ISSN 1064-2307, Journal of Computer and Systems Sciences International, 2013, Vol. 52, No. 5, pp. 677–685. © Pleiades Publishing, Ltd., 2013.
Original Russian Text © M.G. Zotov, 2013, published in Izvestiya Akademii Nauk. Teoriya i Sistemy Upravleniya, 2013, No. 5, pp. 3–11.

**SYSTEMS THEORY
AND GENERAL CONTROL THEORY**

On Constructing Quasi-Optimal Robust Systems

M. G. Zotov

Higher School of Economics, National Research University, ul. Shabolovka 26, Moscow, Russia

Received June 26, 2012; in final form, January 22, 2013

Abstract—For a multi-mode control plant, a system of equations is obtained so that its solution helps construct a robust controller that ensures the quality of system functioning close to optimal for the normal mode and acceptable availability for the emergency mode. An approximate method to solve this problem is given.

DOI: 10.1134/S1064230713030167

INTRODUCTION

The problem of simultaneous stabilization is common to many practical cases when the plant can function in several modes. Transition from one mode to another is arbitrary. The robust controller is to be constructed so that to ensure the quality of system functioning close to optimal for the normal mode and acceptable system availability for the emergency mode. The history of development of this direction and the existing approaches to solve the given class of problems are given in [1–4].

1. STATEMENT OF THE PROBLEM

Without loss of generality, we consider the plant functioning in two modes – normal and emergency. Its transfer function has the form $W_{01}(s)$ in the normal mode and $W_{02}(s)$ in the emergency mode. The quality of functioning is estimated respectively by the criteria $I_1[W_1(\cdot)]$ and $I_2[W_1(\cdot)]$, where $W_1(s)$ is the sought transfer function of the robust controller that ensures, for the given transfer functions of the control plant $W_{01}(s)$ and $W_{02}(s)$, the minimum of the linear combination

$$I[W_1(\cdot)] = I_1[W_1(\cdot)] + \chi I_2[W_1(\cdot)] \tag{1.1}$$

with the weight coefficient χ that estimates the importance of the criterion in the convolution. It is generally chosen so that the quality of system functioning is close to optimal for the normal mode while acceptable availability is ensured for the emergency mode. Unfortunately, functional (1.1) is not quadratic with respect to the sought function $W_1(s)$, which hampers the search for the solution. Below, we give the procedure that helps eliminate this drawback.

We consider the control device with the correction block in the direct circuit common in design practice. Figures 1a and 1b give the schemes for two functioning modes, where $W_{11}(s)$ and $W_{12}(s)$ are the transfer functions of the correction blocks. The transfer functions of these systems with respect to the action $g(t)$ are given by the relation

$$\tilde{H}_k^*(s) = \frac{W_{1k}(s)W_{0k}(s)}{1 + W_{1k}(s)W_{0k}(s)}, \quad k = 1, 2, \tag{1.2}$$

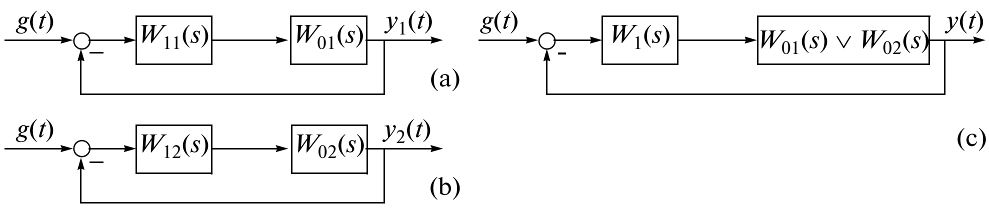

Fig. 1.

and should ensure the quadratic criteria

$$I_k^*[\tilde{H}_k^*(\cdot)] = \frac{1}{2\pi j}\int\limits_{-j\infty}^{j\infty} J_k(s, \tilde{H}_k^*(s))ds, \quad k = 1, 2, \tag{1.3}$$

are optimal with the integrands

$$J_k(s, \tilde{H}_k^*(s)) = \tilde{H}_k^*(s)\tilde{H}_k^*(-s)S_{1k}(s) - \tilde{H}_k^*(-s)S_{2k}(s) - \tilde{H}_k^*(s)S_{2k}(-s) + S_{3k}(s),$$

where $S_{ik}(s)$, $i = 1, 2, 3$, are the initial data for the system functioning in the normal ($k = 1$) and emergency ($k = 2$) mode, respectively. They describe the quality of system functioning from various aspects, in particular how close the sought transfer functions $\tilde{H}_1^*(s)$ and $\tilde{H}_2^*(s)$ are to the desired ones, the quality of transition processes, etc. One of the variants of initial data is given in the example (Section 3).

Since for both modes the robust correction block should have the same transfer function, we have

$$W_{11}(s) - W_{12}(s) = \frac{\tilde{H}_1(s)}{W_{01}(s)(1 - \tilde{H}_1(s))} - \frac{\tilde{H}_2(s)}{W_{02}(s)(1 - \tilde{H}_2(s))} = 0,$$

where $\tilde{H}_1(s)$ and $\tilde{H}_2(s)$ are the transfer functions of the system with the robust controller $W_1(s)$ and the two-mode plant (Fig. 1c)

$$\tilde{H}_k(s) = \frac{W_1(s)W_{0k}(s)}{1 + W_1(s)W_{0k}(s)}, \quad k = 1, 2. \tag{1.4}$$

Finally, we find

$$\frac{D(s)}{W_{01}(s)W_{02}(s)(1 - \tilde{H}_1(s))(1 - \tilde{H}_2(s))} = 0,$$

where the numerator has the form

$$D(s) = \tilde{H}_1(s)\tilde{H}_2(s)\big(W_{01}(s) - W_{02}(s)\big) + \tilde{H}_1(s)W_{02}(s) - \tilde{H}_2(s)W_{01}(s).$$

The obtained relation holds if for all s the restriction is met

$$D(s) = 0, \quad W_{01}(s)W_{02}(s)(1 - \tilde{H}_1(s))(1 - \tilde{H}_2(s)) \neq 0. \tag{1.5}$$

Condition (1.5) is always met – $W_{01}(s) \neq 0$, $W_{02}(s) \neq 0$, $\tilde{H}_1(s) \neq 1$, $\tilde{H}_2(s) \neq 1$, since these are functions describing dynamic properties of the object in one case and of the system in the other case.

Hence, the quality of system functioning in the emergency and normal modes can be estimated by the criterion similar to criterion (1.1)

$$I[\tilde{H}_1(\cdot), \tilde{H}_2(\cdot)] = J_1[\tilde{H}_1(\cdot)] + \chi J_2[\tilde{H}_2(\cdot)] \tag{1.6}$$

under restriction (1.5).

2. SEARCHING FOR OPTIMAL SOLUTION

The stated problem is reduced to the variational problem for conditional minimum. To solve problem (1.6) taking into account (1.3)–(1.5), we form the Lagrange functional

$$I = \frac{1}{2\pi j}\int\limits_{-j\infty}^{j\infty} \{J_1(s, \tilde{H}_1^*(s)) + \chi J_2(s, \tilde{H}_2^*(s)) + \alpha(s)D(s) + \alpha(-s)D(-s)\}ds, \tag{2.1}$$

where $\alpha(s)$ is the undetermined multiplier. Relation (2.1) can be transformed to a standard quadratic functional of form (1.3)

$$I = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\Big(\big(\tilde{H}_1(s)\tilde{H}_1(-s)S_{11}(s) - \tilde{H}_1(-s)\big(S_{21}(s) + \alpha(-s)\big(0.5\tilde{H}_2(-s)(W_{01}(-s) - W_{02}(-s)) + W_{02}(-s)\big)\big)$$
$$- \tilde{H}_1(s)\big(S_{21}(-s) + \alpha(s)\big(0.5\tilde{H}_2(s)(W_{01}(s) - W_{02}(s)) + W_{02}(s)\big)\big) + S_{31}(s)$$
$$+ \chi\big(\tilde{H}_2(s)\tilde{H}_2(-s)S_{12}(s) - \tilde{H}_2(-s)\big(S_{22}(s) + \alpha(-s)\big(0.5\tilde{H}_1(-s)(W_{01}(-s) - W_{02}(-s)) - W_{01}(-s)\big)\big)$$
$$- \tilde{H}_2(s)\big(S_{22}(-s) + \alpha(s)\big(0.5\tilde{H}_1(s)(W_{01}(s) - W_{02}(s)) - W_{01}(s)\big)\big) + S_{32}(s)\Big)ds.$$

Solving ordinary variational problem [5], we have that the sought functions $\tilde{H}_1(s)$ and $\tilde{H}_2(s)$ minimizing functional (2.1) should be found from the system of equations

$$\begin{aligned}&\tilde{H}_1(s)S_{11}(s) - 0.5\alpha(-s)\tilde{H}_2(-s)(W_{01}(-s) - W_{02}(-s)) - (S_{21}(s) - \alpha(-s)W_{02}(-s)) = \Gamma_1(s),\\&\tilde{H}_2(s)\chi S_{12}(s) - 0.5\alpha(-s)\tilde{H}_1(-s)(W_{01}(-s) - W_{02}(-s)) - (\chi S_{22}(s) - \alpha(-s)W_{01}(-s)) = \Gamma_2(s)\end{aligned} \quad (2.2)$$

and restriction (1.5). Here, $\Gamma_1(s)$ and $\Gamma_2(s)$ are the fractional rational functions, with their poles in the right half-plane, that are involved in solving the optimization problem [5] and are due to the fact that the sought transfer functions $\tilde{H}_1(s)$ and $\tilde{H}_2(s)$ used to ensure stability of the closed-loop system are being searched with the poles from the left half-plane only.

We use the transfer functions $\tilde{H}_1(s)$ and $\tilde{H}_2(s)$ to find the transfer function of the robust controller (Fig. 1c)

$$W_1(s) = \frac{\tilde{H}_1(s)}{W_{01}(s)(1 - \tilde{H}_1(s))} = \frac{\tilde{H}_2(s)}{W_{02}(s)(1 - \tilde{H}_2(s))}. \quad (2.3)$$

Whether equality (2.3) holds indicates whether the solution of system (2.2) is correct. Unfortunately, the existing methods of solving matrix Wiener-Hopf equations of form (2.2) do not allow finding this solution. The Lagrange multiplier $\alpha(s)$ in system of equations (2.2) is the principle obstacle.

Practice has shown that generally optimal systems are redundant in terms of complexity. In our case, a differential equation of high order is used to describe the mathematical model of the constructed controller. Practically, we can obtain the system with the value of criterion (2.1) close to optimal using the robust controller described by the differential equation of a lower order. The optimal values of indices I_1^* and I_2^* in (1.3) can be used as a benchmark for potential unattainable capability of the robust system in Fig. 1c. We use this algorithm to construct the algorithm for searching a quasi-optimal robust controller.

3. SEARCHING FOR QUASI-OPTIMAL SOLUTION

Optimal systems have a positive property – significant changes in parameters of the controller do not result in significant decrease of the quality indices, which are the values I_1^* and I_2^* in (1.3) in our case. This is characteristic of optimal system and is a positive factor that allows a designer vary the structure and parameters of the controller in sufficiently wide range [6]. This gives us hope that optimal controllers for each mode can be used to construct one robust controller that ensures satisfactory functioning of the multi-mode system. Without loss of generality, we consider the search algorithm for the system functioning in two modes – normal and emergency. By the algorithm from Section 2, this problem can be solved in three stages.

Stage 1. For each mode, we form the criterion that estimates the quality of functioning of the system in this mode and use the criteria optimum conditions to search for controllers optimal for each mode. The optimal values I_1^* and I_2^* given by relation (1.3) can serve as limiting unattainable benchmarks of the quality of the robust system. The constructed controllers are checked for whether they can be used as robust.

Stage 2. The optimal controllers with the transfer functions $W_{11}(s) = V_{11}(s)/G_{11}(s)$ and $W_{12}(s) = V_{12}(s)/G_{12}(s)$ are used to form the structure of the robust controller

$$W_1(s) = \frac{V_1(s)}{G_1(s)} = \frac{v_m s^m + v_{m-1}s^{m-1} + \ldots + v_1 s + v_0}{g_n s^n + g_{n-1}s^{n-1} + \ldots + g_1 s + g_0}, \quad m = \max_{i=1,2}\deg V_{1i}(s), \quad n = \max_{i=1,2}\deg G_{1i}(s).$$

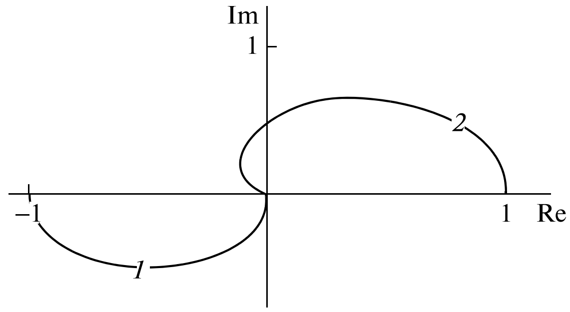

Fig. 2.

Stage 3. We search for the parameters of the robust controller $W_1(s)$ based on the minimum condition of criterion (2.1), omitting two last terms associated with restriction (1.5) in it, i.e., the minimum of

$$I = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\{J_1(s,\tilde{H}_1^*(s)) + \chi J_2(s,\tilde{H}_2^*(s))\}ds, \tag{3.1}$$

with tantamount restriction (2.3) replacing (1.5).

Solving such parametric optimization problems numerically under the restriction is a mathematical programming problem. One can use nonlinear programming methods to solve problems where the optimality criterion and restrictions depend on the sought parameters non-linearly. There are no versatile methods of solving these problems due to their diversity. Gradient methods are quite common among computational algorithms. The initial search point in gradient methods must belong to the admissible set, where we search for optimum [7, 8]. We give an example to complement the algorithm that searches for quasi-optimal robust controller.

Example. The control plant can be in two modes — its transfer function is $W_{01}(s) = Q_{01}(s)/P_{01}(s) = 1/(s-1)$ in the normal mode, becoming $W_{02}(s) = Q_{02}(s)/P_{02}(s) = 6/(s-2)(s-3)$ in the emergency mode. Figure 1a gives the scheme of the system with the robust controller. Figure 2 gives the amplitude phase characteristics of plants (1 is for the hodograph $W_{01}(j\omega)$, 2 is for the hodograph $W_{02}(j\omega)$). We can see their significant difference.

We need to construct the correction block in the direct circuit that ensures the system is robust with respect to its operation modes. With respect to the master control, the transfer functions of the system in both modes are to be close to the sought function $U_1(s) = 1$. In both modes, the system should have first-order astaticism. We solve the problem following the stages stated above.

Stage 1. For each operation mode, we find the transfer functions of optimal controllers. The problem statement leads to two functionals [5]

$$I_1^* = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\left\{\left|(U_1(s) - \tilde{H}_1^*(s))\frac{1}{s}\right|^2 + \lambda_{01}|s^{b_1}\tilde{H}_1^*(s)|^2 - \rho_{11}\left[\frac{\tilde{H}_1^*(-s)}{s+1} + \frac{\tilde{H}_1^*(s)}{-s+1}\right]\right\}ds + 2\rho_{11},$$

$$I_2^* = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\left\{\left|(U_1(s) - \tilde{H}_2^*(s))\frac{1}{s}\right|^2 + \lambda_{12}|s^{b_2}\tilde{H}_2^*(s)|^2\right.$$
$$\left. - \rho_{21}\left[\frac{\tilde{H}_2^*(-s)}{s+2} + \frac{\tilde{H}_2^*(s)}{-s+2}\right] - \rho_{22}\left[\frac{\tilde{H}_2^*(-s)}{s+3} + \frac{\tilde{H}_2^*(s)}{-s+3}\right]\right\}ds + 2(\rho_{21} + \rho_{22}),$$

here λ_{01} and λ_{12} are weight coefficients, the values of which characterize the quality of transition processes, ρ_{11}, ρ_{21}, and ρ_{22}, are the Lagrange coefficients, $\tilde{H}_1^*(s)$ and $\tilde{H}_2^*(s)$ are given by relations (1.2). The first components of the functionals estimate how close the sought functions $\tilde{H}_1^*(s)$ and $\tilde{H}_2^*(s)$ are to the desired function $U_1(s) = 1$ as well as the quality of the transition process. The second component being finite ensures the correction blocks $W_{1i}(s) = V_{1i}(s)/G_{1i}(s)$, $i = 1, 2$ can be implemented. The numerical value

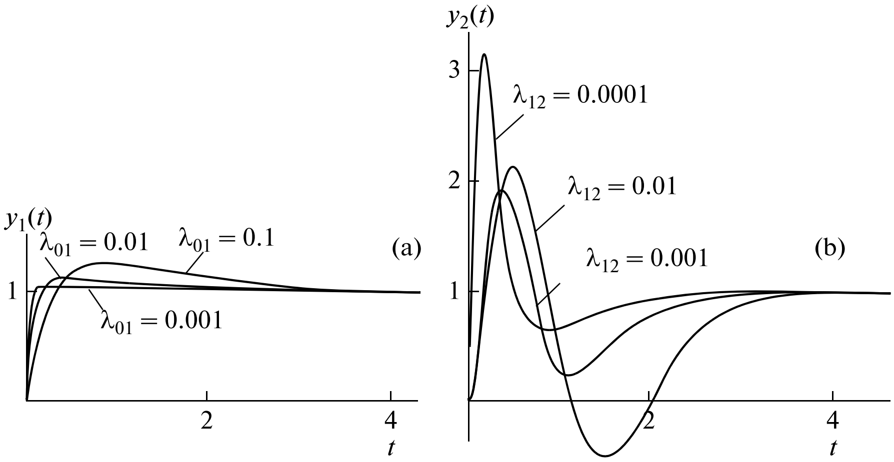

Fig. 3.

of this index reflects the quality of the transition process [9]. The powers $b_1 = 0$ and $b_2 = 1$ are chosen so that the relations $\deg V_{1i}(s) = \deg G_{1i}(s)$, $i = 1, 2$ are met. In this case, the power of the numerator is the power of denominator obtained from the solution to the optimization problem, i.e., the number of variable parameters in the controller will be the greatest [9]. The last components represent the restriction on the correction block compensating the right poles of the transfer function of the control plant.

The functional written above is in line with functional (1.3), where

$$S_{11}(s) = \frac{1}{s(-s)} + \lambda_{01}s^{b_1}(-s)^{b_1}, \quad S_{21}(s) = \frac{U_1(s)}{s(-s)} + \frac{\rho_{11}}{s+1}, \quad S_{31}(s) = \frac{U_1(s)U_1(-s)}{s(-s)};$$

$$S_{12}(s) = \frac{1}{s(-s)} + \lambda_{12}s^{b_2}(-s)^{b_2}, \quad S_{22}(s) = \frac{U_1(s)}{s(-s)} + \frac{\rho_{21}}{s+2} + \frac{\rho_{22}}{s+3}, \quad S_{32}(s) = S_{31}(s).$$

Solving the optimization problem, we arrive at the Wiener-Hopf equations

$$\begin{aligned} &\tilde{H}_1^*(s)\left(\lambda_{01} + \frac{1}{s(-s)}\right) - \left(\frac{1}{s(-s)} + \frac{\rho_{11}}{s+1}\right) = \Gamma_1(s), \\ &\tilde{H}_2^*(s)\left(\lambda_{12}s(-s) + \frac{1}{s(-s)}\right) - \left(\frac{1}{s(-s)} + \frac{\rho_{21}}{s+2} + \frac{\rho_{22}}{s+3}\right) = \Gamma_2(s). \end{aligned} \tag{3.2}$$

Figures 3a and 3b shows the graphs of transition processes $y_1(t)$ and $y_2(t)$ for different values of $\lambda_{01} = 0.1$ and $\lambda_{12} = 0.001$. Comparing the transition processes visually, we choose $\lambda_{01} = 0.1$ and $\lambda_{12} = 0.001$. Figure 4a shows the transition processes for λ_{01} and λ_{12} that are chosen for further synthesis of the control system. The processes in Fig. 4a serve as a benchmark to strive to when designing a robust controller for the given multi-mode plant. Using the standard procedure to solve Wiener-Hopf equation (3.2) [5], we find the following relations.

For the control plant in the normal mode when $\lambda_{01} = 0.1$ in (3.2), we have

$$\tilde{H}_1^*(s) = \frac{5.1623s + 3.1623}{s^2 + 4.1623s + 3.1623}, \quad W_{11}(s) = \frac{\tilde{H}_1^*(s)}{W_{01}(s)(1 - \tilde{H}_1^*(s))} = \frac{5.1623s + 3.1623}{s},$$

$$I_1^* = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\left\{\left|\left(1 - \tilde{H}_1^*(s)\right)\frac{1}{s}\right|^2 + 0.1\left|\tilde{H}_1^*(s)\right|^2\right\}ds = 0.5162,$$

with the first component of the functional $I_{11}^* = 0.1581$ and the second component $I_{12}^* = 3.5811$. These components numerically estimate the transition process $y_1(t)$ given in Fig. 4a.

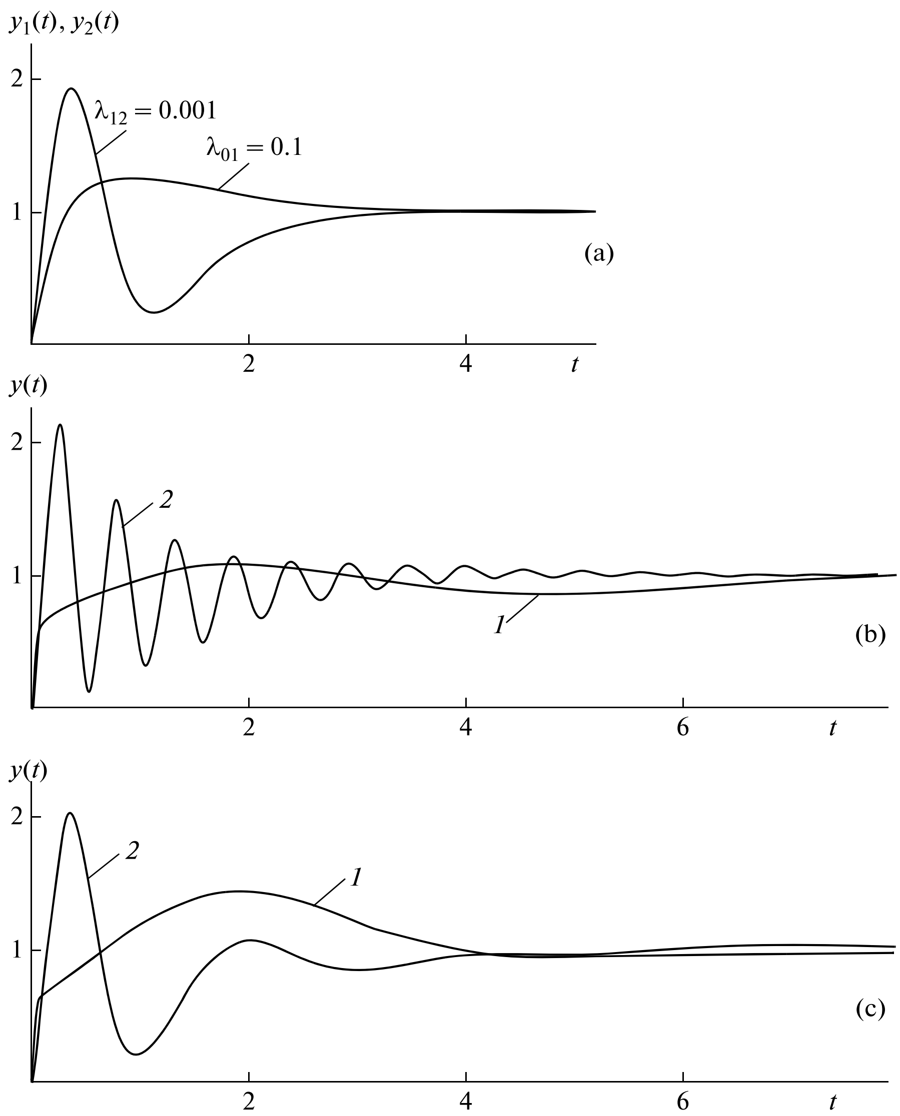

Fig. 4.

For the control plant in the emergency mode when $\lambda_{12} = 0.001$, we have

$$\tilde{H}_2^*(s) = \frac{151.1498s^2 + 98.1139s + 189.7367}{s^4 + 12.9527s^3 + 77.3863s^2 + 205.8301s + 189.7367},$$

$$W_{12}(s) = \frac{\tilde{H}_2^*(s)}{W_{02}(s)(1 - \tilde{H}_2^*(s))} = \frac{26.8583s^2 + 16.3523s + 31.6229}{s^2 + 17.9527s},$$

$$I_2^* = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\left\{\left|(1 - \tilde{H}_2^*(s))\frac{1}{s}\right|^2 + 0.001|s\tilde{H}_2^*(s)|^2\right\}ds = 1.9892,$$

with the first component of the functional $I_{21}^* = 0.7037$ and the second component $I_{22}^* = 1.285 \times 10^3$. These components numerically estimate the transition process $y_2(t)$ given in Fig. 4a.

The constructed controllers are not robust with respect to stability. Indeed,

$$\frac{W_{02}(s)W_{11}(s)}{1 + W_{02}(s)W_{11}(s)} = \frac{30.9737s + 18.9737}{s^3 - 5s^2 + 36.9737s + 18.9737},$$

$$\frac{W_{01}(s)W_{12}(s)}{1+W_{01}(s)W_{12}(s)} = \frac{26.8583s^2+16.3523s+31.6288}{s^3+43.8110s^2-1.6004s+31.6288}.$$

Stage 2. We find the structure and parameters of the controller that ensures the system has robust stability. We assume that the robust controller and the controller $W_{12}(s)$ have the same structure. Moreover, given that even the controller optimal for the emergency mode failed to yield a suitable transition process, we add the structure of the sought robust controller with extra parameters, i.e., we search for its transfer function in the form

$$W_1(s) = \frac{V_1(s)}{G_1(s)} = \frac{v_3s^3+v_2s^2+v_1s+v_0}{g_3s^3+g_2s^2+g_1s}. \tag{3.3}$$

As we said, while searching, the initial values of the parameters of $W_1(s)$ are supposed to ensure the system with the plants $W_{01}(s)$ and $W_{02}(s)$ is stable. With (3.3) in mind, we transform the found transfer functions of the correction blocks

$$\begin{aligned} W_{12}(s) &= \frac{(26.8583s^2+16.3523s+31.6229)(s+1)}{(s^2+17.9527s)(s+1)} \\ &= \frac{26.8583s^3+43.2106s^2+47.9751s+31.6228}{s^3+18.9527s^2+17.9527s}\frac{V_{12}(s)}{G_{12}(s)}. \end{aligned} \tag{3.4}$$

The parameters of the correction block given in (3.4) ensure the plant with the transfer function $W_{02}(s)$ is stable. The correction block with the transfer function

$$W_{11}(s) = \frac{(5.1623s+3.1623)(s+1)^2}{s(s+1)^2} = \frac{5.1623s^3+13.4868s^2+11.48683s+3.1623}{s^3+2s^2+s}\frac{V_{12}(s)}{G_{12}(s)} \tag{3.5}$$

makes the plant with the transfer function $W_{01}(s)$ stable. Note that after these transformations the transfer functions $W_{12}(s)$ and $W_{11}(s)$ coincide in structure. Thus, the robust controller has the form (3.3).

Stage 3. This final stage implies finding the parameters of controller (3.3) so that criterion (3.1) is minimum. Given that the system is stable both in the normal and emergency mode. As we said, there are no versatile methods to solve such problems. Based on experience of solving similar problems [4], we can see it is reasonable to decompose this stage into two substages. At the first substage, we find the parameters of controller (3.3) based on the stability condition of the system both in the normal and emergency modes. At the second substage, the found parameters are refined so that criterion (3.1) is minimum given the restriction on stability of the system both in the emergency and normal modes.

We consider the first substage. We use the coefficients $W_{12}(s)$ and $W_{11}(s)$ to form the function

$$I_3 = (v_{32}-v_{31})^2+(v_{22}-v_{21})^2+(v_{12}-v_{11})^2+(v_{02}-v_{01})^2+(g_{22}-g_{21})^2+(g_{12}-g_{11})^2.$$

We minimize I_3 with respect to parameters, given that the coefficients of the characteristic polynomials

$$T_2(s) = G_{12}(s)P_{02}(s)+V_{12}(s)Q_{02}(s) = (g_{32}s^3+g_{22}s^2+g_{12}s)(s-2)(s-3)+6(v_{32}s^3+v_{22}s^2+v_{12}s+v_{02}),$$

$$T_1(s) = G_{11}(s)P_{01}(s)+V_{11}(s)Q_{01}(s) = (g_{31}s^3+g_{21}s^2+g_{11}s)(s-1)+(v_{31}s^3+v_{21}s^2+v_{11}s+v_{01})$$

and the principal Hurwitz determinants D_2 and D_1 composed of these coefficients are positive. The initial values of the parameters $W_{12}(s)$ and $W_{11}(s)$ are taken from (3.4) and (3.5). For them, all restricting conditions, viz. the coefficients $T_1(s)$ and $T_2(s)$ and the relations $D_2 > 0$, $D_1 > 0$ being positive, are met.

The found, sufficiently small value I_3, which is 3.087 in our example, leads to the solution to the problem for the initial value 4.645×10^3. The resulting controller has the transfer function

$$W_1(s) = \frac{30.166s^3+39.183s^2+47.437s+9.877}{s^3+8.598s^2+1.412s}$$

and makes the system possess robust stability. If we fail to find parameters, we need to increase the power of the numerator and denominator of the transfer functions $W_{12}(s)$ and $W_{11}(s)$.

Unfortunately, the found controller robust in terms of stability does not ensure acceptable values of the quality indices. Figure 4b shows the transition processes $y(t)$—process 1 for the case when the plant has

Table

Controller	Mode	Integral indices			Transition processes
		I_{11}^*	I_{12}^*	I_1^*	
Optimal for the normal mode $W_{11}(s) = \frac{5.1623s + 3.1623}{s}$	Normal	0.1581	3.5811	0.5162	Fig. 4a, curve $\lambda_{01} = 0.1$
	Emergency	∞	∞	∞	
Optimal for the emergency mode	Normal	∞	∞	∞	
$W_{12}(s) = \frac{26.8583s^2 + 16.3523s + 31.6229}{s^2 + 17.9527s}$	Emergency	0.704	1.285×10^3	1.989	Fig. 4a, curve $\lambda_{12} = 0.001$
Robust with respect to stability	Normal	0.094	12.774	1.371	Fig. 4b, curve *1*
$W_1(s) = \frac{30.166s^3 + 39.183s^2 + 47.437s + 9.877}{s^3 + 8.598s^2 + 1.412s}$	Emergency	0.539	9.967×10^3	10.507	Fig. 4b, curve *2*
Robust with respect to stability and quality	Normal	0.301	10.008	1.301	Fig. 4c, curve *1*
$W_1(s) = \frac{28.838s^3 + 46.092s^2 + 62.002s + 5.985}{s^3 + 18.654s^2 + 1.620s}$	Emergency	0.538	1.720×10^3	2.258	Fig. 4c, curve *2*

the transfer function $W_{01}(s)$ and process 2 for the transfer function of the plant $W_{02}(s)$. Process 2 in Fig. 4b differs much from the process $\lambda_{12} = 0.001$ in Fig. 4a, i.e., from the optimal process to be taken as a benchmark.

We move to the second substage that implies correcting the found parameters of the robust controller so that the criterion

$$I = J_1 + \chi J_2,$$

$$I_1 = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\left\{\left|(1 - \tilde{H}_1(s))\frac{1}{s}\right|^2 + 0.1\left|\tilde{H}_1(s)\right|^2\right\}ds,$$

$$I_2 = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\left\{\left|(1 - \tilde{H}_2(s))\frac{1}{s}\right|^2 + 0.001\left|s\tilde{H}_2(s)\right|^2\right\}ds,$$

is minimum, here the transfer functions $\tilde{H}_1(s)$ and $\tilde{H}_2(s)$ are given by (1.4). The value of the weight coefficient χ is chosen using the analysis of the transition processes of the system in the normal and emergency modes. The value $\chi = 1$ turned out to be acceptable. Then,

$$I_1 = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\left\{\left|(1 - \tilde{H}_1(s))\frac{1}{s}\right|^2 + 0.1\left|\tilde{H}_1(s)\right|^2\right\}ds = 1.158,$$

$$I_2 = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\left\{\left|(1 - \tilde{H}_2(s))\frac{1}{s}\right|^2 + 0.001\left|s\tilde{H}_2(s)\right|^2\right\}ds = 2.173,$$

$$W_1(s) = \frac{28.838s^3 + 46.092s^2 + 62.002s + 5.985}{s^3 + 18.654s^2 + 1.620s}.$$

For the functional I_1, the first component is 0.301 and the second component is 10.008; for the functional I_2, the first component is 0.538 and the second component is 1.720×10^3.

Figure 4c gives the graphs of the transition processes $y(t)$ of the system with the controller robust with respect to the quality for the plants $W_{01}(s)$ (curve *1*) and $W_{02}(s)$ (curve *2*). Comparing Figs. 4a and 4c visually, we can see that the transition processes of the constructed system robust with respect to quality are sufficiently close to optimal processes, which is not always true. We put the solution process together in the table.

CONCLUSIONS

We considered the problem of constructing the optimal robust system with a multi-mode plant. We obtained system of Wiener-Hopf equations (2.2), which we can solve under restriction (1.5) to find the robust controller optimal with respect to the quality. The existing methods of solving the matrix Wiener-Hopf equations under the given restrictions do not allow finding the exact solution. We solved the problem using approximate procedures. We gave the example to illustrate the efficiency of the algorithm of finding the approximate solution.

REFERENCES

1. B. T. Polyak and P. S. Shcherbakov, *Robust Stability and Control* (Nauka, Moscow, 2002) [in Russian].
2. D. P. Kim, "Synthesis of systems with maximum robust degree of stability," Comput. Syst. Sci. Int. **46**, 721 (2007).
3. M. G. Zotov, "An approach to robust control design," Autom. Remote Control **71**, 2395–2404 (2010).
4. M. G. Zotov, "On the design of control systems that are robust in terms of performance index," Comput. Syst. Sci. Int. **51**, 477 (2012).
5. M. G. Zotov, *Multicriteria Design of Automatic Control Systems* (BINOM. Lab. znanii, Moscow, 2004) [in Russian].
6. V. S. Pugachev, I. E. Kazakov, and L. G. Evlanov, *Foundations of Statistical Theory of Automatic Systems* (Mashinostroenie, Moscow, 1974) [in Russian].
7. A. V. Attetkov, V. S. Zarubin, and A. N. Kanatnikov, *Introduction to Optimization Methods* (Finansy i statistika, Moscow, 2008) [in Russian].
8. A. A. Zhiglyavskii and A. G. Zhilinskas, *Methods for Global Extremum Search* (Nauka, Moscow, 1991) [in Russian].
9. V. I. Kukhtenko, "On calculation of the correcting circuits of the automatic control systems by the minimum RMS error criterion," Avtom. Telemekh., No. 9, 1180–1187 (1959).

Translated by M. Talacheva

SPELL: OK