

# Simulation Of the Heat Transfer in the Nanocathode

V.G. Daniov

Moscowtechnicaluniversity ofcommunicationsand  
 informatics,  
 Moscowinstitute ofelectronics and mathematics National  
 research university Higher school of economics  
 MTUCI,  
 MIEM NRU HSE  
 Moscow, Russia  
 danilov@miem.edu.ru

V.Yu. Rudnev

Moscowtechnicaluniversity ofcommunicationsand  
 informatics,  
 Moscowinstitute ofelectronics and mathematics National  
 research university Higher school of economics  
 MTUCI,  
 MIEM NRU HSE  
 Moscow, Russia  
 vrudnev78@mail.ru

V.I. Kretov

Moscowinstitute ofelectronics and mathematics National  
 research university Higher school of economics  
 MIEM NRU HSE  
 Moscow, Russia  
 ps-vad@yandex.ru

**Abstract**—The heat transfer process is simulated in a nano-sized cone-shaped cathode. A model of heat transfer is constructed using the phase field system and the Nottingham effect. We consider influence of the free boundary curvature and the Nottingham effect on the heat balance in the cathode.

**Keywords**—thermo-field emission, cathode, Nottingham effect, free boundary, phase field system, Stefan-Gibbs-Thomson problem

## 1. Introduction and Statement of the Problem

Our main goal is to simulate the heat transfer in a doped silicon nanocathode. The cathode has the shape of a blunted cone and the following linear dimensions:

height of the cathode	10–15 $\mu$
diameter of the cathode base	~ 6 $\mu$
radius of the cathode vertex rounding	~ 15 nm
cathode vertex angle	~ 20°

Such a shape of the cathode is specified by the engineering process, see Fig.1. Such cathodes are used in the electron microscope and in other electron devices.

An obstacle for a wide use of this cathode is the instability of electron emission. This instability is in fact caused by the small size of the cathode. The cathode is heated due to the Joule effect. The current in the cathode is very large and the Joule heat can melt the cathode.

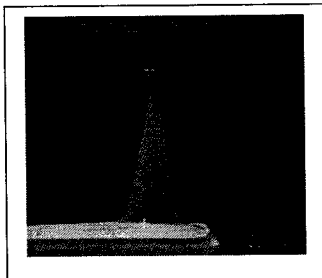


Figure 1. REM image of the silicon nanocathode.

The effect of the cathode melting is confirmed experimentally. Namely, the produced molten (liquid) layer does not contain a small

region near the vertex of the cathode cone. At the same time the cathode material remains solid near the base. So we observe the following sequence of layers: solid, liquid, and gas. It is experimentally known that the liquid layer becomes thicker after some (unknown) time.

We present an explanation of this fact in this paper. The motion of the free boundary (the interface between the plasma and the cathode) depends on the curvature of the free boundary and the Nottingham effect. Namely, the temperature dependence of the free boundary curvature is determined by the Gibbs-Thomson law [8, 9]. The Nottingham effect determines the temperature of the cathode vertex under the thermoemission of electrons. More precisely, the Nottingham effect consists in the following: If the temperature of the cathode vertex is higher than the equilibrium temperature, then the vertex is cooled; if the temperature of the cathode vertex is lower than this equilibrium temperature, then the vertex is heated.

The mathematical model of the heat transfer in the cathode under field emission is known (see [6]),

$$\rho c(T) \frac{\partial T}{\partial t} = \nabla \cdot (\lambda(T) \nabla T) + F, \quad \text{div } j = 0.$$

Here  $T$  is the temperature,  $\rho$  is the density,  $c$  is the specific heat capacity,  $F$  is the power density of the electron emission under the Joule and Thomson effects, and  $j$