Hierarchical Arbitrage-Free Mesomarket Model: A Characteristic Unit of Mesoeconomics

Vasily Lubashevskiy[a,*], Ihor Lubashevsky[b]

[a] *Tokyo International University, 4 Chome-42-31 Higashiikebukuro, Toshima City, Tokyo, 170-0013, Japan*
[b] *HSE Tikhonov Moscow Institute of Electronics and Mathematics, 34 Tallinskaya Ulitsa, 123458, Moscow, Russia*

ARTICLE INFO

Keywords:
Mesoeconomics
Networked markets
Self-regulation
Hierarchical information processing
Complex system integrity

ABSTRACT

We present a hierarchical supply mesomarket model based on a single raw material to illustrate the emergence of mesoeconomic units. The mesomarket is depicted as a tree-like network, with nodes representing micromarkets that balance local supply and demand and edges representing competing firms that produce similar goods. Raw material producers are positioned at the root, while retail outlets are at the leaves. Firm output, measured in raw material units, quantifies mesomarket activity. Firms maximize profit under a no-arbitrage condition linked to external investments at a fixed interest rate. Retail trading is driven by end-consumer demand as a function of prices. As demonstrated, the price pattern is determined solely by production technology and external investment rates, while end-consumer demand influences only the number of firms. The mesomarket is characterized by its integrity and strong connections to both the micro- and macroeconomic levels. It is the intrinsic mathematical structure of the developed model, along with its resulting properties, that elucidates the general features of mesoeconomic units and supports the case for recognizing mesoeconomics as a distinct constituent element of economics.

1. Introduction

Mesoeconomics, positioned as an intermediate level between microeconomics and macroeconomics, has been explored from various perspectives over the past few decades [1, 2, 3, for a review]. Within the concept proposed by Dopfer et al. [4] within the micro-meso-macro framework, *firstly*, meso-units are viewed as integral entities with properties irreducible to those of the micro and macro domains. *Second*, the micro, meso, and macro-units are constitutionally interconnected through ‘bottom-up’ and ‘top-down’ complementarity, representing the hierarchy of labor and technology [5]. *Third*, meso-units are central to the processes of innovation via which entrepreneurs drive economic development [6]. However, the specific mechanisms underlying the emergence of meso-entities, such as the synergy mechanisms in economic activities, are far from being well understood [7, Ch. 5 for a detailed discussion]. The current state of the art in this field has recently been outlined by Potts and Dopfer [8] in the context of evolutionary economics.

This paper provides an example of meso-units illustrating such mechanisms. The proposed model is grounded in the concept of economic sector [e.g., 7], consisting of groups of competing firms producing similar goods. These groups (industries) are interconnected, forming a complex network. The term *mesomarket* will refer to all commodity transactions within a specific sector. Real sectors can be divided into subsectors, and the model represents a single-commodity subsector, such as an agricultural subsector based on the cultivation and processing of a specific crop like wheat, rice, or cotton. Additionally, we consider a quasi-stationary state of the mesomarket to highlight the key role of innovation.

By examining the internal mathematical structure of the constructed model for mesomarkets, viewed as networks of micromarkets, we aim to elucidate the essence of mesoeconomic units. In doing so, we hope to emphasize their integrity and advocate for treating mesoeconomics as a distinct component of economics, alongside microeconomics and macroeconomics.

---

[*] Corresponding author: Vasily Lubashevskiy
✉ `vlubashe@tiu.ac.jp` (V. Lubashevskiy); `ilubashevskii@hse.ru` (I. Lubashevsky)
ORCID(s): 0000-0002-2399-5050 (V. Lubashevskiy); 0000-0002-8932-8023 (I. Lubashevsky)

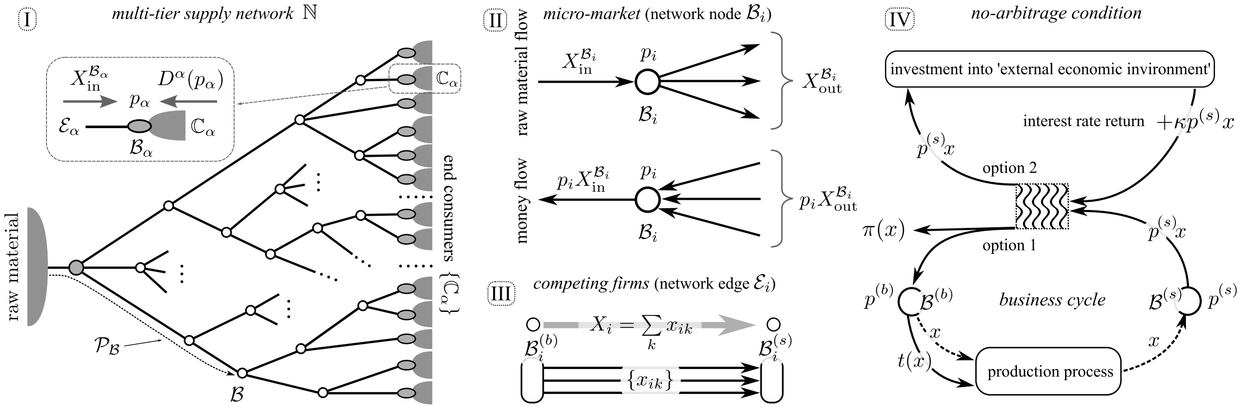

Figure 1: The structure of the analyzed mesomarket and its characteristic features.

2. Mesomarket Model

Figure 1(I) illustrates the proposed mesomarket model as a hierarchically organized supply network ℕ responsible for producing goods from a primary raw material. Supplementary components, added during production, are factored into production costs, including labor, services, transaction costs, etc.

The production process consists of several stages, each stage i is associated with a specific production activity by competing firms $\{f_{ik}\}$. These firms are grouped into the edge $\mathcal{E}_i = \bigcup_k f_{ik}$ of the network ℕ. The branching of the production process into interconnected stages, linked by the flow of raw materials, is represented as a tree-shaped network.[1] The branching points (nodes) are treated as micromarkets $\mathcal{B}_i$. End consumers are grouped into $\{\mathbb{C}\alpha\}$, each purchasing specific goods at the corresponding retail outlets $\mathcal{B}_\alpha$. Overlapping consumer groups are ignored, and goods sold at different outlets are considered distinct.

The mesomarket operates through prices $\{p_i\}$ assigned to micromarkets $\{\mathcal{B}_i\}$, including retail outlets $\{\mathcal{B}_\alpha\}$. These prices correspond to the quantity of the product measured by the unit amount of raw material. Thus, the production activity of a firm f_{ik} is quantified by the amount x_{ik} of product, measured in raw material units per unit time. This common measure ensures that products from different stages are commensurable.

Similarly, we quantify the total demand $D^\alpha(p_\alpha)$ of the consumer group $\mathbb{C}_\alpha$ as a function of the goods price p_α at the retail outlet $\mathcal{B}_\alpha$. The corresponding supply

$$X^{\mathcal{B}_\alpha}_{\mathrm{in}} = \sum_{f_{\alpha k} \in \mathcal{E}_\alpha} x_{\alpha k} = D^\alpha(p_\alpha). \tag{1}$$

represents the cumulative production rate of firms forming the network edge $\mathcal{E}_\alpha$. Equality (1) serves as the 'top boundary condition' for product flux at retail outlets.

2.1. Local equilibrium of micromarkets

At each node $\mathcal{B}_i$, the input and output components of the commodity flow are balanced, described by the continuity of the bottom-up raw material flow (Fig. 1 II)

$$X^{\mathcal{B}_i}_{\mathrm{in}} = \sum_{\mathcal{E}_{i,\mathrm{out}}} X^{\mathcal{B}_i}_{\mathrm{out}}, \tag{2}$$

where the sum runs over all edges $\mathcal{E}_{i,\mathrm{out}}$ leaving node $\mathcal{B}_i$. In the top-down direction, the conjugated money flow is described by the same expression, multiplied by the price p_i (Fig. 1 II).

---

[1] In equilibrium, the mesomarket network can only form cycles in degenerate cases, allowing us to disregard this possibility.

2.2. Perfect competition among firms in micromarkets

Each firm f_{ik} selects a production rate $x^{\rm m}_{ik}$ that maximizes its profit $\pi_{ik}(x_{ik})$, treating the prices $p^{(b)}_i$ and $p^{(s)}_i$ in the micromarkets $\mathcal{B}^{(b)}_i$ and $\mathcal{B}^{(s)}_i$ (Fig. 1 III) as fixed. The firm's profit is expressed as:

$$\pi_{ik}(x_{ik}) = \big[p^{(s)}_i - p^{(b)}_i\big] x_{ik} - t_{ik}(x_{ik}) \,, \tag{3}$$

where the production cost $t_{ik}(x_{ik})$ is convex and positive at $x_{ik} = 0$. Specifically, we use the ansatz:

$$t_{ik}(x_{ik}) = a_{ik} + g_{ik} x^{\beta}_{ik} \,, \tag{4}$$

where the positive constants $\{a_{ik}, g_{ik}\}$ are firm-specific, and $\beta > 1$ is the same for all firms. By virtue of (3), the production rate $x^{\rm m}_{ik}$ is specified by the condition

$$\left.\frac{\partial \pi_{ik}}{\partial x_{ik}}\right|_{x_{ik}=x^{\rm m}_{ik}} = \big[p^{(s)}_i - p^{(b)}_i\big] - \left.\frac{d t_{ik}}{d x_{ik}}\right|_{x_{ik}=x^{\rm m}_{ik}} = 0 \tag{5}$$

and the value $x^{\rm m}_{ik}$, in its turn, specifies the maximum $\pi^{\rm m}_{ik}(\Delta p_i)$ of the profit $\pi_{ik}(x_{ik})$. Thereby, the production rate $x^{\rm m}_{ik}$ and the optimal profit $\pi^{\rm m}_{ik}(\Delta p_i)$ of individual firms are functions of the price difference Δp_i and the adopted technology. For ansatz (4)

$$x^{\rm m}_{ik}(\Delta p_i) = \left(\frac{\Delta p_i}{\beta g_{ik}}\right)^{1/(\beta-1)} \tag{6a}$$

$$\pi^{\rm m}_{ik}(\Delta p_i) = \frac{(\beta-1)}{(g_{ik})^{1/(\beta-1)} \beta^{\beta/(\beta-1)}} \big(\Delta p_i\big)^{\beta/(\beta-1)} - a_{ik} \,. \tag{6b}$$

Expression (6b) can also be used to specify the minimal price difference Δp_i at which production activity is possible, satisfying the condition $\pi^{\rm m}_{ik}(\Delta p_i) > 0$.

For firms extracting or producing raw materials,

$$p^{(b)}_{\rm root} = 0 \tag{7}$$

serves as the 'bottom boundary condition' imposed on the price distribution.

For each edge $\mathcal{E}_i$, the number of firms N_i is assumed to be large, $N_i \gg 1$. Thus, the *variety* of firms $\{f_{ik}\}_{k=1}^{k=N_i}$ should be well-developed and remain stable, despite variations in their number due to changes in end-consumer demand. In this case, the parameters

$$a_i = \frac{1}{N_i} \sum_{1\le k\le N_i} a_{ik} \quad \text{and} \quad \frac{1}{g_i^{1/(\beta-1)}} = \frac{1}{N_i} \sum_{1\le k\le N_i} \frac{1}{g_{ik}^{1/(\beta-1)}} \tag{8}$$

can be treated as constants, leading to the relationship (Fig. 1 III)

$$X^{\rm m}_i \overset{\rm def}{=} \sum_{1\le k\le N_i} x^{\rm m}_{ik}(\Delta_i p) = \frac{N_i}{(\beta g_i)^{1/(\beta-1)}} \Big[p^{(s)}_i - p^{(b)}_i\Big]^{1/(\beta-1)} \tag{9}$$

between the price difference $\Delta_i p$, the number N_i of firms, and the production rate $X^{\rm m}_i$ attributed to edge $\mathcal{E}_i$.

2.3. No-arbitrage condition

The analyzed mesomarket is embedded in a broader economic environment, allowing actors to enter or exit based on financial efficiency. Figure 1(IV) illustrates a firm's business cycle as a sequence of several steps, with its duration as the time unit. The firm sells its product x at price $p^{(s)}$ in the micromarket $\mathcal{B}^{(s)}$, earning revenue $p^{(s)}x$. It then has two options. *Option* 1 is to continue the cycle by purchasing x-amount of commodity at price $p^{(b)}$ in the micromarket $\mathcal{B}^{(b)}$, reserving $t(x)$ for production, and earning the profit per cycle $\pi(x) = \big[p^{(s)} - p^{(b)}\big]x - t(x)$. *Option* 2 is to end production and invest the revenue $p^{(s)}x$ in an external activity at an interest rate κ, determined by the broader economic system, yielding a surplus of $\kappa p^{(s)}x$ per time unit.

In this case, the no-arbitrage condition requires that options 1 and 2 be financially equivalent for all firms $\{f_{ik}\}$, i.e.,

$$\pi_{ik}\big(x^{\rm m}_{ik}\big) = \kappa p^{(s)}_i x^{\rm m}_{ik} \,. \tag{10}$$

Such equality should lead to balanced and stable market functioning [cf. 9].

3. Equilibrium State of Mesomarket

The proposed model specifies the price distribution $\{p_i\}$ in the micromarkets $\{\mathcal{B}\}$ and the distribution of firms $\{f_{ik}\}$ within production processes. The network $\mathbb{N}$ endows the mesomarket with integrity due to going through a node, a micromarket $\mathcal{B}$, the sell price $p_{\mathcal{B}}^{(s)}$ for the parent edge becomes the buy price $p_{\mathcal{B}}^{(b)}$ for its daughter edges. Under steady-state conditions, the analyzed mesomarket exhibits two key properties.

3.1. The price distribution and the firm distribution

The prices in the neighboring micromarkets $\mathcal{B}_i^{(b)}$ and $\mathcal{B}_i^{(s)}$ (Fig. 1 III) are related by the expression

$$\left\{\left[\frac{(\beta-1)}{\beta}-\kappa\right]p_i^{(s)}-\frac{(\beta-1)}{\beta}p_i^{(b)}\right\}\cdot\left[p_i^{(s)}-p_i^{(b)}\right]^{1/(\beta-1)}=a_i(\beta g_i)^{1/(\beta-1)}\,. \tag{11}$$

Condition (7) enables us to integrate the system of equations (11) step-by-step from the network root to the retail outlets, determining the price $p_{\mathcal{B}}$ in each micromarket $\mathcal{B}$ based solely on the technological parameters $\{a_i, g_i\}$ along the path $\mathcal{P}_{\mathcal{B}}$ (Fig. 1 I) and the interest rate κ. Crucially, end-consumer demand does not affect the price distribution. Besides, Expression (11) shows that the interest rate κ for external investment must meet the condition

$$\kappa<\frac{(\beta-1)}{\beta}\,, \tag{12}$$

otherwise, this mesomarket cannot survive in financial competition with its ‘economic environment,’ as market actors would see no reason to participate in the goods production.

This price distribution defines the relationship between the number N_i of firms forming the edge $\mathcal{E}_i$ and the corresponding cumulative commodity flow X_i^{m}, measured as in the amount of raw material going though the edge $\mathcal{E}_i$ per unit time

$$X_i^{\mathrm{m}}=\frac{N_i}{(\beta g_i)^{1/(\beta-1)}}\left[p_i^{(s)}-p_i^{(b)}\right]^{1/(\beta-1)}\,, \tag{13}$$

which should be subjected to the ‘boundary’ condition (1).

3.2. The mesomarket response to consume demand

Due to the independence of price distribution from the end-consumer demands $D^{\alpha}(p_\alpha)$, these demands only determine the number of firms involved in production. Condition (1) specifies the number N_α of firms supplying the retail outlet $\mathcal{B}_\alpha$ as follows:

$$N_\alpha=\left[\frac{\beta g_{\mathcal{E}_\alpha}}{\Delta p_{\mathcal{E}_\alpha}}\right]^{1/(\beta-1)}D^{\alpha}(p_\alpha)\,. \tag{14}$$

Then, according to the conservation law shown in Fig. 1 II, the quantities $\{N_i\}$ are determined through a top-down procedure, that equates the number of firms at the parent (in) and daughter (out) edges at each node $\mathcal{B}_i$:

$$\left[\frac{\Delta p_{\mathrm{in}}^{\mathcal{B}_i}}{\beta g_{\mathrm{in}}^{\mathcal{B}_i}}\right]^{1/(\beta-1)}N_{\mathrm{in}}^{\mathcal{B}_i}=\sum_{\mathcal{E}_{i,\mathrm{out}}}\left[\frac{\Delta p_{\mathcal{E}_i,\mathrm{out}}^{\mathcal{B}_i}}{\beta g_{\mathcal{E}_i,\mathrm{out}}^{\mathcal{B}_i}}\right]^{1/(\beta-1)}N_{\mathcal{E}_i,\mathrm{out}}^{\mathcal{B}_i}\,. \tag{15}$$

4. Conclusion

The results indicate that the analyzed mesomarket qualifies as a mesoeconomic unit. *First*, its integrity stems from the network of bottom-up raw material flows and top-down money flows driven by consumer demand. Crucially, money flow, not prices, conveys demand information and its hierarchical self-processing. The network structure may also serve as an individual parameter.[2] *Second*, the mesomarket is closely linked to micro and macro levels of economics. On the one hand, micromarkets, as its components, cannot be fully understood in isolation. On the other hand, production cost

$t(x)$, including labor, technology, and supplementary materials, along with the interest rate κ reflect macroeconomic factors.

Since the price distribution is independent of end-consumer demand, a change in the demand $D^{\alpha}(p_{\alpha})$ from one consumer group $\mathbb{C}_{\alpha}$ does not disturb the supply-demand equilibrium for other consumer groups $\{\mathbb{C}_{\alpha'}\}_{\alpha'\neq\alpha}$. In this sense, the market response is localized, despite the complex branching structure of the mesomarket itself. This behavior can be classified as perfect self-regulation: when the mesomarket responds to changes in end-consumer demand, it adjusts by altering supply at the necessary retail outlets while maintaining the previous supply levels at other outlets.[3]

The mesomarket state is 'saturated,' meaning firms cannot *extensively* reinvest profits in expanding production, as there is no demand for additional products in micromarkets. To benefit from investment, firms must innovate by improving technology or management to *intensively* reduce the production costs $t(x)$, highlighting the key role of entrepreneurship in driving economic growth.

The 'saturation' of micromarkets should lead to long-term inflation. At a given price level, the money flow through mesomarkets cannot increase without a rise in end-consumer demand. However, money flow from external investment with a fixed interest rate κ or profit accumulation should grow with linear or exponential rate. This proposition aligns with Friedman's [1996] view that inflation is inevitable when the money supply outpaces economic output. In other words, local equilibrium in mesomarkets cannot ensure macroeconomic stability.

The proposed model allows for various dynamic generalizations, including supply-demand fluctuations in micromarkets, entry and exit thresholds, and the reorganization of firm connections, especially when multiple nodes merge into a large firm (reflecting oligarchy). Special attention could be given to extending the model to include multiple embedded tree-like networks, accounting for different raw materials and the production of multi-component goods.

References

[1] W. Elsner, T. Heinrich, H. Schwardt, The size dimension of complex economies—towards a meso-economics: The size of interaction arenas and the emergence of meso-platforms of institutional coordination, in: The Microeconomics of Complex Economies: Evolutionary, Institutional, Neoclassical, and Complexity Perspectives, Elsevier, Oxford, UK, 2015, pp. 419–447. doi:`10.1016/B978-0-12-411585-9.00014-2`, This chapter was coauthored by Claudius Gräbner.

[2] M. Scholz-Wäckerle, Generating meso behaviour, in: R. Frantz, S.-H. Chen, K. Dopfer, F. Heukelom, S. Mousavi (Eds.), Routledge Handbook of Behavioral Economics, Routledge, the Taylor & Francis Group, New York, NY, 2017, pp. 152–167. doi:`10.4324/9781315743479`.

[3] V. Mayevsky, S. Kirdina-Chandler, Mesoeconomics: Elements of a New Paradigm, Institute of Economics of the Russian Academy of Sciences, Moscow, 2020. In Russian.

[4] K. Dopfer, J. Foster, J. Potts, Micro-meso-macro, Journal of Evolutionary Economics 14 (2004) 263–279. doi:`10.1007/s00191-004-0193-0`.

[5] K. Dopfer, J. Potts, A. Pyka, Upward and downward complementarity: The meso core of evolutionary growth theory, in: A. Pyka, U. Cantner (Eds.), Foundations of Economic Change, Springer International Publishing, Cham, Switzerland, 2017, pp. 69–80. doi:`10.1007/978-3-319-62009-1_4`.

[6] K. Dopfer, The origins of meso economics: Schumpeter's legacy and beyond, Journal of Evolutionary Economics 22 (2012) 133–160. doi:`10.1007/s00191-011-0218-4`.

[7] R. Gan, Helix Network Theory: The Dynamic Structure and Evolution of Economy and Society, Springer Nature Singapore, Singapore, 2023. doi:`10.1007/978-981-19-8803-5`.

[8] J. Potts, K. Dopfer, New evolutionary economics, SSRN Electronic Journal (2024) 1–23. doi:`10.2139/ssrn.4837360`.

[9] M. O'Hara, Market Microstructure Theory, Blackwell Publishers, Cambridge, MA, 1995.

[10] V. V. Gafiychuk, I. A. Lubashevsky, Y. L. Klimontovich, Self-regulation in a simple model of hierarchically organized markets, Complex Systems 12 (2000) 103–126.

[11] I. Lubashevsky, V. Gafiychuk, Cooperative mechanism of self-regulation in hierarchical living systems, SIAM Journal on Applied Mathematics 60 (2000) 633–663. doi:`10.1137/S0036139994279177`.

[12] W. Lubashevsky, I. Lubashevsky, R. Mahnke, Distributed self-regulation of living tissue: Beyond the ideal limit, Physical Review E 81 (2010) 021922 (14 pages). doi:`10.1103/PhysRevE.81.021922`.

[13] M. Friedman, The counter-revolution in monetary theory, in: G. E. Wood (Ed.), Explorations in Economic Liberalism: The Wincott Lectures, Macmillan Press, London, 1996, pp. 3–21. doi:`10.1007/978-1-349-24967-1_1`.

---

[2] For this, see [10], which analyzes a similar model under the artificial zero-profit assumption.

[3] A similar behavior has been found for the self-regulation of living tissue, where a peripheral vascular network, supplying nutrients to the corresponding organ in the human body, also responds in a strictly localized manner under ideal conditions [11]. For more realistic characteristics of blood vessels, the vascular network's response remains quasilocal [12], suggesting that the identified properties of this networked mesomarket should not change drastically when the equilibrium state is not significantly disturbed.