Journal of Public Economic Theory

WILEY

JOURNAL OF PUBLIC ECONOMIC THEORY

ORIGINAL ARTICLE

Minorities in Dictatorship and Democracy

Arseniy Samsonov

Department of Economics, HSE University, St. Petersburg, St. Petersburg, Russia

Correspondence: Arseniy Samsonov (ad.samsonov@hse.ru)

Received: 22 May 2024 | **Revised:** 25 March 2025 | **Accepted:** 4 September 2025

ABSTRACT

How does the level of democracy in a country affect the government's treatment of ethnic minorities? I use the Baron–Ferejohn game to model bargaining over government formation and resource division in an ethnically fragmented society. Each ethnic group is a unitary actor, voting weights correspond to ethnic group sizes, and recognition probabilities are proportional to voting weights. The voting quota required to pass a decision is a proxy for the level of democracy. When the majority group exceeds half of the population, the expected payoffs of minorities non-monotonically depend on the voting quota. When the voting quota is small, several minorities may form a winning coalition, so minorities get high expected payoffs. This outcome explains the existence of relatively tolerant autocracies. For intermediate values, a coalition of minorities is insufficient to rule while the majority is sufficient. As a result, it gets most of the surplus, which reflects democracies where minorities are underrepresented in the government and get fewer benefits. Finally, when the voting quota is large, minorities are needed to form a winning coalition, so their expected payoffs are high, too. The latter scenario corresponds to democracies with many constraints on the leader, who needs the support of minorities to get approval from various branches of power.

1 | Introduction

At the beginning of the Syrian civil war, the country's ethnic and religious minorities were reluctant to join the rebellion alongside the Sunni Arab majority (Rafizadeh 2011). Similarly, the Romanians and Serbs supported the Austrian monarchy during the Hungarian Revolution of 1848–1849 (Brubaker and Feischmidt 1848). Could it be that, in some cases, ethnic minorities prefer a dictator or a monarch to a pro-democracy movement? To answer this question, the paper models how ethnic groups divide resources using the Baron–Ferejohn bargaining game. All groups are unitary actors; there is one large group and several small ones of identical size. At each stage, one of the groups is randomly recognized as a proposer and makes an offer to the remaining ones. There is an exogenous voting quota specifying the minimum total size of members of a coalition that must support a division for it to be accepted. It models the level of democracy. The assumption that it is exogenous reflects that state institutions are unlikely to change in the short run (Acemoglu and Robinson 2012). The model predicts that democracy bears a non-monotonic effect on the welfare of minorities, who will be worst off when its level is intermediate.

The case of a small voting quota corresponds to autocracy. In this scenario, several minorities may rule together. Due to their small size, minorities have worse outside options to form a coalition than the majority. Hence, for a minority recognized as a proposer, it is cheaper to form a coalition with other minorities. Therefore, minority coalitions form and minorities' expected payoffs are relatively high. This result explains why there are autocracies where a dictator provides benefits to minorities and includes their members in the inner circle. The other side of the coin is that when the dictator represents a large ethnic group, she can rely on her coethnics and require no coalition partners. Therefore, in autocracies, the outcome for minorities depends on whether the dictator comes from a small or large group.

If the voting quota is medium, minorities cannot form a winning coalition without the majority, but the majority is sufficient. The model predicts that in this situation, their payoffs will be low. This case in the model corresponds to regimes where leaders have to win elections but do not need a broad coalition due to facing few constraints. Unlike in autocracies, in such regimes, minority groups do not influence the government and receive few benefits. For instance, Turkish presidents have rarely attempted to win the votes of the Kurds, although the situation improved somewhat in the early 2010s (Haklai 2013). Moreover, Kurdish members of parliament are sometimes arrested and receive jail terms (Hawramy 2011).

Finally, if the voting quota is large, it is impossible to form a winning coalition without minorities. The model predicts that they receive high payoffs in this case. This scenario corresponds to countries where leaders must win elections to stay in power and are constrained by parliaments, courts, and the possibility of mass demonstrations. Because leaders need broad support, they care about small groups. As a result, such groups tend to get government representation and benefits.

Governance in fragmented societies often requires compromise between leaders of ethnic factions (Francois et al. 2015). Therefore, it is interesting to apply a bargaining model with more than two players to analyze the division of benefits in such a society. The Baron-Ferejohn game is one of the most-studied non-cooperative approaches to this problem[1].

In the model, the stage when one of the players proposes corresponds to a country's leader building support to stay in power by promising an allocation of benefits across ethnic groups. In a presidential democracy, one can think of a presidential candidate trying to win the majority of votes by promising public goods or government jobs to members of various ethnicities. In a parliamentary democracy, one can imagine the leader of an ethnic faction proposing leaders of other ethnic factions to form a ruling coalition. In a non-democracy, one can think of a dictator offering government positions to members of different ethnic groups, which would translate into public goods for their coethnics.

The random proposer assumption reflects that there are no negotiation rules and that various circumstances may enable a representative of a particular group to propose to others at a specific time. The circumstances can be different across political regimes. For example, in a multiparty democracy, the leader of one of the parties may have an opportunity to schedule a meeting with others and propose a coalition. In a dictatorship, a military officer may seize power by force, which would allow him to allocate government positions. Of course, one can develop a more specific and realistic model of coalition formation for each regime type. However, such an approach would make it hard to compare outcomes for minorities in different regimes. By contrast, the Baron–Ferejohn model drops the specifics, and its simplicity allows for analyzing many possible settings. In the Appendix, I show that for the game in this paper, power indices produce patterns similar to the model with a random proposer, so this assumption is not crucial.

I modify the original Baron–Ferejohn model by assuming that players hold different amounts of votes, the quota required to pass the proposal varies, and the recognition probability of a player equals her vote share. For tractability, I assume that there are several minorities with one vote each and one majority with many votes. The different numbers of votes that players hold correspond to the different sizes of ethnic groups. The quota corresponds to the constraints on the leader. Its lower level reflects more autocratic and higher level—more democratic regimes. Finally, the assumption that the recognition probability equals the vote share reflects that bigger groups can supply more leaders who can form the coalition or are otherwise more influential[2]. In the Appendix A1, I show that the model's results are robust to assuming equal recognition probabilities. While the setup is a special case of existing models (Eraslan and McLennan 2013; Montero 2017), it highlights a new result about the relationship of small players' payoffs to the voting quota.

To illustrate the model's implications, consider the consequences of regime change in Ivory Coast in the early 1990s. The economic opportunities in the country attracted a significant number of migrants. The longtime president Felix Houphouet–Boigny encouraged immigration and allowed foreigners to hold dual citizenship. The end of his tenure saw political unrest. In 1990, the first parliamentary elections happened, and in 1993, after Houphouet–Boigny's death, Konan Bedie became president. The country's shift to democracy worsened the opportunities for minorities. Bedie announced a policy of reserving land ownership for Ivorite people, thus discriminating against foreigners, and tried to bar them from voting. The next president, Laurent Gbagbo, was able to restrict foreigners from voting through a discriminatory identity card policy. Subsequent violence against foreigners and clashes between different Ivorian ethnic groups led to a civil war in 2002 (The Economist 2002).

Somalia is also illustrative of the model's logic. Before Siad Barre came to power and after independence (1960–1969), Somalia was a republic where each clan pursued its interests. The largest clans had the most influence. Unlike Barre's rule, the government made no effort to end discrimination against minorities (Lewis 2004). Siad Barre, though blamed for human rights violations, proved to be a relatively good option for minorities. During his rule, the historically marginalized Gaboye people were able to get government and military jobs (Mbanaso and Korieh 2010). His government did not discriminate against the Benadiri minority. The lands of Bantu, though, were confiscated in favor of bigger clans. Still, the situation after Barre proved far worse as the three groups became victims of violence and looting by larger and more powerful clans (Minority Rights Group 2017).

Syria under the Assads was an autocracy that treated ethnic minorities relatively well. The core supporters of Hafiz al-Assad were his fellow Alawites and representatives of other small religious groups (Zisser 1999). Christians, Druze, and Isma'ili Shias were widely tolerated in this generally repressive regime (Minority Rights Group 2017).

On the democratic side, Sri Lanka had a Polity score (Marshall et al. 2002) of 6 in the late 1970s and 5 in the 1980s, on a scale of −10 to 10. The country has had a long and destructive civil war between the Singhalese, who constitute a 75% majority, and the Tamils, who comprise roughly 25%. The Tamils strived for independence as the government was not recognizing their language as official and seizing their lands in favor of the Singhalese (Minority Rights Group 2017).

Türkiye had a Polity score between 6 and 8 from the early 1990s to 2015. Despite the country being relatively democratic, the Kurds have faced arbitrary arrest, torture, and displacement. Authorities targeted their media outlets and activists. Armenians in Türkiye have felt insecure as well. Armenian schools, enterprises, and religious institutions have received threats, and many intellectuals from the community faced trial for expressing antigovernment views (Minority Rights Group 2017).

On a global scale, Western European states are the most democratic and inclusive of minorities. However, democracies that try to fight discrimination exist elsewhere. For example, in India, there are quotas in the government and public service for the historically marginalized castes (The Economist 2012). In those examples, there are constitutional protections for minority groups. While changing the constitution and abolishing these protections is possible, doing so requires a supermajority, which corresponds to a large voting quota in the model.

2 | Literature

The model builds on the idea of minimum winning coalitions due to Riker (1975), who observed that in politics, the smallest coalitions sufficient to govern will form. In this paper, given different voting quotas, it is optimal for the proposer to form different coalitions that may or may not include minorities. The voting quota models the political regime type. A smaller quota corresponds to a more autocratic regime, and a larger quota represents a more democratic regime. Thus, this paper also builds on the selectorate theory (Bueno de Mesquita et al. 2005), where the size of the minimum winning coalition that a leader must assemble to stay in power models the regime type. While the idea of minimum winning coalitions is old, to the best of my knowledge, there are no models that apply it to understanding minority welfare under different political regimes.

In this paper, autocracy corresponds to the parameter region such that the voting quota is less than half of the population. At the same time, no player is restricted from being in the winning coalition. As a result, multiple disjoint sets of players may be winning coalitions. Bueno de Mesquita et al. (2005) also model autocracy as an institution where a few people rule, but many may potentially enter the elite, replacing some or all of its current members. This assumption aligns with empirical evidence in Geddes et al. (2014), who show that most autocratic regime breakdowns occur when one leadership group replaces another, but the new government is also autocratic. A case in point is the Iranian revolution that replaced the secular autocratic regime of the Shah with the new Islamist elite.

In this paper, an ethnic group can form a winning coalition and participate in the sharing of benefits if it persuades sufficiently many other ethnic groups to support it. Otherwise another ethnic group gets a chance to form the coalition. Similarly, in the model from Bueno de Mesquita et al. (2005), a leader assembles a winning coalition that keeps him in power. A challenger tries to unseat the current leader by proposing benefits to the current members of the winning coalition. If at least one of them agrees, the challenger replaces the old leader and provides the benefits he promised for one period. However, in the future, the new leader may assemble a new winning coalition. This possibility gives an extra incentive for the current coalition members to not support the challenger. If all the current coalition members support the current leader, outsiders cannot overthrow him, and he stays in power. Hence, people who are already in power are more powerful than outsiders. This assumption reflects the fact that elite members control the key government, army, and security service positions.

Acemoglu et al. (2010) make similar assumptions about autocracy. In their paper, autocracy is a situation when few people form the government compared to the total population size. Potentially, any citizen may enter the government, and its current members may lose their positions. However, a change in government requires the consent of its current members.

Several papers analyze the impact of changing the quota in weighted voting games on players' power, as given by the Shapley-Shubik and Banzhaf indices. Barthelemy et al. (2021) focus on games with up to six players, a large total number of votes, and random allocation of votes. In this context, they show that the probability of there being a player with no voting power non-monotonically depends on the quota. Zuckerman et al. (2012) provide an example, which is a special case of my model: there are $n-1$ players with one vote each and one player with n votes; the quota is either 1 or n. Moving from a quota of 1 to a quota of n makes each small player powerless. Unlike this paper, they do not consider intermediate quotas or different voting weights in a setting with one large and several identical small players. Finally, Zick et al. (2011) show that a player's voting power is the highest when the quota equals her weight. Using simulations, they demonstrate that the quota that equals the number of players' votes plus one is likely bad for her if she is a small player. By contrast, in this paper, where each small group has one vote, a quota equal to two votes yields relatively high expected payoffs to each of them, whereas they get low payoffs when the quota is considerably larger than two. The main contribution of this paper to that literature is that it not only demonstrates that a voting quota non-monotonically affects small players' payoffs but also analytically derives the comparative statics in a natural setting[3]. The comparative statics are intuitive and robust to different modeling approaches: payoffs of small groups are nondecreasing and then nonincreasing in the quota. In the Appendix, the paper presents results for the Baron-Ferejohn model with equal recognition probabilities and the Shapley-Shubik index. Interestingly, in these models, when the quota is relatively small, small players' payoffs monotonically decrease with respect to it, but these players have nonzero voting power. This finding complements the first two papers, which focus on the instances when a player becomes a dummy, i.e., loses all voting power.

There is a vast literature on the Baron-Ferejohn model that I use to study coalition formation and resource division (Eraslan and Evdokimov 2019). Most papers focus on expected payoffs that players get in a subgame-perfect equilibrium such that strategies do not depend on history. Similar to this paper, Montero (2017) analyzes a game with different voting weights

TABLE 1 | Feasible winning coalitions and minimum winning coalitions.

Condition on the voting quota W	Winning coalitions	Minimum winning coalitions
$W \leq m$	Any set that contains the majority group or at least W minorities	The majority group or W minorities
$m + 1 \leq W \leq k$	Any set that contains the majority group	The majority group alone
$k + 1 \leq W \leq m + k$	Any set that contains the majority group and at least $W - k$ minorities	The majority group and $W - k$ minorities

and recognition probabilities equal to them. She shows that the stationary subgame perfect equilibrium payoffs are proportional to voting weights if the set of winning coalitions is *weakly balanced*, i.e. there is a probability distribution over winning coalitions with minimum total weight such that each player is included in one with the same probability. It is easy to show that in my game, the set of winning coalitions is never balanced, so the result does not apply[4]. Montero does not analyze the parameterization I focus on with one large group and several small ones.

Table 1

Eraslan and Merlo (2017) analyze a model where the bargaining surplus depends on the proposer. Each player has one vote. These assumptions are different from this paper, where the surplus is the same, but voting weights are different. Similar to this paper, Eraslan and Merlo study the impact of the voting quota on the distribution of payoffs. They show that a larger quota can make the distribution more unequal. While this conclusion is similar to the one in this paper, the logic is different. In their model, the result occurs because more inclusive voting rules protect the player with the highest surplus, so she has an incentive to wait until she becomes the proposer. In my model, a higher voting quota may reduce the payoff of a minority because it may lead to a situation when minority-only winning coalitions cannot form.

Like in the classic Baron–Ferejohn model, I assume that once players reach an agreement, the game ends. In another variation of this game, Kalandrakis (2004) assumes that the outcome of each round of bargaining becomes the next round's status quo. His motivation for such a model is a legislature in a democracy where the old legislation is in place until politicians agree on a new one. Like in Baron and Ferejohn (1989), in Kalandrakis (2004), players must divide a dollar. The model makes a stark prediction that, at some point, the proposer will extract the whole surplus. The comparative politics literature that I survey below shows that distributive politics in ethnically divided societies does not produce such stark outcomes. While there are improvements for a leader's coethnics, other ethnic groups do not lose all public spending. Moreover, some of these papers show that having a coethnic in a government position other than the leader improves one's material welfare too. In the same scenario, Kalandrakis' model would predict a very high inequality between the coethnics of the leader and others. Hence, the Baron-Ferejohn model agrees with this evidence more than Kalandrakis'. The same logic applies to other bargaining models with an endogenous status quo (e.g. Dziuda and Loeper 2016).

While Kalandrakis' model applies to lawmaking, the classic Baron-Ferejohn model I use is a more accurate description of bargaining over the budget or government formation. When a group of politicians negotiates the distribution of resources, the alternative to agreeing is often deadlock. For example, in the US, the government may enter a shutdown if it fails to pass the budget (Committee for a Responsible Federal Budget 2024). In Israel, there are new elections if the ruling coalition fails to form (Kingsley 2022). Hence, in many situations, negotiation continues until there is a decision, and nobody benefits before that.

Several theoretical papers study the link between democracy and minority rights. To start with, Mukand and Rodrik (2020) build a model where both the regime type and minority rights protection are endogenous and depend on income inequality between elites and non-elites, and the identity cleavage between the majority and minority. In particular, if income inequality is low, the elite may forestall revolution by co-opting the minority and establishing a liberal autocracy. If income inequality is high, then to forestall a revolution, the elites concede to transform the regime into an electoral democracy, which does not protect minority rights. Compared to their paper, this project's main contribution is to look at how a fixed regime type affects the welfare of minorities. The comparative politics literature that I survey later in this section shows that in the same country with the same regime type, the welfare of ethnic groups varies depending on who is in the government. This observation aligns with my model and differs from that of Mukand and Rodrik, where minorities' rights may or may not be guaranteed after the revolution and are not renegotiated later.

Fernandez and Levy (2008) model redistribution between groups in a democratic country and look at the impact of diversity on redistribution. In their model, the regime type does not vary. Francois et al. (2015) model transfers between ethnic groups in Africa made by leaders under the revolution constraint. These two papers only analyze redistribution given a single regime type. In contrast, the current study compares different regime types. However, the above two papers are similar to this article in that they model groups in society as unitary actors seeking to maximize their share of resources.

The assumption that ethnic groups behave as unitary actors is a strong one. However, theories that utilize this approach help find and explain empirical regularities. Using the Ethnic Power Relations data set, Beiser–McGrath and Metternich (2021) find that smaller ethnic groups often choose other minorities as coalition partners. This finding aligns with my model, which predicts that minority-minority coalitions are likely. Martinez Machain and Rosenberg (2018) show that leaders of minority groups strategically avoid conflict when the country's leader may use diversionary repression against them.

In the model, an ethnic group gains when its representative joins the government. There is mixed empirical evidence on whether ethnic favoritism exists and significantly affects welfare outcomes. Franck and Rainer (2012) analyze data from 18 African countries and find that having a coethnic as a country's leader improves primary education and infant mortality rates. While the effects they find are small in magnitude, they are statistically significant and comparable to the impact of policy interventions. Kramon and Posner (2016) focus on the effect of ethnic favoritism on education outcomes in Kenya. They report that if a child's coethnic serves as president or minister of education during her primary school years, she gets an extra third of a year of education on average. In line with the model's assumptions, their paper suggests that people in government besides the leader can provide favors to coethnics. Another finding is that ethnic favoritism persists regardless of the regime type, consistent with the model. While Africa is known for ethnic favoritism (Kramon and Posner 2016), there are similar findings for other countries. Duflo (2005) reviews several studies showing that in India, government seat reservations for members of historically disadvantaged groups increase the value of public goods they get. Also, De Luca et al. (2018) find cross-national evidence that the intensity of nighttime light increases by 7%–8% and regional GDP—by 2% in a country leader's ethnic homeland. The authors show that their result holds for non-African countries.

The empirical results on the impact of democracy on minority rights are mixed. Bertrand and Haklai (2013) summarize case studies across countries to conclude that democratization may bring about ethnic exclusion and intra-ethnic violence. Haklai (2013) highlights that minority rights in democracies depend on their status before democratization and on whether state-building occurs together with it. He focuses on three case studies: Bulgaria, Estonia, and Macedonia. According to his analysis, Bulgaria accommodated the Turkish minority after the fall of Communism because it was already an established state facing no existential threat. Estonia and Macedonia were more hostile to the Russian and Albanian minorities, respectively. Unlike Bulgaria, they were previously parts of larger federations, and the ethnic majority in each country tried to reassert control over the state. In Estonia, the Russian minority enjoyed higher status before democratisation because of the Soviet state's policy. Thus, Haklai provides a qualitative theory focusing on different factors than this paper.

There are several gaps in Haklai's analysis that my model addresses. First, Haklai focuses on what happens after democratization and does not analyze states that have existed for a long time or emerged as a result of a retreat of European powers. Second, he restricts attention to "ethnic democracies," where democratic procedures work but the state mainly serves the interests of the majority. Hence, he does not ask why some states become "ethnic democracies" and others do not. For instance, he does not list India as one but does not explain what made it different. Third, there is no discussion of how stronger or weaker constraints on the leader among democracies could affect minority rights. Finally, there is no theory of how minorities fare under dictatorship.

Sorens (2010) tests several hypotheses, of which two are the most relevant to this paper. His first hypothesis is that higher executive constraints make the initiation and termination of repression against minorities less likely. The intuition for this hypothesis is that the presence of veto players makes any change less feasible. His second hypothesis is that competitive political participation and executive selection decrease the probability of repression because its targets have more say in policy choice. Empirical analysis reveals that executive constraints make new repression less likely but have no effect on their continuation. Competitive political participation decreases the likelihood of both. However, contrary to the hypothesis, competitive executive selection makes repression more likely. Sorens' results partially align with the conclusion in this paper that a higher level of democracy does not always benefit minorities.

Fox and Sandler (2003) report that, compared to full democracies and autocracies, semi-democracies are least likely to discriminate against ethnoreligious minorities. They do not find any effect of democracy on the discrimination of ethnic groups without a strong religious identity.

Several papers use bargaining models and power indices to understand the distribution of resources and influence in international organizations (Blaydes 2004; Fertő et al. 2020) and parliaments (Carreras and Owen 1988). In such papers, the players are members of an institutionalized body. This paper applies a bargaining model to a broad set of countries, including those lacking a powerful legislature or dominant party. Such an approach is valid because bargaining and coalition formation among ethnic groups may occur outside such institutions. For example, Hafez al-Assad built a ruling coalition of minorities in a country without parliamentary institutions (Geddes et al. 2018). To give an example from a democracy, the Civil Rights Movement in the United States had a sizable political effect while its leaders had no formal government roles (Hall 2007).

3 | Model

3.1 | Environment

The players are m small groups and one large group: a total of $m + 1$ players. Each group is a unitary actor. In the rest of the text, I will use the terms "large group" and "the majority" ('small group' and 'a minority') interchangeably for the sake of style. Each group i has size $w_i = 1$ if it is a minority and $w_i = k$ if it is the majority. The groups have to divide a dollar among themselves. The division is implemented if a set of players S votes for it such that $\sum_{i \in S} w_i \geq W$. In other words, W is the voting quota. W is exogenous. A set of players S is called a *winning coalition* if $\sum_{i \in S} w_i \geq W$. I assume that:

$$2 \leq W \leq m + k$$

The assumption that $W \geq 2$ means that a single minority group cannot rule alone. I assume that $W \leq m + k$ because $m + k$ is the total size of all groups, so a coalition of size $W > m + k$ cannot form.

I also assume that $m < k$. In other words, the majority size is greater than the size of all minorities combined. This assumption allows focusing on societies with a clear distinction between the majority and minorities [5].

It is useful to define the *minimum winning coalition* as a winning coalition such that any of its proper subsets has a size strictly less than W. Coalitions that form in equilibrium will be either minimum winning coalitions or consist of a minimum winning coalition plus the proposer. Table 1 below represents the possible winning coalitions and minimum winning coalitions under different conditions.

3.2 | The Political Process

The game proceeds at a possibly infinite set of time periods. At each period, one group is randomly selected to propose the division of the dollar to other players. A group's probability to be chosen equals its share in the population. Specifically, for each minority group, the probability of being selected is $\frac{1}{m+k}$, and for the majority group it equals $\frac{k}{m+k}$. The draws in each period are independent of each other. Hence, a player drawn as a proposer in one of the periods can become a proposer in another period. After a proposal has been made, all other players vote Yes or No. If the size of those who voted Yes, including the proposer, is greater than or equal to W, the game ends, and the proposal is implemented. Otherwise, the game is repeated and the dollar is discounted by δ, where $0 < \delta < 1$.

3.3 | Timing

The sequence of play is as follows.

1. A proposer is randomly selected, with the probability for each group to be selected equal to its share of the population.
2. The selected group proposes a division of a dollar.
3. The remaining groups vote Yes or No.
4. If the total size of those groups that voted Yes, including the proposer, is greater than or equal to W, the proposal is implemented. Otherwise, the prize is discounted and the game goes back to stage 1.

3.4 | Strategies and Solution Concept

I restrict attention to equilibria in strategies with several properties. First, I follow the literature by focusing on *stationary* strategies (see e.g. Montero 2017). Such strategies do not depend on history and are time-independent. Formally, define $X = \left\{x \in \mathbb{R}_+^{m+1} \middle| \sum_{i=1}^{m+1} x_i = 1\right\}$ to be the set of feasible divisions of the dollar, where the ith coordinate of a vector $x \in X$ represents the share that player i gets. A mixed *stationary strategy* is a sequence $\sigma_i = (\sigma_i^t)_{t=1}^{\infty}$ where σ_i^t is the set of actions for player i in the tth round that involves:

- if i is the proposer, randomly selecting a proposal $x \in X$ according to probability distribution p_i.
- if i is not the proposer, accepting or rejecting the proposal according to the response function $\phi_i : X \to \{Yes, No\}$.

I restrict attention to *finite* strategies so that $\hat{X} \equiv \{x \in X : p_i(x) > 0 \text{ for some } i\}$ is a finite set.

Finally, I restrict attention to *symmetric stationary* strategies. For those, when the proposer identity is fixed, each minority group that is not the proposer in round t is equally likely to get a strictly positive proposal in round t. Formally, let players $1, ..., m$ be minorities and player $m+1$ be the majority. Define $\hat{X}_i = \{x \in \hat{X} : x_i > 0\}$. Then, a *symmetric strategy* is such that for any non-proposers $i, j \leq m$ and any proposer $l \neq i, j$:

$$\sum_{x \in \hat{X}_i} p_l(x) = \sum_{x \in \hat{X}_j} p_l(x)$$

The solution concept is the subgame-perfect equilibrium in symmetric, finite, and stationary strategies. I will simply use the word "equilibrium" in the rest of the text.

I restrict attention to stationary strategies because according to them, players behave the same way in the same situations. The symmetry assumption is for tractability, and finiteness makes it easier to define. Eraslan and McLennan (2013) show that expected payoffs are the same in all subgame-perfect equilibria in a class of games that includes the one in this paper. Proposition 1 shows that equilibria of the type I describe exist. Hence, they are sufficient to analyze payoffs, which is the focus of this paper.

3.5 | Payoffs

For each player, the payoff equals the share of the dollar that she obtains once the division is implemented. Future payoffs are discounted by δ. A *continuation value* of a player is the expected payoff that she obtains when the game moves to the next round. Define v_{maj} to be the continuation value of the majority and v_{min} - the continuation value of any minority. These continuation values are well-defined because the expected payoffs are unique across all subgame stationary equilibria and because all minorities get the same equilibrium expected payoff. Note that these payoffs are endogenous. The *equilibrium expected payoff* of a player is her expected payoff at the beginning of the game. I define the equilibrium expected payoffs for the majority and for any minority as $\hat{v}_{maj}$ and $\hat{v}_{min}$, respectively. The payoff $\hat{v}_{min}$ measures the welfare of minorities. In an equilibrium in stationary strategies, each player is in the same situation in each round before the proposer identity is realized. If the game continues to the next round, each player's payoff is discounted by δ. Hence, $v_{maj} = \delta \hat{v}_{maj}$ and $v_{min} = \delta \hat{v}_{min}$.

In the model, a random draw decides who becomes the proposer. Also, when indifferent, a proposer randomizes between several winning coalitions that she may form. Because the payoff that a generic minority receives depends on random factors, it is natural to use the equilibrium expected payoff of the minority $\hat{v}_{min}$ as a proxy for the welfare of a minority.

4 | Analysis

My main result, Proposition 1, presents the equilibrium expected payoff for a generic minority $\hat{v}_{min}$. Recall that k defines the size of the majority group, m defines the number of minorities and δ is the discount factor.

Table 2

Proposition 1. *The equilibrium expected payoff for each minority group, $\hat{v}_{min}$, is as in Table 2 below.*

The following corollary describes the comparative statics. With a slight abuse of notation, write $\hat{v}_{min}(W)$ as a function that maps the voting quota to the equilibrium expected payoff of a minority.

Corollary 1.

i. For any W_0, W_1 such that $W_0 \leq m$ and $m + 1 \leq W_1 \leq k$, $\hat{v}_{min}(W_0) > \hat{v}_{min}(W_1)$.

ii. For any W_1, W_2 such that $m + 1 \leq W_1 \leq k$ and $W_2 \geq k + 1$, $\hat{v}_{min}(W_2) > \hat{v}_{min}(W_1)$.

iii. For any $W \geq k + 1$, $\hat{v}_{min}(W)$ strictly increases in W.

iv. For any $W_0 \leq m$, $\hat{v}_{min}(W_0) = \hat{v}_{min}(m + k)$.

TABLE 2 | The equilibrium expected payoff of a minority given different values of the voting quota W.

Condition on the voting quota W	The equilibrium expected payoff of a minority $\hat{v}_{min}$
$W \leq m$	$\frac{1}{m+k}$
$m + 1 \leq W \leq k$	$\frac{1-\delta}{k+(1-\delta)m}$
$k + 1 \leq W \leq m + k$	$\frac{(1-\delta)m}{\delta k^2 + k(m-\delta W)+(1-\delta)m^2}$

The corollary implies that, as the voting quota W changes from the minimum value of 2 to the maximum value of $m + k$, the equilibrium expected payoff of each minority player first stays constant, then drops and stays constant, then increases up to the point where it was for the lowest values of the voting quota. The graph on Figure 1 below illustrates the comparative statics.

Figure 1

To see the intuition, consider the case when $W \leq m$, so either a coalition of minorities or the largest group alone can form a minimum winning coalition. The largest group has a high expected payoff from turning down an offer. So, it is a better choice for a minority proposer to make an offer to a coalition of minorities because they will demand less. As the voting quota W increases, this calculation stays true, until W is greater than m, so a coalition of minorities cannot be winning anymore. Suppose now that $W \geq m + 1$. In this case, a minority proposer chooses the largest group as a coalition partner, while the largest group rules alone if it becomes the proposer. The reason the largest group accepts an offer from a minority is that the resource is discounted after each round, so the bargaining range is not empty. Because all winning coalitions include the majority, each minority proposer makes a large offer to it. Minority groups are not necessary for a winning coalition to form, so each minority group gets a positive payoff if and only if it becomes a proposer. As a result, the expected payoff for each minority group is small. Finally, if $W \geq k + 1$, the largest group cannot rule alone. If the largest group is the proposer, it makes offers to minority groups. If a minority becomes the proposer, it makes an offer to both the largest group and several other

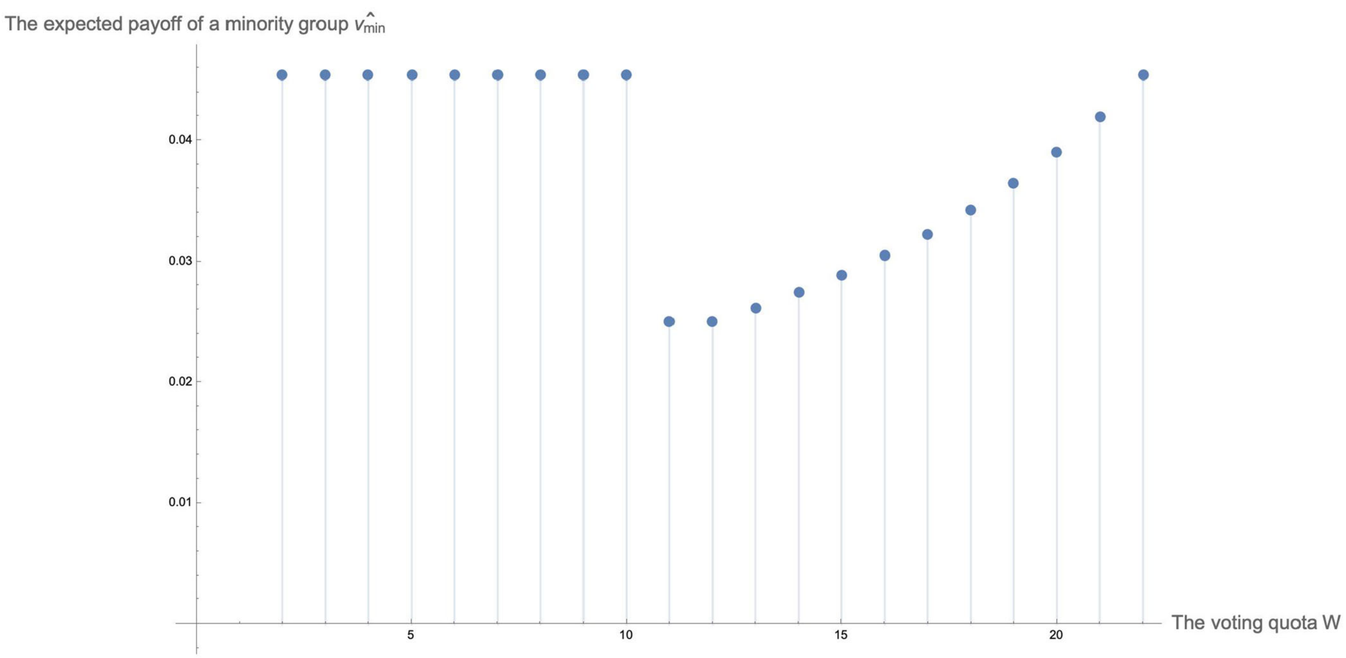

FIGURE 1 | The equilibrium expected payoff of a minority group $\hat{v}_{min}$. In this example, the number of minorities is $m = 10$, the size of the majority group is $k = 12$ (hence, the majority is 12 times larger than any minority), and the discount factor is $\delta = 0.6$.

minorities. As W increases, more minorities are needed to complete the winning coalition, so their expected payoffs increase. The formal proof of Proposition 1 and Corollary 1 is below.

Proof. To begin with, note that the result in Eraslan and McLennan (2013) implies that the equilibrium expected payoffs in our game are unique. Since one can relabel the minorities in an arbitrary way, a permutation of their equilibrium payoffs is also an equilibrium payoff vector. Hence, each minority must get the same equilibrium expected payoff and continuation payoff, which I define as $\hat{v}_{min}$ and v_{min}, respectively.[6]

Suppose that $W \le m$. Hence, a winning coalition that consists only of minorities is sufficient. Because $m < k$, if the majority group becomes a proposer, it does not need to offer anything strictly more than zero to any other players. Hence, if the majority becomes a proposer, it gets 1 and all the minorities get 0. Consider an equilibrium where each minority only proposes to other minorities. Each minority i becomes a proposer with probability $\frac{1}{m+k}$. In this case, it offers the expected continuation payoffs to $W-1$ other minorities and obtains $1-(W-1)v_{min}$. With probability $\frac{m-1}{m+k}$, one of the minorities except for i becomes a proposer. It proposes to any given minority, including i, with probability $\frac{W-1}{m-1}$. In that case, minority i obtains exactly v_{min}.

Each minority's expected payoff in each next round is:

$$v_{min} = \delta\left(\frac{1}{m+k}(1-(W-1)v_{min}) + \frac{(m-1)}{(m+k)}\frac{(W-1)}{(m-1)}v_{min}\right) = \frac{\delta}{m+k}$$

The majority is selected as a proposer with probability $\frac{k}{m+k}$. Hence, its expected payoff in the next round is: $v_{maj} = \delta\frac{k}{m+k}$. Now, this is an equilibrium if and only if it is better for a minority to propose to $W-1$ other minorities instead of proposing to the majority. In other words:

$$1-(W-1)\frac{\delta}{m+k} \ge 1-\delta\frac{k}{m+k}$$

which is true given that $k > m \ge W$. By Eraslan and McLennan (2013), players get these expected payoffs in any subgame-perfect stationary equilibrium.

Suppose now that $m+1 \le W \le k$. Hence, each winning coalition must include the majority. However, once the majority is included, no minorities are necessary. Therefore, the majority proposes 0 to all minorities. Because $m+1 \le W \le k$, each minority proposes to the majority. The majority is selected as a proposer with probability $\frac{k}{m+k}$, in which case it gets 1 and each minority gets 0. A given minority i is selected to be a proposer with probability $\frac{1}{m+k}$. In this case, minority i gets $1-v_{maj}$, the majority gets v_{maj}, and all other minorities get 0. Thus, v_{min} and v_{maj} satisfy:

$$v_{maj} = \delta\left(\frac{k}{m+k} + \frac{m}{m+k}v_{maj}\right) \Rightarrow v_{maj} = \frac{\delta k}{m(1-\delta)+k}$$

and

$$v_{min} = \frac{\delta}{m+k}(1-v_{maj}) = \frac{\delta(1-\delta)}{k+(1-\delta)m}$$

Finally, let $W \ge k+1$. The winning coalition must include the majority and $W-k$ minorities. Each minority i becomes a proposer with probability $\frac{1}{m+k}$ and offers $v_{min}(W-k-1)$ to other minorities and v_{maj} to the majority. With probability $\frac{m-1}{m+k}$ some other minority becomes a proposer and offers v_{min} to a given minority with probability $\frac{W-k-1}{m}$. Finally, with probability $\frac{k}{m+k}$ the majority becomes the proposer and offers v_{min} to a given minority with probability $\frac{W-k}{m}$. Hence, the expected payoff in the next round for a minority group is:

$$v_{min} = \delta\left(\frac{1}{m+k}(1-v_{maj}-v_{min}(W-k-1)) + \frac{(m-1)}{(m+k)}\frac{(W-k-1)}{(m-1)}v_{min} + \frac{k}{(m+k)}\frac{(W-k)}{m}v_{min}\right)$$

The majority group becomes the proposer with probability $\frac{k}{m+k}$. In this case, it offers $v_{min}(W-k)$ to a total of $W-k$ minorities. One of the minorities becomes the proposer with probability $\frac{m}{m+k}$ and always offers v_{maj} to the majority. Hence, the expected payoff in the next round for the majority group is:

$$v_{maj} = \delta\left(\frac{k}{m+k}(1-v_{min}(W-k)) + \frac{m}{m+k}v_{maj}\right)$$

Solving the two equations above yields:

$$v_{min} = \frac{\delta(1-\delta)m}{\delta k^2 + k(m-\delta W) + (1-\delta)m^2}$$

We can write:

$$v_{min} = \begin{cases} \dfrac{\delta}{m+k} & \text{if } W \le m \\ \dfrac{\delta(1-\delta)}{k+(1-\delta)m} & \text{if } m+1 \le W \le k \\ \dfrac{\delta(1-\delta)m}{\delta k^2 + k(m-\delta W) + (1-\delta)m^2} & \text{if } W \ge k+1 \end{cases}$$

Using the fact that $v_{min} = \delta\hat{v}_{min} \Leftrightarrow \hat{v}_{min} = \frac{v_{min}}{\delta}$, we get

$$\hat{v}_{min} = \begin{cases} \dfrac{1}{m+k} & \text{if } W \le m \\ \dfrac{1-\delta}{k+(1-\delta)m} & \text{if } m+1 \le W \le k \\ \dfrac{(1-\delta)m}{\delta k^2 + k(m-\delta W) + (1-\delta)m^2} & \text{if } W \ge k+1 \end{cases}$$

This proves Proposition 1.

To prove the corollary, first, notice that:

$$\frac{1}{m+k} > \frac{1-\delta}{k+(1-\delta)m}$$

This proves point (i) of the corollary. Hence, the minority payoff decreases when W switches from m to $m+1$.

If $W \geq k+1$, we have:

$$\frac{(1-\delta)m}{\delta k^2 + k(m-\delta W) + (1-\delta)m^2} > \frac{1-\delta}{k+(1-\delta)m}$$

Hence, the minority payoff increases when W switches from k to $k+1$.

Taking the derivative shows that if $W \geq k+1$, then:

$$\frac{d}{dW}\hat{v}_{min} = \frac{(1-\delta)\delta km}{(\delta k^2 + k(m-\delta W) + (1-\delta)m^2)^2} > 0$$

This proves (ii) and (iii). Direct computation verifies (iv). This completes the proof. □ □

Observe that, because $m < k$, it must be that $m < \frac{m+k}{2} < k$. Hence, $W = \frac{m+k}{2}$, which is half of the population, falls under the worst scenario for minorities. This feature of the model translates into a real-world prediction that to be protected, minorities require more than fair elections. Additional constraints on the leader, such as bicameral parliaments or high courts, are needed. In our model, such constraints translate into very high values of the voting quota W.

Observe also that for any $W \geq k+1$

$$\frac{(1-\delta)m}{\delta k^2 + k(m-\delta W) + (1-\delta)m^2} \leq \frac{1}{m+k}$$

and therefore in expectation high values of the voting quota W are never strictly better for minorities compared to very low values. This feature of the model is not realistic. However, the main insight of the model is not a comparison between perfect dictatorship and democracy, but rather a comparison between the medium versus very high and very low values of W. One should also note that Proposition 1 describes the expected payoff for a minority. Another relevant metric is the probability that a given minority receives a nonzero payoff. Recall that if $W \leq m$, this probability is given by:

$$\frac{1}{m+k} + \frac{(m-1)(W-1)}{(m+k)(m-1)} = \frac{W}{m+k}$$

If $W \geq k+1$, this probability becomes:

$$\frac{1}{m+k} + \frac{(m-1)(W-1-k)}{(m+k)(m-1)} + \frac{k(W-k)}{(m+k)m} = \frac{W-k}{m}$$

Observe that if $k > m^2 - m$ then:

$$\frac{W}{m+k}\Big|_{W=m} < \frac{W-k}{m}\Big|_{W=k+1}$$

and therefore a given minority is strictly more likely to be offered a nonzero payment under high compared to low values of W. Intuitively, if the majority group is sufficiently large, a given minority group is more likely to enter the winning coalition under democracy compared to dictatorship.

4.1 | Robustness

To show that the result is robust to the choice of the model, I analyze the Baron-Ferejohn bargaining model with equal recognition probabilities, and the Shapley-Shubik (Shapley and Shubik 1954), Banzhaf, and normalized Banzhaf power indices (Van den Brink and Van der Laan 1998). The first two approaches predict the same relationship between the voting quota W and the payoff of a minority group as the main model. The normalized Banzhaf power index provides an ambiguous relationship. However, plotting its value against the voting quota W suggests the same comparative statics as the Shapley–Shubik index and the main model. The Banzhaf index yields a highly non-monotonic relationship that is different from other models. For a detailed discussion, see Appendix A1.

Power indices are a valuable tool to measure the influence of different players in collective decision-making. However, they do not suggest which coalitions the players decide to form. Also, they do not reflect that players can pay others to support their decisions. By contrast, a bargaining model assumes that the player who forms a coalition chooses partners that are cheapest to buy and pays them off for their support. This feature of the model makes cheap coalitions more likely and expensive ones less likely. It also allows to factor in the coalition formateur's loss from paying off her coalition partners. Such properties of the bargaining model make it relevant to analyzing ethnic politics, which involves negotiations and the exchange of favors.

4.2 | Empirical Patterns

The highly stylized model in this paper suggests a non-monotonic relationship between the level of democracy and the welfare of ethnic minorities. The voting quota in the model represents the number of people whose support a leader needs to stay in power. I proxy this parameter using the “checks” variable in the Database of Political Institutions that measures checks and balances (Scartascini et al. 2021). The value of the variable depends on how competitive are the executive and legislative elections and on whether agreement between competing political factions is needed to maintain the government. I refer the reader to the codebook of the data set for a detailed definition.

To proxy the welfare of ethnic minorities, I compute the share of ethnic minorities that were represented in power for each

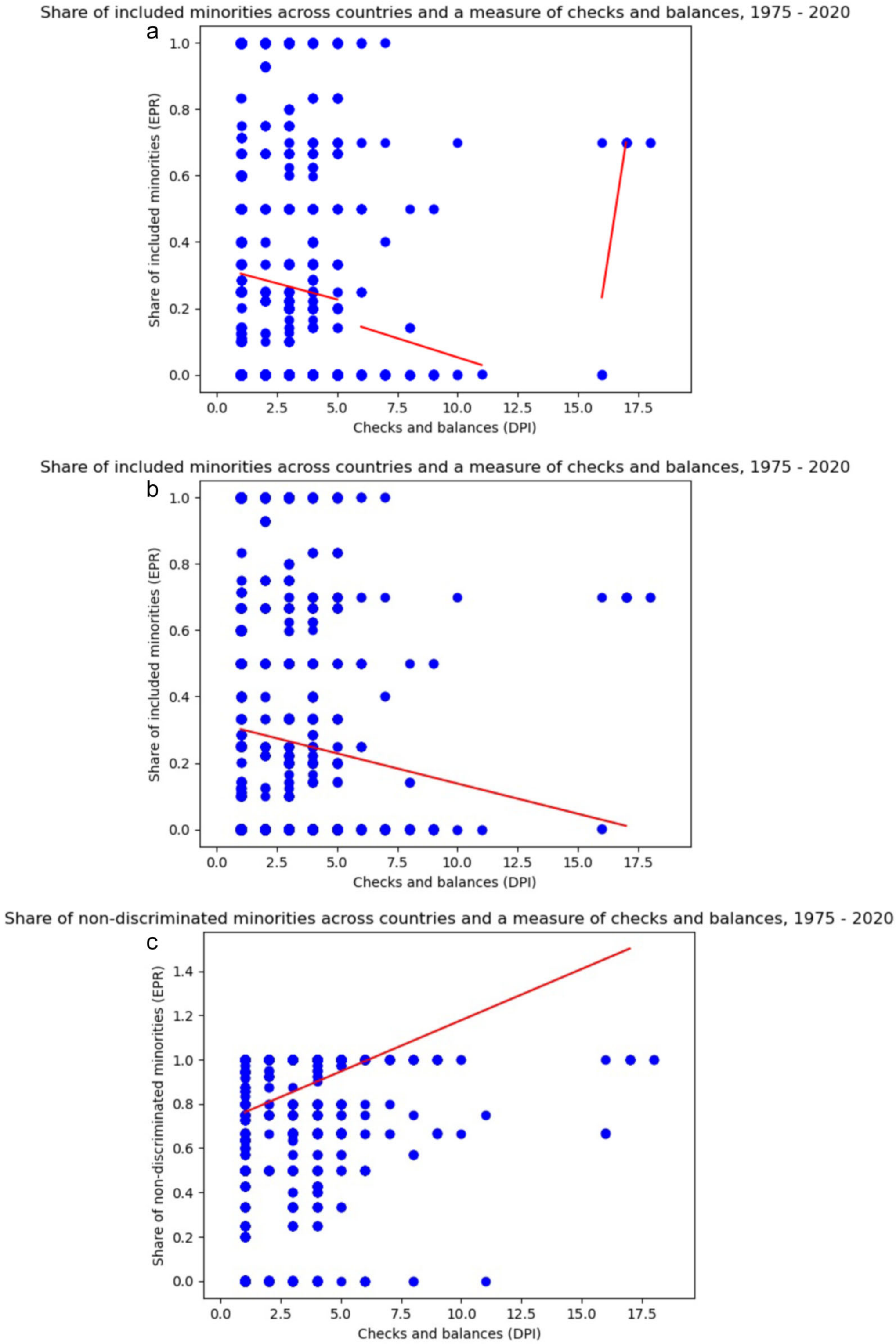

FIGURE 2 | (a) The scatterplot and predicted relationship between the measure of checks and balances and the share of included minorities. Linear fits are made separately for three value regions of the dependent variable. (b) The scatterplot and predicted relationship between the measure of checks and balances and the share of included minorities. There is one linear fit for the whole data. (c) The scatterplot and predicted relationship between the measure of checks and balances and the share of non-discriminated minorities. There is one linear fit for the whole data.

country and year using the Ethnic Power Relations data set (Vogt et al. 2015). I code any ethnic group that comprises less than 25% of the population as a minority. Following the model, I only include countries where the majority comprises more than 50%. I do not include countries where there are no minorities.

Figure 2a below provides the scatter plot of the data and linear approximations for three value regions of the independent variable. The slopes of linear fits are negative for the values of the independent variable lower than 12. Therefore, the plot partially supports the model's conclusion that increasing the level of democracy from low to medium can make minorities worse off. There is little variation in the independent variable for values higher than 12, so the graph provides no conclusions in this case. Figure 2b provides one linear trend, which has a negative slope. Figure 2c does the same exercise for the share of minorities that are not discriminated against, according to the Ethnic Power Relations data set. The slope of the linear fit is positive on the latter graph, which contradicts the model's predictions. It follows that the regime type can make a different impact on different aspects of the welfare of ethnic minorities.

5 | Conclusion

The model in this paper suggests a nonlinear effect of democracy on the welfare of minorities in countries with a large majority ethnic group. On average, minorities are better off in autocracies than in semi-democracies. Full democracies are more favorable than the latter as well. Of course, some dictators are brutal to minorities. Still, there are many cases when they protect small groups and share benefits with them. This outcome typically occurs if the dictator comes from a small group that cannot rule alone. Therefore, if a dictator who includes minorities loses power, these groups can face bad consequences, as the examples of Bashar al-Assad in Syria and Siad Barre in Somalia show. Another implication concerns semi-democracies. In such countries, the leader must build a large winning coalition, so relying only on minorities would not be enough. On the other hand, the support of the largest ethnic group is sufficient, so minorities are not needed. Minorities will likely be excluded and discriminated against. For instance, Türkiye and Sri Lanka are semi-democracies that exhibit long-lasting ethnic tensions.

In the model, ethnic groups are unitary actors that divide resources. This setup assumes that a government representative of an ethnic group provides benefits to its members. There are studies that support this assumption with empirical evidence across the globe. Nevertheless, these studies show that ethnic favoritism accounts for only a fraction of welfare outcomes. It is also clear that in the real world, there are policy disagreements aside from ethnic politics. While the model is a useful approximation for how the regime type could affect minority welfare, a more detailed analysis would be an exciting direction for future work. Because politics differs across countries, having a more narrow focus could be a valuable exercise. For example, one could analyze outcomes in a specific country or region using a model similar to Fernandez and Levy (2008) that takes economic and ethnic divisions into account.

The paper models politics as a division of the dollar. As examples in the paper show, this approximation can help describe ethnic politics. However, the model is less applicable to other social cleavages. For instance, if there is a conflict over economic inequality, redistributive taxation can help poorer citizens at the expense of richer ones. As a result, the poor and middle class can align to expropriate the rich. However, it is harder to imagine a coalition between the rich and the poor against the middle class (Acemoglu and Robinson 2006). Another type of competition is the one between special interest groups. However, such agents usually achieve their goals by offering policymakers rewards, not by mobilizing popular support (Grossman and Helpman 2001). Hence, a model where society makes decisions through voting cannot accurately describe their activity. Finally, an interesting type of social cleavage is one between informed and uninformed voters (Guriev and Treisman 2020). However, a meaningful model of politics where such division is central should involve information transmission, which is not present in the current paper.

Another important assumption is that group identities are fixed: groups cannot merge or partition into smaller ones. In reality, people's understanding of their ethnicity varies over time. However, such changes are slow, and it is hard to imagine ethnic group leaders quickly forging them even if this could bring about political gains. Of course, alliances between ethnic groups are possible, but this is precisely what happens in our game when a winning coalition forms. In other contexts, merges and partitions of groups of players are more realistic. For example, it is common for political parties to merge or split. An interesting topic for future research would be to apply sophisticated models of coalition formation, such as Ray and Vohra (1999), to study such scenarios.

Empirical patterns in the paper show that, as the model suggests, a higher level of democracy can reduce the share of minorities that have some government role. However, this conclusion is not robust to choosing other measures of minority welfare. A more involved empirical analysis is beyond the scope of this paper, but it is needed to check the predictions of this and other bargaining models of ethnic politics. Besides the issue of measurement, there is a challenge of finding an exogenous variation for the level of democracy to establish a causal claim.

Acknowledgments

I would like to thank the Editor, the Associate Editor, three anonymous Referees, Marina Agranov, Michael Chwe, Barbara Geddes, Jennifer Hamilton, Leslie Johns, Natalia Lamberova, Jeffrey Lewis, Barry O'Neill, Steven Kivinen, Daniel Posner, Marcel Roman, Arturas Rozenas, Konstantin Sonin, Anton Suvorov, Daniel Treisman, Eyal Winter, Leeat Yariv, Alexei Zakharov, and the participants at APSA, MPSA, UCLA and Higher School of Economics for great discussions and helpful comments. Financial support from the Institute for Humane Studies at George Mason University (2016–2017), received when the author was a PhD student at UCLA, is gratefully acknowledged. This paper is one of the chapters in my PhD dissertation "The problems facing democracy".

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Endnotes

[1] An example of an alternative non-cooperative approach is Pérez–Castrillo and Wettstein (2001).

[2] Diermeier and Merlo (2004) find that coalition formation in parliamentary democracies is consistent with the assumption that larger parties are more likely to be coalition formateurs.

[3] There is a well-known class of similar voting games called "apex games" that suggests that the setting in this paper is indeed important. In such games, there are two types of minimum winning

coalitions: all "minority" players or one "majority" plus one "minority" player. While similar, these games are not a special case of the game in this paper because they are equivalent to a voting game where one player has $n - 2$ votes, $n - 1$ players have 1 vote each, and the quota is $n - 1$ votes, that is, the "majority" is smaller than all "minorities" collectively (Montero 2002).

[4]Suppose that the voting quota is less than or equal to the number of minorities. My model assumes that the weight of each minority is 1 and the weight of the majority is greater than the weight of all minorities combined. Hence, the minimum-weight winning coalition only consists of minorities, so the majority is never part of one. Otherwise, the majority is part of any winning coalition (see Table 1) while a given minority is never part of all of them. Hence, for any probability distribution over such coalitions, the probability to be included is 1 for the majority player and strictly less than 1 for any minority.

[5]Because the model assumes that the largest group comprises more than 50% of the population, it only applies to such societies. Conflict and bargaining can be different in settings where the first- and second-largest groups have similar sizes (see e.g. Bleaney and Dimico 2017). Figure A3 in the Appendix depicts the shares of the first- and second-largest ethnic groups cross-nationally and shows that in most countries, the largest ethnic group exceeds half of the population.

[6]I would like to thank an anonymous Reviewer for this argument.

References

Acemoglu, D., G. Egorov, and K. Sonin. 2010. "Political Selection and Persistence of Bad Governments." *Quarterly Journal of Economics* 125, no. 4: 1511–1575. https://doi.org/10.1162/qjec.2010.125.4.1511.

Acemoglu, D., and J. A. Robinson. 2006. *Economic Origins of Dictatorship and Democracy*. Cambridge University Press. https://doi.org/10.1017/CBO9780511510809.

Acemoglu, D., and J. A. Robinson. 2012. *Why Nations Fail: The Origins of Power, Prosperity, and Poverty*. Currency.

Baron, D. P., and J. A. Ferejohn. 1989. "Bargaining in Legislatures." *American Political Science Review* 83, no. 4: 1181–1206. https://doi.org/10.2307/1961664.

Barthelemy, F., D. Lepelley, and M. Martin, et al. 2021. "Dummy Players and the Quota in Weighted Voting Games." *Group Decision and Negotiation* 30, no. 1: 43–61. https://doi.org/10.1007/s10726-020-09705-y.

Beiser-McGrath, J., and N. W. Metternich. 2021. "Ethnic Coalitions and the Logic of Political Survival in Authoritarian Regimes." *Comparative Political Studies* 54, no. 1: 144–178. https://doi.org/10.1177/0010414020920656.

Bertrand, J. and Haklai, O., ed. 2013. *Democratization and Ethnic Minorities: Conflict or Compromise?* Routledge.

Blaydes, L. 2004. "Rewarding Impatience: A Bargaining and Enforcement Model of OPEC." *International Organization* 58, no. 2: 213–237. https://doi.org/10.1017/S0020818304582012.

Bleaney, M., and A. Dimico. 2017. "Ethnic Diversity and Conflict." *Journal of Institutional Economics* 13, no. 2: 357–378. https://doi.org/10.1017/S1744137416000369.

Brubaker, R., and M. Feischmidt. 2002. "1848 in 1998: The Politics of Commemoration in Hungary, Romania, and Slovakia." *Comparative Studies in Society and History* 44, no. 4: 700–744. https://doi.org/10.1017/S0010417502000336.

Bueno de Mesquita, B., A. Smith, and R. M. Siverson, et al. 2005. *The Logic of Political Survival*. MIT Press.

Carreras, F., and G. Owen. 1988. "Evaluation of the Catalonian Parliament, 1980–1984." *Mathematical Social Sciences* 15, no. 1: 87–92. https://doi.org/10.1016/0165-4896(88)90032-7.

Committee for a Responsible Federal Budget. 2024. "Government Shutdowns Q & A: Everything You Should Know.", 26 February 2024, Committee for a Responsible Federal Budget. https://www.crfb.org/papers/government-shutdowns-qa-everything-you-should-know.

Diermeier, D., and A. Merlo. 2004. "An Empirical Investigation of Coalitional Bargaining Procedures." *Journal of Public Economics* 88, no. 3–4: 783–797. https://doi.org/10.1016/S0047-2727(03)00035-5.

Dziuda, W., and A. Loeper. 2016. "Dynamic Collective Choice With Endogenous Status Quo." *Journal of Political Economy* 124, no. 4: 1148–1186. https://doi.org/10.1086/686747.

De Luca, G., R. Hodler, and P. A. Raschky, et al. 2018. "Ethnic Favoritism: An Axiom of Politics?" *Journal of Development Economics* 132: 115–129. https://doi.org/10.1016/j.jdeveco.2017.12.006.

Duflo, E. 2005. "Why Political Reservations?" *Journal of the European Economic Association* 3, no. 2–3: 668–678. https://doi.org/10.1162/jeea.2005.3.2-3.668.

Eraslan, H., and K. S. Evdokimov. 2019. "Legislative and Multilateral Bargaining." *Annual Review of Economics* 11: 443–472. https://doi.org/10.1146/annurev-economics-080218-025633.

Eraslan, H., and A. McLennan. 2013. "Uniqueness of Stationary Equilibrium Payoffs in Coalitional Bargaining." *Journal of Economic Theory* 148, no. 6: 2195–2222. https://doi.org/10.1016/j.jet.2013.09.007.

Eraslan, H., and A. Merlo. 2017. "Some Unpleasant Bargaining Arithmetic?" *Journal of Economic Theory* 171: 293–315. https://doi.org/10.1016/j.jet.2017.08.004.

Fernandez, R., and G. Levy. 2008. "Diversity and Redistribution." *Journal of Public Economics* 92, no. 5: 925–943. https://doi.org/10.1016/j.jpubeco.2007.12.012.

Fertő, I., et al. 2020. "The Power Ranking of the Members of the Agricultural Committee of the European Parliament." *European Review of Agricultural Economics* 47, no. 5: 1897–1919. https://doi.org/10.1093/erae/jbaa011.

Fox, J., and S. Sandler. 2003. "Regime Types and Discrimination Against Ethnoreligious Minorities: A Cross-Sectional Analysis of the Autocracy–Democracy Continuum." *Political Studies* 51, no. 3: 469–489. https://doi.org/10.1111/1467-9248.00436.

Franck, R., and I. Rainer. 2012. "Does the Leader's Ethnicity Matter? Ethnic Favoritism, Education, and Health in Sub-Saharan Africa." *American Political Science Review* 106, no. 2: 294–325. https://doi.org/10.1017/S0003055412000172.

Francois, P., I. Rainer, and F. Trebbi. 2015. "How Is Power Shared in Africa?" *Econometrica* 83, no. 2: 465–503. https://doi.org/10.3982/ECTA11237.

Geddes, B., J. Wright, and E. Frantz. 2018. *How Dictatorships Work: Power, Personalization, and Collapse*. Cambridge University Press.

Geddes, B., J. Wright, and E. Frantz. 2014. "Autocratic Breakdown and Regime Transitions: A New Data Set." *Perspectives on Politics* 12, no. 2: 313–331. https://doi.org/10.1017/S1537592714000851.

Grossman, G. M., and E. Helpman. 2001. *Special Interest Politics*. MIT Press.

Guriev, S., and D. Treisman. 2020. "A Theory of Informational Autocracy." *Journal of Public Economics* 186: 104158. https://doi.org/10.1016/j.jpubeco.2020.104158.

Hall, J. D. 2007. "The Long Civil Rights Movement and the Political Uses of the Past." In *The Best American History Essays 2007*, 235–271. Palgrave MacMillan. https://doi.org/10.1007/978-1-137-06439-4_11.

Haklai, O. 2013. "Regime Transition and the Emergence of Ethnic Democracies." In *Democratization and Ethnic Minorities*, 18–38. Routledge. https://doi.org/10.4324/9781315882642.

Hawramy, F. 2011. *Suppressing Kurdish Identity Has Failed. Turkey Must Take a New Approach*. The Guardian.

Kalandrakis, A. 2004. "A Three-Player Dynamic Majoritarian Bargaining Game." *Journal of Economic Theory* 116, no. 2: 294–314. https://doi.org/10.1016/S0022-0531(03)00259-X.

Kramon, E., and D. N. Posner. 2016. "Ethnic Favoritism in Education in Kenya." *Quarterly Journal of Political Science* 11, no. 1: 1–58. http://dx.doi.org/10.1561/100.00015005.

Kingsley, P. 2022. *Why Does Israel Keep Having Elections?* The New York Times 1 Nov. 2022.

Lewis, I. 2004. "Visible and Invisible Differences: The Somali Paradox." *Africa* 74, no. 4: 489–515. https://doi.org/10.3366/afr.2004.74.4.489.

Marshall, M. G., K. Jaggers, and T. R. Gurr. 2002. *Polity IV Project: Political Regime Characteristics and Transitions, 1800-2002*. [Dataset]. Version p4v2002. Center for International Development and Conflict Management (CIDCM), University of Maryland. https://www.systemicpeace.org/polity/polity4.htm.

Martinez Machain, C., and L. Rosenberg. 2018. "Domestic Diversion and Strategic Behavior by Minority Groups." *Conflict Management and Peace Science* 35, no. 5: 427–450.

Mbanaso, M. U., and C. J. Korieh. 2010. *Minorities and the State in Africa*. Cambria Press.

Minority Rights Group International. 2017. "Syria." [Country profile]. World Directory of Minorities and Indigenous Peoples. http://minorityrights.org/country/syria/.

Minority Rights Group International. 2017. "Somalia." [Country Profile]. World Directory of Minorities and Indigenous Peoples. http://minorityrights.org/country/somalia/.

Minority Rights Group International. 2017. "Sri Lanka." [Country Profile]. World Directory of Minorities and Indigenous Peoples. http://minorityrights.org/country/sri-lanka/.

Minority Rights Group International. 2017. "Turkey." [Country Profile]. World Directory of Minorities and Indigenous Peoples. http://minorityrights.org/country/Turkey/.

Montero, M. 2002. "Non-Cooperative Bargaining in Apex Games and the Kernel." *Games and Economic Behavior* 41, no. 2: 309–321. https://doi.org/10.1016/S0899-8256(02)00501-8.

Montero, M. 2017. "Proportional Payoffs in Legislative Bargaining With Weighted Voting: A Characterization." *Quarterly Journal of Political Science* 12, no. 3: 325–346. http://dx.doi.org/10.1561/100.00016019.

Mukand, S. W., and D. Rodrik. 2020. "The Political Economy of Liberal Democracy." *Economic Journal* 130, no. 627: 765–792. https://doi.org/10.1093/ej/ueaa004.

"Nepal's Minorities. Constituting a Nationality." The Economist - Banyan, May 24, 2012. The Economist Newspaper Limited. https://www.economist.com/blogs/banyan/2012/05/nepals-minorities.

Pérez-Castrillo, D., and D. Wettstein. 2001. "Bidding for the Surplus: A Non-Cooperative Approach to the Shapley Value." *Journal of Economic Theory* 100, no. 2: 274–294. https://doi.org/10.1006/jeth.2000.2704.

Rafizadeh, M. 2011. *For Syria's Minorities, Assad Is Security*. Al Jazeera.

Ray, D., and R. Vohra. 1999. "A Theory of Endogenous Coalition Structures." *Games and Economic Behavior* 26, no. 2: 286–336. https://doi.org/10.1006/game.1998.0648.

Riker, W. H. 1975. *The Theory of Political Coalitions*. Yale University Press.

Scartascini, C., C. Cruz, and P. Keefer. 2021. *The Database of Political Institutions 2020 (dpi2020)*. Banco Interamericano de Desarrollo. https://doi.org/10.60966/gn3s-at31.

Sorens, J. 2010. "The Politics and Economics of Official Ethnic Discrimination: A Global Statistical Analysis, 1950–2003." *International Studies Quarterly* 54, no. 2: 535–560. https://doi.org/10.1111/j.1468-2478.2010.00598.x.

Shapley, L. S., and M. Shubik. 1954. "A Method for Evaluating the Distribution of Power in a Committee System." *American Political Science Review* 48, no. 3: 787–792. https://doi.org/10.2307/1951053.

"The Melting-Pot Cracks." The Economist (Middle East & Africa), October 3, 2002. The Economist Newspaper Limited. http://www.economist.com/node/1372126.

Van den Brink, R., and G. Van der Laan. 1998. "Axiomatizations of the Normalized Banzhaf Value and the Shapley Value." *Social Choice and Welfare* 15: 567–582. https://doi.org/10.1007/s003550050125.

Vogt, M., N. -C. Bormann, S. Ruegger, L. -E. Cederman, P. Hunziker, and L. Girardin. 2015. "Integrating Data on Ethnicity, Geography, and Conflict: The Ethnic Power Relations Data Set Family." *Journal of Conflict Resolution* 59, no. 7: 1327–1342. https://doi.org/10.1177/0022002715591215.

Winter, E. 2002. "The Shapley Value." in *Handbook of Game Theory With Economic Applications*, Vol. 3, 2025–2054. https://doi.org/10.1016/S1574-0005(02)03016-3.

Zick, Y., A. Skopalik, and E. Elkind. 2011. "The Shapley Value as a Function of the Quota in Weighted Voting Games." 22nd International Joint Conference on Artificial Intelligence, IJCAI 2011. https://doi.org/10.5591/978-1-57735-516-8/IJCAI11-089.

Zisser, E. 1999. "Hafiz Al-Asad Discovers Islam." *Middle East Quarterly* 6, no. 2: 49.

Zuckerman, M., P. Faliszewski, and Y. Bachrach, et al. 2012. "Manipulating the Quota in Weighted Voting Games." *Artificial Intelligence* 180: 1–19. https://doi.org/10.1016/j.artint.2011.12.003.

Appendix A

Appendix

Robustness of the Model's Conclusions for the Baron–Ferejohn Model With Equal Recognition Probabilities

In this section, I show that the qualitative prediction remains the same in the Baron–Ferejohn model with equal recognition probabilities.

Table A1

Proposition 2. *In the Baron–Ferejohn game with equal recognition probabilities, the equilibrium expected payoff for each minority group,* $\hat{v}_{min}$*, is as in Table A1 below.*

The Proposition implies that the payoff of the minority $\hat{v}_{min}$ *first strictly decreases in* W*, then stays constant, and then strictly increases. The intuition is somewhat different from Proposition 1. When* $2 \leq W \leq \frac{2-\delta}{1-\delta}$*, the voting quota* W *is small, so for a minority proposer, it is cheap to form a coalition with other minorities. Hence, there cannot be an equilibrium when all proposals go to the majority. However, the majority is the proposer with probability* $\frac{1}{m+1}$*, which is smaller compared to the main model. Hence, if it gets no proposals, it is also a cheap coalition partner.*

TABLE A1 | The equilibrium expected payoff of a minority given different values of the voting quota W.

Condition on the voting quota W	The equilibrium expected payoff of a minority $\hat{v}_{min}$
$2 \leq W \leq \frac{2-\delta}{1-\delta}$	$\frac{1}{m+W-1}$
$\frac{2-\delta}{1-\delta} < W \leq k$	$\frac{(1-\delta)}{1+(1-\delta)m}$
$k+1 \leq W \leq m+k$	$\frac{(1-\delta)m}{(1-\delta)m^2+m-\delta(W-k)}$

Therefore, the only possibility is a mixed-strategy equilibrium in which the minority randomizes between forming a coalition with other minorities or the majority. When W increases, forming a coalition with minorities becomes more costly. For a mixed strategy to exist, the proposer must be indifferent. Therefore, the minimum offer to each minority should decrease, or the one to the majority should increase. Because the minimum offer to each minority equals the expected continuation payoff v_{min}, the first scenario means that v_{min} decreases. The second scenario implies the same because now a minority must propose more to the majority and therefore gets a smaller surplus. When W exceeds $\frac{2-\delta}{1-\delta}$, it becomes cheaper to buy the support of the majority than $W-1$ minorities. As a result, the minority always proposes a positive amount only to the majority, so minority expected payoffs are low and W becomes irrelevant.

If $\frac{2-\delta}{1-\delta} < W \le m$, forming a minority coalition is possible but costly, so a minority proposer only offers a nonzero payment to the majority. When $m+1 \le W \le k$, a coalition of minorities is insufficient while the majority is enough, so the minority only proposes to the majority. When $k+1 \le W \le m+k$, the majority and $W-k$ minorities are necessary to form a winning coalition. When W increases, more minorities are needed for the winning coalition. As a result, each minority is more likely to be included. Also, when the majority is the proposer, it needs to pay off more minorities, so its expected payoff decreases. For these two reasons, the minority payoff increases in W.

Proof. Recall that W defines the exogenous voting quota, k is the size of the majority group, the size of each minority group is 1, and there are m minority groups. Let $W \le m$. Consider the following equilibrium. When a minority is a proposer, it makes a strictly positive offer to the majority with probability p, to $W-1$ other minorities - with probability $1-p$, and is indifferent between the two options. Analogous to the proof of Proposition 1, the continuation values v_{min} of a given minority and v_{maj} - of the majority satisfy the following conditions:

$$v_{min} = \delta\left(\frac{1}{m+1}(1 - pv_{maj} - (1-p)(W-1)v_{min}) + \frac{m-1}{m+1}(1-p)\frac{W-1}{m-1}v_{min}\right)$$

and

$$v_{maj} = \delta\left(\frac{1}{m+1} + \frac{m}{m+1}pv_{maj}\right)$$

Because a minority must be indifferent between proposing a strictly positive amount to $W-1$ minorities or the majority, it must be true that:

$$(W-1)v_{min} = v_{maj}$$

Solving this system of equations yields:

$$v_{min} = \frac{\delta}{m+W-1}$$

and

$$v_{maj} = \frac{\delta(W-1)}{m+W-1}$$

and

$$p = \frac{W-2}{\delta(W-1)}$$

The condition $p \in [0,1]$ can be rewritten as $2 \le W \le \frac{2-\delta}{1-\delta}$.

Setting $W=2$ yields $p=0$, which implies that the minority always proposes a strictly positive amount to one other minority and never—to the majority. In that case, $v_{min} = v_{maj} = \frac{\delta}{m+1}$. The indifference condition is still satisfied, so this outcome is an equilibrium.

Consider an equilibrium where each minority always proposes a strictly positive amount only to the majority. Then:

$$v_{min} = \delta\left(\frac{1-v_{maj}}{m+1}\right)$$

and

$$v_{maj} = \delta\left(\frac{1}{m+1} + \frac{m}{m+1}v_{maj}\right)$$

which yields:

$$v_{min} = \frac{(1-\delta)\delta}{1+(1-\delta)m}$$

and

$$v_{maj} = \frac{\delta}{1+m-\delta m}$$

The minority proposer strictly prefers to offer a strictly positive amount to the majority if and only if:

$$(W-1)v_{min} > v_{maj} \Leftrightarrow W > \frac{2-\delta}{1-\delta}$$

Suppose now that $m+1 \le W \le k$. Now for the minority proposer, the only option is to partner with the majority. The majority player does not propose more than zero to anyone because its size is sufficient to win. Hence, payoffs are the same as in the previous case. All deviations would involve proposing a positive amount to more players than necessary to pass a decision. Hence, this outcome is an equilibrium.

Finally, suppose that $k+1 \le W \le m+k$. Now, the majority and $W-k$ minorities are needed to pass a decision. Payoffs satisfy:

$$v_{min} = \delta\left(\frac{1}{m+1}(1 - v_{maj} - (W-1-k)v_{min}) + \frac{1}{m+1}\frac{W-k}{m}v_{min} + \frac{m-1}{m+1}\frac{W-k-1}{m-1}v_{min}\right)$$

and

$$v_{maj} = \delta\left(\frac{1}{m+1}(1-(W-k)v_{min}) + \frac{m}{m+1}v_{maj}\right)$$

which yields:

$$v_{min} = \frac{(1-\delta)\delta m}{(1-\delta)m^2 + m - \delta(W-k)}$$

and

$$v_{maj} = \frac{\delta(m+\delta(k-W))}{(1-\delta)m^2 + m - \delta(W-k)}$$

We can write:

$$v_{min} = \begin{cases} \dfrac{\delta}{m+W-1} & \text{if } 2 \le W \le \dfrac{2-\delta}{1-\delta} \\ \dfrac{(1-\delta)\delta}{1+(1-\delta)m} & \text{if } \dfrac{2-\delta}{1-\delta} < W \le k \\ \dfrac{(1-\delta)\delta m}{(1-\delta)m^2+m-\delta(W-k)} & \text{if } W \ge k+1 \end{cases}$$

and therefore:

$$\hat{v}_{min} = \begin{cases} \dfrac{1}{m+W-1} & \text{if } 2 \le W \le \dfrac{2-\delta}{1-\delta} \\ \dfrac{(1-\delta)}{1+(1-\delta)m} & \text{if } \dfrac{2-\delta}{1-\delta} < W \le k \\ \dfrac{(1-\delta)m}{(1-\delta)m^2+m-\delta(W-k)} & \text{if } W \ge k+1 \end{cases}$$

Observe that:

$$\frac{1}{m+W-1}|_{W=\frac{2-\delta}{1-\delta}} = \frac{(1-\delta)}{1+(1-\delta)m}|_{W=\frac{2-\delta}{1-\delta}}$$

and

$$\frac{(1-\delta)}{1+(1-\delta)m}|_{W=k} = \frac{(1-\delta)m}{(1-\delta)m^2+m-\delta(W-k)}|_{W=k}$$

so $\hat{v}_{min}$ is continuous as a function of W. Finally, observe that

$$\frac{d}{dW}\frac{1}{m+W-1} = -\frac{1}{(m+W-1)^2} < 0$$

and

$$\frac{d}{dW}\frac{(1-\delta)}{1+(1-\delta)m} = 0$$

and

$$\frac{d}{dW}\frac{(1-\delta)m}{(1-\delta)m^2+m-\delta(W-k)} = \frac{(1-\delta)\delta m}{((1-\delta)m^2+m-\delta(W-k))^2} > 0$$

which yields the same comparative statics as the main model. This completes the proof. □

6.1.1 | Robustness of the Model's Conclusions for Power Indices

In the model, players' payoffs depend on the amount of votes that they have. To see if the model's predictions are robust to assuming a Baron-Ferejohn bargaining procedure, it is natural to analyze the same situation using classic power indices. A power index applies to a *votinggame*. Formally, a *voting game* $v = (N, \Omega)$ consists of a set of players N and set of winning coalitions Ω. In our setting it is natural to set $\Omega = \left\{S \subset N : \sum_{i \in S} w_i \ge W\right\}$ where $w_i = k$ if i is the majority group and $w_i = 1$ if i is a minority group. In other words, as in the non-cooperative bargaining framework, a winning coalition is any coalition for which the total size of its members is weakly greater than W and which can, therefore, pass a decision on how to divide the dollar.

The Shapley–Shubik power index: The Shapley–Shubik index applies the Shapley value to voting games. It is the probability for a given player to be pivotal if players form a coalition by arriving one by one randomly and with equal probability. Formally, in a voting game v, the Shapley–Shubik power index for player i is:

$$\phi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(|N| - |S| - 1)!}{|N|!}(z(S \cup i) - z(S))$$

where $z(S) = 1$ if $S \in \Omega$ and $z(S) = 0$ otherwise. I refer the reader to Winter (2002) for a thorough exposition. The following Proposition computes the Shapley-Shubik power index for a given minority and describes the comparative statics with respect to the voting quota W.

Proposition 3. *The voting power of a minority ϕ_{min} according to the Shapley–Shubik power index is given by Table A2 below.*

Moreover, according to the Shapley–Shubik power index, as W changes from the minimum to the maximum value, the voting power of a minority first decreases, then drops and stays constant, and then increases. The following proposition proves both the result in the table and its consequences.

Table A2

Proof. First, note that by the Anonymity and Efficiency properties of the Shapley value:

$$\phi_{min} = \frac{1 - v_{maj}}{m}$$

Recall that the payoff of a player in the Shapley value of a voting game is the probability that her vote is pivotal given that players are entering a coalition in a random order. Let $W \le m$. Hence, the majority player makes a coalition winning (becomes pivotal) if and only if her position is from 1 to W. The number of such orderings is:

$$W \times m!$$

while the total number of orderings is:

TABLE A2 | The voting power of a minority according to the Shapley–Shubik power index.

Condition on the voting quota W	The voting power of a minority according to the Shapley–Shubik power index ϕ_{min}
$2 \le W \le m$	$\frac{1+m-W}{m(1+m)}$
$m+1 \le W \le k$	0
$k+1 \le W \le m+k$	$\frac{W-k}{m(1+m)}$

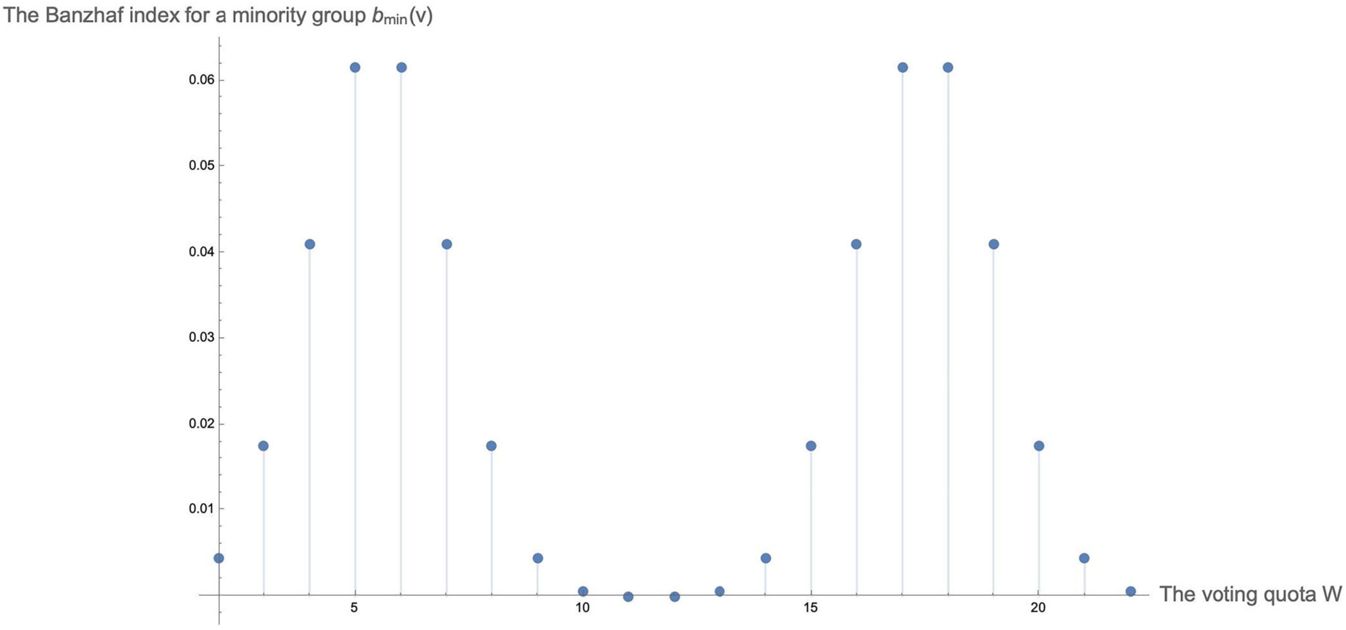

FIGURE A1 | The Banzhaf index for a generic minority group $b_{min}(v)$. In this example, the number of minorities is $m = 10$ and the size of the majority group is $k = 12$ (hence, the majority is 12 times larger than any minority).

$$(m+1)!$$

So, the probability of being pivotal for the majority player is:

$$\phi_{maj} = \frac{W \times m!}{(m+1)!} = \frac{W}{m+1}$$

Hence the voting power of each minority player is:

$$\phi_{min} = \frac{1 - v_{maj}}{m} = \frac{1 + m - W}{m(1+m)}$$

which decreases in W.

Let $m + 1 \leq W \leq k$. Then no minority group is ever pivotal. Hence, $\phi_{maj} = 1$ and $\phi_{min} = 0$.

Let $W \geq k + 1$. Then the majority is pivotal if and only if her position is from $W - k + 1$ to $m + 1$. There are $m + 1 - (W - k + 1) + 1 = m - W + k + 1$ such positions and $m!$ orderings of minorities which correspond to each, which gives:

$$(m - W + k + 1)m!$$

combinations. Since there are $(m + 1)!$ possible orderings, the probability for the majority to be pivotal is:

$$\frac{(m - W + k + 1)m!}{(m+1)!} = \frac{m - W + k + 1}{m+1}$$

Hence the minority voting power is:

$$\phi_{min} = \frac{1 - \phi_{maj}}{m} = \frac{W - k}{m(1+m)}$$

which increases in W. So, similar to the Baron-Ferejohn case, the power of any given minority first decreases, then stays constant, then increases in the voting quota W. This completes the proof. □

The ordinary and normalized Banzhaf power indices: In voting games, the Banzhaf power index measures a player's ability to change the outcome. Formally, define a *swing* for player i to be a pair $(S, S \cup i)$ such that in the voting game $v = (N, \Omega)$, $S \cup i \in \Omega$ and $S \notin \Omega$. In words, a swing is a pair consisting of a player i and a set S such that S is not a winning coalition, but adding i to S makes it one. Let $\eta_i(v)$ be the number of swings for player i. Then the value of the *Banzhafindex* for player i equals $b_i(v) \equiv \frac{\eta_i(v)}{2^{|N|}}$. The value of the *normalizedBanzhafindex* is $\beta_i(v) = \frac{\eta_i(v)}{\sum_i \eta_i(v)}$ (Van den Brink and Van der Laan 1998).

To compute the ordinary and normalized Banzhaf indices for our game, it is useful to consider the same three cases as before. First, suppose that $2 \leq W \leq m$. Recall that, by assumption, $m < k$, so $W < k$. For a generic minority group i, the coalition $S \cup i$ in a swing is a set of W minority groups that includes i. For the majority group, such a coalition consists of the majority group itself and fewer than W (possible 0) minority groups. Hence, for a generic minority group:

$$\eta_{min}(v) = \binom{m-1}{W-1}$$

and for the majority group:

$$\eta_{maj}(v) = \sum_{l=0}^{W-1} \binom{m}{l}$$

Therefore, for a generic minority group:

$$b_{min}(v) = \frac{\binom{m-1}{W-1}}{2^{m+1}}$$

and

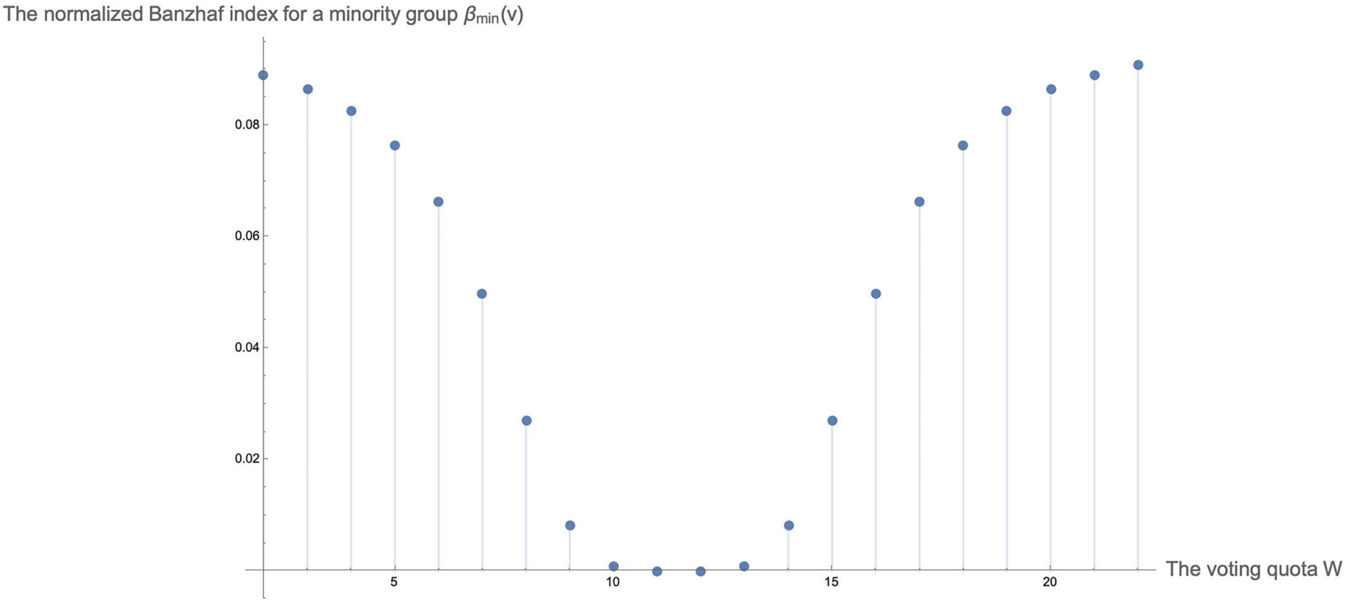

FIGURE A2 | The normalized Banzhaf index for a generic minority group $\beta_{min}(v)$. In this example, the number of minorities is $m = 10$, and the size of the majority group is $k = 12$ (hence, the majority is 12 times larger than any minority).

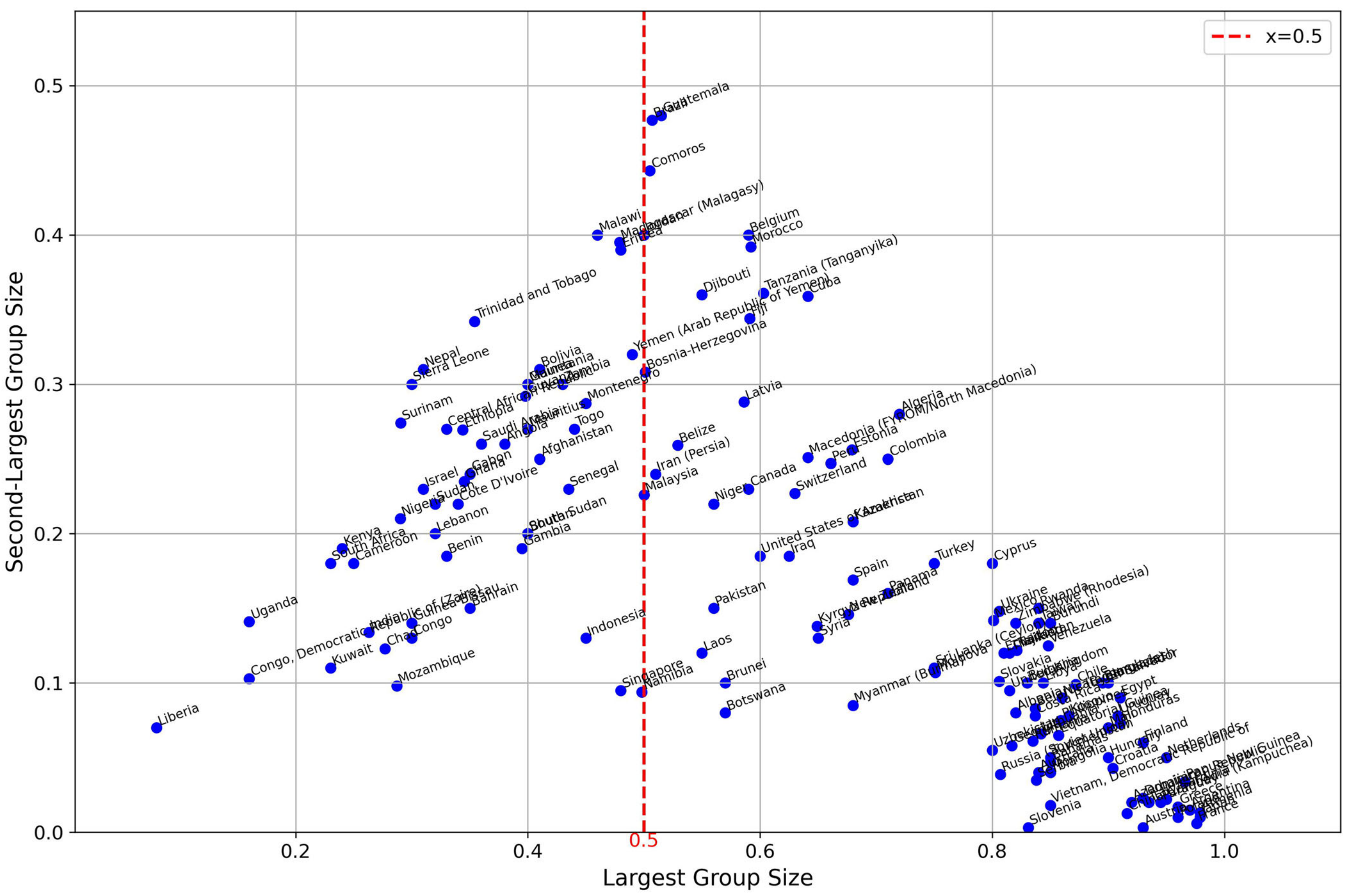

FIGURE A3 | Sizes of the first- and second-largest politically relevant ethnic groups according to the Ethnic Power Relations 2021 data set.

$$\beta_{min}(v) = \frac{\binom{m-1}{W-1}}{m\binom{m-1}{W-1} + \sum_{l=0}^{W-1}\binom{m}{l}} = \frac{1}{m + \frac{\sum_{l=0}^{W-1}\binom{m}{l}}{\binom{m-1}{W-1}}}$$

Suppose now that $m + 1 \leq W \leq k$. Clearly, a minority group cannot be decisive, so there are no swings for it, and its ordinary and normalized Banzhaf indices are 0. Finally, suppose that $k + 1 \leq W \leq m + k$. For a minority group i, a set $S \cup i$ in a swing is a set consisting of the majority group and $W - k$ minorities, including i. There are $\binom{m-1}{W-k-1}$ such sets.

Because the majority group is a part of every winning coalition, a set $S \cup i$ in a swing for the majority group is any set that includes it and weakly more than $W - k$ minorities. The number of such sets is

$$\sum_{l=W-k}^{m} \binom{m}{l}$$

Hence, for a generic minority group

$$b_{min}(v) = \frac{\binom{m-1}{W-k-1}}{2^{m+1}}$$

and

$$\beta_{min}(v) = \frac{\binom{m-1}{W-k-1}}{m\binom{m-1}{W-k-1} + \sum_{l=W-k}^{m}\binom{m}{l}} = \frac{1}{m + \frac{\sum_{l=W-k}^{m}\binom{m}{l}}{\binom{m-1}{W-k-1}}}$$

The graph on Figure A1 below plots the ordinary Banzhaf index for a generic minority group against the voting quota W. The relationship is highly non-monotonic. The peaks arise when W is in the middle of the range $[2, m]$ or in the middle of the range $[k + 1, m + k]$. In the first range, a minority group is decisive for coalitions that include it and $W - 1$ other minorities. In the second range, a minority is decisive if it belongs to a coalition of $W - k$ minorities and the majority. Recall that the function $\binom{n}{l}$ is the largest when l is close to $\frac{n}{2}$. This result concerning the factorial function explains why, in both ranges, the number of such coalitions is large when W is medium and small when W is small or large.

The next graph (Figure A2) plots the normalized Banzhaf index for a generic minority group against the size of the minimum winning coalition W. While the mathematical relationship is ambiguous, the graph suggests the same comparative statics as the Baron-Ferejohn model and the Shapley value.

The normalized Banzhaf index depends on the ratio between the number of winning coalitions for which the majority is decisive to the analogous number for a generic minority. Intuitively, a generic minority is powerful when the majority is not very powerful. When W is close to its minimum value of 2, the majority is decisive in coalitions that include it and fewer than W minorities. When W is small, there are few such coalitions, which explains the high value of the index. When W is close to the maximum value of $m + k$, the majority is decisive for coalitions that include it and weakly more than W minorities. For large values of W, the number of such coalitions is small, which, again, explains the high value of the index for a generic minority. Like the Shapley–Shubik index, both Banzhaf indices assign a value of 0 to a generic minority group when $m + 1 \leq W \leq k$ and therefore it cannot be a decisive voter. The same would be true for any index that assigns zero power to players that are not decisive for any coalition. This property of the voting game partially drives the non-monotonicity of the payoff of a generic minority group.

6.1.2 | Sizes of the First- and Second-Largest Groups Cross-Nationally

The model in the paper applies to countries where the largest ethnic group exceeds 50%. Figure A3 below shows that this is not the case in relatively few countries. In many countries, concentrated in the lower-right corner, the largest group is over 80%. However, in many, the largest group size is between 50% and 80%, while the second-largest is over 10%. Hence, in the latter countries, minorities comprise a sizable part of the population collectively, and there is at least one large minority, so minorities may play an important political role. Such countries are not restricted to a specific region, which makes the framework in the paper widely applicable.