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Oligopolistic markets with sequential search and production cost uncertainty

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This article analyzes a sequential search model where firms face identical but stochastic production costs, the realizations of which are unknown to consumers. We characterize a perfect Bayesian equilibrium satisfying a reservation price property and provide a sufficient condition for such an equilibrium to exist. We show that (i) firms set on average higher prices and make larger profits compared to the scenario where consumers observe production costs, (ii) expected prices and consumer welfare can be non-monotonic in the number of firms, and (iii) the impact of production cost uncertainty vanishes as the number of firms becomes very large.

1. Introduction

■ Consider a consumer who observes the price of gasoline at a gas station. Knowing that prices between different stations may vary considerably, the consumer must decide whether to buy at the observed price or search for a better deal elsewhere. When making this decision, she must estimate how much of the observed price is due to common factors affecting all gasoline stations in a similar way, such as the price of crude oil, and how much is due to idiosyncratic factors affecting the particular seller being visited. If the consumer believes that common factors are more relevant in determining the price, she might consider searching for a cheaper gas station

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not worthwhile and, hence, buy at the observed price. Conversely, if she believes that the station charges a particularly high price compared to other stations, she will probably find it optimal to look for a better deal. A key feature of this problem is that the consumer must take her decision not knowing the gas station's input (production) cost. Moreover, information is *asymmetric*, as gasoline retailers are obviously aware of this cost.

In this article, we study how asymmetric information between firms and consumers affects market outcomes in situations like the one described above. To this end, we introduce this feature into the standard sequential search model with homogeneous goods, as developed by Stahl (1989). In our model, finitely many firms sell a homogeneous product on an oligopolistic market, with all firms facing the same stochastic production cost and being aware of its realization. Consumers have inelastic demand and engage in sequential search for low prices. Unlike Stahl's sequential search model and unlike most search literature, consumers do not observe the firms' production cost realization. Instead, they hold prior beliefs about the distribution of production costs and update these beliefs as they observe prices. This environment captures not only important features of the retail gasoline market but also of retail markets for mortgages and other financial services, where retailers (i) are subject to important exogenous, common shocks in their main cost factor and (ii) are better informed about these shocks than final consumers.[1]

In this environment, we examine the properties of a *perfect Bayesian equilibrium satisfying a reservation price property* (PBERP) where consumers buy if the observed price is at or below their current reservation price, and search for a lower price otherwise. At the relevant reservation price, the consumer is indifferent between buying and searching for a better deal. Notice that a consumer who observes a price in the first search round and continues to search will update her beliefs about the firms' underlying production cost on the basis of her first price observation, and therefore she will (generically) have a different reservation price in the second search round. Thus, a consequence of production cost uncertainty within a sequential search framework is that, if there exists an equilibrium where optimal search behavior in each search round is characterized by a reservation price, the reservation price must depend on the history of price observations. This history dependence renders the analysis considerably more complicated compared to the case where consumers know the production cost. This may have led the literature to restrict attention to other, often less plausible, search protocols when studying information asymmetry in consumer search models.[2] Our article attempts to fill this gap in the literature by examining a *sequential* search framework within the context of production cost uncertainty. Within this framework, we focus on three questions. What are the characteristics of a PBERP? Under which conditions does such an equilibrium exist? And what is the role of asymmetric information within a sequential search framework?

Concerning the first question, we first show that in a PBERP no firm will set a price larger than the consumers' first-round reservation price. To gain intuition for this finding, assume a firm would charge the upper bound of the equilibrium price distribution, and that this upper bound is higher than the consumers' first-round reservation price. This firm would obviously not have any sales in the first search round; moreover, in later search rounds, potential customers visiting the firm already have lower price observations in their pockets and, thus, the firm would not have any sales in later search rounds either. This behavior cannot be optimal. The central step in characterizing equilibrium, thus, turns out to be the characterization of the consumers' first-round reservation price. This is not a straightforward task, as consumers may in principle

[1] This is not to say that there are no cost differences between individual firms. What seems to be true, however, in most of these markets—and certainly in the retail gasoline market—is that the idiosyncratic component of costs—such as the rental rate of the shop location—is much more stable than the variations of the main cost component, whether it is the world oil price or the interest rate in financial markets.

[2] For example, Dana (1994) considers a model of asymmetric information with *newspaper* search, that is, uninformed consumers can learn all prices at once by paying a search cost. He emphasizes, however, that the "assumption of newspaper search is clearly restrictive and is not as realistic as the assumption that search is sequential (...). However, analyzing sequential search equilibrium under incomplete information is extremely difficult because consumers could in principle search more than once and hence more than one instance of Bayesian updating could occur" (p. 747).

search more than once, leading to multiple instances of updating of beliefs about the underlying production cost. One important theoretical result of this article is that this cannot happen in a PBERP. Importantly, a consumer who observes the first-round reservation price in a PBERP and, being indifferent, continues to search has a strictly higher reservation price in round two than in round one. This implies that the first-round reservation price is the price at which a consumer is indifferent between buying now and continuing to search exactly one more round, similar to the complete information model.

Concerning our second research question, we show that existence of a PBERP is not trivially guaranteed. This finding is closely related to Rothschild (1974), who shows that an optimal consumer search strategy does not need to satisfy a reservation price property if the price distribution is unknown to the consumer, and thus obtaining price observations also has an informational value as consumers update their beliefs about the true price distribution from which prices are sampled. In an environment where the price distribution is exogenously given, Rothschild shows that the consumer search strategy satisfies a reservation price property if for any two prices the difference between these two prices is smaller than the difference in their informational content. Using similar ideas in an environment where the price distribution is endogenously determined, and using specific properties the equilibrium price distribution has to satisfy, we specify a sufficient condition for the existence of a PBERP. In particular, we show that a PBERP exists in markets with either (i) a small support of the production cost distribution, (ii) relatively large search costs, (iii) relatively many firms, or (iv) relatively few shoppers. If a PBERP exists, we show that it is necessarily unique.

We extend Rothschild's logic in our framework by arguing that our condition is also "necessary" in the sense that, if the condition fails to hold, then one can always find distributions of the production cost such that a PBERP does not exist. Moreover, we provide an example showing that even when the production cost is uniformly distributed, a reservation price equilibrium fails to exist if our condition is not satisfied. Thus, production cost uncertainty introduces significant changes to the sequential search model with respect to the existence of reservation price equilibria.

Concerning our third question, we have the following observations. First, in the cost certainty benchmark[3] (which is essentially the Stahl model with unit demand), we show that the expected minimum price in the market is independent of the number of firms when this number is not too large, whereas the expected price is increasing. Interestingly, when consumers are uncertain about the underlying cost level, the expected price and total consumer surplus over informed and uninformed consumers can be nonmonotonic in the number of firms. Second, examining equilibrium price strategies used by firms, we show that the lower bound of the price distribution is increasing in the cost level whereas its upper bound is independent of the cost level.[4] Thus, the extent of equilibrium price dispersion under production cost uncertainty decreases as the cost level rises. This constitutes an important and interesting difference from the case where consumers are aware of the production cost where the extent of price dispersion is independent of the production cost realization. Third, studying the welfare effects of production cost uncertainty, we show that, from an *ex ante* perspective, consumer welfare is unambiguously lower under production cost uncertainty and profits are unambiguously higher as compared to the environment where consumers know the production cost. Both informed and uninformed consumers pay in expected terms higher prices. Finally, when the number of firms in the industry becomes large, the importance of production cost uncertainty vanishes; both the expected price and the expected minimum price in the cost uncertainty model converge to their counterparts in the cost certainty benchmark. The main reason is that, with many firms, prices close to the reservation price do not convey information to consumers about the underlying cost of firms.

[3] We use the term "cost certainty" when we refer to the situation that consumers and firms all know the relevant production cost. "Cost uncertainty" is used when we refer to the situation that consumers do not know the relevant production cost but firms do.

[4] In a PBERP, the upper bound must be equal to the first-round reservation price of consumers. Because the latter cannot depend on the cost realization which is unknown to consumers, this property carries over to the upper bound.

This article contributes to a large and growing literature on equilibrium consumer search models starting from seminal contributions by Reinganum (1979), Varian (1980), Burdett and Judd (1983), and Stahl (1989). In terms of the research question being addressed, the articles most closely related to the present one are Benabou and Gertner (1993) and Dana (1994).[5] Both articles consider, however, simplified search protocols. Benabou and Gertner analyze a duopoly market where production costs are subject to correlated shocks. Half the consumers observe one price and the other half observes the other price at no cost. The only decision consumers have to make is whether to also observe the price of the firm they have not yet observed at a search cost. Their analysis is restricted to the case where the correlation between the production shocks of individual firms is not too large or the search costs are not too large. We consider the case where production costs are perfectly correlated and where search costs and the number of firms can take on any value. Dana (1994) considers a model with two types of consumers (informed and uninformed) where the uninformed consumers are engaged in *newspaper search*. These consumers get a first price quote for free and, on the basis of this price, they decide whether or not to become fully informed about all prices by paying a search cost. Papers by Fershtman and Fishman (1992) and Fishman (1996), and the recent contributions by Yang and Ye (2008) and Tappata (2009), use frameworks similar to Dana (1994), but extend them to a dynamic setting. In such environments, these articles study asymmetric price adjustment to cost shocks, the so-called rockets-and-feathers pattern. To the best of our knowledge, our article is the first to introduce production cost uncertainty into a *sequential* consumer search model.[6]

In a broader sense, the current article is related to recent work that elaborates on the role of information gathering and information processing in consumer search.[7] This literature focuses on obfuscation (Ellison and Wolitzky, 2009; Ellison and Ellison, 2009), boundedly rational agents (see, e.g., Spiegler, 2006), or information gatekeepers on the Internet (see, e.g., Baye and Morgan, 2001). Another strand of the literature makes progress on the policy implications of the consumer search literature on consumer protection policies (see, e.g., Armstrong, Vickers, and Zhou, 2009a) or on the empirical implementation of consumer search models (see, e.g., Lach, 2007; Hortaçsu and Syverson, 2004; Moraga-González and Wildenbeest, 2008).

The remainder of this article is organized in the following way. In Section 2, we briefly discuss the standard sequential search model where consumers observe production costs, establishing a theoretical benchmark for comparison of our cost uncertainty model. This section also contains some new results (on the expected price paid by consumers who know all prices in the market) and some new characterizations making it easier to work with the model. In Section 3, we develop our model in which the production cost realization is uncertain to consumers, define a perfect Bayesian equilibrium, and characterize its properties. In Section 4, we summarize the effects of production cost uncertainty on consumer and producer welfare, assess the effects of entry by comparing the models when the number of firms becomes large, and provide some other comparative statics analysis. Section 5 contains a discussion and Section 6 concludes. Proofs are provided in the Appendix.

2. Sequential search under production cost certainty

■ We start our analysis by describing a sequential search model where there is no uncertainty regarding the underlying production cost along the lines of Stahl (1989). This model will, at a later stage, serve as our benchmark to assess the implications of production cost uncertainty

[5] Earlier work by Diamond (1971) and Rothschild (1974) analyzes optimal search behavior in a world where the price distribution is unknown but exogenously given. Recently, Gershkov and Moldovanu (2009) uncover some formal relations between optimal stopping rules in the consumer search literature and the problem of ensuring monotone allocation rules in dynamic allocation problems.

[6] Strictly speaking, Benabou and Gertner (1993) analyze a case of sequential search, but as their analysis is restricted to duopoly markets, consumers effectively know the prices of *all* firms once they engage in search (beyond their first free price observation). With two firms, the difference between sequential search and newspaper search disappears.

[7] Baye, Morgan, and Scholten (2006) survey a wide range of consumer search models.

within the sequential search framework. Essentially, we modify Stahl's model along only two dimensions. First, to simplify the analysis, we consider a model of inelastic demand as in Janssen, Moraga-González, and Wildenbeest (2005) and, second, do not normalize marginal costs to zero. Solving the model for positive marginal costs is inevitable for our purposes, because later we want to analyze and compare situations with different marginal cost levels.

□ **Model.** We consider an oligopolistic market where $N \geq 2$ firms sell a homogeneous good and compete in prices. Each firm $n \in \{1, \ldots, N\}$ faces the same production technology and the same marginal production cost, denoted by c. Without loss of generality, we normalize fixed costs to zero. Each firm's objective is to maximize profits, taking the prices charged by other firms and the consumers' behavior as given.

On the demand side of the market, we have a continuum of risk-neutral consumers with identical preferences. Each consumer $j \in [0, 1]$ has inelastic demand normalized to one unit, and holds the same constant valuation $v > 0$ for the good. Observing a price below v, consumers will, thus, either buy one unit of the good or search for a lower price. In the latter case, they have to pay a search cost s to obtain one additional price quote, that is, search is *sequential*. A fraction $\lambda \in [0, 1]$ of consumers, the *shoppers*, have zero search costs. These consumers sample all prices and buy at the lowest price. The remaining fraction of $1 - \lambda$ consumers—the *nonshoppers*—have positive search costs $s > 0$. These consumers face a nontrivial problem when searching for low prices, as they have to trade off the search cost with the (expected) benefit from search. Consumers can always come back to previously visited firms, incurring no additional cost, that is, we are considering a model of *costless recall*.[8] We assume that v is large relative to c and s so that v is not binding. In this section, consumers are informed about the cost realization c.

□ **Equilibrium.** In this model, there exists a unique symmetric Nash equilibrium where consumer behavior satisfies a reservation price property. Kohn and Shavell (1974) and Stahl (1989) argue that reservation prices are stationary, that is, the consumers' reservation prices are independent from the history of price observations and the number of firms left to be sampled (provided there is still at least one firm left) and can therefore simply be denoted by $\rho^k(c)$.[9]

To characterize this equilibrium, it is useful to introduce some further notation: we denote, for a given production cost c, the distribution of prices charged by firms by $F^k(p \mid c)$, its density by $f^k(p \mid c)$, and the lower and upper bound of its support by $\underline{p}^k(c)$ and by $\bar{p}^k(c)$, respectively.

It is well known that the presence of both shoppers and nonshoppers, $\lambda \in (0, 1)$, implies that there does not exist an equilibrium in pure strategies and that there are no mass points in the equilibrium price distribution. The main reason behind this observation is that firms face a tradeoff between setting low prices to cater to the shoppers and setting high prices to extract profits from the nonshoppers. Also, the upper bound of the equilibrium price distribution must satisfy $\bar{p}^k(c) = \rho^k(c)$, that is, in a symmetric equilibrium, no firm will set a price higher than the reservation price $\rho^k(c)$. Given these two observations, the equilibrium price distribution can be characterized by the following.

Proposition 1. For $\lambda \in (0, 1)$ and $\rho^k(c)$, the equilibrium price distribution for the cost realization c is given by

$$F^k(p \mid c) = 1 - \left(\frac{1 - \lambda}{\lambda N} \frac{\bar{p}^k(c) - p}{p - c} \right)^{\frac{1}{N-1}} \tag{1}$$

respectively

[8] Janssen and Parakhonyak (2007) analyze the case where this assumption is replaced by costly recall.

[9] We use the superscript k to indicate variables and parameters of the model where consumers know the production cost realization c.

$$f^k(p \mid c) = \frac{1}{N-1}\frac{\bar{p}^k(c)-c}{(p-c)^2}\left(\frac{1-\lambda}{\lambda N}\right)^{\frac{1}{N-1}}\left(\frac{\bar{p}^k(c)-p}{p-c}\right)^{\frac{2-N}{N-1}} \tag{2}$$

with support on $[\underline{p}^k(c), \bar{p}^k(c)]$ with $\underline{p}^k(c) = \frac{\lambda N}{\lambda N+1-\lambda}c + \frac{1-\lambda}{\lambda N+1-\lambda}\bar{p}^k(c)$ and $\bar{p}^k(c) = \rho^k(c)$.

The proof follows essentially Stahl (1989) and is therefore omitted. The result mainly follows from the fact that given the mixed-strategy distribution chosen by the competitors and its own price p, a firm's profit is given by

$$\left[\lambda(1 - F^k(p \mid c))^{N-1} + \frac{1-\lambda}{N}\right](p-c),$$

and that in a mixed-strategy equilibrium, this profit has to be equal to the profit the firm gets if it sets a price equal to the upper bound of the price distribution, which is equal to $\frac{1-\lambda}{N}(\bar{p}^k(c) - c)$.

Having characterized the Nash equilibrium price distribution conditional on the reservation price $\rho^k(c)$, we turn to optimal consumer behavior. Given a distribution of prices $F^k(p \mid c)$ and an observed price p', it is straightforward to argue that the reservation price $\rho^k(c)$ is implicitly determined by

$$v - \rho^k(c) = v - s - \int_{\underline{p}^k(c)}^{\rho^k(c)} p f^k(p \mid c)\, dp.$$

Using the result that the equilibrium price distribution satisfies $\bar{p}^k(c) = \rho^k(c)$, this condition boils down to

$$\rho^k(c) = s + E^k(p \mid c). \tag{3}$$

As the next lemma shows, this relation between reservation price and expected price allows us to derive a simple formula for the expected price conditional on the cost realization c, $E^k(p \mid c)$.

Lemma 1. The expected price conditional on the cost realization c, $E^k(p \mid c)$, is given by

$$E^k(p \mid c) = c + \frac{\alpha}{1-\alpha}s, \tag{4}$$

where $\alpha = \int_0^1 \frac{1}{1+\frac{\lambda N}{1-\lambda}z^{N-1}}dz \in [0, 1)$.

Janssen and Moraga-González (2004) have shown that α is increasing in N. It follows that the expected price is increasing in N as well.

Note that (3) and (4) imply the following simple expression for the reservation price,

$$\rho^k(c) = \bar{p}^k(c) = c + \frac{s}{1-\alpha}. \tag{5}$$

The reservation price is, thus, a constant markup over the cost, with the size of the markup being determined by the model's parameters s and α which, in turn, depends on N and λ. Note that α approaches one as the number of firms grows without bound, $N \to \infty$, or the fraction of informed consumers shrinks, $\lambda \to 0$, and therefore the reservation price grows without bound. At some point, the willingness to pay v becomes binding. In order not to complicate notation, we restrict the analysis to the parameter values where v is not binding. Note further that, by Proposition 1, $\underline{p}^k(c)$ is a weighted average of c and $\rho^k(c)$. Consequently, it immediately follows that, provided $s > 0$, the lower bound satisfies $\underline{p}^k(c) > c$. Thus, firms make positive profits when charging prices according to $F^k(p \mid c)$. Furthermore, the following result obtains.

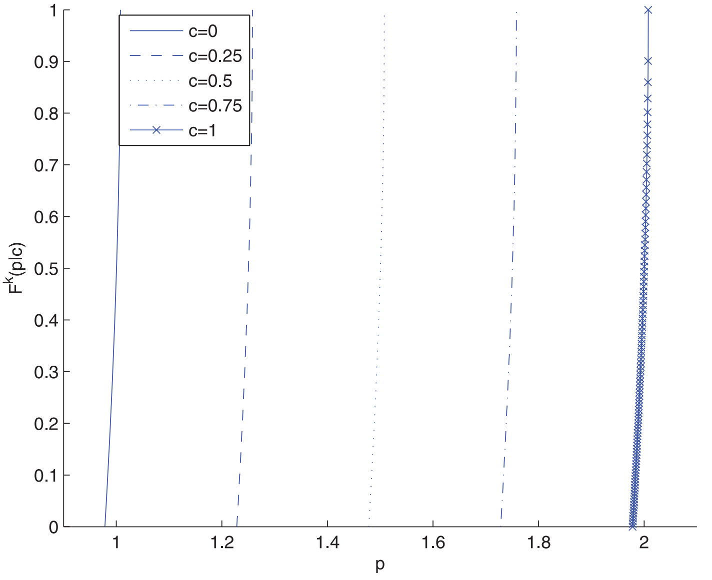
Corollary 1. The equilibrium price spread, that is, the difference between the upper bound and the lower bound of the price distribution, is independent of the realized cost level c and given by

$$\bar{p}^k(c) - \underline{p}^k(c) = \frac{\lambda N}{\lambda N + 1 - \lambda}\frac{s}{1-\alpha}. \tag{6}$$

The proof follows from (5) and Proposition 1. What is interesting about Corollary 1 is that a change in c leads to a one-to-one shift in the price distribution, leaving the extent of price

FIGURE 1

PRICE DISTRIBUTIONS



Parameters: $N = 3$, $s = 0.01$, $\lambda = 0.01$, $c \sim U(0, 1)$

dispersion unaffected. In Figure 1, we highlight this point by plotting price distributions for various realizations of cost.

Note at this stage that, conditional on the cost c, the average price paid by a fraction $1 - \lambda$ of consumers, that is, the nonshoppers, is equal to $E^k(p \mid c)$ as given in (4). This is, however, not the (average) price paid by the λ shoppers who observe all prices in the market and buy at the cheapest firm. This latter price is given by $E^k(p_\ell \mid c)$, with $p_\ell = \min\{p_1, p_2, \ldots, p_N\}$. As firms choose prices randomly and independently from each other, it follows that the distribution of p_ℓ is given by

$$F_l^k(p_\ell \mid c) = 1 - [1 - F^k(p \mid c)]^N. \tag{7}$$

Stahl (1989), and recently Morgan, Orzen, and Sefton (2006) and Waldeck (2008), suggest that in their models the shoppers observing all prices are better off with entry. In our model featuring unit demand and an endogenous reservation price this is, however, not the case. In particular, the expected minimum price is independent of N:

Proposition 2. The expected minimum price conditional on the cost realization c, $E^k(p_\ell \mid c)$, is given by

$$E^k(p_\ell \mid c) = c + \frac{1 - \lambda}{\lambda} s, \tag{8}$$

and therefore independent of N.

In our model, entry has three different effects on the expected minimum price. First, the reservation price is increasing in the number of firms through its effect on α. As this implies that prices tend to be higher with more firms, this effect unambiguously makes shoppers worse off. A second effect is that shoppers sample more firms when there are more firms available in the market. This has the unambiguous effect that for a given price distribution, shoppers are better off. Finally, price dispersion increases with more firms in the industry, implying that firms concentrate more on lower and higher prices at the expense of moderate prices. In fact, the lower bound of the price distribution decreases in the number of firms. The proposition shows that the net result of these three effects is zero.

The difference with earlier results can be explained as follows. Stahl's (1989) result that shoppers are better off with entry holds true when there are already sufficiently many firms. In that case, the willingness to pay v imposes a binding constraint on the upper bound of the price distribution and the first effect mentioned above does not apply anymore. For the special case of unit demand, our model is identical to Stahl (1989) and our result holds when N is small enough so that the willingness to pay is not binding. Morgan, Orzen, and Sefton (2006) consider Varian's (1980) model where the consumer reservation price is exogenously given by the willingness to pay and consumers have unit demand. This, again, implies that the first effect mentioned above does not apply.

Finally, we compute the unconditionally expected prices $E^k(p) = \int E^k(p \mid c)dc$ and $E^k(p_\ell) = \int E^k(p_\ell \mid c)dc$. These expected prices will later on be important to assess consumer welfare in the economy, as $v - E^k(p_\ell)$ is the expected equilibrium consumer surplus attained by shoppers whereas $v - E^k(p)$ is the expected equilibrium surplus of the $1 - \lambda$ nonshoppers.[10] Formally, we obtain the following.

Corollary 2. The unconditionally expected prices $E^k(p)$ and $E^k(p_\ell)$ are given by

$$E^k(p) = E(c) + \frac{\alpha}{1-\alpha}s, \quad \text{and} \tag{9}$$

$$E^k(p_\ell) = E(c) + \frac{1-\lambda}{\lambda}s, \tag{10}$$

where $E(c) = \int_{\underline{c}}^{\bar{c}} cg(c)\,dc$.

The proof follows trivially from Lemma 1 and Proposition 2. Note that both *ex ante* expected prices take the form of a markup over the *ex ante* expected cost, with the size of the respective markup being determined by the model's parameters. It is interesting to note that the markup for the expected minimum price is independent of N, and the markup of the expected price is increasing in N.

3. Sequential search under production cost uncertainty

■ We now turn to the analysis of production cost uncertainty and modify the model presented in Section 2 by postulating that consumers are uninformed about the firms' production cost. Let nature randomly draw c from a continuous distribution $g(c)$ with compact support on $[\underline{c}, \bar{c}]$. Consumers do not know the cost realization and they all hold correct prior beliefs about the production cost distribution and update their beliefs according to Bayes' rule after observing prices.

□ **On out-of-equilibrium beliefs.** Under production cost uncertainty, the exact specification of out-of-equilibrium beliefs plays an important role in determining reservation prices. To see this point, assume that consumers hold out-of-equilibrium beliefs that are such that, if a price above their reservation price is observed, they think that the lowest cost level has been realized with probability one and therefore continue to search. In such a case, in equilibrium no firm would set a price above the reservation price (more details will be given shortly) and therefore such a price observation is clearly an out-of-equilibrium event. Note that under these particular beliefs, one can support a (first-round) "reservation price" with the property that a consumer who observes it will strictly prefer to buy instead of actually being indifferent between buying and searching for a lower price. In a setting where consumers know the production cost, this could never be

[10] We will compute expected consumer surplus in the very beginning of the game, that is, before the cost level c is drawn from $g(c)$.

a reservation price, as consumers would then also be willing to buy at a slightly higher price. However, in the case of production cost uncertainty, under these particular out-of-equilibrium beliefs, a consumer would prefer to continue searching for lower prices, thinking that the lowest cost level has been realized and thus that prices should be low. Consequently, there would be a discontinuity in the willingness of consumers to buy around this "reservation price."

However, we think that such a discontinuity is difficult to defend in a consumer search model and we certainly do not want the comparison between the cases where consumers know and do not know the production cost to depend on the arbitrary choice of out-of-equilibrium beliefs. We therefore insist that, if at a reservation price consumers strictly prefer to buy, out-of-equilibrium beliefs should be such that consumers also should buy at a slightly higher price.[11] This effectively defines the first-round reservation price as the price at which the consumer is indifferent between buying and continuing to search, in a way similar to the familiar case where consumers know the production cost. In the following, we limit attention to equilibria satisfying such a reservation price property.

□ **Equilibria with a reservation price property.** To provide a formal definition of what we mean by *equilibria satisfying a reservation price property* requires us to introduce some more notation. In particular, we denote by $\rho_t(p_1, \dots, p_{t-1})$ the reservation price of a consumer in search round t who has observed prices $p_1, \dots, p_{t-1}$ in the $t-1$ previous search rounds. Note that, unlike the case where consumers know the production cost, any reservation price $\rho_t(p_1, \dots, p_{t-1})$ held by consumers has to be independent of the production cost. As the reservation price ρ_1 in the first round cannot be conditioned on any price observation, we write $\rho_1 = \rho$. We have the following.[12]

Definition 1. A perfect Bayesian equilibrium satisfying a reservation price property is characterized by

(a) each firm $n \in \{1, \dots, N\}$ uses a price strategy that maximizes its (expected) profit, given the competing firms' price strategies and the search behavior of consumers;
(b) given the (possibly degenerate) distribution of firms' prices, consumers search optimally given their beliefs, and update their beliefs given their price observations (if possible) and formulate out-of-equilibrium beliefs whenever they observe a nonequilibrium price; moreover, optimal consumer search is of the following form:
   (i) after observing $p_t = \rho_t(p_1, \dots, p_{t-1})$ in round t and $p_1, \dots, p_{t-1}$ in previous rounds, the consumer is indifferent between buying and continuing to search;
   (ii) after observing any $p_t < \rho_t(p_1, \dots, p_{t-1})$ in round t and $p_1, \dots, p_{t-1}$ in previous rounds, the consumer buys.

In what follows, we concentrate on the characterization of this type of equilibrium and determine conditions for existence.

□ **Properties of PBERP.** We first examine the properties of a PBERP, assuming that such an equilibrium exists. Later, we consider the existence question. A first observation is that in a PBERP, the upper bound of the price distribution has to be equal to the reservation price of consumers in the first search round, that is, $\bar{p}(c) = \bar{p} = \rho$ for all $c \in [\underline{c}, \bar{c}]$. Suppose this was not the case and that for some c, $\bar{p}(c) > \rho$. If a firm charges $\bar{p}(c)$, it will not sell to shoppers in any PBERP, as $\bar{p}(c)$ does not have positive probability and therefore shoppers observe lower prices with probability one. Furthermore, a firm setting $\bar{p}(c)$ will not sell to nonshoppers either, as these consumers will continue to search after observing $\bar{p}$ in the first search round, and will then find a lower price in a subsequent search round with probability one. On the other hand, it can also

[11] We do not impose further constraints on the out-of-equilibrium beliefs. The continuity requirement we impose is consistent with, but not implied by, the Intuitive Criterion as defined by Cho and Kreps (1987).

[12] This definition is an adaptation of the reservation price equilibrium defined by Dana (1994) to the case of sequential search.

not be the case that for some c, $\bar{p}(c) < \rho$, as firms could profitably deviate to a price equal to ρ because nonshoppers would continue to buy.

Under production cost uncertainty, firms face virtually the same maximization problem as in the benchmark case where consumers know the production cost. The only major difference is that the upper bound of the price distribution is now constant at $\bar{p} = \rho$ for all realizations of the cost c. Formally,

Proposition 3. In any PBERP, the equilibrium price distribution for the cost realization c is given by

$$F(p \mid c) = 1 - \left(\frac{1-\lambda}{\lambda N} \frac{\bar{p} - p}{p - c} \right)^{\frac{1}{N-1}} \tag{11}$$

respectively

$$f(p \mid c) = \frac{1}{N-1} \frac{\bar{p} - c}{(p-c)^2} \left(\frac{1-\lambda}{\lambda N} \right)^{\frac{1}{N-1}} \left(\frac{\bar{p} - p}{p - c} \right)^{\frac{2-N}{N-1}} \tag{12}$$

with support on $[\underline{p}(c), \bar{p}]$ with $\underline{p}(c) = \frac{\lambda N}{\lambda N + 1 - \lambda} c + \frac{1-\lambda}{\lambda N + 1 - \lambda} \bar{p}$ and $\bar{p} = \rho$.

The proof is omitted, as it is a straightforward extension of Proposition 1. Essentially (11) is identical to (1), the main difference being that now the upper bound is independent of c as consumers cannot condition their search behavior on the unknown cost parameter. Inspection of (11) reveals that $F(p \mid c)$ first–order stochastically dominates (FOSD) $F(p \mid c')$ whenever $c > c'$. Furthermore, we have that $\underline{p}(c)$ is increasing in c, implying that consumers who observe prices below $\underline{p}(\bar{c})$ can rule out certain (high-) cost realizations. Employing the techniques used in Lemma 1, we can write $E(p \mid c)$ as

$$E(p \mid c) = (1 - \alpha)c + \alpha \bar{p}. \tag{13}$$

Figure 2 visualizes all these observations by plotting the price distributions $F(p \mid c)$ for different realizations of the production cost c. This figure points to the fact that the price spread is decreasing in the cost level, as stated in the following corollary.

Corollary 3. If consumers are uninformed about the firms' cost realization, the price spread in a PBERP is equal to

$$\bar{p} - \underline{p}(c) = \frac{\lambda N}{\lambda N + 1 - \lambda} (\bar{p} - c)$$

and therefore is decreasing in the cost level c.

Corollary 3 establishes a sharp contrast to the case with production cost certainty where the price spread is independent of the realized cost level.

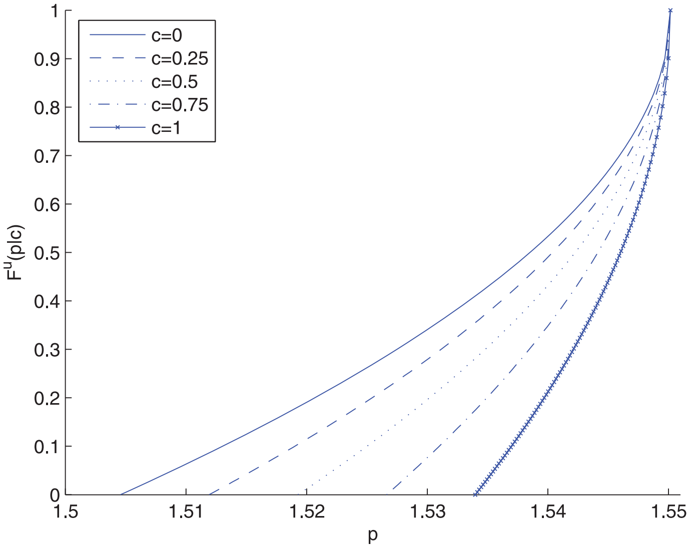
We next focus on consumers' search behavior. Assuming consumers have correct prior beliefs about the production cost and use Bayesian updating after observing a price p, let $\delta(c \mid p)$ be the (posterior) probability density function of the production cost c conditional on a price observation p . By Bayes rule, we have that

$$\delta(c \mid p) = \frac{g(c) f(p \mid c)}{\displaystyle\int_{\underline{c}}^{\bar{c}} g(c') f(p \mid c')\, dc'}. \tag{14}$$

It remains to specify consumers' out-of-equilibrium beliefs, that is, beliefs on the cost level for price observations not in the support of the equilibrium price distribution. As argued above, we want to avoid out-of-equilibrium beliefs that create a discontinuity in the willingness of consumers to buy around the reservation price. To this end, we assume that for a price observation above the upper bound of the price distribution, consumers hold the same beliefs on the cost level

FIGURE 2

PRICE DISTRIBUTIONS



Parameters: $N = 3$, $s = 0.01$, $\lambda = 0.01$, $c \sim U(0, 1)$

as if they had observed the upper bound, that is, $\delta(c \mid p) = \delta(c \mid \bar{p})$ for $p > \bar{p}$.[13]

In the following, we derive several lemmas that will prove useful to examine the properties of PBERP. First, we identify an important feature of Bayesian updating in our framework that plays a key role in our main results: a consumer who has observed a price $p \in [\underline{p}(\bar{c}), \bar{p}]$ will put more probability mass on higher realization of the production cost and less mass on lower realizations of the production cost than under the prior distribution $g(c)$.

Lemma 2. For any $p \in [\underline{p}(\bar{c}), \bar{p}]$, the posterior distribution of cost levels $\delta(c \mid p)$ first-order stochastically dominates the prior distribution $g(c)$.

Lemma 2 implies that nonshoppers who have observed any $p \in [\underline{p}(\bar{c}), \bar{p}]$ expect a higher cost level $E(c \mid p)$ than if they had not observed any price, that is,

$$E(c \mid p) = \int_{\underline{c}}^{\bar{c}} \delta(c \mid p) c dc > \int_{\underline{c}}^{\bar{c}} g(c) c dc = E(c).$$

Moreover, we find that the higher the price observed in the interval $p \in [\underline{p}(\bar{c}), \bar{p}]$, the more optimistic the consumer is about the possibility of finding low prices if she continues searching. Formally,

Lemma 3. For all $p \in [\underline{p}(\bar{c}), \bar{p})$, there is a unique cost level $\hat{c}$ such that

$$\frac{\partial \delta(c \mid p)}{\partial p} \begin{cases} > 0 & \text{if} \quad c < \hat{c} \\ = 0 & \text{if} \quad c = \hat{c} \\ < 0 & \text{if} \quad c > \hat{c}. \end{cases}$$

Consequently, for all $p, p' \in [\underline{p}(\bar{c}), \bar{p})$ with $p' > p$, the posterior distribution of cost levels $\delta(c \mid p)$ FOSD $\delta(c \mid p')$.

[13] As at prices below the lower bound of the price distribution in the lowest cost scenario consumers buy regardless of their beliefs on the realized cost level, beliefs in these states are irrelevant.

Lemma 3 appears puzzling at first sight: it asserts that the larger the price that is observed, the more consumers believe that the underlying production cost is low. However, the lemma can be understood as follows: in the interval $[\underline{p}(\bar{c}), \bar{p}]$, the ratio of endogenously chosen densities $f(p \mid c')/f(p \mid c)$ as implied by (12) is increasing in p when $c' < c$. This implies that higher prices in $[\underline{p}(\bar{c}), \bar{p}]$ are *relatively* more likely under low costs than under high costs, which in turn explains why higher price observations in $p \in [\underline{p}(\bar{c}), \bar{p}]$ lead the consumer to become more optimistic that the cost realization is actually low.

We now move on to the characterization of reservation prices under production cost uncertainty. Recall that if the production cost is known to consumers, the reservation price is defined as the price at which the consumer is indifferent between buying now and continuing to search one more time. In the present context, this would translate into defining the (first-round) reservation price ρ by the indifference condition

$$v - \rho = v - s - E(p \mid \rho),$$

with $E(p \mid \rho) = \int_{\underline{c}}^{\bar{c}} E(p \mid c)\delta(c \mid \rho)\, dc$. The reservation price ρ would thus be implicitly given by

$$\rho = s + \int_{\underline{c}}^{\bar{c}} E(p \mid c)\delta(c \mid \rho)\, dc. \tag{15}$$

However, under production cost uncertainty, prices are not stationary and, due to Bayesian updating of beliefs, do depend on the search history. The arguments used above may hence not be valid, especially not if the second-round reservation price could be lower than the first-round reservation price. In that case, the consumer who continues to search after observing the first-round reservation price may optimally want to search beyond the second round and then it is not obvious that the first-round reservation price should satisfy (15). In what follows, we however prove that (15) still provides a proper characterization of the first-round reservation price, as this reservation price is strictly below the consumer's round *two* reservation price. Lemma 4 establishes this result.

Lemma 4. In any PBERP, after observing the upper bound of the price distribution in the first search round, a consumer's reservation price in the second search round satisfies $\rho_2(\rho) > \rho$.

It follows that there does *not* exist a PBERP where consumers follow a stationary reservation price search rule. This is an important difference with the model where consumers know the underlying production cost. Using equation (13) and $\rho = \bar{p}$ gives us

$$\begin{aligned}\bar{p} &= s + \int_{\underline{c}}^{\bar{c}} ((1-\alpha)c + \alpha\bar{p})\delta(c \mid \bar{p})\, dc \\ &= s + (1-\alpha)\int_{\underline{c}}^{\bar{c}} c\delta(c \mid \bar{p})\, dc + \alpha\bar{p}\int_{\underline{c}}^{\bar{c}} \delta(c \mid \bar{p})\, dc.\end{aligned}$$

As $\int_{\underline{c}}^{\bar{c}} \delta(c \mid \bar{p})\, dc = 1$ and $\int_{\underline{c}}^{\bar{c}} c\delta(c \mid \bar{p})\, dc = E(c \mid \bar{p})$, we further have that the reservation price is implicitly defined by

$$\rho = \bar{p} = E(c \mid \bar{p}) + \frac{s}{1-\alpha}. \tag{16}$$

Substituting (16) into (13), we arrive at the following result concerning the conditional expected price. The claim concerning the conditional expected minimum price is a little more difficult to arrive at, but essentially follows from the fact that

$$(1-\lambda)(E(p \mid c) - c) + \lambda(E(p_\ell \mid c) - c) = (1-\lambda)(\rho - c), \tag{17}$$

that is, the sum of the expected industry profits over shoppers and nonshoppers has to be equal to N times the expected individual profit, which equals $(1-\lambda)(\rho - c)$.

Proposition 4. In any PBERP, the conditionally expected prices $E(p \mid c)$ and $E(p_\ell \mid c)$ in a PBERP are given by

$$E(p \mid c) = c + \frac{\alpha}{1-\alpha}s + \alpha[E(c \mid \bar{p}) - c], \tag{18}$$

$$E(p_\ell \mid c) = c + \frac{(1-\lambda)}{\lambda}(s + (1-\alpha)[E(c \mid \bar{p}) - c]). \tag{19}$$

From Proposition 4 it immediately follows that the unconditionally expected prices $E(p)$ and $E(p_\ell)$ are given by

$$E(p) = E(c) + \frac{\alpha}{1-\alpha}s + \alpha[E(c \mid \bar{p}) - E(c)], \tag{20}$$

$$E(p_\ell) = E(c) + \frac{(1-\lambda)}{\lambda}(s + (1-\alpha)[E(c \mid \bar{p}) - E(c)]). \tag{21}$$

As α in (21) depends on N, the expected minimum price is now no longer independent of N as was the case under production cost certainty.

□ **Existence of PBERP.** Having established some properties any PBERP should satisfy, we now discuss the existence question. Note that, so far, we have implicitly assumed that non-shoppers would like to buy at all prices below ρ. Although this is straightforward to establish when consumers know the production cost, it is not obvious when they do not. For consumers update their beliefs about the true cost as they observe prices. In particular, after observing a price $p < \underline{p}(\overline{c})$, a consumer may suddenly think that the cost is very low and thus may decide to continue searching. Moreover, we need to verify that for all cost realizations, firms find it optimal to set the prices implicitly specified above. In particular, we need that $\underline{p}(c) > c$ for all values of c. Again, under production cost uncertainty, this condition is not automatically satisfied, as the reservation price (and thereby the upper bound of the price distribution) is independent of the cost realization.

The next proposition establishes a sufficient condition for the existence of PBERP with non-stationary reservation prices. Rothschild (1974) already observed that if consumers sample from an unknown distribution, it may happen that they prefer to buy at high prices, whereas they continue to search (and do not buy) at lower prices. He also provides a sufficient condition under which it is optimal for a consumer to actually follow a reservation price strategy. The sufficient condition he states is that the difference between any two prices should be smaller than the difference in informational content of these prices. Rothschild focuses, however, on the consumer search problem for a given (but unknown) price distribution. We show that these considerations also arise in consumer search models where firms are strategically choosing prices. Importantly, our sufficient condition is in terms of the exogenous parameters of the model.

Proposition 5. If

$$\overline{c} - \underline{c} \leq \frac{\lambda N}{\lambda N + 1 - \lambda}\left(\frac{s}{1-\alpha}\right), \tag{22}$$

then a unique PBERP exists.

The proof is based on the following considerations. We first show that observing a price p with $\underline{p}(\overline{c}) < p < \rho$, an uninformed consumer prefers to buy instead of continuing to search and buy in a later round. Note that at these prices, consumers assign positive probability to any cost realization c and by Lemma 2 become more pessimistic about the possibility of finding lower prices when continuing to search after observing such a price than before observing any price. We then examine lower price observations $p' < \underline{p}(\overline{c})$, where consumers can rule out certain high-cost realizations. For a PBERP to exist, consumers must still find it optimal to buy at such prices. This, in turn, requires that consumers do not infer from observing a price $p' < \underline{p}(\overline{c})$ that the cost is low enough so that continued search pays off. To rule out this case, we exploit the idea that a consumer who finds it optimal to buy at a price p' if he *knows* the cost realization is $\underline{c}$, that is, $p' \leq \rho^k(\underline{c})$, certainly has to find it optimal to buy in the unknown cost case at the same price. Consequently, by imposing $\underline{p}(\bar{c}) \leq \rho^k(\underline{c})$, we can guarantee that a consumer will find it optimal to buy at all price observations smaller than the reservation price ρ. This condition translates into inequality (22) characterizing the existence of a PBERP. Furthermore, inequality (22) also is sufficient to ensure that firms will set prices as specified above, that is, $\underline{p}(c) > c$ holds for all $c \in [\underline{c}, \bar{c}]$. Note that the existence condition we impose bears no relation to the discussion on out-of-equilibrium beliefs. The prices in the middle range of the distributions are in the support of the equilibrium prices of the firms. The main question is whether consumers prefer really to buy at these prices, or whether *using Bayes' rule* they believe that costs may be quite low and would prefer to continue to search. The main question is whether consumers do indeed prefer to buy at these prices, or whether, using Bayes' rule, they believe that costs are quite low and hence continue to search.

It is interesting to see when the condition in Proposition 5 holds. Clearly, this is the case when the support of the cost distribution $\bar{c} - \underline{c}$ is small or s is large. More interestingly, it is also the case when N is large enough (for any given values of the other parameters). To see this, note that both $\frac{\lambda N}{\lambda N+1-\lambda}$ and α approach one as N approaches infinity; the right-hand side of inequality (22) therefore approaches infinity as well. Finally, note that when $\bar{c} \leq \underline{c} + Ns$, a PBERP exists also for small values of λ. To arrive at this observation, we evaluate

$$\frac{\lambda N}{\lambda N + 1 - \lambda} \frac{1}{1 - \displaystyle\int_0^1 \frac{1}{1 + \frac{\lambda N}{1-\lambda} z^{N-1}} dz}$$

when λ is close to zero. Applying l'Hopital's rule, we get that in a neighborhood of $\lambda = 0$,

$$\frac{\overline{\dfrac{N}{(\lambda N + 1 - \lambda)^2}}}{\displaystyle\int_0^1 \frac{N z^{N-1}/(1-\lambda)^2}{\left(1 + \frac{\lambda N}{1-\lambda} z^{N-1}\right)^2} dz} = \frac{N}{\displaystyle\int_0^1 N z^{N-1} dz} = \frac{1}{\displaystyle\int_0^1 z^{N-1} dz} = N.$$

For λ close to 0, the right-hand side of our inequality is thus approximately equal to $\underline{c} + Ns$.

We summarize our findings regarding the existence of a PBERP in the following corollary.

Corollary 4. A PBERP exists in environments with

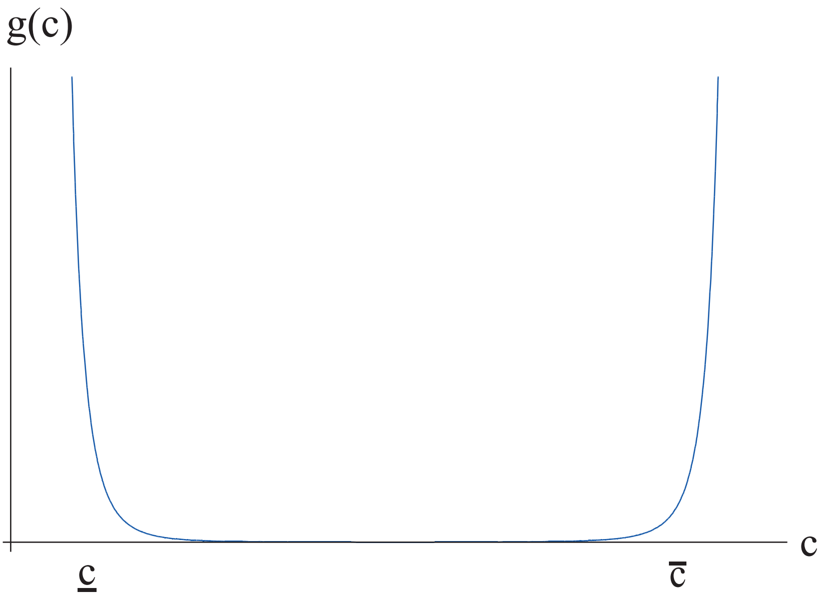
(i) a sufficiently small support of the cost distribution, $\bar{c} - \underline{c}$, and/or
(ii) sufficiently large search costs s, and/or
(iii) sufficiently many firms N, and/or
(iv) a sufficiently small fraction of shoppers λ, provided that $\bar{c} \leq \underline{c} + Ns$ holds.

For general functions $g(c)$, the condition established in Proposition 5 is "almost necessary" in the following sense. If $\underline{p}(\bar{c}) > \rho^k(\underline{c})$, then one can construct a density function of the cost parameter, $g(c)$, that is concentrated on values close to the two extremes $\underline{c}$ and $\bar{c}$ (see Figure 3) such that, after observing a price smaller than $\underline{p}(\bar{c})$, consumers suddenly consider it extremely likely that the cost is close to $\underline{c}$. In particular, if a price observation p is in the interval $(\rho^k(\underline{c}), \underline{p}(\bar{c}))$, consumers will then prefer to continue to search.

One may then wonder whether, if we restrict the prior cost distribution, existence of a PBERP may always be guaranteed. Considering a uniform distribution of production costs, the next example demonstrates that this is not the case. Figure 4 displays the net benefits of search in a duopoly market with search costs equal to $s = 0.00675$, a shopper share equal to $\lambda = 0.025$, and production costs drawn from the uniform distribution $U(0, 1)$.

FIGURE 3

A COST DISTRIBUTION CONCENTRATED AROUND THE TWO EXTREMES



As can easily be seen, for this parameter constellation no PBERP exists: the consumer does not prefer to buy for all prices below the potential reservation price defined by equation (16). Whereas for prices between $\rho = 1.0260$ and $\underline{p}(\bar{c}) = 1.0248$ the consumer strictly prefers to buy, when observing prices slightly below $\underline{p}(\bar{c})$, the net benefits of search are increasing rapidly. Indeed, the net search benefits become positive for an interval of prices a bit below $\underline{p}(\bar{c})$. The reason is that when the consumer observes prices just below $\underline{p}(\bar{c})$, she infers that the expected production cost is relatively low and so is the expected price. When she would observe even lower prices, the search benefits increase rapidly and it becomes profitable not to buy at the observed price but to search for a lower price, even though the consumer would have bought had she observed a slightly higher price.

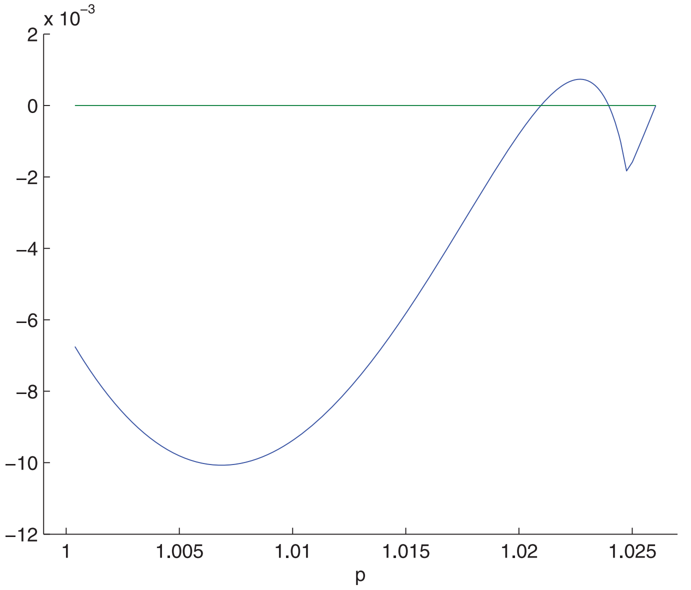
In case a PBERP does not exist, it is important to know what type of equilibrium does exist. Unfortunately, it turns out this is a very difficult issue to resolve. For example, it can be shown that allowing for reservation prices which are more general than the ones in Definition 1 does not overcome the nonexistence problem. Note that a more general reservation price strategy is a strategy according to which consumers buy if, and only if, they observe a price at or below a certain cutoff price, but where consumers are not indifferent between buying and continuing searching at the reservation price. As explained before, such reservation prices could be supported in our framework by specifying out-of-equilibrium beliefs in such a way that after observing prices higher than the upper bound of the price support (the reservation price), consumers believe that the underlying costs are very low and therefore strictly prefer to continue searching. The next result establishes that there are parameter values for which equilibria where consumers follow such strategies do not exist.

Proposition 6. If s is relatively small or $\bar{c} - \underline{c}$ is relatively large and $g(c)$ has a relatively high probability mass close to $\underline{c}$, then an equilibrium where consumers follow a reservation price strategy does not exist.

Together with the obvious fact that there cannot be a hole in the price distribution at which the nonshoppers decide to buy with probability one, Proposition 6 implies that for some parameter

FIGURE 4

NET BENEFITS OF SEARCH



Parameters: $N = 2, s = 0.00675, \lambda = 0.025, c \sim U(0, 1)$

values, consumers have to follow a mixed strategy in equilibrium. We leave it for further research to fully characterize these equilibria.

4. Welfare implications and the impact of entry

■ We are now ready to compare the two models more formally. The most important basis for this comparison is the examination of the split of welfare between uninformed consumers, informed consumers, and firms generated in the two models, and how this split of welfare depends on the number of firms in the industry. First, we assess the impact of production cost uncertainty on (i) the *ex ante* expected price, (ii) the *ex ante* expected lowest price, (iii) the *ex ante* expected price spread, and (iv) the *ex ante* expected profit of firms by comparing the two information scenarios.

Proposition 7. In the PBERP of the sequential search model under production cost uncertainty,

- the *ex ante* expected price paid by nonshoppers, $E(p)$,
- the *ex ante* expected price paid by shoppers, $E(p_\ell)$,
- the *ex ante* expected price spread, $E(\bar{p} - \underline{p})$, and
- the *ex ante* expected profit made by firms

are higher than in the known cost model. Consequently, consumer surplus is lower and producer surplus is higher.

Proposition 7 illustrates that consumer welfare is higher when consumers are informed about the firms' production cost, suggesting that policy interventions inducing observability of production cost benefit consumers. Note that this result is in contrast with one of the findings in Benabou and Gertner (1993), where more uncertainty in the market leads to increased benefit of search and thereby imposes a downward pressure on prices. The main reason for our result is the following. As uninformed consumers update their beliefs about the underlying cost level upon observing the reservation price, their cost expectation $E(c \mid \bar{p})$ in the unknown cost case is larger than the unconditional *a priori* expected cost $E(c)$ (which is relevant in the known cost case). The expected prices in equations (20) and (21) are therefore larger than the ones in (9) and (10). By the same argument, the *ex ante* expected price spread under production cost uncertainty, $E(\bar{p} - \underline{p}(c))$, is larger than the *ex ante* expected price spread if consumers know the production cost, $E(\bar{p}^k(c) - \underline{p}^k(c))$. Yet, it is important to note that the *ex ante* variation of prices is much *larger* in the known cost case than in the unknown cost case, as a quick inspection of Figures 1 and 2 reveals. This is because the upper bound of the price distribution in the known cost case shifts with the cost parameter, whereas this is not the case in the unknown cost case. For the welfare comparison in our model, this is of no relevance, as consumers are supposed to be risk neutral. However, in case consumers would be risk averse, it may be that the welfare comparison seriously depends on how consumers value the *ex ante* price variation.

The result that the price spread is smaller when consumers are uninformed about the underlying production cost can also be obtained in a much simpler setting, where one firm is prominent in the sense that all consumers first visit that firm before they go elsewhere (see, e.g., Armstrong, Vickers, and Zhou, 2009b). In the case of prominence with known production costs, there is an equilibrium where the prominent firm sets a price equal to $c + s$, and all other firms set a price equal to c. It is clear that in this equilibrium, all prices move up and down with the underlying production cost so that the price spread is insensitive to the production cost. When consumers are not informed about the production cost, there is an equilibrium for $\bar{c} < E(c) + s$, where the prominent firm sets a price $\hat{p}$, with $\bar{c} < \hat{p} \leq E(c) + s$ for all cost realizations, and all other firms choose a price equal to c. In this equilibrium, the price spread is smaller when production costs are high. Thus, the prominence model would predict the same effect of information asymmetry on the price spread.

Further, we have the following.

Proposition 8. The conditionally expected profits of firms are decreasing in the cost level c when consumers are uninformed about the cost realization, whereas these profits are independent of c when consumers are perfectly informed. In particular, conditionally expected profits under production cost uncertainty are higher (lower) for low- (high-) cost realizations compared to the case where consumers know the production cost.

In light of Proposition 8, one may wonder whether firms would have an incentive to disclose their production cost (if they had the possibility to do so truthfully) if the cost realization is high. To investigate this question, consider a game in which, in a first stage, firms have the possibility to disclose their information after observing the cost realization. If firms do so, then the subsequent game is the traditional one analyzed in Section 2. Otherwise, the subsequent game is the one analyzed in Section 3. Proposition 8 implies that, when the cost of disclosure is negligible, firms always have an incentive to disclose information if cost is sufficiently high, as this will lead to higher profits. As this argument holds true for any highest cost that "pools" with lower costs, there cannot be a threshold level of cost such that it is optimal to disclose only if the cost is above this level. The unraveling argument (see, e.g., Grossman and Hart, 1980) implies that there will be full disclosure. Does this mean that the model of Section 3 where consumers do not know the relevant production cost is self-defeating? We think it is not. First, the above argument relies on the fact that firms can credibly and truthfully communicate their production cost. As firms always have an incentive to pretend cost is high, truthful communication may be difficult in practice. Second, disclosure is often costly, especially if one wants to reach all or many consumers. If disclosure costs are sufficiently high, firms will not want to disclose. Third, Proposition 7 shows that on average, firms are better off when consumers are uninformed. If firms can commit not to disclose in all cost states, then they will certainly do so.

Finally, we study the effect of changes in the number of firms on the welfare evaluation of the different groups in the market and especially how the differences between the two models depend on N. For the known cost case, these results are already summarized at the end of Section 2. It is difficult to evaluate the relevant expressions for the unknown cost case, as it is difficult to evaluate the impact of N on the reservation price. We are able, however, to ascertain the following two results. First, when the number of firms in the industry is large, the difference between the two models becomes negligibly small, that is, the known cost model provides a good approximation of the more complicated model where consumers are unaware of the underlying production costs.[14] This is the content of the next proposition.

Proposition 9. For all $\epsilon > 0$, there exist N and v large enough such that $\rho(N) < v$ and $\rho^k(c; N) < v$ for all c, and

$$(\rho(N) - E(\rho^k(c; N))) < \epsilon,$$

$$(E(p; N) - E^k(p; N)) < \epsilon,$$

$$(E(p_\ell; N) - E^k(p_\ell; N)) < \epsilon.$$

The proof of this proposition exploits the fact that, in the cost uncertainty case, when N is very large and v is large enough such that $\rho(N) < v$, the parts of the price distributions just below the upper bound $\bar{p}$ are almost identical for different cost realizations. A consumer who observes the upper bound $\bar{p}$ of the price distribution therefore gains little additional information concerning the production cost realization, and therefore her cost estimate $E(c \mid \bar{p})$ will be close to her unconditional estimate $E(c)$. Thus, for large N, the reservation price in the unknown cost case is very close to the *ex ante* expected reservation price in the known cost case. A comparison of (1) and (11) reveals that the upper bound is the main difference between the price distributions of the two cases, and hence the results in Proposition 9 follow. Once N is so large that v becomes binding for all values of c in the known cost case, the comparison becomes uninteresting, as the distributions for both cases are identical for each value of c. It is striking that the difference between the two models becomes negligible in competitive markets where N is large. This is in contrast to results in Janssen and Roy (2010) where, in a context where prices signal quality, the difference between the complete and incomplete information models remains for large N.

A second result is that the effects of increased competition in the unknown cost case may be nonmonotonic. As can be verified from Figure 5, there are numerical examples where the expected price, $E(p)$, is first decreasing in the number of firms and starts to increase only above a certain number of firms in the market.[15] The same is true for the reservation price (although not depicted). The fact that the expected price $E(p)$ may be decreasing in N for small N is in sharp contrast with the comparative statics results in other search models in this direction, where expected prices are typically increasing in N. As α is increasing in N it is clear from (20) that this result is due to the fact that $E(c \mid \bar{p})$ is decreasing in N. Thus, the consumers' updating of their beliefs about the underlying production cost may have very important qualitative implications for the behavior of prices in relatively concentrated markets. The profits in our industry are given by $(1 - \lambda)(\rho - c)$. Thus, the industry's total profits reach a minimum in this example when there are five firms in the market. As consumer surplus is just the reverse of industry total profits in this model, total consumer surplus reaches a maximum for five firms, as Figure 6 illustrates. This is in contrast to the known cost case (also depicted in Figure 6) where it can be easily verified that consumer welfare is decreasing in the number of firms. The nonmonotonicity encountered in the unknown cost case points to the fact that it is extremely difficult to obtain analytic comparative statics results for small N.

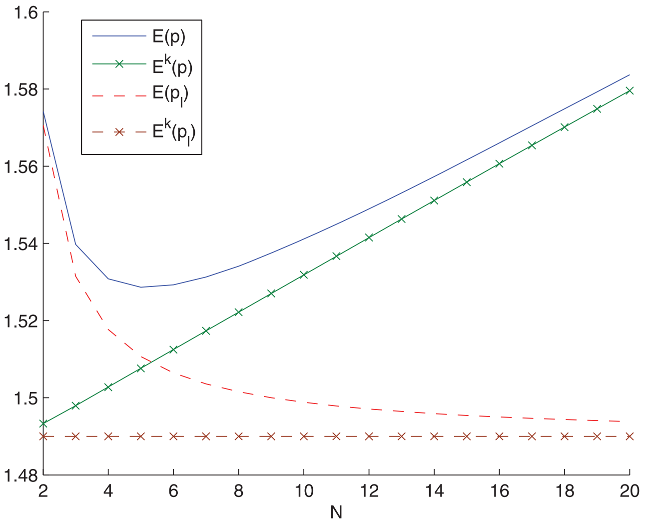
Finally, note that inspection of (9) and (10) reveals that in the known production cost benchmark, the expected prices $E^k(p)$ and $E^k(p_\ell)$ are both increasing in the search cost s and decreasing in the fraction of informed consumers λ. Numerical evidence points to the fact that

[14] We already know from Section 3 that the existence condition for PBERP is satisfied for large N.

[15] Notice that the relationship between $E(p)$ and N is nonmonotonic also in a newspaper search framework, as shown by Chandra and Tappata (2008).

FIGURE 5

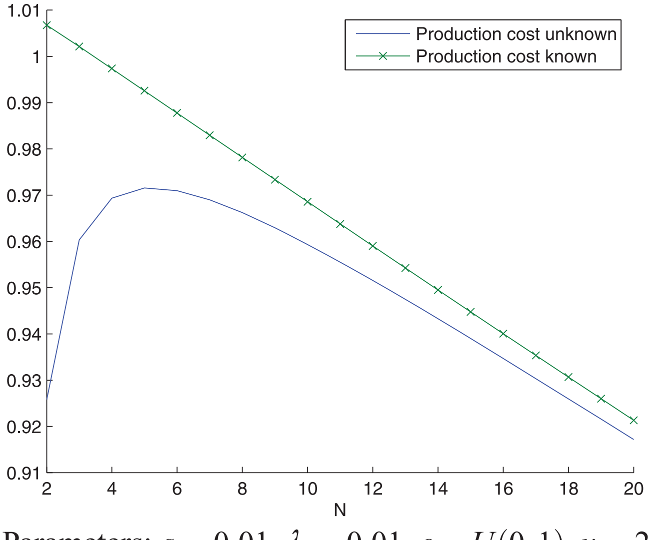
THE EFFECTS OF ENTRY ON EXPECTED (MINIMUM) PRICES



Parameters: $s = 0.01, \lambda = 0.01, c \sim U(0, 1)$

FIGURE 6

THE EFFECTS OF ENTRY ON CONSUMER WELFARE



Parameters: $s = 0.01, \lambda = 0.01, c \sim U(0, 1), v = 2$

these properties carry over to the case when the production cost is unknown. For this latter case, comparative statics results are, however, hard to obtain analytically; the expected prices in (20) and (21) depend on the expected cost after observing the reservation price $\bar{p}$, which in turn is only an implicitly defined function of s and λ as described by (16).

5. Discussion

■ There are many interesting aspects that could be explored further in more detail. From an empirical perspective, it would be nice to know how well our theory fares in explaining some aspects of retail gasoline or mortgage markets. We provide a first step in this direction in a web appendix to this article where we test certain implications of the sequential search framework on retail gasoline price data for Vienna, Austria. Several interesting observations can be made. First, we find that random pricing seems to capture an important part of the observed price variations, which supports the mixed-strategy equilibrium present in the search theoretic framework employed. Moreover, the data support a further central implication of the cost uncertainty model, namely that the spread between the lowest and highest prices in the market observed on a particular day is decreasing in the underlying cost. This empirical finding is also reported by Chandra and Tappata (2008), who provide a detailed econometric analysis of the retail gasoline market in four major U.S. states. Finally, we also discuss that the well-known empirical phenomenon of *rockets and feathers* in gasoline prices may arise in an extension of our model where consumers have systematically biased beliefs about the underlying production costs. Our preliminary empirical analysis delivers several promising findings, yet an important caveat is that it is based on a simple (static) model and uses simple econometric techniques. A rigorous examination of whether the sequential search model with production cost uncertainty fits retail gasoline markets would require the use of more elaborate techniques. Providing such an examination is beyond the scope of the present article and is therefore left for future research.

From a theoretical perspective, the present article does not answer the question of which type of equilibria do exist in the case where a PBERP does not exist. It would clearly be a major theoretical innovation to characterize equilibria that do not have a reservation price property. Such equilibria are, however, inherently difficult to characterize, as they involve some consumers searching more than once. A further line of research that would be interesting from both a theoretical and an empirical angle is to extend the model discussed here by introducing idiosyncratic shocks, and study the signal extraction problem in a real sequential search framework. One drawback of the present model is that there is no real search in the sense that no consumer with positive search cost searches beyond the first firm. In a recent contribution, Goldmanis et al. (2010) present a relatively simple model that overcomes this problem, and it would be interesting to investigate the impact of information asymmetry in that context.

6. Conclusion

■ In this article, we have analyzed a sequential consumer search model with uncertainty concerning the underlying production costs of firms. We have characterized a perfect Bayesian equilibrium of this model satisfying a reservation price property. In this equilibrium, firms sample prices from an optimal distribution and, in each search round, consumers buy if they observe a price below their current reservation price and search for a better deal otherwise. Unlike in the standard consumer search model, the consumers' reservation prices under production cost uncertainty are not stationary but differ across search rounds. This is due to consumers updating their beliefs about the production cost level when observing prices. We have further shown that an equilibrium with the properties just outlined exists for a relevant range of parameter values, but there are cases where a reservation price equilibrium does not exist.

Comparing our environment to the standard search model, we have shown that both the average price and the expected lowest price in the market are higher, and consumer welfare is accordingly lower, when consumers are uncertain about production costs. This difference between the two models becomes, however, negligibly small when the number of firms in the industry is large. When the number of firms is relatively small, the unknown cost model also may give rise to a nonmonotonic relationship between expected consumer welfare and the number of firms. In particular, consumer surplus is first increasing in the number of firms and then decreasing, so that, from a consumer perspective, there is an optimal number of firms in the industry. We have furthermore demonstrated that the average profit margin charged by firms and the extent of equilibrium price dispersion are decreasing in the cost level, which is not the case in the known cost model.

Appendix

Proof of Lemma 1. The expected price $E^k(p \mid c)$ is given by

$$E^k(p \mid c) = \int_{\underline{p}}^{\rho^k(c)} p f^k(p \mid c)\, dp = c + \int_0^1 (p - c)\, dF^k(p \mid c). \tag{A1}$$

Introducing

$$z = 1 - F^k(p \mid c) = \left(\frac{1-\lambda}{\lambda N} \frac{\rho^k(c) - c}{p - c} \right)^{\frac{1}{N-1}},$$

we have that

$$p - c = (\rho^k(c) - c) \frac{1}{1 + \dfrac{\lambda N}{1-\lambda} z^{N-1}}.$$

This allows us to rewrite expression (A1) as

$$E^k(p \mid c) = (1-\alpha)c + \alpha \rho^k(c),$$

with $\alpha = \int_0^1 \frac{1}{1+\frac{\lambda N}{1-\lambda} z^{N-1}} dz \in [0, 1]$. Substituting (3) allows us then to write the expected price as in the statement of the lemma.

Proof of Proposition 2. Firms expect to make a profit of $E^k(p \mid c) - c$ on the $(1 - \lambda)$ nonshoppers and a profit of $E^k(p_\ell \mid c) - c$ on the λ shoppers. Moreover, we know that in a mixed-strategy equilibrium, all prices in the support of $F^k(p \mid c)$ have to yield the same profit and that this profit has to equal the profit made when charging the reservation price. Hence, we have that

$$(1-\lambda)(E^k(p \mid c) - c) + \lambda(E^k(p_\ell \mid c) - c) = (1-\lambda)(\rho^k(c) - c). \tag{A2}$$

Substituting the reservation price defined in (5) and the expected price, the expression for the expected minimum price follows as given in the statement of the proposition. As c, λ, and s are exogenous parameters, it follows that $E^k(p_\ell \mid c)$ is independent of N.

Proof of Lemma 2. Recall that the probability density function of the production cost c conditional on a price observation p is given by

$$\delta(c \mid p) = \frac{g(c) f(p \mid c)}{\int_{\underline{c}}^{\bar{c}} g(c') f(p \mid c')\, dc'} = \frac{g(c)}{\int_{\underline{c}}^{\bar{c}} g(c') \dfrac{f(p \mid c')}{f(p \mid c)} dc'} \equiv \frac{g(c)}{y(c; p)}.$$

Note that if $y(c; p)$ is monotonically decreasing in c, then $\delta(c \mid p)$ first-order stochastically dominates $g(c)$. In the following, we show that $y(c; p)$ is indeed monotonically decreasing in c for price observations in the interval $[\underline{p}(\bar{c}), \bar{p}]$. To this end, note that for $p \in [\underline{p}(\bar{c}), \bar{p}]$, the function $y(c; p)$ is given by

$$\begin{aligned} y(c; p) &= \int_{\underline{c}}^{\bar{c}} g(c') \frac{\bar{p} - c'}{\bar{p} - c} \left(\frac{p - c}{p - c'} \right)^{\frac{N}{N-1}} dc' \\ &= \int_{\underline{c}}^{\bar{c}} g(c') \frac{\bar{p} - c'}{(p - c')^{\frac{N}{N-1}}} \frac{(p - c)^{\frac{N}{N-1}}}{\bar{p} - c} dc'. \end{aligned}$$

Its derivative with respect to c is hence given by

$$\begin{aligned} \frac{\partial y(c; p)}{\partial c} &= \int_{\underline{c}}^{\bar{c}} g(c') \frac{\bar{p} - c'}{(p - c')^{\frac{N}{N-1}}} \left[\frac{-\dfrac{N}{N-1}(p - c)^{\frac{N}{N-1}-1}(\bar{p} - c) + (p - c)^{\frac{N}{N-1}}}{(\bar{p} - c)^2} \right] dc' \\ &= \int_{\underline{c}}^{\bar{c}} g(c') \frac{\bar{p} - c'}{(p - c')^{\frac{N}{N-1}}} \frac{(p - c)^{\frac{N}{N-1}}}{(\bar{p} - c)^2} \left[1 - \frac{N}{N-1} \frac{(\bar{p} - c)}{(p - c)} \right] dc' < 0 \end{aligned}$$

as $[1 - \frac{N}{N-1} \frac{(\bar{p}-c)}{(p-c)}] < 0$.

Proof of Lemma 3. Taking the derivative of $\delta(c \mid p)$ with respect to p yields

$$\frac{\partial \delta(c \mid p)}{\partial p} = -\frac{[\delta(c \mid p)]^2}{g(c)} \int_{\underline{c}}^{\bar{c}} g(c') \frac{\partial \frac{f(p \mid c')}{f(p \mid c)}}{\partial p} dc'.$$

Restricting attention to prices in the interval $[\underline{p}(\overline{c}), \bar{p}]$, we have that

$$\frac{\partial \frac{f(p \mid c')}{f(p \mid c)}}{\partial p} = \frac{\bar{p} - c'}{\bar{p} - c} \frac{N}{N-1} \frac{c - c'}{(p - c')^2} \left(\frac{p - c}{p - c'} \right)^{\frac{1}{N-1}},$$

such that we obtain

$$\begin{aligned} \frac{\partial \delta(c \mid p)}{\partial p} &= -\frac{[\delta(c \mid p)]^2}{g(c)} \int_{\underline{c}}^{\bar{c}} g(c') \frac{\bar{p} - c'}{\bar{p} - c} \frac{N}{N-1} \frac{c - c'}{(p - c')^2} \left(\frac{p - c}{p - c'} \right)^{\frac{1}{N-1}} dc' \\ &= -\frac{[\delta(c \mid p)]^2}{g(c)} \frac{N}{N-1} \frac{(p - c)^{\frac{1}{N-1}}}{\bar{p} - c} \int_{\underline{c}}^{\bar{c}} g(c') \frac{(c - c')(\bar{p} - c')}{(p - c')^{\frac{2N-1}{N-1}}} dc'. \end{aligned}$$

Because $\delta(c \mid p)$ is a density function for every p, we have that $\int_{\underline{c}}^{\bar{c}} \delta(c \mid p)\, dc = 1$ for all p and, consequently,

$$\frac{\partial [\int_{\underline{c}}^{\bar{c}} \delta(c \mid p)\, dc]}{\partial p} = \int_{\underline{c}}^{\bar{c}} \frac{\partial \delta(c \mid p)}{\partial p} dc = 0. \tag{A3}$$

This in turn implies that $\frac{\partial \delta(c \mid p)}{\partial p}$ can neither be positive nor negative for all values of c.

In particular, because $\delta(c \mid p)$ is continuously differentiable, it follows that for all prices $p \in [\underline{p}(\overline{c}), \bar{p}]$ there exists (at least) one cost level $\hat{c}$ such that

$$\frac{\partial \delta(\hat{c} \mid p)}{\partial p} = 0.$$

Consequently, at this cost level,

$$\int_{\underline{c}}^{\bar{c}} g(c') \frac{(\hat{c} - c')(\bar{p} - c')}{(p - c')^{\frac{2N-1}{N-1}}} dc' = 0.$$

For notational simplicity, let us introduce the function

$$\phi(p, c) = \int_{\underline{c}}^{\bar{c}} g(c') \frac{(c - c')(\bar{p} - c')}{(p - c')^{\frac{2N-1}{N-1}}} dc',$$

such that the above statement boils down to

$$\phi(p, \hat{c}) = 0.$$

To prove the lemma, it basically remains to show that (i) there exists only one unique $\hat{c}$ that satisfies $\phi(p, \hat{c}) = 0$; and (ii) $\phi(p, c) < 0$ for $c < \hat{c}$ and $\phi(p, c) > 0$ for $c > \hat{c}$. The last part is due to the fact that $\partial \delta(c \mid p)/\partial p$ and $\phi(p, c)$ have opposing signs.

Assume that there exist more than one value of $\hat{c}$ that satisfy $\phi(p, \hat{c}) = 0$. In such a case, at least one of these cost levels would have to satisfy

$$\left. \frac{\partial \phi(p, c)}{\partial c} \right|_{c=\hat{c}} \leq 0.$$

This, however, cannot be true, as

$$\frac{\partial \phi(p, c)}{\partial c} = \int_{\underline{c}}^{\bar{c}} g(c') \frac{(\bar{p} - c')}{(p - c')^{\frac{2N-1}{N-1}}} dc' > 0.$$

Consequently, there can only be a unique cost level $\hat{c}$ that satisfies $\phi(p, \hat{c}) = 0$, and $\phi(p, c) < 0$ for $c < \hat{c}$ and $\phi(p, c) > 0$ for $c > \hat{c}$ obtain trivially. Thus,

$$\frac{\partial \delta(c \mid p)}{\partial p} \begin{cases} > 0 & \text{if} \quad c < \hat{c} \\ = 0 & \text{if} \quad c = \hat{c} \\ < 0 & \text{if} \quad c > \hat{c}. \end{cases}$$

For $p' > p$ the posterior $\delta(c \mid p')$ puts more weight on low values of c and less weight on high values of c as compared to $\delta(c \mid p)$. Put differently, $\delta(c \mid p)$ first-order stochastically dominates $\delta(c \mid p')$.

Proof of Lemma 4. Define $\rho_2(\overline{p})$ as the (hypothetical) price at which a consumer in round two would be indifferent between buying at that price and continuing to search after having observed $\overline{p}$ in the first search round. There are two cases to consider: (i) $\rho_2(\overline{p}) < \rho = \overline{p}$, and (ii) $\rho_2(\overline{p}) \geq \rho_1 = \rho = \overline{p}$. In the remainder of this proof, we argue that case (i) leads to an inconsistency, whereas case (ii) leads to a consistent procedure with ρ being indeed defined by

$$v - \rho = v - E(p \mid \rho) - s.$$

Case (i). Assume that $\rho_2(\overline{p}) < \rho$, and let us introduce for notational simplicity $\widehat{p} = \rho_2(\overline{p})$. Note that if a consumer observes $\widehat{p}$ in the first search round, she would prefer to buy rather than continue to search. Formally, we have that $v - \widehat{p} \geq \pi^s(\widehat{p}; N-1)$, where we denote by $\pi^s(\widehat{p}; N-1)$ the payoff of a consumer who has observed $\widehat{p}$ in the first search round and continues to search optimally given that there are potentially still $N-1$ other firms to sample.

Next consider the hypothetical situation that there are in total $N+1$ firms in the market, but all of these set their prices according to the equilibrium price distribution in the market with N firms. Assume that a consumer has already observed two prices, $\widehat{p}$ and $\overline{p}$. If this consumer continues to search optimally after this hypothetical situation, her payoff is given by $\pi^s(\widehat{p}, \overline{p}; N-1)$. Furthermore, denote by $\pi^s(\widehat{p}, \overline{p}; N-2)$ the payoff if the consumer continues to search optimally after having observed $\widehat{p}$ and $\overline{p}$ and there are potentially only $N-2$ other firms to sample, as is true in our original market with N firms. Note further that, as $\rho_2(\overline{p}) = \widehat{p} < \rho$, we have that $v - \widehat{p} = \pi^s(\widehat{p}, \overline{p}; N-2)$.

At this stage, note that the benefits of search as defined above must satisfy

$$\pi^s(\widehat{p}; N-1) > \pi^s(\widehat{p}, \overline{p}; N-1) \geq \pi^s(\widehat{p}, \overline{p}; N-2).$$

The second inequality is obvious: a consumer can never get a higher payoff of searching if she has the same price observations in her pocket and she has fewer search alternatives left. Regarding the first inequality, note that $\delta(c \mid \widehat{p}, \overline{p})$ is the posterior distribution which is obtained by updating the belief $\delta(c \mid \widehat{p})$ using the price observation $\bar{p}$. As $\overline{p} > \underline{p}(\overline{c})$, we can apply (a modified version of) Lemma 2, taking $\delta(c \mid \widehat{p})$ instead of $g(c)$ as prior belief distribution, to obtain that $\delta(c \mid \widehat{p}, \overline{p})$ FOSD $\delta(c \mid \widehat{p})$. Hence, consumers become more pessimistic about the underlying cost level having observed the price $\bar{p}$. Further, recall that we have that $F(p \mid c)$ FOSD $F(p \mid c')$ whenever $c > c'$, such that consumers expect higher prices when they expect higher costs. Consequently, it is strictly less attractive to continue searching after having observed both $\widehat{p}$ and $\overline{p}$ compared to a situation where only $\widehat{p}$ had been observed. Thus, we arrive at an inconsistency, because

$$v - \widehat{p} \geq \pi^s(\widehat{p}; N-1) > \pi^s(\widehat{p}, \overline{p}; N-2) = v - \widehat{p}.$$

Case (ii). Now assume that $\rho_2(\overline{p}) \geq \rho = \overline{p}$. A consumer who has observed the upper bound ρ and continues to search will now buy in the next period at a price below ρ with probability one. Thus, a consumer who has observed the upper bound $\bar{p}$ and continues to search optimally will get a payoff of $\pi^s(\bar{p}; N-1) = v - E(p \mid \overline{p}) - s$. It follows that $v - \rho = v - E(p \mid \rho) - s$.

Proof of Proposition 5. For a PBERP to exist, a consumer must necessarily find it optimal to buy if she observes a price lower than the reservation price ρ, and firms must not make negative profits when choosing prices from the PBERP price distribution. In the following, we provide restrictions on the model's parameters such that these two conditions are satisfied. To this end, we proceed in two steps. In part (i), we first show that at prices p with $\underline{p}(\overline{c}) < p < \rho$, an uninformed consumer prefers to buy instead of continuing to search and buy necessarily in the next round. Later on, we allow for more general search behaviors. In part (ii), we provide conditions such that a consumer also finds it optimal to buy when she observes a price in $[\underline{p}(\underline{c}), \underline{p}(\overline{c})]$. Finally, we show that these conditions already guarantee that firms make positive profits in equilibrium.

Part (i). Consider for the time being the following hypothetical scenario. A consumer observes the price p' and must decide between buying at p' immediately and visiting one more firm, provided that she must *necessarily* buy after having obtained this additional price quote. In such a situation, the consumer is indifferent between buying and searching if she observes the reservation price, that is, if $p' = \rho$, as this price equates her net benefits of search to zero. Now assume that the consumer has observed a lower price p' in $(\underline{p}(\overline{c}), \rho)$. By Lemma 3, she is now more pessimistic about the cost and, thus, the possibility of finding lower prices compared to if she had observed ρ. Consequently, she must find it optimal to buy rather than search one more firm.

So far, we have argued that for prices p such that $\underline{p}(\overline{c}) < p < \rho$, the uninformed consumer prefers to buy instead of continuing to search and buy necessarily in the next round. We now consider more general search behaviors. In particular, it may easily be the case that the consumer, after continuing to search, may not want to buy after observing the next price, but instead prefers to continue searching at least one more time. We will now show that this cannot be optimal either if consumers observe prices p with $\underline{p}(\overline{c}) < p \leq \rho$.

If a consumer has observed t prices with $p' = \min(p_1, \ldots, p_t) \geq \underline{p}(\overline{c})$, then her payoff from searching is given by

$$v - s - F(p' \mid p_1, \ldots, p_t) \cdot \int_{\underline{c}}^{\bar{c}} E(p \mid p < p', c)\delta(c \mid p_1, \ldots, p_t)\, dp\, dc - (1 - F(p' \mid p_1, \ldots, p_t))p',$$

where $F(p' \mid p_1, \ldots, p_t) = \int_{\underline{c}}^{\bar{c}} F(p' \mid c)\delta(c \mid p_1, \ldots, p_t)\, dc$ is the subjective probability of finding a price lower than p' of a consumer who has observed prices $p_1, \ldots, p_t$. By Bayes rule, we have that

$$\delta(c \mid p_1, \ldots, p_t) = \frac{\delta(c \mid p_1, \ldots, p_{t-1}) f(p \mid c)}{\displaystyle\int_{\underline{c}}^{\bar{c}} \delta(c' \mid p_1, \ldots, p_{t-1}) f(p \mid c')\, dc'}. \tag{A4}$$

In this sense, $\delta(c \mid p_1, \ldots, p_t)$ is the distribution obtained from updating $\delta(c \mid p_1, \ldots, p_{t-1})$ after the price observation p_t. We can again apply (a modified version of) Lemma 2, taking $\delta(c \mid p_1, \ldots, p_{t-1})$ instead of $g(c)$ as prior distribution, to obtain that if $p_t \geq \underline{p}(\bar{c})$, then $\delta(c \mid p_1, \ldots, p_t)$ first-order stochastically dominates $\delta(c \mid p_1, \ldots, p_{t-1})$. By induction, $\delta(c \mid p_1, \ldots, p_t)$ FOSD $\delta(c \mid p')$. Thus, as $E(p \mid c)$ is increasing in c and $F(p' \mid c)$ is decreasing in c, we have that

$$\begin{aligned} & v - s - F(p' \mid p_1, \ldots, p_t) \cdot \int_{\underline{c}}^{\bar{c}} E(p \mid p < p', c)\delta(c \mid p_1, \ldots, p_t)\, dc - (1 - F(p' \mid p_1, \ldots, p_t))p' \\ & < v - s - F(p' \mid p_1, \ldots, p_t) \cdot \int_{\underline{c}}^{\bar{c}} E(p \mid p < p', c)\delta(c \mid p')\, dc - (1 - F(p' \mid p_1, \ldots, p_t))p' \\ & < v - s - F(p') \int_{\underline{c}}^{\bar{c}} E(p \mid p < p', c)\delta(c \mid p')\, dc - (1 - F(p'))p', \end{aligned}$$

where the last inequality follows from the fact that $E(p \mid p < p', c) < p'$ and that $F(p' \mid p_1, \ldots, p_t) < F(p')$. Thus, as

$$v - p' > v - s - \int_{\underline{c}}^{\bar{c}} E(p \mid p < p', c)\delta(c \mid p')\, dc - (1 - F(p'))p',$$

it follows that if $p' = \min(p_1, \ldots, p_t) \geq \underline{p}(\bar{c})$,

$$v - p' > v - s - F(p' \mid p_1, \ldots, p_t) \cdot \int_{\underline{c}}^{\bar{c}} E(p \mid p < p', c)\delta(c \mid p_1, \ldots, p_t)\, dc - (1 - F(p' \mid p_1, \ldots, p_t))p'.$$

Consequently, the consumer does not want to continue searching and then buy immediately in the next round after having observed t prices with $\underline{p}(\bar{c}) \leq p' = \min(p_1, \ldots, p_t) \leq \rho$ for any t.

Let us finally consider the following, alternative search strategy: the consumer decides to continue searching in round t and, after having observed one more price, does not buy at any of the prices observed up to that moment if the newly observed price is larger than $\underline{p}(\bar{c})$. It is easy to see that, if the consumer searches in this way and then buys at a later moment, her payoff evaluated from period t onward is smaller than

$$v - s - F(\tilde{p} \mid p_1, \ldots, p_{t+1}) \int_{\underline{c}}^{\bar{c}} E(p \mid p < \tilde{p}, c)\delta(c \mid p_1, \ldots, p_{t+1})\, dc - (1 - F(\tilde{p} \mid p_1, \ldots, p_{t+1}))\tilde{p}$$

for some $\tilde{p} \geq \underline{p}(\bar{c})$. Using the argument given above, it follows that this is not optimal as well. By induction, it follows that it is also not optimal to wait more than one period. Taken together, the arguments we have used so far show that a consumer will indeed buy if she observes a price in the interval $(\underline{p}(\bar{c}), \rho)$.

Part (ii). Let us now consider consumer behavior if a price below $\underline{p}(\bar{c})$ is observed. By assumption, we have $\underline{p}(\bar{c}) \leq \rho^k(\underline{c})$, so that all these prices are below the reservation price in a model where the consumers (i) are informed about the cost realization and (ii) know this realization is equal to $\underline{c}$. We will argue that consumers should always buy at such prices.

As we consider prices $p' \leq \rho^k(\underline{c})$, it easily follows that

$$\begin{aligned} v - p' &\geq v - \rho^k(\underline{c}) = v - s - E^k(p \mid \underline{c}) \\ &> v - s - F^k(p' \mid \underline{c})E^k(p \mid p < p', \underline{c}) - (1 - F^k(p' \mid \underline{c}))p' \\ &\geq v - s - F(p') \int_{\underline{c}}^{\bar{c}} E(p \mid p < p', c)\delta(c \mid p')\, dc - (1 - F(p'))p'. \end{aligned}$$

Thus, the consumer prefers to buy at prices $p' \leq \rho^k(\underline{c})$ instead of continuing to search and then buy immediately. Furthermore, arguments similar to the ones used in part (i) of this proof can be applied to establish that it does not pay off to continue searching and then not to buy immediately after observing some price. Thus, under our assumption $\underline{p}(\bar{c}) \leq \rho^k(\underline{c})$, a reservation price strategy is optimal for the consumer.

To complete the existence part of the proof, we need to rewrite the condition $\underline{p}(\bar{c}) \leq \rho^k(\underline{c})$ in terms of the model's exogenous parameters and examine the profits made by firms given the consumers' behavior. Note first that we have

$$\begin{aligned} \underline{p}(\bar{c}) &= \frac{\lambda N}{\lambda N + 1 - \lambda}\bar{c} + \frac{1 - \lambda}{\lambda N + 1 - \lambda}\left(\frac{s}{1 - \alpha} + \int_{\underline{c}}^{\bar{c}} c\delta(c \mid \bar{p})\, dc\right) \\ &< \bar{c} + \frac{1 - \lambda}{\lambda N + 1 - \lambda}\left(\frac{s}{1 - \alpha}\right), \\ \rho^k(\underline{c}) &= \underline{c} + \frac{s}{1 - \alpha} \end{aligned}$$

such that certainly $\underline{p}(\bar{c}) \le \rho^k(\underline{c})$ if

$$\bar{c} \le \underline{c} + \frac{\lambda N}{\lambda N + 1 - \lambda}\left(\frac{s}{1-\alpha}\right).$$

To check that firms' profits are positive, it is sufficient to check that for all c, $\underline{p}(c) > c$, this $\underline{p}(c) = \frac{\lambda N}{\lambda N+1-\lambda}c + \frac{1-\lambda}{\lambda N+1-\lambda}\bar{p}$, this is the case if $\bar{p} > \bar{c}$. As $\bar{p} = \frac{s}{1-\alpha} + \int_{\underline{c}}^{\bar{c}} c\delta(c \mid \bar{p})\,dc \ge \frac{s}{1-\alpha} + \underline{c}$, this inequality is automatically satisfied if $\bar{c} \le \underline{c} + \frac{\lambda N}{\lambda N+1-\lambda}(\frac{s}{1-\alpha})$. Uniqueness of the equilibrium is proved by showing that the reservation price is uniquely defined by

$$\overline{p} = \frac{s}{1-\alpha} + \int_{\underline{c}}^{\bar{c}} c\delta(c \mid \bar{p})\,dc.$$

To show that this equation has a unique solution, we show that the right-hand side is decreasing in $\overline{p}$, which together with the fact that the left-hand side is increasing in $\overline{p}$, suffices. For this purpose, we have to evaluate the sign of

$$\frac{\partial \delta(c \mid \overline{p})}{\partial \overline{p}} = -\frac{[\delta(c \mid \overline{p})]^2}{g(c)} \int_{\underline{c}}^{\bar{c}} g(c') \frac{\partial \dfrac{f(\overline{p} \mid c')}{f(\overline{p} \mid c)}}{\partial p} dc'.$$

We have that

$$\frac{f(\overline{p} \mid c')}{f(\overline{p} \mid c)} = \left(\frac{\overline{p} - c}{\overline{p} - c'}\right)^{\frac{1}{N-1}}$$

and therefore

$$\frac{\partial \dfrac{f(\overline{p} \mid c')}{f(\overline{p} \mid c)}}{\partial \overline{p}} = \frac{1}{N-1} \frac{c - c'}{(\overline{p} - c')^2} \left(\frac{\overline{p} - c}{\overline{p} - c'}\right)^{\frac{2-N}{N-1}}.$$

It follows that if $\frac{\partial \frac{f(\overline{p} \mid c')}{f(\overline{p} \mid c)}}{\partial \overline{p}} > 0$ for some $\widetilde{c}$ it is positive for all $c > \widetilde{c}$. Moreover, as $\delta(c \mid \overline{p})$ is a density function, we have $\int_{\underline{c}}^{\bar{c}} \delta(c \mid \overline{p})\,dc = 1$, and consequently we have

$$\frac{\partial \left[\int_{\underline{c}}^{\bar{c}} \delta(c \mid \overline{p})\,dc\right]}{\partial \overline{p}} = \int_{\underline{c}}^{\bar{c}} \frac{\partial \delta(c \mid \overline{p})}{\partial \overline{p}} dc = 0. \tag{A5}$$

This in turn implies that

$$\frac{\partial \delta(c \mid \overline{p})}{\partial \overline{p}} \begin{cases} > 0 & \text{if} \quad c < \hat{c} \\ = 0 & \text{if} \quad c = \hat{c} \\ < 0 & \text{if} \quad c > \hat{c}. \end{cases}$$

Thus, the posterior $\delta(c \mid \overline{p})$ puts relatively more weight on low values of c the larger the values $\overline{p}$. Thus, the right-hand side is decreasing in $\overline{p}$.

Proof of Proposition 6. In a reservation price equilibrium, there is some ρ' such that consumers buy in the first round of search if, and only if, $p \le \rho'$. It is clear that $\rho' \ge \rho^k(\underline{c})$, as otherwise consumers will buy even if they observe a price (slightly) above ρ'. Also, $\rho' \le \rho$. This latter claim follows from the following observations. First, for all $\overline{p} \le \rho^k(\underline{c}), v - \overline{p} < v - s - E(p \mid \overline{p})$, that is, if the upper bound of the price distribution is relatively low, consumers would prefer to buy at the upper bound rather than continuing to search. Second, for all $\overline{p} \ge \rho^k(\underline{c})$, $v - \overline{p} > v - s - E(p \mid \overline{p})$, that is, if the upper bound of the price distribution is relatively high, consumers would prefer to continue to search if they observe a price equal to the upper bound rather than buy. Third, ρ is uniquely defined by $v - \rho = v - s - E(p \mid \rho)$. Thus, for all $\rho' > \rho$, $v - \rho > v - s - E(p \mid \rho)$, that is, consumers prefer to buy immediately instead of continuing to search. Thus, it follows that the upper bound of the price distribution $\bar{p} = \rho'$ and $\rho^k(\underline{c}) \le \bar{p} \le \rho$.

Let us then consider the profits of firms. In an equilibrium it has to be the case that these profits are positive for all c. This is the case if, and only if, for all c, $\underline{p}(c) > c$. As $\underline{p}(c) = \frac{\lambda N}{\lambda N+1-\lambda}c + \frac{1-\lambda}{\lambda N+1-\lambda}\bar{p}$, this is the case if, and only if, $\bar{p} > \bar{c}$. A reservation price equilibrium therefore does not exist if $\bar{p} = \frac{s}{1-\alpha} + \int_{\underline{c}}^{\bar{c}} c\delta(c \mid \bar{p})\,dc < \bar{c}$. This is the case if (i) s is relatively small enough or (ii) $\bar{c}$ and $\underline{c}$ are relatively far apart and $g(c)$ has a relatively high probability mass close to $\underline{c}$.

Proof of Proposition 7. By equations (9), (10), (20), and (21), it is obvious that both $E(p) > E^k(p)$ and $E(p_\ell) > E^k(p_\ell)$ hold if $E(c \mid \bar{p}) = \int \delta(c \mid \bar{p})\,dc > E(c) = \int g(c)\,dc$. By Lemma 2, $\delta(c \mid \bar{p})$ FOSD $g(c)$, such that $E(c \mid \bar{p}) > E(c)$ obtains trivially. As the expected price of each consumer type is higher under production cost uncertainty, expected profits are higher and thus producer welfare is higher. Furthermore, as consumer welfare is inversely related to the expected price, we have that consumer welfare of both shoppers and nonshoppers is lower under production cost uncertainty. Finally, note that by Corollary 1 the *ex ante* price spread under complete information is given by

$$E(\bar{p}^k(c) - \underline{p}^k(c)) = \frac{\lambda N}{\lambda N + 1 - \lambda} \frac{s}{1-\alpha}.$$

Corollary 3 and (16), in turn, imply that the *ex ante* price spread under production cost uncertainty is equal to

$$E(\bar{p} - \underline{p}(c)) = \frac{\lambda N}{\lambda N + 1 - \lambda}\left(\frac{s}{1-\alpha} + E(c \mid \bar{p}) - E(c)\right).$$

By Lemma 2, $E(c \mid \bar{p}) > E(c)$ such that $E(\bar{p} - \underline{p}(c)) > E(\bar{p}^k(c) - \underline{p}^k(c))$ obtains.

Proof of Proposition 8. Under production cost uncertainty, each firm's conditionally expected profit $E(\Pi \mid c)$ amounts to

$$E(\Pi \mid c) = \frac{1-\lambda}{N}(\bar{p} - c) = \frac{1-\lambda}{N}\left(\frac{s}{1-\alpha} + \int_{\underline{c}}^{\bar{c}} c\delta(c \mid \bar{p})\,dc - c\right).$$

To see this, note that in a mixed-strategy equilibrium, each firm's expected profit must be equal to the profit resulting from charging the upper bound of the price distribution. Obviously, $E(\Pi \mid c)$ is a decreasing function of the cost realization c. Under complete information, we have that

$$E^k(\Pi \mid c) = \frac{1-\lambda}{N}\frac{s}{1-\alpha},$$

which clearly is independent of c. Finally, note that the difference in profits between the cost uncertainty and the cost certainty models is given by

$$E(\Pi \mid c) - E^k(\Pi \mid c) = \frac{1-\lambda}{N}\left(c - \int_{\underline{c}}^{\bar{c}} c'\delta(c' \mid \bar{p})\,dc'\right),$$

establishing that conditional expected profits under production cost uncertainty are higher for low-cost realizations, and lower for high-cost realizations, compared to conditional expected profits in the known cost case.

Proof of Proposition 9. Janssen and Moraga-González (2004) have shown that α is increasing in N and that $\alpha \to 1$ as $N \to \infty$. Consider the reservation price as defined in (16),

$$\rho = E(c \mid \bar{p}) + \frac{s}{1-\alpha}.$$

Note that we have that $E(c \mid \bar{p}) \in [\underline{c}, \bar{c}]$. Hence, we have that $\rho \to \infty$ as $N \to \infty$. Now, consider $E(c \mid \bar{p}) = \int_{\underline{c}}^{\bar{c}} \delta(c \mid \bar{p}) c\,dc$. We have that

$$\delta(c \mid \bar{p}) = \frac{g(c) f(\bar{p} \mid c)}{\int_{\underline{c}}^{\bar{c}} g(c') f(\bar{p} \mid c')\,dc'} = \frac{g(c)}{\int_{\underline{c}}^{\bar{c}} g(c')\frac{f(\bar{p} \mid c')}{f(\bar{p} \mid c)}\,dc'}.$$

Further, note that

$$\frac{f(\bar{p} \mid c')}{f(\bar{p} \mid c)} = \left(\frac{\bar{p} - c}{\bar{p} - c'}\right)^{\frac{1}{N-1}}.$$

As we have that $\bar{p} = \rho \to \infty$, as $N \to \infty$, it follows that the previous expression converges to one. This in turn implies that the posterior distribution $\delta(c \mid p)$ converges to the prior distribution $g(c)$ and, provided that v is not binding, we have that

$$\lim_{N\to\infty} (E(c \mid \bar{p}) - E(c)) = 0.$$

From equations (5), (16), (9), (20), (10), and (21), it then follows that

$$\begin{aligned} \rho - E(\rho^k(c)) &= E(c \mid \bar{p}) - E(c), \\ E(p) - E^k(p) &= \alpha[E(c \mid \bar{p}) - E(c)], \quad \text{and} \\ E(p_\ell) - E^k(p_\ell) &= (1-\alpha)[E(c \mid \bar{p}) - E(c)]. \end{aligned}$$

It follows that the above expressions converge to 0 as $N \to \infty$. Note that, no matter how large v is, it eventually becomes binding as $N \to \infty$. We have formulated the proposition such that the respective differences between the relevant model characteristics in the two scenarios vanishes for large N and v.

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