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Research paper

Multi-product firms in monopolistic competition: The role of scale-scope spillovers[☆]

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ABSTRACT

We develop a monopolistic competition model where firms are multi-product, and the elasticity of substitution on the consumption side is variable. The cost function, otherwise very general, is such that expanding firm-level product range (scope) reduces marginal costs of production of existing varieties. This captures *scale-scope spillovers*, i.e. within-firm spillovers between the scale at which firms operate and their choices of scope. Firm-level product ranges and the mass of firms are endogenously determined. We show how an increase in market size affects the market outcome. A larger market leads to lower prices, larger outputs, and a wider industry-level product range. Firm-level product ranges expand (shrink) under sufficiently strong (weak) scale-scope spillovers. Last, under strong (weak) spillovers, the number of firms increases less (more) than proportionally to the market size.

1. Introduction

Monopolistic competition models based on Spence (1976) and Dixit and Stiglitz (1977) do not account for the existence of *multi-product firms*, which play a significant role in modern economies. As reported by Bernard et al. (2010), multiproduct firms account for 91 percent of manufacturing sales in the United States. These authors also report a strong positive correlation between product ranges (extensive margins) and per-variety outputs (intensive margins) in firm-level data for US manufacturers. Goldberg et al. (2010) and Manova and Zhang (2009) find the same relationship in the Indian and Chinese data, respectively. However, most of existing theoretical models of multiproduct firms, including (Allanson and Montagna, 2005; Feenstra and Ma, 2007), Dhingra (2013) and (Nocke and Yeaple, 2014), predict the opposite: due to love for variety on the consumer's side, expanding the product range reduces demand for existing varieties. This effect is often referred to as *the cannibalization effect*: there is a firm-level tradeoff between creating new varieties (which dilutes demand and reduces the market shares of existing varieties) and keeping higher demands for varieties already produced (Brander and Eaton, 1984; Spence, 1976). One possible way to reconcile theory with data is to take into account heterogeneity in firm-level productivities, like in Mayer et al. (2014).

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In this paper, we suggest an alternative explanation, which focuses on the interaction between scale economies and scope economies. More specifically, we propose a model of monopolistic competition in which the cost function is defined non-parametrically, and need not even satisfy conditions like additive separability. It is only assumed to possess the property of *scale-scope spillovers*: firms choosing a wider product range tend to be more efficient, that is, they have lower marginal production costs. This assumption is consistent with empirical evidence. For example, Henderson and Cockburn (1996) and Cockburn and Henderson (2001) find a positive correlation between firm size and the efficiency of R&D projects in the pharmaceutical industry. We show that economies of scope in production combined with variable elasticity of substitution can outweigh the diseconomies of scope, thus providing a theoretical justification for the empirical regularities mentioned above.

We consider a one-sector economy with one production factor (labor), in which a continuum of multiproduct firms operates. Firms produce a horizontally differentiated good, and each firm produces a continuum of varieties. The mass of firms is endogenized by imposing the standard assumption of free entry.

On the demand side, each variety-loving consumer is endowed with directly additive preferences.[1] Moreover, any two varieties - no matter whether they are produced by the same firm or by different firms – enter symmetrically the utility function. Thus, we follow (Mayer et al., 2014) in eliminating the cannibalization effect. Otherwise, our approach is conceptually close to the two-tier nested CES setting used by Allanson and Montagna (2005). However, it is well known that CES models of monopolistic competition suffer from some severe drawbacks, e.g. markups and firm sizes are independent of the market size. To obviate these difficulties, we work with the whole non-parametric class of symmetic directly additive preferences, like in Zhelobodko et al. (2012). This allows us to uncover new effects that cannot be captured when preferences are CES.

On the supply side, the technology of each firm is described by a cost function that depends on the total output (firm's size) and the firm-level product range (scope). Unlike most existing papers, we do not assume any specific functional form for the variable cost. Moreover, until Section 4, we do not even assume that the variable cost possesses separability of any kind. We only assume that the variable cost is monotone and convex in both output and product range. More importantly, the variable cost displays positive scale-scope spillovers: firms with wider product ranges have lower marginal production costs. In other words, the cross-derivative of variable cost is negative.

Our main findings can be summarized as follows. First, the firm's choice of scope is determined by two factors: (i) market size, captured by the population size, and (ii) the intensity of scale-scope spillovers. On the one hand, both love for variety and within-firm spillovers push producers towards expanding their product ranges. On the other hand, however, an increase in market size invites more firms to the market, thus triggering a competition effect that forces firms to supply narrower product lines. As a consequence, sufficiently strong spillovers dominate the competition effect, and firm-level product lines expand. However, when spillovers are weak, the competition effect becomes the dominating force, which leads to a contraction of firm-level product lines.

Second, due to love for variety, the mass of varieties in the whole economy, defined as the equilibrium mass of firms times the equilibrium firm-level product range, increases with the market size. As a result, under relatively weak spillovers, the number of firms increases, but their product ranges shrink, when the market grows. Hence, there is a tendency to "monopolization" in smaller markets. However, this ceases to hold if scale-scope spillovers are strong enough, as both the number of firms and firm-level product ranges increase with the market size.

Third, and last, we show that a larger market leads to lower prices and higher per-variety outputs. Indeed, as the total number of varieties in the economy increases, each consumer's per-variety expenditure shrinks. This, in turn, leads to a pointwise reduction in the consumption pattern. In particular, when the inverse demand elasticity is an increasing function of the individual consumption level, an increase in the market size drives markups and prices downwards, while per-variety outputs increase.

Our analysis shows that, whether the relationship between firm' s per-variety output and its product range positive or negative, relies on the strength of scale-scope spillovers. More precisely, when such spillovers are weak, an increase in market size shifts the outputs (intensive margins) and scopes (extensive margins) in the opposite directions. However, these variables move in the same direction under strong spillovers, which concurs with the empirical evidence.

To sum up, our results explain differences in outputs and firm-level product ranges across industries using a very simple, yet general, symmetric setting. Moreover, our analysis suggests a new perspective for future research. For example, some industries involve a large number of small firms, while in other industries (such as tobacco industry) a few dozens of leading firms produce thousands of varieties. Our analysis suggests that these differences may stem from the strength of scale-scope spillovers.

A remark is in order. Our model, although quite general, is tractable enough to obtain a full characterization of equilibrium behavior. Because the model is both flexible and easy to handle, it can be used as a building block for more complex settings. This is to be contrasted to models of multiproduct oligopoly (Johnson and Myatt, 2003, 2006), which are of limited applied use due to their technical complexity.

[1] We say "directly additive" to make a distinction with *indirectly additive* preferences used by Bertoletti and Etro (2017).

1.1. Related literature

There is growing literature on multi-product firms, especially with applications to trade (Dhingra, 2013; Mayer et al., 2014). In what follows, we only mention contributions closely related to ours and compare the results. The main distinctive feature of our approach is that we work with a general symmetric cost function, which allows us to fully characterize the market outcomes in terms of relationships between scale-scope spillovers in production and the elasticity of substitution in consumption.

Ottaviano and Thisse (1999) consider oligopolistic competition of multiproduct firms and find that both firm-level and industry-level product varieties increase with the size of the market. In contrast, Anderson and de Palma (2006) and Feenstra and Ma (2007), who also use oligopolistic settings, find negative relationships between the equilibrium firm-level product range and the number of firms. Our model captures both these regimes of equilibrium behavior: firm-level product variety increases (decreases) with the size of the market under strong (weak) spillover.

Allanson and Montagna (2005) develop a monopolistic competition model with CES preferences, which imply independence of market prices, outputs, and firm-level product ranges from market size. In contrast, our analysis does not rely on any parametric specification of preferences and yields non-trivial and intuitive comparative statics with respect to market size. Eckel and Neary (2010) stress the role of multi-product firms' heterogeneity. Each firm has a core competence in producing a particular variety. They find that an increase in market size does not affect firm-level product ranges, although other globalization effects do. In our model, high (low) scale-scope spillovers strengthen (suppress) benefits from expanding product ranges as the market grows. The borderline case is when variable production costs are per-variety additive.

Mayer et al. (2014) propose a model with linear-quadratic preferences, marginal-cost random draws a Melitz (2003), and core competencies. They find that an increase in market size shifts prices and firm-level product ranges down. Again, this need not be the case in our model: under sufficiently strong spillovers, firms can expand their product range on larger markets.

Nocke and Yeaple (2014) consider an economy where multi-product firms are heterogeneous in their organizational capabilities. These authors find that trade liberalization (defined as a reduction in tariffs) always shifts outputs and scopes in opposite directions. In our model, firm-level product ranges and per-variety outputs both increase in response to a hike in the market size when scale-scope spillovers are sufficiently strong. This echoes the empirical results obtained by Bernard et al. (2010).

Finally, recent contributions by Bertoletti and Etro (2016) and Parenti et al. (2017) stress the importance of the demand side in imperfect competition models with free entry. Using parsimonious settings with single-product firms but very general symmetric preferences, these authors show that the fundamental role in shaping the market outcomes belongs to the properties of the *elasticity of substitution across varieties*, namely, how it behaves with respect to the mass of firms and individual consumption levels. Although we focus here on additive preferences, our results clearly show that the behavior of the elasticity of substitution keeps its relevance in the multi-product case. However, we also show that, when firms are multi-product, the cost side plays an equally important role. This is due to the presence of scale-scope spillovers, i.e. the cross-effects between firm-level outputs and product ranges, captured by the (otherwise very general) cost function. Thus, the market outcomes are eventually shaped by the *interplay* between the elasticity of substitution, on the demand side, and the scale-scope spillovers, on the supply side.

The rest of the paper is organized as follows. Section 2 describes the model. In Section 3, we formulate the equilibrium conditions and prove existence and uniqueness of a free -entry equilibrium. We also provide comparative statics with respect to the market size captured by the population size. Section 4 provides more insight into the role of spillovers, dealing with some benchmark cases. Section 5 concludes. Most technical proofs can be found in the Appendix.

2. The model

We consider an economy involving one production factor – labor – and one sector producing a horizontally differentiated good. The set of varieties supplied on the market is a two-dimensional continuum: each variety, ij, is labeled by its producer, $j \in [0, N]$, and the index $i \in [0, n_j]$ of a variety within firm j's product line. Here N is the number - or, more precisely, the mass - of active firms, while n_j is firm j's product range, or scope.[2] Fig. 1 illustrates a possible pattern of product ranges.

Each active firm j's behavior is fully described by (i) firm j's *scope*, n_j, which is defined as the size of its product range, and (ii) firm j's output vector, or *production pattern*, $\mathbf{q}_j \equiv (q_{ij})_{i \in [0, n_j]}$. The commodity is horizontally differentiated, both across firms and within the product lines of the firms.

2.1. Consumers

The economy is endowed with L identical consumers/workers. Each consumer inelastically supplies one unit of labor. Given the price vector $\mathbf{p} \equiv (p_{ij})_{i \le n_j,\, j \le N}$, each consumer chooses her individual consumption vector $\mathbf{x} \equiv (x_{ij})_{i \le n_j,\, j \le N}$ so that

[2] More formally, n_j is the density of firm-level product ranges defined over the set $[0, N]$ of firms.

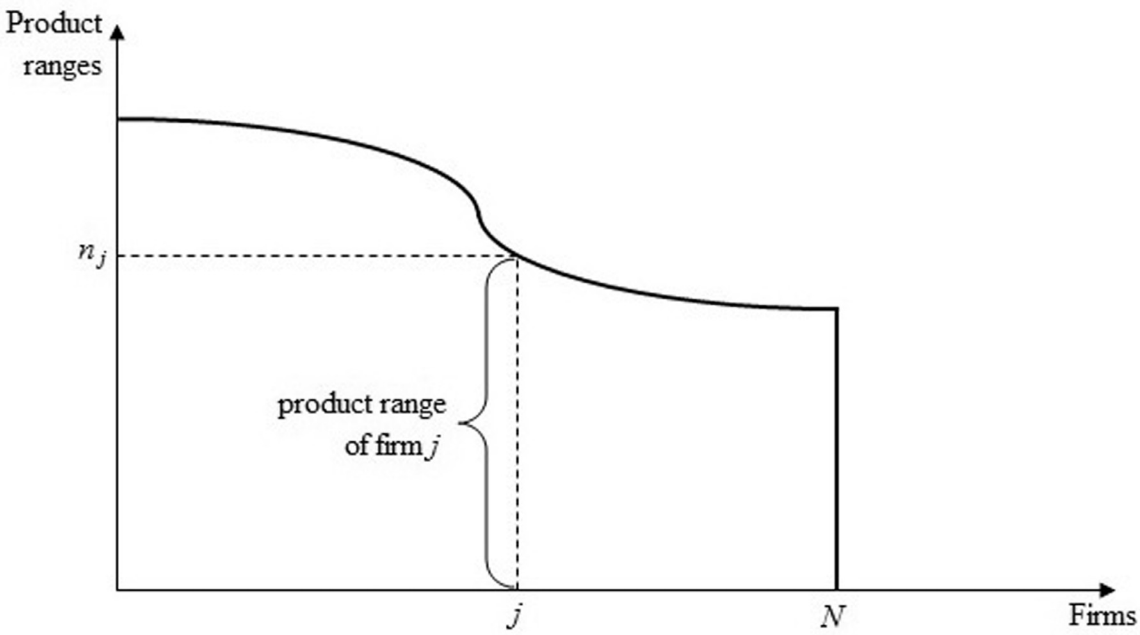

Fig. 1. A possible pattern of product ranges.

the following utility maximization problem is solved:

$$\max_{\mathbf{x}} \mathcal{U}(\mathbf{x}) \equiv \int_0^N \int_0^{n_j} u(x_{ij}) di dj \qquad \text{s.t.} \qquad \int_0^N \int_0^{n_j} p_{ij} x_{ij} di dj \leq \mathcal{I},$$

where $u(\cdot)$ is the *subutility function*, its value $u(x_{ij})$ capturing the utility of consuming variety i produced by firm j, while $\mathcal{I} > 0$ is consumer's individual income.

Two comments are in order. First, the utility function $\mathcal{U}$ is assumed to be *directly additive across all varieties*, both within and across firms. As a consequence, all varieties are equally substitutable, regardless of whether they are produced by the same firm or by different firms. Contrast to this, Allanson and Montagna (2005) assume within-firm varieties to be closer substitutes than varieties of different firms. This approach is appealing, because it captures both toughness of competition across firms and the cannibalization effect by means of two independent parameters.[3] However, extending this modeling strategy to the general case of variable elasticity of substitution results in a non-tractable model. Moreover, it is outside the main focus of this paper, which is the impact of within-firm spillovers on the market outcome.

Second, consumer's income $\mathcal{I}$ includes wages and shares in profits. Choosing homogeneous labor to be the numéraire, we can normalize wages to 1. Moreover, profits equal to zero in a free-entry equilibrium. Therefore, in what follows, we set $\mathcal{I} = 1$.

The subutility function $u(\cdot)$ is assumed to be strictly increasing (either over the whole $\mathbb{R}_+$ or at least over a compact interval $[0, \bar{x}]$, where $\bar{x} > 0$), strictly concave, thrice continuously differentiable, and satisfying $u(0) = 0$. These assumptions are not too restrictive, as they allow encompassing a broad range of widely used preference specifications, including (i) the CES preferences, where $u(x) = x^{\rho}$, (ii) the CARA preferences (Behrens and Murata, 2007), where $u(x) = 1 - \exp(-ax)$, (iii) the Stone-Geary-type preferences (Simonovska, 2015), where $u(x) = \ln(b + x) - \ln b$, $b > 0$, etc.

Due to additive separability of $\mathcal{U}$, it is relatively easy to determine the inverse demand for variety i produced by firm j:

$$p_{ij} = \frac{u'(x_{ij})}{\lambda}, \tag{1}$$

where λ is the Lagrange multiplier of consumer's problem, which captures the marginal utility of income and is given by

$$\lambda = \int_0^N \left(\int_0^{n_j} x_{ij} u'(x_{ij}) di \right) dj. \tag{2}$$

As pointed out below (see Section 2.2), λ can be viewed as a market aggregate, which has a nature similar to the ideal price index under CES preferences (Dixit and Stiglitz, 1977). Another object crucial for the subsequent analysis is the

[3] Dhingra (2013) develops a similar approach but works with linear-quadratic preferences instead of CES.

behavior of the *inverse demand elasticity* $r(x)$, which is given by

$$r(x) \equiv -\frac{xu''(x)}{u'(x)}. \tag{3}$$

Note that $r(x)$ is also the inverse measure of substitutability across varieties. Namely, as shown by Zhelobodko et al. (2012), if $\sigma(x)$ is the elasticity of substitution associated with an additive preference and evaluated at a symmetric consumption pattern, then the following identity holds:

$$r(x) = \frac{1}{\sigma(x)}. \tag{4}$$

Eq. (4) explains the important role of the elasticity of substitution in our analysis.

Although we do not impose any parametric specification of preferences, we still have to make some additional assumptions which guarantee that the profit-maximization problem (to be defined in Section 2.2) is well behaved. First, assume that

$$0 < r(x) < 1, \tag{5}$$

at least over an open interval $(0, \widehat{x})$ where $\widehat{x} > 0$. Second, to ensures global concavity of each firm's revenue with respect to outputs, we assume that the following inequality holds:

$$-\frac{xu'''(x)}{u''(x)} < 2. \tag{6}$$

Third, we focus mainly (although not totally) on the case when the inverse demand elasticity increases in individual consumption level: $r'(\cdot) > 0$. This property is appealing, as it delivers a necessary and sufficient condition for a pro-competitive effect of a larger market: an increase in market size L leads unambiguously to a reduction in the equilibrium price and equilibrium markup[4] (see Proposition 2 below). In contrast, when $r'(\cdot) < 0$, the anti-competitive effect becomes a dominating force, which is a less realistic result. The CES case, where $r(x)$ is constant, is a limiting case. Under CES preferences, comparative statics with respect to market size turn out to be redundant. See Zhelobodko et al. (2012) for more discussion on this issue.

2.2. Producers

Each firm $j \in [0, N]$ chooses the size n_j of its product range, and a production pattern $\mathbf{q}_j = (q_{ij})_{i\in[0,n_j]}$, where q_{ij} is the output of variety $i \in [0, n_j]$. In other words, firms make decisions about how many varieties to produce and how much of each variety is produced.

Firms share the same technology: each firm incurs a fixed cost $F > 0$ and a variable cost $V(\mathbf{q}_j, n_j)$, which is assumed to be increasing and convex with respect to both arguments. To capture the idea that variable costs are symmetric across varieties, we also assume that V satisfies the following *symmetry condition*:

$$V\left(\mathbf{q}^1, n\right) = V\left(\mathbf{q}^2, n\right), \tag{7}$$

where $\mathbf{q}^1$ and $\mathbf{q}^2$ are two production plans which can be obtained from one another by reshuffling varieties.

Variable costs satisfying (7) are widely used in the literature. For example, Allanson and Montagna (2005) and Feenstra and Ma (2007) assume that

$$V(\mathbf{q}, n) = c\int_0^n q_i \mathrm{d}i + \phi n,$$

while Nocke and Yeaple (2014) consider a variable cost given by

$$V(\mathbf{q}, n) = c(n)\int_0^n q_i \mathrm{d}i + \phi n, \qquad c'(n) > 0. \tag{8}$$

In both cases, $\phi > 0$ stands for the per product line fixed cost, while $c > 0$ and $c(n)$ stand for the marginal production costs, which are the same for all varieties. We stress here that, contrast to the vast majority of other authors, we impose no parametric restrictions on variable cost.

[4] There is ample empirical evidence in favor of pro-competitive effects in most of the real-world markets. See, e.g., Handbury and Weinstein (2015), who observe that the price level for food products falls with city size.

2.2.1. Profit maximization

Each firm j seeks to maximize its profit with respect to the scope n_j and the production pattern $\mathbf{q}_j$. Each firm is a price-maker at the market of each variety it chooses to produce. The marginal utility of income λ is taken by firms as given, because each producer is negligible to the market.[5]

Firm j seeks to solve the profit-maximization program:

$$\max_{\mathbf{q}_j, n_j} \Pi(\mathbf{q}_j, n_j) \equiv \mathcal{R}(\mathbf{q}_j, n_j) - V(\mathbf{q}_j, n_j) - F, \tag{9}$$

where $\mathcal{R}(\mathbf{q}_j, n_j)$ is firm j's revenue. Using the inverse demands given by (1), firm j's total revenue can be written as follows:

$$\mathcal{R}(\mathbf{q}_j, n_j) = \frac{1}{\lambda}\int_0^{n_j} u'(q_{ij}/L)\, q_{ij}\mathrm{d}j. \tag{10}$$

Because all firms face the same Lagrange multiplier λ, the profit functions Π_j are identical for all $j \in [0, N]$. For this reason, we drop the firm's index j in the rest of the paper.

Clearly, the revenue concavity condition (6) and convexity of variable cost $V(\mathbf{q}, n)$ with respect to $\mathbf{q}$ imply strict concavity of profits with respect to $\mathbf{q}$ under a given n. Furthermore, combining the symmetry property (7) of variable cost with Eqs. (9) and (10), we find that the profit function $\Pi(\mathbf{q}, n)$ is symmetric in $\mathbf{q}$. As a consequence, the profit-maximizing production pattern of each firm is symmetric, i.e. $q_i = q$ for all $i \in [0, n]$. Hence, the profit maximization program (9) can be simplified by reformulating it as a two-dimensional optimization problem:

$$\max_{y, n \geq 0} \pi(y, n) \equiv R(y, n) - v(y, n) - F. \tag{11}$$

Here y is firm's *total output*, $y \equiv \int_0^n q_i \mathrm{d}i = nq$, while $R(y, n)$ and $v(y, n)$ stand for, respectively, firm's revenue and variable costs evaluated at a symmetric outcome:

$$R(y, n) \equiv \frac{1}{\lambda} u'\left(\frac{y}{Ln}\right) y, \tag{12}$$

$$v(y, n) \equiv V\left(\frac{y}{n}\mathbb{I}_{[0,n]}, n\right), \tag{13}$$

where $\mathbb{I}_A$ is the indicator function of $A \subseteq \mathbb{R}$. Because profit functions are symmetric and identical across firms, it must be that the market outcome is also symmetric. Thus, referring to $v(y, n)$ and $R(y, n)$ as firm's variable cost and revenue without further qualifications should not be confusing. In what follows, we assume that v is differentiable, increasing and convex in both arguments. In other words, the following conditions are satisfied:

$$v_y > 0, \quad v_n > 0, \quad v_{yy} \leq 0, \quad v_{yy}v_{nn} - v_{yn}^2 \geq 0. \tag{14}$$

2.2.2. Scale-scope spillovers

It remains to describe the interaction between scale and scope economies. Most theoretical papers impose some separability of variable costs with respect to output and scope. However, as discussed in the Introduction, the existence of positive spillovers between scale and scope economies has empirical support. In our context, this means that marginal production costs decrease with the size of the product range:

$$v_{yn} \leq 0,$$

which we assume throughout.[6] By introducing such spillovers, we capture the effects found in the data by Henderson and Cockburn (1996) and Cockburn and Henderson (2001). These authors report positive correlation between firm sizes and efficiency of design projects in pharmaceutical industry.

To illustrate where positive spillovers may come from, consider the following example:

$$V(\mathbf{q}, n) = n^{\alpha}\int_0^n q_i^{\beta}\mathrm{d}i + n^{\gamma}, \tag{15}$$

where $\beta, \gamma > 1$, while α can be either positive or negative. The first term in (15) accounts for the costs of production, whereas the second term captures the cost of developing new varieties. The parameter α can be interpreted, in the spirit of

[5] This point requires clarification. Even though we work with monopolistic competition, how can a firm producing a continuum of varieties be negligible to the market? To understand the intuition behind this, note that (i) firm j's output q_{ij} of product $i \in [0, n_j]$ can be viewed as the density of the bivariate distribution of outputs, and (ii) using (2), the market aggregate λ can be expressed as follows:

$$\lambda = \frac{1}{L}\int_0^N \left(\int_0^{n_j} q_{ij}u'(q_{ij}/L)di\right)dj.$$

Clearly, if firm j changes its production plan, while all the other firms choose to maintain the status quo, then the value of λ remains unchanged. This is because firm j's product line $[0, n_j]$ has zero measure with respect to the double integral $\int_0^N \left(\int_0^{n_j} q_{ij}u'(q_{ij})di\right)dj$, even though it has positive "length" n_j. This justifies the claim that each firm is negligible to the market.

[6] We only require that weak inequality holds to that encompass the limiting case where $v_{yn} = 0$, which most of the literature focuses on.

Ethier (1982), as a measure of interaction between specialization effect and complexity effect: $\alpha < 0$ ($\alpha > 0$) means that the former (latter) effect dominates. If $\alpha = 0$, then the two effects exactly outbalance each other, which corresponds to additive separability of production cost across varieties (see more on the role of this property in Section 4).

At a symmetric outcome, variable cost (15) amounts to

$$v(y, n) = n^{1+\alpha-\beta} y^{\beta} + n^{\gamma}. \tag{16}$$

From (16) we find $v_{yn} = (1 + \alpha - \beta)\beta n^{\alpha-\beta} y^{\beta-1}$, which implies that v meets the assumption of positive scale-scope spillovers ($v_{yn} \le 0$) if and only if $\alpha < \beta - 1$, i.e. when the “complexity effect” is not too strong.

2.3. Symmetric free-entry equilibrium

Using (12) and (13), the first-order conditions to the symmetrized profit maximization program (11) are given by

$$\frac{p - v_y}{p} = r(x), \tag{17}$$

$$\frac{R/n - v_n}{R/n} = 1 - r(x), \tag{18}$$

where x is the per-variety individual consumption level pinned down by the market-clearing condition

$$Lnx = y, \tag{19}$$

while $r(x)$ is the inverse demand elasticity defined by (3).

Eq. (17) is the standard monopoly-pricing rule: the profit-maximizing markup equals the inverse demand elasticity. Eq. (18), although it looks less familiar, has similar intuitive content: the percentage difference between per-variety revenue, $R/n = pq = pLx$, and the marginal cost v_n of developing a new variety equals the elasticity of per-variety revenue with respect to x.

Free entry implies that profits are zero in equilibrium, i.e. total revenue equals total cost:

$$R(y, n) = v(y, n) + F. \tag{20}$$

To close the model, we impose the labor balance condition, which is given by

$$L = N(v(y, n) + F). \tag{21}$$

The meaning of (21) is that the total labor supply in the economy must equal the total labor demand.[7] Note that, due to the zero-profit condition (20), labor balance (21) is equivalent to the budget constraint.

We define a *symmetric free-entry equilibrium (SFE)* as a bundle (p^*, y^*, n^*, N^*) solving (17)–(21).

3. Market outcome

This section contains our main results. Namely, we prove existence and uniqueness of the SFE, and provide comparative statics with respect to the market size L.

3.1. Reduced equilibrium conditions and equilibrium existence

We start with simplifying the equilibrium equations and providing conditions for existence and uniqueness of the SFE.

Combining (17) and (18) yields

$$R(y, n) = yv_y + nv_n,$$

which, together with the zero-profit condition (20), implies

$$yv_y + nv_n = v(y, n) + F. \tag{22}$$

Dividing (17) over (18), we obtain

$$\frac{nv_n}{yv_y} = \frac{r(y/nL)}{1 - r(y/nL)}. \tag{23}$$

The system (22) – (23) contains only two endogenous variables: total output y and the product range n. Thus, given that (y^*, n^*) solves (22) and (23), the equilibrium price p^* and the equilibrium mass of firms N^* are uniquely determined from (20) and (21):

$$N^* = \frac{L}{v(y^*, n^*) + F}, \qquad p^* = \frac{v(y^*, n^*) + F}{y^*}.$$

[7] Recall that labor is the only production factor in our model, which implies that both fixed and variable costs are expressed in labor.

As a consequence, proving existence and uniqueness of an SFE boils down to showing that system (22) – (23) has a unique solution. This is true under minor additional restrictions on variable cost $v(\cdot,\ \cdot)$ and fixed cost F. More specifically, the following proposition holds.

Proposition 1. *Assume that there exists a positive (but arbitrarily small) number δ, such that*

$$\frac{yv_y}{v} + \frac{nv_n}{v} > 1 + \delta \text{ for all } (y, n) \in \mathbb{R}_+^2. \tag{24}$$

Then, (i) there exists a SFE; (ii) there exists a positive (possibly infinite) threshold value $\overline{F}$ of fixed costs, such that the SFE is unique for any $F < \overline{F}$; (iii) if variable cost v is positive homogeneous of some degree exceeding 1, then $\overline{F} = \infty$.

Proof. In the Appendix. □

Intuitively, (24) means variable cost v should be "a little more convex" then a linear function. The need for condition (24) stems from assuming away the cannibalization effect. The condition (24) would not matter if we worked with the following more complex utility function:

$$\mathcal{U}(\mathbf{x}) \equiv \int_0^N \psi\left(\int_0^{n_j} u(x_{ij})di\right)dj,$$

where $\psi(\cdot)$ is an increasing and strictly concave function defined over $\mathbb{R}_+$. However, this approach increases drastically the complexity of the problem and fails to generate clear-cut predictions.

3.2. Comparative statics: prices, per-variety outputs, and industry-level product variety

The key qualitative feature of the system (22) and (23), which makes it possible to obtain clear-cut comparative statics results without specifying the subutility $u(\cdot)$ and the variable cost $v(\cdot,\ \cdot)$ parametrically, is that a change in L does not shift the locus of (22), shifting only the curve defined by (23).

The following Proposition characterizes comparative statics of equilibrium with respect to the market size L.

Proposition 2. *Assume that $r'(\cdot) > 0$. Then, in response to an increase in the market size L, the equilibrium price p^* decreases, the equilibrium per-variety output q^* increases, and the equilibrium industry-level product variety n^*N^* increases.*

Proof. In the Appendix. □

The intuition behind Proposition 2 is as follows. As the market becomes larger, per-variety profits increase. This, in turn, invites new varieties to the market, due to both entry of new firms and expanding product ranges of the incumbent firms. As a result, the industry-level product range n^*N^* increases. Moreover, due to the binding budget constraint and love for variety, it must be that the equilibrium per-variety consumption level x^* decreases. Consequently, the value of the inverse demand elasticity $r(x^*)$ also decreases. Hence, the elasticity of substitution increases, and product differentiation becomes less deep. This, in turn, makes competition tougher and leads to a reduction in prices and an increase in quantities supplied. Finally, one can show that the results of Proposition 2 get reversed under $r'(\cdot) < 0$.

3.3. Comparative statics: firm-level product ranges and firm-level total outputs

We now study how an increase in the market size L affects the firm-level scope n^* and firm-level total output $y^* = n^*q^*$. To conveniently formulate our results, we introduce additional terminology. We say that the elasticity of marginal production cost with respect to product range $-nv_{yn}/v_y$ captures *scale-to-scope spillovers*, i.e. spillovers that the production department of a firm gains from the R&D department. Similarly, we say that $-yv_{ny}/v_y$ captures *scope-to-scale spillovers*. Note that both these cross-elasticities are non-negative because we impose $v_{yn} \leq 0$. Furthermore, the marginal cost elasticities yv_{yy}/v_y and nv_{nn}/v_n, which are also non-negative due to monotonicity and convexity of variable cost v, can be viewed as, respectively, measures of diseconomies of scale and diseconomies of scope.

Proposition 3. *An increase in market size L shifts the total output y^* and the equilibrium firm-level product range n^* in the same direction if and only if spillovers are sufficiently strong, namely, if*

$$-v_{yn} > \frac{\min\{y^2v_{yy},\ n^2v_{nn}\}}{yn}. \tag{25}$$

Otherwise, y^* and n^* move in the opposite directions in response to a positive shock in L.

Proof. In the Appendix. □

The result of Proposition 3 highlights the role of scale-scope spillovers. If they are sufficiently strong, then a larger market will motivate firms to both increase scales of operation and expand product ranges. In contrast, when spillovers are weak, tougher competition on a growing market will result in a reduction of firm-level product variety, but not to a decrease in total output, which will increase due to a hike in per-variety outputs (see Proposition 2 above).

Proposition 3 compares the changes in y^* and n^* in the SFE under an increase in the market size, but it is agnostic about the direction of these changes. In what follows, we focus on comparative statics of the scope n^*, as the behavior of y^* will follow immediately from Proposition 3. We now provide a characterization of the equilibrium firm-level product range behavior under a population shock.

Proposition 4. *Assume that $r'(x) > 0$. Then, under an increase in the market size L, the SFE firm-level product range n^* increases if and only if scale-to-scope spillovers dominate diseconomies of scale in equilibrium, i.e. the following condition holds:*

$$-\frac{nv_{yn}}{v_y} > \frac{yv_{yy}}{v_y}. \tag{26}$$

Proof. In the Appendix. □

One can show that, when $r'(\cdot) < 0$, the result of Proposition 3 is reversed: the condition (26) becomes a necessary and sufficient condition for the scope to *shrink* (rather than expand) in response to an increase in the market size L. Finally, when $r'(\cdot) = 0$ (i.e. when we fall back to the CES case), the scope of each firm remains unchanged as the market size increases.

The intuition behind Proposition 4 is as follows. Under relatively weak scale-scope spillovers (i.e., when v_{yn} is close to zero), firms have no incentive to expand their product range, because an increase in L shifts individual consumption x down. As a consequence, the elasticity of substitution $1/r(x)$ increases, which makes varieties less differentiated and results in tougher competition. Hence, firms don't gain much from scope economies. However, if the spillovers are strong, then an expansion of firm's product line leads to a substantial reduction in firm's marginal production costs. Thus, on a larger market producers are motivated to develop new varieties, even if the degree of their monopoly power (which stems from product differentiation) shrinks.

3.4. Two examples

Propositions 2 and 4 imply that, under sufficiently strong spillovers, both q^* and n^* increase at a growing market, which is in line with the empirical evidence provided by Bernard et al. (2010). To gain more intuition about the effects triggering such an outcome, we illustrate them with two examples.

Example 1. Consider again variable cost of type (16). It is readily verified that

$$-\frac{nv_{yn}}{v_y} = \beta - \alpha - 1, \qquad \frac{yv_{yy}}{v_y} = \beta - 1.$$

Thus, (26) is equivalent $\alpha < 0$, that is, when specialization effect is stronger than the complexity effect. But according to Propositions 2 and 3, the inequality (26) is the exact condition when an increase in L triggers hikes in both $q^*$and n^*. On the contrary, when complexity effect exceeds specialization effect, i.e. when $\alpha > 0$, we fall back to the case when n^* and q^* are shifted in the opposite direction as the market size increases. Finally, when $\alpha = 0$, i.e. when the cost function is additively separable across varieties, firm-level product ranges in SFE remain unchanged. It is shown below (see Section 4) that this is always the case under per-variety additive cost, more general than (15).

Example 2. Our second example is a numerical one. Consider the Stone-Geary-type subutility (Simonovska, 2015) given by $u(x) = \ln(a + x) - \ln a$, and set $a = 1$. Then, it is readily verified that the corresponding inverse demand elasticity is given by $r(x) = x/(x+1)$, and that $r'(x) > 0$ for all $x > 0$. Furthermore, consider a linear-quadratic variable cost with an interaction term:

$$v(y, n) = cy + \phi n + \frac{y^2}{2} + \frac{n^2}{2} - \theta ny,$$

where c, $\phi > 0$, $0 < \theta < 1$. Finally, set $c = 15.89$, $\phi = 4$, $\theta = 0.6$, $F = 9$. Fig. 2 demonstrates the behavior of total output and scope under market size variations.

One can see from Fig. 2 that, if L is not too large, both firm-level total output y^* and scope n^* increase when L grows. Moreover, per-variety output q^*, which equals the reciprocal slope of a ray connecting the origin and the equilibrium point, also increases. Thus, a positive shock in L shifts both q^* and n^* upwards. The dotted cone on Fig. 2 is the domain of *low scale-scope spillovers*, i.e. where y^* and n^* are driven by a hike in L to the opposite directions, which is in accordance with Proposition 3. Outside the dotted cone, the scale-scope spillovers are sufficiently strong, i.e. condition (25) is met.

3.5. Comparative statics: the mass of firms

Finally, we study how the mass of firms N^* behaves under a positive shock in the market size L. The answer to this question is as follows.

Proposition 5. *Assume that $r'(\cdot) > 0$. Then the equilibrium mass of firms N^* increases more than proportionally in response to a hike in L if and only if the following inequality holds:*

$$\frac{yv_{yy}}{v_y} + \frac{nv_{yn}}{v_y} > \frac{nv_{nn}}{v_n} + \frac{yv_{ny}}{v_n}. \tag{27}$$

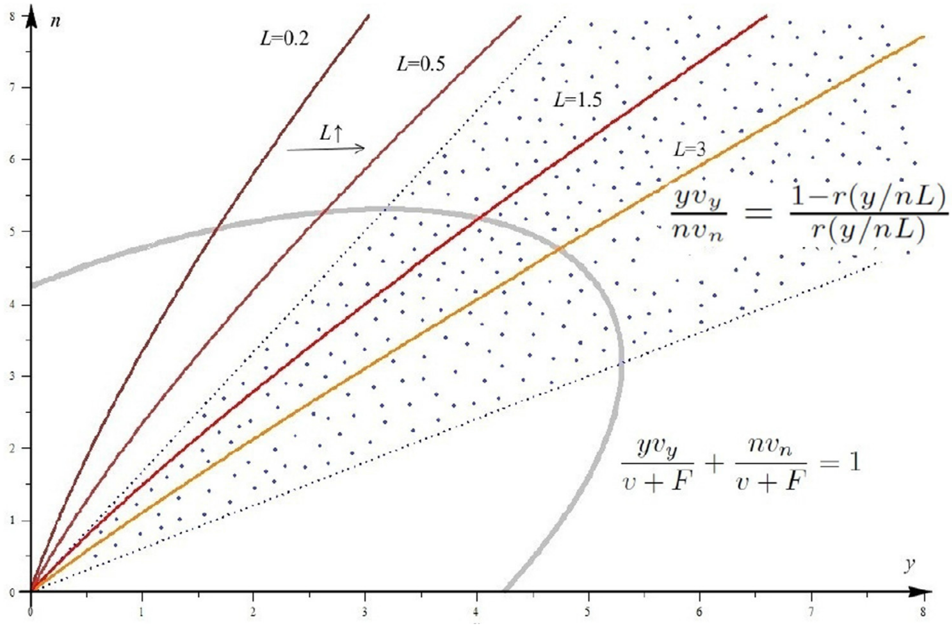

Fig. 2. Total output and scope behavior under market size growth: the case of linear-quadratic variable cost.

Proof. In the Appendix. □

The left(right)-hand side of (27) can be interpreted as a measure of diseconomies of scale (scope) *net of scale-scope spillovers*. In other words, showing how the marginal production cost v_y (respectively, the innovation cost v_n) changes under a simultaneous 1% increase in y and n, or, equivalently, a 1% increase in n when per-variety output q remains unchanged. Thus, Proposition 5 suggests a test of whether the mass of firms changes more or less than proportionately to an increase in the population size L. The key-factor is the relative role of scale and scope, i.e. whether diseconomies of scale are higher or lower than diseconomies of scope. If the impact of scope expansions on the reduction of marginal production costs is low, then an increase in L results in more firms rather than more varieties per firm.

4. Separable cost functions

To better understand the role of scale-scope spillovers, we consider in this section two special cases: (i) the case where there are *no scale-scope spillovers*, and (ii) the case where production costs are *additive across varieties*.

4.1. No spillovers

Consider first a variable cost given by

$$V(\mathbf{q}, n) \equiv \vartheta\left(\int_0^n q_i di\right) + S(n), \tag{28}$$

where the functions $\vartheta(\cdot)$ and $S(\cdot)$ are strictly increasing and strictly convex. The cost function given by (28) suggests perfect substitutability of varieties from the technological viewpoint. Moreover, it exhibits zero scale-scope spillovers. Indeed, at a symmetric outcome (28) becomes

$$v(y, n) = \vartheta(y) + S(n), \tag{29}$$

which implies $v_{yn} = 0$ for all $y, n \geq 0$. Apparently, (29) is the most general variable cost that does not exhibit any spillovers.

Proposition 6. *Assume that there are no scale-scope spillovers, and $r'(x) > 0$. Then, an increase in market size L triggers (i) a hike in firm-level total output y^*; (ii) a reduction in the firm-level product range n^*; (iii) an increase in the mass of firms N^*.*

Proof. In the Appendix. □

Several comments are in order. First, Proposition 6 implies that, when population L increases, the market always become less concentrated in the sense that additional entry is invited, while firm-level product ranges shrink. Note that this result is the reverse of what Feenstra and Ma (2007) find.

Second, Proposition 6 also highlights the role of spillovers in our analysis. Namely, parts (i) and (ii) immediately imply that, when no scale-scope spillovers are at work, changes in firm-level product ranges (extensive margins) n^* decrease at a growing market. Contrast to this, per-variety outputs (intensive margins) q^* increase with L by Proposition 1. In other words, changes in equilibrium values of these two variables are negatively associated. As noted above, this is not in the line with the empirical findings, particularly those of Bernard et al. (2010).

Third, and last, observe that, in the absence of scale-scope spillovers, condition (27) takes the form:

$$\frac{y\vartheta''(y)}{\vartheta'(y)} > \frac{nS''(n)}{S'(n)}.$$

In other words, the number of new firms entering the growing market is more than proportional to the degree of market expansion if and only if *diseconomies of scale exceed diseconomies of scope*.

4.2. Per-variety additive costs

We now consider another extreme case, namely, when the variable cost is additively separable across varieties. To illustrate, recall the variable cost

$$V(\mathbf{q}, n) = \int_0^n q_i^\beta \mathrm{d}i + n^\gamma, \tag{30}$$

which is a special case of (15) where $\alpha = 0$. Consider the following extension of (30):

$$V(\mathbf{q}, n) = \int_0^n \nu(q_i)\mathrm{d}i + S(n), \tag{31}$$

where $\nu(\cdot)$ and $S(\cdot)$ are both positive, increasing, and convex functions. The cost structure given by (31) captures the case when each variety is produced by a separate branch of the firm, and there are no spillovers between different. Accordingly, $\nu(q_i)$ may be viewed as a variable cost of branch $i \in [0, n]$, while $S(n)$ is the cost of designing n varieties. Strict convexity of $\nu(\cdot)$ describes diseconomies of scale in producing each variety, while convexity of $S(\cdot)$ means that the marginal labor requirement $S'(n)$ for designing a new variety increases with the product range n. In other words, as the range of varieties produced by a firm expands, finding ideas for further innovation becomes more effort-consuming.

The following Proposition provides a characterization of comparative statics with respect to the market size under the cost function satisfying (31).

Proposition 7. *Assume that the variable cost $V(\mathbf{q}, n)$ is per-variety additive, i.e. it satisfies (31), while the demand side is such that $r'(\cdot) > 0$. Then, in response to an increase in the market size L, the scope n^* remains unchanged, while the mass of firms N^* increases.*

Proof. In the Appendix. □

The most striking result in Proposition 7 is that the equilibrium firm-level product range is *neutral to the market size*. The intuition behind this neutrality is as follows: when variable cost is additive across varieties, the manager who is in charge of a specific variety maximizes per-variety profit regardless of what other managers do. In other words, everything works *as if firms were single-product*.

5. Concluding remarks

We have developed a model of a monopolistically competitive industry with multi-product firms, under unspecified technologies which possess the property of scale-scope spillovers. Our goal was to study the implications of an increase in market size. We found that the patterns followed by firm-level product ranges and the number of firms in the industry are triggered by the interaction between the scale-scope spillovers and the shape of the demand schedule. Namely, under increasing inverse demand elasticity, the equilibrium firm-level product range expands (shrinks) while the number of firms increases less (more) than proportionately to the increase in the market size if and only if the intensity of scale-scope spillovers is sufficiently high (low). Another finding is that prices decrease and quantities supplied increase in response to a market size increase under increasing inverse demand elasticity, due to more severe competitive pressure. When inverse demand elasticities decrease, this result is reversed.

Our model is in line with empirical evidence provided by Bernard et al. (2010), who report significant positive correlation between intensive and extensive margins of U.S. manufacturing firms. In particular, we have found that intensive margins (quantities supplied) and extensive margins (scopes) can co-move in response to an increase in the market size, provided scale-scope spillovers are highly intensive.

At least three extensions of our model seem to be relevant. First, heterogeneity among firms can be introduced in order to study the distribution of product ranges, as well as the interactions of heterogeneity and scale-scope spillovers. There are at least two modeling strategies to achieve this. The first option is “horizontal” heterogeneity: each firm has a core competency in producing a specific variety, as in Eckel and Neary (2010). This approach is appealing because most of the results depending crucially on symmetry assumptions are still likely to hold. Moreover, it allows us to transform our model of non-localized competition to a model with semi-localized competition, thus bringing space into the story. An alternative modeling strategy is to introduce "vertical" heterogeneity, i.e. to assume that firms are ordered with respect to their randomly drawn productivity, like in Melitz (2003). This strategy has already proven a powerful tool for studying the impact of market size on firms' size distribution and product range distribution (Mayer et al., 2014).

Second, an extension of our model to the open economy case could provide new insights about the impact of multi-product firms and within-firm spillovers on international trade. From the international trade perspective, we have only compared autarky to free trade, as an increase in the market size can be interpreted as opening borders between two countries (Krugman, 1979). A possible way to say more on the impact of trade liberalization on firm-level product ranges is to study comparative statics with respect to iceberg trade costs. Furthermore, our approach is potentially applicable to the "outsourcing vs FDI" trade-off, multi-national firms, and exporting behavior.

Finally, endogenizing firm-level choices between producing either a single product or a variety of products within our setting also seems both doable and appealing. Again, there are two possible approaches: either to model explicitly the process of formation of multi-product firms as a result of cooperation/collusion, or to focus on endogenous investment in productivity, quality, and/or product diversity, as in Fauli-Oller et al. (2012) and Eckel et al. (2011). We leave these tasks for future research.

Appendix

Proof of Proposition 1

To prove Proposition 1, it suffices to show that the system (22) - (23) has a unique solution. A convenient way to do this is to make the following change of variables:

$$x \equiv \frac{y}{Ln}, \quad v \equiv v(y, n).$$

We first show that, for each given level $v = v_0$ of the variable cost, Eq. (23) has a unique solution $\hat{x}(v_0)$. Indeed, (23) can be restated as

$$k(x, v_0) = \frac{Lxr(x)}{1 - r(x)}, \tag{32}$$

where $k(x, v)$ is defined by

$$k(x, v) \equiv \frac{v_n(y(x, v),\ n(x, v))}{v_y(y(x, v),\ n(x, v))}.$$

The value of $k(x, v)$ has the following geometric interpretation. Consider an isocost $v(y, n) = v_0$ on the (n, y)-plane, and determine the intersection point of this isocost with a ray starting from the origin and such that its slope equals Lx. Then, $k(x, v_0)$ is the absolute value of the slope of the isocost at the intersection point. As v is assumed to be increasing and convex, it is clear that $k(x, v_0)$ decreases with x for any $v_0 > 0$.

On the other hand, the expression on the right-hand side of (32) increases in x. To show this, we differentiate $xr(x)/(1 - r(x))$ with respect to x and obtain

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{xr(x)}{1 - r(x)}\right) = \frac{xr'(x) + r(x)(1 - r(x))}{(1 - r(x))^2}. \tag{33}$$

Using the identity

$$xr'(x) = r(x)\left(1 + r(x) + \frac{xu'''(x)}{u''(x)}\right) \text{ for all } x > 0, \tag{34}$$

(33) can be rewritten as follows:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{xr(x)}{1 - r(x)}\right) = \frac{r(x)}{(1 - r(x))^2}\left(2 + \frac{xu'''(x)}{u''(x)}\right), \tag{35}$$

which is positive by the revenue concavity condition (6). Hence, by the intermediate value theorem, there exists a unique solution $\hat{x}(v_0)$ to (32). Furthermore, since $v(y, n)$ is increasing and convex in (y, n), the system

$$v(y, n) = v_0, \qquad y = Ln\hat{x}(v_0)$$

uniquely determines $\big(\hat{y}(v_0),\ \hat{n}(v_0)\big)$.

It remains to pin down v_0. Plugging $\big(\hat{y}(v_0),\ \hat{n}(v_0)\big)$ into (22), we come to

$$\varphi(v_0) = F, \tag{36}$$

where

$$\varphi(v_0) \equiv \hat{y}(v_0)v_y\big(\hat{y}(v_0),\ \hat{n}(v_0)\big) + \hat{n}(v_0)v_n\big(\hat{y}(v_0),\ \hat{n}(v_0)\big) - v\big(\hat{y}(v_0),\ \hat{n}(v_0)\big).$$

Note that φ is continuous and such that $\varphi(0) = 0$. Moreover, condition (24) implies that $\varphi(v_0) > \varepsilon v_0$, hence the function φ is not bounded from above over $\mathbb{R}_+$. Hence, (36) has at least one solution for each $F > 0$, which proves part (i). Furthermore, $\varphi(v_0) > \varepsilon v_0$ implies that $\varphi'(0) > \varepsilon > 0$, which means that φ is strictly increasing at least in the vicinity of the origin. This proves part (ii). Finally, if variable cost v is homogeneous of some degree $\gamma > 1$, then $\varphi(v) \equiv (\gamma - 1)v$. Hence, (36) has a unique solution for all $F > 0$, which proves part (iii) and completes the proof of the whole Proposition.

Proof of Proposition 2

In what follows, we denote by $\mathcal{E}_z$ the elasticity of an endogenous variable z (more precisely, of its SFE level z^*) with respect to the market size L:

$$\mathcal{E}_z \equiv \frac{\partial z}{\partial L}\frac{L}{z}.$$

We start with studying how the equilibrium price p^* and the per-variety equilibrium output level q^* vary in response to an increase in L.

Lemma 1. *The elasticities of the equilibrium price p^* and per-variety output q^* with respect to L are related as follows:*

$$\mathcal{E}_p = -r(x)\mathcal{E}_q. \tag{37}$$

Proof of Lemma 1. Differentiating both sides of the zero-profit condition (20) with respect to L, we obtain:

$$\frac{\partial p}{\partial L}y + \frac{\partial y}{\partial L}p = v_y\frac{\partial y}{\partial L} + v_n\frac{\partial n}{\partial L},$$

multiplying both sides of which by L/py yields

$$\mathcal{E}_p + \mathcal{E}_y = \frac{v_y}{p}\mathcal{E}_y + \frac{v_n}{R/n}\mathcal{E}_n, \tag{38}$$

where $R = py$ is the revenue. Combining (38) with (17) and (18) and using the identity $\mathcal{E}_y - \mathcal{E}_n = \mathcal{E}_q$, we obtain (37).

Using Lemmas 1 and (5), we find that (i) an increase in market size always shifts prices and per-variety outputs quantities supplied in opposite directions, and (ii) the relative change of per-variety outputs caused by a market expansion is always higher in absolute value than that of prices.

Lemma 2. *The elasticities $\mathcal{E}_q$ of equilibrium per-variety output level q^* and the equilibrium scope n^* with respect to the market size L are given, respectively, by*

$$\mathcal{E}_q = \frac{xr'(x)}{1 - r(x)}\frac{v_{yy}y^2 + 2v_{yn}yn + v_{nn}n^2}{yn\frac{v_{yy}v_{nn} - v_{yn}^2}{v_y/n} + \frac{r(x)}{1-r(x)}\left(2 + \frac{xu'''(x)}{u''(x)}\right)\left(v_{yy}y^2 + 2v_{yn}yn + v_{nn}n^2\right)}, \tag{39}$$

$$\mathcal{E}_n = -\frac{xr'(x)}{1 - r(x)}\frac{v_{yy}y/n + v_{yn}}{\frac{v_{yy}v_{nn} - v_{yn}^2}{v_y/n} + \frac{r(x)}{1-r(x)}\left(2 + \frac{xu'''(x)}{u''(x)}\right)\left(v_{yy}y^2 + 2v_{yn}yn + v_{nn}n^2\right)}. \tag{40}$$

Proof of Lemma 2. Differentiating both sides of (17) with respect to L, we get after simplifications:

$$\mathcal{E}_p = \frac{v_{yy}y}{v_y}\mathcal{E}_y + \frac{xr'(x)}{1 - r(x)}\mathcal{E}_x.$$

combining which with (37) and using the identities $\mathcal{E}_y = \mathcal{E}_q + \mathcal{E}_n$, $\mathcal{E}_x = \mathcal{E}_q - 1$ yields

$$\left(\frac{v_{yy}y}{v_y} + \frac{xr'(x)}{1 - r(x)} + r(x)\right)\mathcal{E}_q + \frac{v_{yy}y}{v_y}\mathcal{E}_n = \frac{xr'(x)}{1 - r(x)}. \tag{41}$$

Similarly, one can show by differentiating both sides of (22) with respect to L and using the identity $\mathcal{E}_y = \mathcal{E}_q + \mathcal{E}_n$ that the following relationship holds in the SFE for all $L > 0$:

$$\left(y^2v_{yy} + ynv_{yn}\right)\mathcal{E}_q + \left(n^2v_{nn} + 2nyv_{ny} + y^2v_{yy}\right)\mathcal{E}_n = 0. \tag{42}$$

Finally, solving (41) and (42) for $\mathcal{E}_q$ and $\mathcal{E}_n$ and using (34), we obtain (39) and (40).

The second term of the product in the right-hand side of (39) is positive due to (6) and (14). Thus, under increasing (decreasing) inverse demand elasticity $r(x)$, an increase in L triggers an upward (downward) shift of per-variety output q^*.

Lemma 3. *The elasticity of equilibrium per-variety output level q^* with respect to L is lower than one: $\mathcal{E}_q < 1$.*

Proof of Lemma 3. Observe that

$$xr'(x) = r(x)\left(1 + r(x) + \frac{xu'''(x)}{u''(x)}\right) < r(x)\left(2 + \frac{xu'''(x)}{u''(x)}\right),$$

combining which with (39) and (34) yields $\mathcal{E}_q < 1$.

Consider now the industry-level product range n^*N^*. Combining zero-profit condition (20) with labor balance (21), we find that $nN = L/pq$, which yields together with (37):

$$\mathcal{E}_{nN} = 1 - \mathcal{E}_p - \mathcal{E}_q = 1 - (1 - r(x))\mathcal{E}_q. \tag{43}$$

As $\mathcal{E}_q < 1$ by Lemma 3, (43) implies that an increase in the market size L leads unambiguously to an increase in the industry-level product range. Furthermore, as $\mathcal{E}_q > 0$ under increasing inverse demand elasticity, we conclude that $\mathcal{E}_{nN} < 1$, i.e. an increase in industry-level product variety is less than proportional to a hike in L.

This completes the proof of Proposition 2.

Proof of Proposition 3

Differentiating (22) with respect to L yields

$$(yv_{yy} + nv_{yn})\frac{\partial y}{\partial L} + (nv_{nn} + yv_{ny})\frac{\partial n}{\partial L} = 0.$$

which amounts to

$$\left(y^2 v_{yy} + ynv_{yn}\right)\mathcal{E}_y + \left(n^2 v_{nn} + nyv_{ny}\right)\mathcal{E}_n = 0, \tag{44}$$

Note that $y^2 v_{yy} + 2ynv_{yn} + n^2 v_{nn} > 0$ by convexity of the variable cost $v(\cdot, \cdot)$. As a consequence, $y^2 v_{yy} + ynv_{yn}$ and $n^2 v_{nn} + nyv_{ny}$ cannot be negative simultaneously. Furthermore, both these terms are positive if and only if $-ynv_{yn} < \min\{y^2 v_{yy}, n^2 v_{nn}\}$, which is equivalent to (25). In this case, (44) implies that $\mathcal{E}_y$ and $\mathcal{E}_n$ have the opposite signs, otherwise they have the same sign. This proves the result.

Proof of Proposition 4

Using the revenue concavity condition (6) and the cost convexity conditions (14), it is readily verified that the denominator of the second term in the right-hand side of (40) is positive. Moreover, the assumption that $r'(\cdot) > 0$ proves the result.

Proof of Proposition 5

As implied by (20) and (21), we have $N = L/npq$. Hence, the elasticity of N^* with respect to L is given by $\mathcal{E}_N = 1 - \mathcal{E}_p - \mathcal{E}_q - \mathcal{E}_n$. Using (37) and (39), we obtain

$$\mathcal{E}_N = 1 + \frac{xr'(x)\frac{v_y v_n}{v+F}\left[\left(\frac{yv_{yy}}{v_y} + \frac{nv_{yn}}{v_y}\right) - \left(\frac{nv_{nn}}{v_n} + \frac{yv_{ny}}{v_n}\right)\right]}{(1 - r(x))\frac{v_{yy}v_{nn} - v_{yn}^2}{v_y/n} + r(x)\left(2 + \frac{xu'''(x)}{u''(x)}\right)\left(v_{yy}y^2 + 2v_{yn}yn + v_{nn}n^2\right)}. \tag{45}$$

As the denominator in (45) is positive, it follows immediately from (45) that $\mathcal{E}_N > 1$ if and only if (27) is satisfied. This completes the proof.

Proof of Proposition 6

As there are no spillovers when the variable cost satisfies (29), we have $v_{yn} = 0$. Then, as implied by Proposition 4, the equilibrium firm-level product range shrinks as population grows: $\mathcal{E}_n < 0$. Furthermore, combining this with Proposition 3, we find that $\mathcal{E}_y > 0$. Finally, using (43), we also have $\mathcal{E}_{nN} > 0$, which implies that $\mathcal{E}_N = \mathcal{E}_{nN} - \mathcal{E}_n > 0$. This proves the result.

Proof of Proposition 7

As implied by the expression (40) for $\mathcal{E}_n$ in Lemma 2, we have $\mathcal{E}_n = 0$ if and only if $yv_{yy} = -nv_{yn}$. Under per-variety additive variable costs, we have

$$v(y, n) = nv\left(\frac{y}{n}\right) + S(n),$$

whence

$$yv_{yy} = \frac{y}{n}v''\left(\frac{y}{n}\right) = -nv_{yn}.$$

This completes the proof.

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