. (7.13) as a function of $\dot{\theta}/T$ for different values of τ is presented in Fig. 4, in the left panel. It is interesting that the dependence on τ is nonmonotonic. This may be understood by diminishing the impact of $\ddot{\theta}t^2$ at a smaller time interval.

To check the dependence on $\ddot{\theta}$ we calculated again the rhs of Eq. (7.13) but now as a function of $\ddot{\theta}/T^2$ presented in the right panel of Fig. 4 for fixed time of integration and different values of $\dot{\theta}/T$. We see that the equilibrium point oscillates as a function of $\ddot{\theta}$.

C. Oscillating $\theta(t)$

If the potential of θ were nonvanishing, its evolution would be more complicated. The potential $U(\theta)$ should be a periodic function of the angle θ and so it is often taken as $m^2 \cos\theta$. We assume that the field θ is initially near the minimum of the potential, which in this case can be approximated as $U = m^2\theta^2/2$, where m is the mass of the theta field. In the absence of backreaction of the produced baryons $\theta(t)$ should evolve as

$$\theta(t) = \theta_0 \cos(mt + \phi). \tag{8.5}$$

Unfortunately the integral (7.3) cannot be taken analytically and the numerical calculations with two-dimensional integrals are quite time consuming. However, the integrand can be expanded as

$$e^{i\theta(t)} = 1 + i\theta_0 \cos(mt + \phi). \tag{8.6}$$

In this approximation the integral (7.3) can be easily taken analytically. Thus also in this case we can reduce the calculation of the deviation of the algebraic sum of dimensionless chemical potentials from $\dot{\theta}/T$ (8.2) to the numerical calculation of the one-dimensional integral. However, to be sure of the safely of the procedure it is desirable to compare the time integrated exact amplitude with the approximate expanded one. Numerical comparison shows indeed that even for $\theta_0 = 1$ the corrections are negligible, while for $\theta_0 \leq 0.5$ they are practically indistinguishable (see Fig. 5).

The deviation of the rhs of Eq. (7.13) from $\dot{\theta}/T$ is demonstrated in Fig. 6. The difference with the standard predictions of SBG can be significant if the mass of θ is not negligible, so the oscillations of θ manifest themselves during time τ. So the standard SBG, for which the baryonic chemical potential is proportional to $\dot{\theta}$, is not accurate at

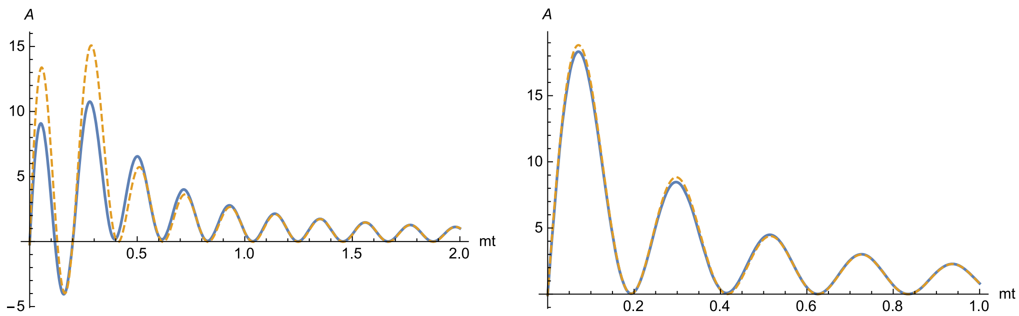

FIG. 5. Exact (thick) and approximate (dashed) expressions for the amplitude A (7.3) with $\theta(t)$ determined by Eq. (8.6) as functions of mt for $\theta_0 = 1$ (left) and $\theta_0 = 0.3$ (right).

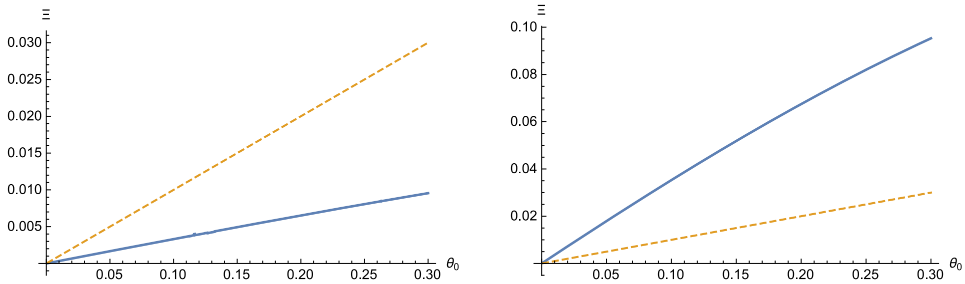

FIG. 6. Left: Thick curve, the rhs of Eq. (7.13), Ξ, for $m/T = 0.1$ and the maximal time of interation $\tau = 30$; dashed curve, $\dot{\theta}/T$ as functions of θ_0; see Eq. (8.5). Right: The same as the maximal time of integration $\tau = 3$.

large times or, better to say, for large $mt_{\max}$. On the other hand, as we see in these figures, for small τ the deviations are also quite noticeable, but now the effect is related to the energy spread because of the finite time integration. As it is seen in the figures, the effect changes sign—the relative positions of thick and dashed curves interchange.

IX. CONCLUSION

To summarize, we have clarified the relation between the Lagrangian and Hamiltonian in the SBG scenario. We argue that in the standard description $\dot{\theta}$ is not formally the chemical potential, though in thermal equilibrium $\dot{\theta}$ may tend to the chemical potential with the numerical coefficient that depends upon the model. However, this result is not always true but depends upon the chosen representation of the quark fields. In the theory described by the Lagrangian (2.5) that appears "immediately" after the spontaneous symmetry breaking, $\theta(t)$ directly enters the interaction term and in equilibrium $\mu_B \sim \dot{\theta}$ indeed. On the other hand, if we transform the quark field, so that the dependence on θ is shifted to the bilinear product of the quark fields (2.7), then chemical potential in equilibrium does not tend to $\dot{\theta}$, but to 0. Still, the magnitude of the baryon asymmetry in equilibrium is always proportional to $\dot{\theta}$.

It can be seen, according to the equation of motion of the Goldstone field, that $\dot{\theta}/T$ drops down in the course of the cosmological cooling as T^2, so the baryon number density in the comoving volume decreases in the same way. So to avoid the complete vanishing of n_B the baryoviolating interaction should switch off at some nonzero and not very small temperature. The dependence of the baryon asymmetry on the interaction strength is nonmonotonic. Too strong and too weak interactions lead to small baryon asymmetry, as is presented in Fig. 1.

The assumption of a constant or slowly varying $\dot{\theta}$, which is usually made in the SBG scenario, may not be fulfilled and to include the effects of an arbitrary variation of $\theta(t)$, as well as the effects of the finite time integration, we transformed the kinetic equation in such a way that it becomes operative in nonstationary background. A shift of the equilibrium value of the baryonic chemical potential due to this effect is numerically calculated.

In spite of these corrections to the standard SBG scenario, it remains a viable mechanism for creating the observed cosmological excess of matter over antimatter. However, this mechanism is not particularly efficient in the case of pure spontaneous symmetry breaking, when the potential of the θ field is absent. Nonzero potential $U(\theta)$, which can appear as a result of an explicit breaking of the baryonic $U(1)$-symmetry in addition to the spontaneous breaking, may grossly enhance the efficiency of the spontaneous baryogenesis. The evaluation of the efficiency demands a numerical solution of the ordinary differential equation of motion for the θ field together with the integral kinetic equation. In the case of thermal equilibrium the kinetic equation is reduced to an algebraic one and the system is trivially investigated. The out-of-equilibrium situation is much more complicated technically and is studied elsewhere.

We assumed that the symmetry breaking phase transition in the early Universe occurred instantly. It may be a reasonable approximation, but still the corrections can be significant. This can also be a subject of future work.

There remains the problem of the proper definition of the fermionic Hamiltonian but presumably it does not have an important impact on the problems considered here and thus is neglected.

ACKNOWLEDGMENTS

We thank A. I. Vainshtein for stimulating criticism. The work of E. A. and A. D. was supported by the RSF Grant No. 16-12-10037. V. N. acknowledges the support of the RFBR Grants No. 14-02-00995 and No. 16-02-00342.

[1] A. D. Sakharov, Pis'ma ZhETF **5**, 32 (1967) [JETP Lett. **5**, 24 (1967)].
[2] A. Cohen and D. Kaplan, Phys. Lett. B **199**, 251 (1987).
[3] A. Cohen and D. Kaplan, Nucl. Phys. **B308**, 913 (1988).
[4] A. G. Cohen, D. B. Kaplan, and A. E. Nelson, Phys. Lett. B **263**, 86 (1991).
[5] A. D. Dolgov, Phys. Rep. **222**, 309 (1992); V. A. Rubakov and M. E. Shaposhnikov, Usp. Fiz. Nauk. **166**, 493 (1996) [Phys.Usp. **39**, 461 (1996)]; A. Riotto and M. Trodden, Annu. Rev. Nucl. Part. Sci. **49**, 35 (1999); M. Dine and A. Kusenko, Rev. Mod. Phys. **76**, 1 (2003).
[6] A. D. Dolgov, Surveys in High Energy Physics **13**, 83 (1998).
[7] L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics, Vol. 5: Statistical Physics, Part 1* (Nauka, Moscow, 1964; Pergamon, Oxford, 1969).
[8] A. D. Dolgov and Ya. B. Zeldovich, Uspekhi Fizicheskih Nauk. **130**, 559 (1980); Rev. Mod. Phys. **53**, 1 (1981).
[9] A. D. Dolgov, Phys. At. Nucl. **73**, 588 (2010).
[10] A. D. Dolgov and K. Freese, Phys. Rev. D **51**, 2693 (1995); A. D. Dolgov, K. Freese, R. Rangarajan, and M. Srednicki, Phys. Rev. D **56**, 6155 (1997).
[11] H. Davoudiasl, R. Kitano, G. D. Kribs, H. Murayama, and P. J. Steinhardt, Phys. Rev. Lett. **93**, 201301 (2004).
[12] G. Lambiase, S. Mohanty, and A. R. Prasanna, Int. J. Mod. Phys. D **22**, 1330030 (2013).
[13] G. Lambiase and G. Scarpetta, Phys. Rev. D **74**, 087504 (2006); L. Pizza, arXiv:1506.08321.
[14] D. G. Phillips II *et al.*, Phys. Rep. **612**, 1 (2016).
[15] C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C **40**, 100001 (2016).
[16] J. C. Pati, A. Salam, and U. Sarkar, Phys. Lett. **133B**, 330 (1983).
[17] A. D. Dolgov and K. Kainulainen, Nucl. Phys. **B402**, 349 (1993).
[18] E. Kolb and M. Turner, *The Early Universe* (Addison-Wesley, Reading, MA, 1989).
[19] A. D. Dolgov and S. H. Hansen, Nucl. Phys. **B548**, 408 (1999).