Distributed Self-Regulation of Hierarchical Arbitrage-Free Mesomarkets: Towards Knowledge Problem and Informational Role of Money Flow

Vasily Lubashevskiy[a,*], Ihor Lubashevsky[b]

[a] *Tokyo International University, 1-13-1 Matoba-Kita, Kawagoe, Saitama, 350-1197, Japan*
[b] *HSE Tikhonov Moscow Institute of Electronics and Mathematics, 34 Tallinskaya Ulitsa, 123458, Moscow, Russia*

Abstract

We present a model for the equilibrium state of a hierarchical supply mesomarket based on a single type of raw material. This model illustrates Hayek's knowledge problem, covering dispersed knowledge, the role of the price system and entrepreneurship, and spontaneous order. The mesomarket is depicted as a tree-like network with nodes representing micromarkets balancing local supply and demand. Edges consist of competing firms at the same production stage, with a large number of firms assumed. The tree root represents raw material producers, and the upper level (tree leaves) represents retail outlets. The output of each firm is measured by the raw material used, enabling the quantification of the mesomarket in terms of raw material flow. Firms maximize profit under the no-arbitrage condition linked to fixed interest rate external investments. Retail outlet trading is driven by end-consumer demand as functions of prices. The mesomarket shows perfect self-regulation, meaning changes in demand for one type of goods do not affect the supply-demand balance for others. This self-regulation is due to the price pattern being dependent only on production process technology and external investment rates. End-consumer demand impacts the number of firms in production. Our findings illustrate the implementation of Hayek's knowledge problem mechanisms. Notably, the price pattern does not directly reflect end-consumer demand; instead, money flow through the network carries this information and enables its self-processing, which is not accessible to individual firms. Lastly, we discuss mesoeconomics as a bridge between micro- and macroeconomics

Keywords: Complex systems, Hierarchical information processing, Mesoeconomics, Networked markets, Self-regulation, Hayek's knowledge problem, Theory of money

1. Introduction

In this paper, we address a longstanding challenge in market theory known as the knowledge problem, which is typically associated with Hayek's seminal article [1]. In our discussion, we focus on the following key points of the the knowledge problem (for a review see [2, 3, 4]).

– *Dispersed Knowledge and Limits of Central Planning*:[1] The knowledge required for economic decision-making is dispersed among many market actors. Each actor holds unique information about their own circumstances, preferences, and opportunities, which cannot be fully known by others, especially a central planner. As a result, centralized planning is inherently flawed because a central authority cannot access and process all relevant information.

– *Contextual Complexity of Knowledge*: Knowledge relevant to the coordination of market actors comprises at least two constituent components different in nature. One component is *information* about market functioning, which refers to raw data or factual details that can be collected, stored, and transmitted. This information includes prices, quantities, market demands, supply levels, and other measurable data points that can be systematically recorded

*Corresponding author: Vasily Lubashevskiy
Email addresses: `vlubashe@tiu.ac.jp` (Vasily Lubashevskiy), `ilubashevskii@hse.ru` (Ihor Lubashevsky)

[1] The concept 'dispersed knowledge' has been elaborated by Sowell [5].

and communicated. The other component is *tacit knowledge* [6], which does not exist outside the market context and can be only partly consciously recognized. Based on personal experience, tacit knowledge includes intuition and specific situational awareness, which can hardly be expressed directly and aggregated.

– *The Price System and Entrepreneurship*: Prices aggregate the dispersed information about supply and demand, allowing each market participant to act based on their local knowledge and circumstances by responding to relevant prices. Entrepreneurs utilize their specific knowledge of local conditions, consumer preferences, and emerging trends to create new products and services, thereby disturbing and subsequently restoring market equilibrium. Put differently, the price system, by transmitting essential information through prices, and entrepreneurship, by leveraging local knowledge and driving innovation, together enhance market efficiency. This synergy creates a dynamic environment where information is continually processed and acted upon, leading to optimal resource allocation and economic growth [7, 8].

– *Spontaneous Order*: The market creates a spontaneous order where individual participants, acting in their own interest, unknowingly contribute to the overall economic order. This order emerges naturally and should be more efficient and adaptable than any centrally designed system because it harnesses the diverse and dispersed knowledge throughout society (see, e.g., [9] for a review).

In recent years, the specific mechanisms of market economics underpinning the knowledge problem, as well as the feasibility of these mechanisms, have been debated from various perspectives. In particular, these include (i) arguments for and against the mechanisms at a general level [e.g., 10, 11, 12]; (ii) the efficient markets hypothesis, which posits that market prices fully reflect all available information [e.g., 13], and is also discussed in the contexts of human psychology [e.g., 14] and biological evolution, including the adaptive markets hypothesis [15] (see [13] for a review); and (iii) the spontaneous emergence of order as a result of self-organization described through agent-based simulations [e.g., 16, 17].

Concerning the modern state of economics, where large-scale companies, especially multinational and transnational corporations, play crucial roles, the essence of Hayek's knowledge problem remains relevant due to the deep parallelism between the functioning of large-scale companies and the knowledge problem as understood within market economics. Their similarities include [e.g., 18, 19, 20]:

– *Decentralization*: Both operate through decentralized decision-making units; in firms, departments or divisions function semi-independently, similar to individual actors in a market.
– *Information coordination*: Large firms and markets use pricing mechanisms and information flow to coordinate activities. Internal pricing in firms resembles market price signals.
– *Competition*: Divisions within firms compete for resources and incentives, much like companies compete in a market.
– *Adaptability*: Both systems rely on flexibility to respond to changes, using innovation and structural adjustments to maintain efficiency.

In this paper, we consider the equilibrium problem for a networked mesomarket that forms a branching multi-tier supply chain system.[2] This equilibrium problem focuses on price formation and resource allocation within a specific type of goods production sector. First, it manages the commodity flow from one sort of raw material to meet the end consumers' demand for the corresponding goods. Second, this production sector—referred to as a *mesomarket*—is assumed to be composed of a network of interdependent micromarkets, where firms that are sellers in one micromarket become buyers in another. The integrity of the mesomarket is due to the raw material being the main ingredient of the produced goods, such as wheat in bread-and-bakery manufacturing. Other ingredients are sourced from corresponding sectors of the economy, acting as a 'thermostat' for the analyzed mesomarket.[3] Moreover, the mesomarket itself is treated as an open system, which means that its members can leave or enter the system, depending on the state of affairs in the given economic sector.

We want to emphasize, leaping ahead, that the model to be developed exemplifies a plausible mechanism through which market economics, addressing all aspects of Hayek's knowledge problem, can become efficient. This means

[2] For an introduction to the general and partial equilibrium of networked markets, readers may refer to Nagurney [21, 22]. To review similar problems confined to supply networks, readers may refer, for example, to Wiedmer and Griffis [23], Jiang et al. [24]."

[3] The concept of a thermostat is widely used in statistical physics, where a complex system is assumed to be embedded in a surrounding medium whose properties are practically independent of the states of the analyzed system.

that (i) prices can adequately reflect the current state of technology and (ii) uncorrelated variations in demand among different consumer groups do not gives rise to chaotic behavior. The key feature of this model is the presence of two counter-current flows: one is the commodity flow from raw material extraction to the end consumers, and the other is the money flow in the opposite direction. It is precisely the money flow that directly aggregates dispersed information, enabling market participants to plan their activities in response to prices without needing access to all relevant information. Such self-regulation mechanisms do not seem unique to market economics; at least, living tissue self-regulation is also governed by a similar mechanism [25, 26]. The self-regulation of mesomarkets was also discussed in [27]; however, the accepted formalism for describing perfect competition—a key point in this model—has remained rather problematic for its justification.

2. Mesomarket Model

The analyzed model for the multi-tier supply network is illustrated in Fig. 1. This system is assumed to produce goods of a certain type using one raw material as the main component. Other supplementary components are added during the production process, which will be taken into account in terms of production cost, including also labor, services, transaction costs, asset depreciation, etc. The production process, comprising several stages, starts from either extracting raw materials such as minerals and oil or producing raw materials through the cultivation of crops and livestock breeding, and continues to transporting goods to the final retail outlets $\{\mathcal{B}_\alpha\}$. Each stage is treated as a specific production activity of a market member, referred to as a firm. The branching of the production process into connected stages, joined through the flow of raw material, underlies the representation of this process as a network in the form of a tree.[4] These branching points (nodes) are treated as micromarkets $\{\mathcal{B}_i\}$, and transactions between different firms in these micromarkets are regarded as trading activities. Firms $\{f_{ik}\}$ whose production activities can be categorized as one stage k of the production process, and whose trading activities are based on the same pair of micromarkets, are grouped into one edge $\mathcal{E}_i$ of this network, $\mathcal{E}_i = \bigcup_k f_{ik}$. These firms directly compete with one another. The collection of end consumers is divided into groups $\{\mathbb{C}_\alpha\}$ that purchase specific goods (of the type in question) at the corresponding retail outlets $\{\mathcal{B}_\alpha\}$. The overlapping of these consumer groups is ignored, and the goods sold at different outlets are considered distinct.

In mathematical terms, the functioning of the mesomarket is characterized by the prices $\{p_i\}$ assigned to the micromarkets $\{\mathcal{B}_i\}$, which include the retail outlets $\{\mathcal{B}_\alpha\}$. The prices $\{p_i\}$, as defined, correspond to the quantity of the respective product measured by the unit amount of raw material. Consequently, the production activity of a firm f_{ik} is quantified by the amount x_{ik} of its product, which is measured in the same units of raw material produced per unit time.

In the same way, we quantify the total demand $D^\alpha(p_\alpha)$ of the consumer group $\mathbb{C}_\alpha$, which is considered a given function of the goods prince p_α at the retail outlet $\mathcal{B}_\alpha$. The corresponding supply S^α is the cumulative production rate of the firms forming to the network edge $\mathcal{E}_\alpha$, connected directly to the retail outlet $\mathcal{B}_\alpha$, i.e.,

$$S^\alpha = \sum_{f_{\alpha k}\in\mathcal{E}_\alpha} x_{\alpha k}. \tag{1}$$

It is necessary to emphasize that the use of the raw material unit as the common measure makes the products of different stages, as well as the goods in question, commensurable.

Let us confine our analysis to the equilibrium state of this networked mesomarket. The concept of its equilibrium encompasses several aspects: (i) some are general and widely covered in microeconomics textbooks (e.g., [28, Ch. 5]), (ii) others reflect the characteristics of network economics (e.g., [22]), and (iii) the final aspects account for mesomarkets being small parts of a macroeconomic system, where individual changes in their properties have minor effects on the overall economy (cf. [28, Ch. 14] and [29, 30]).

[4] In considering the equilibrium market state, the market network can have cycles only in degenerate cases, allowing us to ignore this possibility. Market dynamics inevitably leads to time variations in the network topology, including the emergence of cycles, which, however, requires individual investigation.

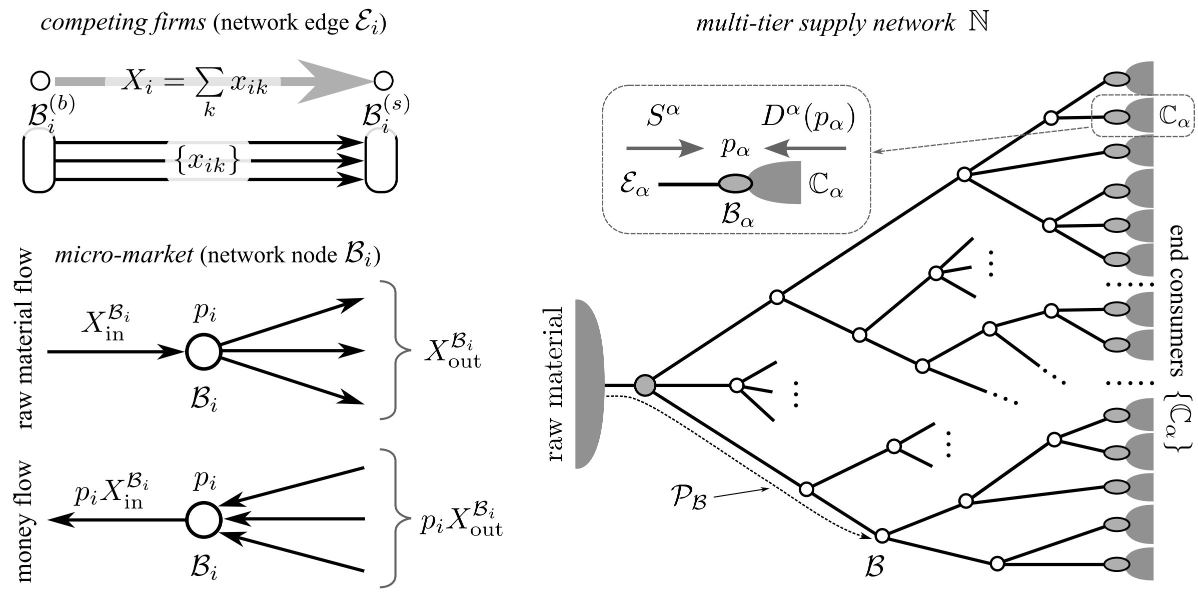

Figure 1: The analyzed hierarchical supply network $\mathbb{N}$ being of the tree form. It represents the production process from one kind of raw materials to the related types of goods meeting the demand of different groups of end consumers $\{\mathbb{C}_\alpha\}$. This demand is quantified as a given before function $D^\alpha(p_\alpha)$ whose argument is the price p_α at the corresponding retail outlet $\mathcal{B}_\alpha$. The connectivity of this networked market reflects conservation of the raw material being the main component of the produced goods. The network edges represent individual stages of the production process, the network nodes correspond to the branching of this process and play the role of micromarkets $\{\mathcal{B}_i\}$ with prices $\{p_i\}$. Firms whose production activity is assume to be confined to one stage and whose trading transactions are implemented in the same pair of micromarkets are grouped in one edge. The left bottom fragments illustrate the conservation of raw material in its bottom-up flow and the counter-current flow of money, which underlies the self-processing of information about consumer demand.

2.1. Local equilibrium of micromarkets

At each node $\mathcal{B}_i$ of the network $\mathbb{N}$, there is a balance between the input and output components of the commodity flow, which is reduced to the flow of raw material due to the units of measurement used (Fig. 1)

$$X_{\text{in}}^{\mathcal{B}_i} = \sum_{\mathcal{E}_{i,\text{out}}} X_{\text{out}}^{\mathcal{B}_i}, \tag{2a}$$

where the sum runs over all the edges $\mathcal{E}_{i,\text{out}}$ going out of the note $\mathcal{B}_i$. Equation (2a) represents conservation of raw material within its bottom-up flow from the raw material extraction (the root firms) to the final retail outlets. In the opposite top-down direction, there is the conjugated flow of money, which is actually described by the same expression

$$p_i X_{\text{in}}^{\mathcal{B}_i} = p_i \sum_{\mathcal{E}_{i,\text{out}}} X_{\text{out}}^{\mathcal{B}_i}. \tag{2b}$$

We have included equation (2b) here to emphasize that the two counter-current flows though the mesomarket accompany the production process.

Equality (1), which can be written as

$$X_{\text{in}}^{\mathcal{B}_\alpha} = D^\alpha(p_\alpha), \tag{3}$$

serves as the 'top boundary condition' imposed on the product flux at the retail outlets, where consumer demand specifies the product flow through the corresponding edges $\{\mathcal{E}_\alpha\}$.

2.2. Perfect competition among firms in micromarkets

The concept of perfect competition among the firms making up one network edge, e.g., $\mathcal{E}_i$, is understood as the combination of two propositions. *Firstly*, under steady state conditions, each firm f_{ik} selects a production rate x_{ik}^{m} that maximizes its profit $\pi_{ik}(x_{ik})$, considering the prices $p_i^{(b)}$ and $p_i^{(s)}$ in the micromarkets $\mathcal{B}_i^{(b)}$ and $\mathcal{B}_i^{(s)}$ (Fig. 1) as fixed external quantities. The firm's profit can be written as

$$\pi_{ik}(x_{ik}) = [p_i^{(s)} - p_i^{(b)}]x_{ik} - t_{ik}(x_{ik})\,, \tag{4}$$

where the production cost $t_{ik}(x_{ik})$, which accounts for all the factors noted above and represents the production technology including labor, is convex and takes a positive value at $x_{ik} = 0$. To illustrate the results to be obtained, we will use the following ansatz for the production cost:

$$t_{ik}(x_{ik}) = a_{ik} + g_{ik}x_{ik}^{\beta}\,. \tag{5}$$

Here, the positive constants $\{a_{ik}, g_{ik}\}$ are individual parameters characterizing the production processes of different firms, whereas the exponent $\beta > 1$ is assumed to be the same for all firms.

By virtue of (4), the production rate x_{ik}^{m} is specified by the condition

$$\left.\frac{\partial \pi_{ik}}{\partial x_{ik}}\right|_{x_{ik}=x_{ik}^{\mathrm{m}}} = [p_i^{(s)} - p_i^{(b)}] - \left.\frac{dt_{ik}}{dx_{ik}}\right|_{x_{ik}=x_{ik}^{\mathrm{m}}} = 0\,, \tag{6}$$

which enables us to treat the value of x_{ik}^{m} as a function of the price difference $\Delta p_i = p_i^{(s)} - p_i^{(b)}$ and the technological process used by the firm f_{ik}, denoted as $x_{ik}^{\mathrm{m}}(\Delta p_i)$. In its turn, the latter function specifies the maximum $\pi_{ik}^{\mathrm{m}}(\Delta p_i)$ of the profit $\pi_{ik}(x_{ik})$ is a function of the price difference Δp_i. Figure 2 illustrates the production rate x_{ik}^{m} that maximizes the profit $\pi_{ik}(x_{ik})$ and the related quantities characterizing the dependence $\pi_{ik}(x_{ik})$.

In particular, for ansatz (5)

$$x_{ik}^{\mathrm{m}}(\Delta p_i) = \left(\frac{\Delta p_i}{\beta g_{ik}}\right)^{1/(\beta-1)} \tag{7a}$$

$$\pi_{ik}^{\mathrm{m}}(\Delta p_i) = \frac{(\beta-1)}{\beta}\Delta p_i x_{ik}^{\mathrm{m}} - a_{ik} = \frac{(\beta-1)}{(g_{ik})^{1/(\beta-1)}\beta^{\beta/(\beta-1)}}(\Delta p_i)^{\beta/(\beta-1)} - a_{ik}\,. \tag{7b}$$

The latter expression can also be used to specify the minimal price difference Δp_i at which production activity is possible, satisfying the condition $\pi_{ik}^{\mathrm{m}}(\Delta p_i) > 0$.

For the firms responsible for extracting or producing raw materials, the value

$$p_{\mathrm{root}}^{(b)} = 0 \tag{8}$$

represents the 'bottom boundary condition' imposed on the price distribution over the network $\mathbb{N}$.

Secondly, for each network edge, e.g., $\mathcal{E}_i$, the number of firms N_i making it up is quite large, $N_i \gg 1$. Therefore, we may treat the equality

$$\sum_{1\leq k\leq N_i} x_{ik}^{\mathrm{m}}(\Delta_i p) = X_i^{\mathrm{m}} \tag{9}$$

as the relationship between the price difference $\Delta_i p$, the number N_i of firms, and the production rate X_i^{m} attributed to the edge $\mathcal{E}_i$ as a whole in the case of perfect competition, ignoring small deviations from the equality due to the value N_i being an integer.

2.3. Arbitrage-free mesormarket

The analyzed mesomarket is not an isolated system; it is embedded in the 'economic environment,' and its actors can leave or enter the mesomarket based on its financial efficiency. Confining the analysis of these opportunities to the equilibrium case, we focus on the 'interaction' between the mesomarket and the 'economic environment,' as illustrated in Fig. 2. This interaction can be seen as a specific implementation of the no-arbitrage condition.

Generally, arbitrage refers to the opportunity, e.g., to borrow and immediately lend at two different fixed interest rates, a situation that cannot persist for along time because arbitrageurs' activity will equalize the rates [e.g., 31]. In the analyzed case, the no-arbitrage condition is understood as the requirement that the mesomarket and the 'economic environment' be in financial equilibrium. Put differently, it implies that for all actors in the market, the choice between

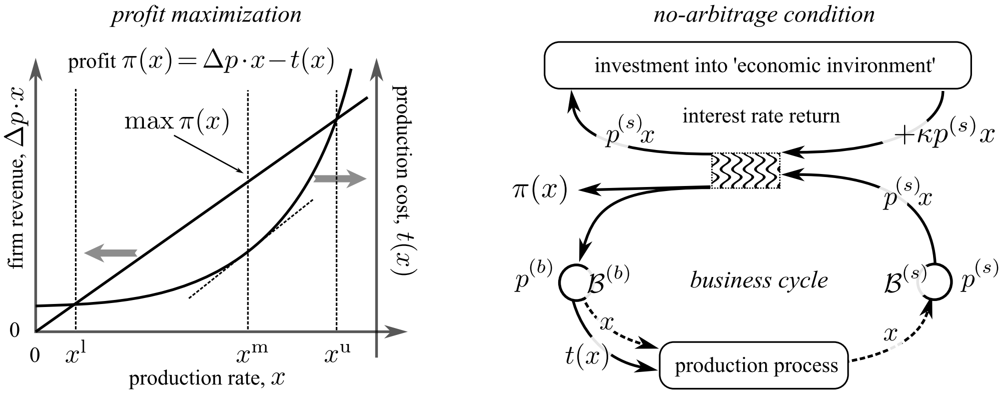

Figure 2: Two conditions characterizing the equilibrium state of the mesomarket. The left fragment illustrates the maximization of a firm's individual profit $\pi(x)$ achieved by adjusting its production rate x assuming the price difference Δp to be fixed. The production rates x^{l} and x^{u} define the range $(x^{\mathrm{l}}, x^{\mathrm{u}})$ within which production activity is meaningful. The right fragment illustrates the mechanism underlying the accepted no-arbitrage condition. One component of this mechanism is the business cycle, which represents the production activity of a characteristic firm. This cycle includes (i) selling its product x for the price $p^{(s)}$ in the micromarket $\mathcal{B}^{(s)}$, (ii) purchasing the x-amount of material for production for the price $p^{(b)}$ in the micromarket $\mathcal{B}^{(b)}$, and (iii) investing $t(x)$-amount of money into the production process. The difference $\pi(x) = (p^{(s)} - p^{(b)})x - t(x)$ forms the firm's profit gained within this business cycle. Here, the solid and dashed directed lines represent the flow of money and materials, respectively. The other component of this mechanism is illustrated in the upper part of the left fragment. It involves investing the revenue $p^{(s)}x$ with an interest rate of κ into some external activity. This investment is expected to yield a surplus of $\kappa p^{(s)}x$ over the period of the business cycle. The dashed rectangle denotes the possibility of choosing either of the two types of firm activities, which underpins the arbitrage opportunities. In these plots, we omitted the indices i and k to emphasize the generality of the represented fragments.

staying in the market or investing externally appears to be equivalent, as both options are expected to yield identical returns. Such equality should lead to balanced and stable market functioning [cf. 32].

Figure 2 (right fragment) illustrates this situation. The business cycle of firm f is represented as a sequence of several steps, with the duration of the business cycle treated as the time unit. The firm sells its product x for the price $p^{(s)}$ in the micromarket $\mathcal{B}^{(s)}$, receiving revenue equal to $p^{(s)}x$. Subsequently, two opportunities arise. One opportunity is to continue the business cycle by purchasing the x-amount of material for the price $p^{(b)}$ in the micromarket $\mathcal{B}^{(b)}$, reserving $t(x)$ for the production process. The remaining amount, $\pi(x) = [p^{(s)} - p^{(b)}]x - t(x)$, represents the firm's profit within one business cycle. The other opportunity is to terminate the production process and invest the received $p^{(s)}x$ in an external activity at an interest rate κ. The value of κ is determined by the overall economic system and, for simplicity, is assumed to be constant. This investment must yield a surplus amount of money equal to $\kappa p^{(s)}x$ per unit of time.

Naturally, in discussing the financial equilibrium between the networked mesomarket and its 'economic environment,' we consider the network edges fundamental elements representing various stages of the production process. Therefore, we impose the no-arbitrage condition on each individual edge by integrating (averaging) the profit π^{m}_{ik} across all firms comprising the edge. For instance, for an edge $\mathcal{E}_i$ this can be expressed as follows:

$$\sum_{1\leq k\leq N_i} \pi^{\mathrm{m}}_{ik} = \kappa p^{(s)}_i \sum_{1\leq k\leq N_i} x^{\mathrm{m}}_{ik} = \kappa p^{(s)}_i X^{\mathrm{m}}_i\,, \tag{10}$$

where we have used Exp. (9) introducing the value X^{m}_i. In particular, for ansatz (5), this condition, due to (7), takes the form

$$\frac{(\beta-1)}{\beta}\Delta p_i\, X^{\mathrm{m}}_i(\Delta_i p) - A_i = \kappa p^{(s)}_i\, X^{\mathrm{m}}_i(\Delta_i p) \tag{11a}$$

or, by virtue of $\Delta p_i = p^{(s)}_i - p^{(b)}_i$, the latter equality can be rewritten as

$$\left\{\left[\frac{(\beta-1)}{\beta} - \kappa\right]p^{(s)}_i - \frac{(\beta-1)}{\beta}p^{(b)}_i\right\}X^{\mathrm{m}}_i\left(p^{(s)}_i - p^{(b)}_i\right) = A_i\,. \tag{11b}$$

Here, the quantity A_i, introduced via the expression

$$A_i = \sum_{1 \le k \le N_i} a_{ik}\,, \tag{12}$$

characterizes the cumulative loss of the firms $\{f_{ik}\}_{k=1}^{k=N_i}$ when their production activity is halted.

It should be emphasized that Eq. (11b) imposes a restriction on the interest rate for external investment, specifically, the inequality

$$\kappa < \frac{(\beta - 1)}{\beta} \tag{13}$$

must hold. Otherwise, this mesomarket cannot survive in financial competition with its 'economic environment,' as market actors would see no reason to participate in the goods production. In addition, by virtue of (11b), the relationship between the prices $p_i^{(s)}$ and $p_i^{(b)}$ in the micromarkets $\mathcal{B}_i^{(s)}$ and $\mathcal{B}^{(s)}$ corresponding to the edge $\mathcal{E}_i$ meets the inequality

$$p_i^{(s)} > \frac{p_i^{(b)}}{1 - \kappa\beta/(1-\beta)}\,, \tag{14}$$

which illustrates that prices for goods should grow with the interest rate increasing. A discussion of the appropriate values of the interest rate and their influence on economics can be found, for example, in [33, 34].

3. Equilibrium State of Mesomarket

The governing equations (2), (6), and (10), or their specific instantiation for ansatz (5) (i.e., equations (7) and (11)), combined with the 'boundary conditions' (3) and (8), specify the distribution of the prices $\{p_i\}$ in the micromarkets $\{\mathcal{B}\}$ and the firms $\{f_{ik}\}$ involved in the production process. The network $\mathbb{N}$ endows this system of equations with integrity which is due to going through a node, a micromarket $\mathcal{B}$, the sell price $p_{\mathcal{B}}^{(s)}$ for the parent edge becomes the buy price $p_{\mathcal{B}}^{(b)}$ for its daughter edges.

Let us analyze the characteristic features of this networked mesomarket by turning to ansatz (5). Additionally, due to $N_i \gg 1$, we assume that for each edge $\mathcal{E}_i$, the variety of firms $\{f_{ik}\}_{k=1}^{k=N_i}$ is well-developed and remains stable despite possible changes in the end-consumer demands $\{\mathcal{D}^\alpha(p_\alpha)\}$. Therefore, for each edge $\mathcal{E}_i$, we can introduce a typical firm f_i whose technological parameters a_i and g_i (see ansatz (5)) represent the characteristic features of the corresponding stage of the production process and are not sensitive to variations in $\{\mathcal{D}^\alpha(p_\alpha)\}$. These variations affect the number of firms $\{f_{ik}\}$ in the mesomarket rather than the shape of the firms' distribution. Taking into account Exp. (7a), relationship (9), and definition (12), we specify the parameters a_i and g_i as

$$a_i = \frac{A_i}{N_i} = \frac{1}{N_i}\sum_{1 \le k \le N_i} a_{ik} \qquad \text{and} \qquad \frac{1}{g_i^{1/(\beta-1)}} = \frac{1}{N_i}\sum_{1 \le k \le N_i} \frac{1}{g_{ik}^{1/(\beta-1)}}\,, \tag{15}$$

which finalizes the desired description of the given networked mesomarket.

3.1. *The price distribution*

In the introduced terms, relationship (9) is read

$$X_i^{\mathrm{m}} = \frac{N_i}{(\beta g_i)^{1/(\beta-1)}}\left[p_i^{(s)} - p_i^{(b)}\right]^{1/(\beta-1)}, \tag{16}$$

and, as a result, equations (11) are reduced to

$$\left\{\left[\frac{(\beta-1)}{\beta} - \kappa\right]p_i^{(s)} - \frac{(\beta-1)}{\beta}p_i^{(b)}\right\} \cdot \left[p_i^{(s)} - p_i^{(b)}\right]^{1/(\beta-1)} = a_i(\beta g_i)^{1/(\beta-1)}\,. \tag{17}$$

The 'bottom boundary condition' (8), i.e., the equality $p_{\mathrm{root}}^{(b)} = 0$, enables us to integrate the system of equations (17) step-by-step from the network root to the top level of the tree, the retail outlets. In this way, we determine the price $p_{\mathcal{B}}$ established in each micromarket $\mathcal{B}$ (including the retail outlets $\{\mathcal{B}_\alpha\}$) as a function of only the technological parameters $\{a_i, g_i\}$ distributed along the path $\mathcal{P}_{\mathcal{B}}$ leading from the network root to the node $\mathcal{B}$ (Fig. 1). The interest rate κ of external investment should also be including in the list of this function arguments.

The direct dependence of the prices $\{p_{\mathcal{B}}\}$ solely on the technological characteristics of the production process justifies the assumed tree form of the network $\mathbb{N}$. In this context, the emergence of cycles in the network would only be possible in a degenerate case. Naturally, such cycles can emerge during the mesomarket dynamics, which, however, is a separate issue.

3.2. *The mesomarket response to consume demand*

The constructed distribution of the prices $\{p_{\mathcal{B}}\}$ leads to the conclusion that the price difference $\Delta p_{\mathcal{E}}$ for each edge $\mathcal{E}$ does not depend on the consumer demands $\{\mathcal{D}^\alpha(p_\alpha)\}$. This allows us to determine the number $N_{\mathcal{E}}$ of firms involved in the production process at each edge $\mathcal{E}$. Indeed, the 'top boundary condition' (3) specifies the number N_α of firms supplying the retail outlet $\mathcal{B}_\alpha$ in response to the demand $D^\alpha(p_\alpha)$ of the consumer group $\mathbb{C}_\alpha$ (Fig. 1). Namely, by virtue of (3) and (16),

$$N_\alpha = \left[\frac{\beta g_{\mathcal{E}_\alpha}}{\Delta p_{\mathcal{E}_\alpha}}\right]^{1/(\beta-1)} D^\alpha(p_\alpha)\,. \tag{18}$$

Then, using the conservation law (2a) at each network node together with Exp. (16), we determine the quantities $\{N_{\mathcal{E}}\}$ step-by-step from the retail outlets to the tree root. This step-by-step top-down procedure is illustrated by the equality holding at each node $\mathcal{B}_i$

$$\left[\frac{\Delta p^{\mathcal{B}_i}_{\text{in}}}{\beta g^{\mathcal{B}_i}_{\text{in}}}\right]^{1/(\beta-1)} N^{\mathcal{B}_i}_{\text{in}} = \sum_{\mathcal{E}_{i,\text{out}}} \left[\frac{\Delta p^{\mathcal{B}_i}_{\mathcal{E}_i,\text{out}}}{\beta g^{\mathcal{B}_i}_{\mathcal{E}_i,\text{out}}}\right]^{1/(\beta-1)} N^{\mathcal{B}_i}_{\mathcal{E}_i,\text{out}}\,. \tag{19}$$

This equality, stemming from Exps. (2) and (16), relates the number of firms at the parent (in) and daughter (out) edges. It is the pattern $\{N_{\mathcal{E}}\}$ that bears the information about the end-consumer demand.

It should be emphasized that the obtained conclusions about the basic properties of the price distribution $\{p_{\mathcal{B}}\}$ and the firm number distribution $\{N_{\mathcal{E}}\}$ do not depend on a particular form $t(x)$ of the production cost. We have used ansatz (5) only to elucidate the constructions.

4. Conclusion and Discussion

The developed model for the equilibrium state of the interconnected mesomarket illustrates characteristics that enable us to regard these types of markets as an efficient mechanism for satisfying consumer demand for specific goods within the framework of market-based mesoeconomics. In fact, in the analyzed ideal scenario, prices are dictated by the prevailing state of production technology, thereby rendering them 'equitable.' Hence, it follows that the mesomarket response to a change in the demand $D^\alpha(p_\alpha)$ from one consumer group $\mathbb{C}_\alpha$ does not disrupt the supply-demand equilibrium for other consumer groups $\{\mathbb{C}_{\alpha'}\}_{\alpha'\neq\alpha}$. In this context, the market response is localized, despite the complex branching architecture of the mesomarket itself. This behavior can be categorized as perfect self-regulation; when the mesomarket responds to variations in end-consumer demands, it adjusts itself so that the supply changes at the necessary retail outlets while maintaining its previous level at the other outlets.

A similar behavior has been found for the self-regulation of living tissue, where a peripheral vascular network, which supplies the corresponding organ of the human body with nutrients, also responds strictly locally in the ideal case [25]. For realistic characteristics of blood vessels, the vascular network response remains quasilocal [26], which allows us to expect that the identified properties of a networked mesomarket should not change drastically when the equilibrium state is not significantly disturbed.

By comparing the analyzed properties of the given mesomarket with similar properties of the peripheral vascular networks in the human body, we can conclude that the perfect (quasi-perfect) self-regulation of mesomarkets and peripheral vascular networks is based on the same mechanism. The fundamental components of this mechanism are two counter-current flows. In mesomarkets, these are the bottom-up flow of raw materials (the primary material component involved in the production process) and the top-down flow of money. In living tissue, they correspond to the bottom-up flow of nutrients with blood via arterial bed and the top-down flow of metabolic waste components with blood via venous bed. It is the top-down flow of money or metabolic waste components that is responsible for the hierarchical self-processing of information about the state of consumer demand or the condition of living tissue. In the context of the mesomarket, the merging of money 'streams' at the network nodes (micromarkets), which is reflected in the effective summation (19) of the number of firms corresponding to the daughter edges, represents the aggregation of information about end-consumer demand carried separately by the daughter edges.

This hierarchical self-processing of information directly represents a mechanism by which the issue of *Dispersed Knowledge*—one of the central aspects of Hayek's knowledge problem (see Introduction)—can be addressed. The information about end-consumer demands is distributed in an aggregated form among all the firms involved in the production process. None of them holds this information in its entirety; moreover, even fragments of this information can be useless for them, as is the case for firms extracting or producing raw materials. Each firm maximizes its own profit based on the trading activity between the corresponding micromarkets dealing with its product. Nonetheless, the interaction of firms through the micromarket networks enhances the efficiency of the production process, which is reflected in the term *distributed self-regulation* to emphasize the essence of dispersed knowledge.

The issue of *Contextual Complexity* in economic knowledge is also pertinent to the developed model. The component of this knowledge—information—that can be represented quantitatively was discussed in the previous paragraph. Here, its second component—tacit knowledge—is in focus. Tacit knowledge is embodied in the expertise required to manage a specific stage of the production process, which is reflected in the production cost $t(x)$. Tacit knowledge cannot propagate through market networks; it is crucial only for firms engaged in similar production activities. In other words, tacit knowledge is strictly localized to specific stages of the production process and becomes 'invisible' at the global level.

The significance of *Entrepreneurship* also becomes evident within the developed model. The issue is that the constructed state of the mesomarket is 'saturated' in the sense that firms cannot invest their profits extensively into their own production activities, such as purchasing additional units of equipment. In the micromarkets, there is simply no demand for extra quantities of firm products.[5] In this case, a firm can benefit from its investment only by modifying its technological process or enhancing management practices through innovations that reduce the production cost $t(x)$ in an intensive manner, which implies increasing efficiency or productivity within the existing scale of production.

Now, let us compare the role of the *Price System*, as articulated within the framework of Hayek's knowledge problem, with the characteristics of the price pattern described by the developed model for mesomarkets. Prices certainly aggregate dispersed information about the state of the economy in some manner. Therefore, firms focus on the prices in micromarkets where they operate as either sellers or buyers. However, their individual outputs only implicitly reflect consumer demand which determines mainly the quantity of their *cumulative* output passing through these micromarkets. This cumulative output is beyond their individual control. Therefore, prices can only *implicitly* reflect dispersed information about consumer demand. In contrast, it is the flow of money that underlies the self-processing of information and *explicitly* aggregates this information into blocks related to individual stages of the production process. This emphasizes the informational role of money flow rather than the price pattern, at least at the mesoeconomic level. Such information blocks, however, are not directly accessible to firms based on their individual production activities, supporting the relevance of Smith's metaphor of the 'invisible hand' [36, for a modern review] even today.

For the given model of a mesomarket, we can quantify the money flow directly through the networked mesomarket as the amount of money F_M passing through the root edge $\mathcal{E}_{\text{root}}$ per unit time, equivalent to the duration of a business cycle. The total amount of money entering the networked mesomarket through the system of retail outlets is the cumulative demand D_C from all end-consumers for goods derived from the given raw material, multiplied by the mean price P of these goods averaged across all retail outlets, i.e., $D_C P$. However, not all money remains within the mesomarket; some amount 'leaves' the mesomarket during the production process as production costs and firms' profits. In contrast, the flow of raw materials, used to quantify the amount of goods sold at retail outlets, is conserved in its movement through the market network. This conservation enables us to write down

$$F_M = \varkappa_{\mathbb{N}}\, D_C P\,, \tag{20a}$$

[5] In economic literature, such market states are characterized as Cournot competition, which describes an industry structure where firms compete on the quantity of output they produce, assuming their competitors' output levels and the cumulative demand for their products are fixed in some manner [e.g., 35].

where the coefficient $\varkappa$ is specified by the expression

$$\varkappa_{\mathbb{N}} = \frac{P_{\mathcal{E}_{\text{root}}}}{P} \tag{20b}$$

relating the price $P_{\mathcal{E}_{\text{root}}}$ in the raw material micromarket to the mean price at the retail outlets. We have singled out the price ratio $\varkappa_{\mathbb{N}}$ in Exp. (20a) as an individual parameter of the networked mesomarket $\mathbb{N}$ because it is a characteristic of the mesomarket structure and the technological process. The end-consumer demand does not affect it at all, at least within the given model. Besides, by turning to the self-regulation problem in living tissue with non-ideal vessel behavior [26] as an analogy, we can assume that the price distribution along paths from the raw material micromarket to the retail outlets should be characterized by uniform growth for the mesomarket self-regulation to be efficient in non-ideal cases. This assumption enables us to estimate the coefficient $\varkappa_{\mathbb{N}}$ as $\varkappa_{\mathbb{N}} \sim 1/\mathcal{N}$, where $\mathcal{N}$ is the mean number of hierarchy levels in the network $\mathbb{N}$. Expressions (20) may be treated as the quantity equation of the Quantity Theory of Money [e.g., 37] confined to mesoeconomics.

Another issue, stemming from the identified property of the price pattern and the 'saturation' of micromarkets concerning the cumulative production rate, is inflation as an inherent feature of the market economy. The matter is that for a given level of prices, the flow of money passing through the analyzed mesomarket cannot grow without an increase in end-consumer demand. In contrast, the flow of money related to external investment with a fixed interest rate κ or profit accumulation should give rise to growth, shifting from a linear to an exponential rate. This allows us to hypothesize that, at least in the long term, inflation must be an inherent property of market economics. This hypothesis aligns with Friedman's [38] proposition that when the money supply grows faster than the economy's output, inflation is inevitable. For a modern discussion of this issue, readers can refer to [39].

In concluding this section, we highlight the concept of mesomarkets in the context of *Spontaneous Order*, a crucial aspect of Hayek's knowledge problem. As shown by the developed model, the network structure of mesomarkets should be considered an individual parameter of economics, alongside the commonly analyzed price distribution and the variety of agents operating in markets. Turning to the analyzed model, it becomes clear that such mesomarkets may be regarded as distinct structural entities. In bridging microeconomics and macroeconomics, mesomarkets can be considered structural units that macroeconomics deals with, which is typical in self-organized complex systems; for further discussion, see [40, 41, 42, 43, 44], [28, Ch. 14 for a review], and collections [29, 45, 46].

As a plausible extension of the proposed model, we suggest a dynamic generalization addressing several aspects. This includes supply-demand perturbations in micromarkets; introducing thresholds for entering or leaving these micromarkets, which disrupts the no-arbitrage condition; spontaneous rearrangement of firms' connections, including cases where multiple network nodes become part of a large firm (reflecting the emergence of oligarchy). Additionally, the temporal formation of cycles in mesomarket networks should be considered. Special attention could be given to generalizing the model to include multiple tree-like networks embedded within each other to account for different types of raw materials and the production of multi-component goods.

References

[1] F. A. Hayek, The use of knowledge in society, The American Economic Review 35 (1945) 519–530.

[2] L. Kiesling, The knowledge problem, in: C. J. Coyne, P. Boettke (Eds.), The Oxford Handbook of Austrian Economics, Oxford University Press, New York, NY, 2015, pp. 45–64. doi:10.1093/oxfordhb/9780199811762.013.3.

[3] F. Sautet, Market theory and the price system, in: C. J. Coyne, P. Boettke (Eds.), The Oxford Handbook of Austrian Economics, Oxford University Press, New York, NY, 2015, pp. 65–93. doi:10.1093/oxfordhb/9780199811762.013.4.

[4] O. Al-Ubaydli, P. Boettke, B. C. Albrecht, Testing the Hayek hypothesis: Recent theoretical and experimental evidence, PLOS ONE 17 (2022) e0270489. doi:10.1371/journal.pone.0270489.

[5] T. Sowell, Knowledge and Decisions, Basic Books, New York, NY, 1996. First BasicBooks edition published 1980, reissued 1996 with a new preface ty the author.

[6] M. Polanyi, The Tacit Dimension, University of Chicago Press, Chicago, 2009. First edition published 1966, reissued2009 with a new foreword by Amartya Sen.

[7] I. M. Kirzner, Market Theory and the Price System, D· Van Nostrand Co., Inc., New York, NY, 1963.

[8] I. M. Kirzner, Competition and Entrepreneurship, University of Chicago Press, Chicago, 1973.

[9] D. J. D'Amico, Spontaneous order, in: C. J. Coyne, P. Boettke (Eds.), The Oxford Handbook of Austrian Economics, Oxford University Press, New York, NY, 2015, pp. 115–142. doi:10.1093/oxfordhb/9780199811762.013.6.

[10] P. Leiashvily, Self-Regulation of Market Economy: Interdisciplinary Analysis, Nova Science Publishers, Inc., New York, NY, 2015.

[11] P. De Grauwe, The Limits of the Market: The Pendulum Between Government and Market, Oxford University Press, Oxford, UK, 2017. doi:10.1093/acprof:oso/9780198784289.001.0001.
[12] J. Gerber, A Great Deal of Ruin: Financial Crises since 1929, Cambridge University Press, 2019. doi:10.1017/9781108608589.
[13] A. W. Lo, Efficient markets hypothesis, in: S. N. Durlauf, L. E. Blume (Eds.), The New Palgrave Dictionary of Economics, 3rd ed., Palgrave Macmillan, London, 2018, pp. 3543–3560. doi:10.1057/978-1-349-95121-5_42-2.
[14] A. Lewis (Ed.), The Cambridge Handbook of Psychology and Economic Behaviour, 2nd ed., Cambridge University Press, Cambridge, UK, 2018. doi:10.1017/9781316676349.
[15] A. W. Lo, The adaptive markets hypothesis, The Journal of Portfolio Management 30 (2004) 15–29. doi:10.3905/jpm.2004.442611.
[16] J. Rouchier, Agent-based simulation as a useful tool for the study of markets, in: B. Edmonds, R. Meyer (Eds.), Simulating Social Complexity: A Handbook, Springer-Verlag, Berlin, 2013, pp. 617–650. doi:10.1007/978-3-540-93813-2_23.
[17] M. Steinbacher, M. Raddant, F. Karimi, E. Camacho Cuena, S. Alfarano, G. Iori, T. Lux, Advances in the agent-based modeling of economic and social behavior, SN Business & Economics 1 (2021) 99. doi:10.1007/s43546-021-00103-3.
[18] N. J. Foss, V. Mahnke, Knowledge creation in firms: An organizational economics perspective, in: M. Easterby-Smith, M. A. Lyles (Eds.), Handbook of Organizational Learning and Knowledge Management, 2nd ed., John Wiley & Sons Ltd., Chichester, West Sussex, UK, 2011, pp. 125–151. doi:10.1002/9781119207245.ch7.
[19] N. J. Foss, P. G. Klein, S. Linder, Organizations and markets, in: C. J. Coyne, P. Boettke (Eds.), The Oxford Handbook of Austrian Economics, Oxford University Press, New York, NY, 2015, pp. 272–295. doi:10.1093/oxfordhb/9780199811762.013.12.
[20] M. Machaj, A Market Process Theoryof the Firm: An Alternative to the Neoclassical Model, Routledge, the Taylor & Francis Group, New York, NY, 2021.
[21] A. Nagurney, Network Economics: A Variational InequalityApproach, 2nd (revised) ed., Springer Science+Business Media, Dordrecht, 1999. doi:10.1007/978-1-4757-3005-0.
[22] A. Nagurney, Network economics, in: D. A. Belsley, E. J. Kontoghiorghes (Eds.), Handbook of Computational Econometrics, John Wiley & Sons, Chichester, West Sussex, UK, 2009, pp. 429–486. doi:10.1002/9780470748916.ch11.
[23] R. Wiedmer, S. E. Griffis, Structural characteristics of complex supply chain networks, Journal of Business Logistics 42 (2021) 264–290. doi:10.1111/jbl.12283.
[24] T. Jiang, Y. Lin, T. Nguyen, Market equilibrium in multi-tier supply chain networks, Naval Research Logistics (NRL) 69 (2022) 355–370. doi:10.1002/nav.22022.
[25] I. Lubashevsky, V. Gafiychuk, Cooperative mechanism of self-regulation in hierarchical living systems, SIAM Journal on Applied Mathematics 60 (2000) 633–663. doi:10.1137/S0036139994279177.
[26] W. Lubashevsky, I. Lubashevsky, R. Mahnke, Distributed self-regulation of living tissue: Beyond the ideal limit, Physical Review E 81 (2010) 021922 (14 pages). doi:10.1103/PhysRevE.81.021922.
[27] V. V. Gafiychuk, I. A. Lubashevsky, Y. L. Klimontovich, Self-regulation in a simple model of hierarchically organized markets, Complex Systems 12 (2000) 103–126.
[28] W. Elsner, T. Heinrich, H. Schwardt, The Microeconomics of Complex Economies: Evolutionary, Institutional, Neoclassical, and Complexity Perspectives, Elsevier, Oxford, UK, 2015. doi:10.1016/c2012-0-06498-8.
[29] S. Mann (Ed.), Sectors Matter!: Exploring Mesoeconomics, Springer-Verlag, Berlin, 2011. doi:10.1007/978-3-642-18126-9.
[30] O. Gomes, M. Gubareva, Complex systems in economics and where to find them, Journal of Systems Science and Complexity 34 (2021) 314–338. doi:10.1007/s11424-020-9149-1.
[31] P. H. Dybvig, S. A. Ross, Arbitrage, in: J. Eatwell, M. Milgate, P. Newman (Eds.), The New Palgrave Dictionary of Economics, 3rd ed., Palgrave Macmillan, London, 2018, pp. 377–388. doi:10.1057/978-1-349-95189-5_449.
[32] M. O'Hara, Market Microstructure Theory, Blackwell Publishers, Cambridge, MA, 1995.
[33] K.-W. Li, Redefining Capitalism in Global Economic Development, Elsevier, London, 2017. doi:10.1016/b978-0-12-804181-9.00009-4.
[34] R. Kozlov, The effect of interest rate changes on consumption: An age-structured approach, Economies 11 (2023) 23. doi:10.3390/economies11010023.
[35] A. F. Daughety, Cournot competition, in: S. N. Durlauf, L. E. Blume (Eds.), The New Palgrave Dictionary of Economics, 3rd ed., Palgrave Macmillan, London, 2018, pp. 2408–2416. doi:10.1057/978-1-349-95121-5_2345-1.
[36] T. Aspromourgos, Invisible hand, in: M. Vernengo, E. P. Caldentey, J. Ghosh (Eds.), The New Palgrave Dictionary of Economics, 4th ed., Palgrave Macmillan, London, 2020. doi:10.1057/978-1-349-95121-5_3108-1, online.
[37] M. Friedman, Quantity theory of money, in: S. N. Durlauf, L. E. Blume (Eds.), The New Palgrave Dictionary of Economics, 3rd ed., Palgrave Macmillan, London, 2018, pp. 11090–11120. doi:10.1057/978-1-349-95121-5_1640-2.
[38] M. Friedman, The counter-revolution in monetary theory, in: G. E. Wood (Ed.), Explorations in Economic Liberalism: The Wincott Lectures, Macmillan Press, London, 1996, pp. 3–21. doi:10.1007/978-1-349-24967-1_1.
[39] M. Parkin, Inflation, in: S. N. Durlauf, L. E. Blume (Eds.), The New Palgrave Dictionary of Economics, 3rd ed., Palgrave Macmillan, London, 2018, pp. 6433–6446. doi:10.1057/978-1-349-95121-5_888-2.
[40] K. Dopfer, J. Foster, J. Potts, Micro-meso-macro, Journal of Evolutionary Economics 14 (2004) 263–279. doi:10.1007/s00191-004-0193-0.
[41] K. Dopfer, Mesoeconomics: Bridging micro and macro in a Schumpeterian key, in: S. Mann (Ed.), Sectors Matter!: Exploring Mesoeconomics, Springer-Verlag, Berlin, 2011, pp. 103–114. doi:10.1007/978-3-642-18126-9_5.
[42] K. Dopfer, The origins of meso economics: Schumpeter's legacy and beyond, Journal of Evolutionary Economics 22 (2012) 133–160. doi:10.1007/s00191-011-0218-4.
[43] P. Boettke, J. Dirmeyer, Spontaneous order, in: S. N. Durlauf, L. E. Blume (Eds.), The New Palgrave Dictionary of Economics, 3rd ed., Palgrave Macmillan, London, 2018, pp. 12825–12828. doi:10.1057/978-1-349-95121-5_2488-1.
[44] R. Gan, Helix Network Theory: The Dynamic Structure and Evolution of Economy and Society, Springer Nature Singapore, Singapore, 2023. doi:10.1007/978-981-19-8803-5.

[45] V. Mayevsky, S. Kirdina-Chandler, Mesoeconomics: Elements of a New Paradigm, Institute of Economics of the Russian Academy of Sciences, Moscow, 2020. In Russian.
[46] P. C. Kostis (Ed.), Bridging Microeconomics and Macroeconomics and the Effects on Economic Development and Growth, IGI Global, Hershey, PA, 2021. doi:10.4018/978-1-7998-4933-9.