NATIONAL RESEARCH UNIVERSITY
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THE USE OF POLYNOMIAL TRANSFORMATIONS IN ORGANIZATIONAL RESEARCH: REVIEW AND RECOMMENDATIONS

BASIC RESEARCH PROGRAM

WORKING PAPERS

SERIES: MANAGEMENT
WP BRP 09/MAN/2013

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THE USE OF POLYNOMIAL TRANSFORMATIONS IN ORGANIZATIONAL RESEARCH: REVIEW AND RECOMMENDATIONS[4]

Polynomial variable transformations have become increasingly popular in organizational studies to help deal with a variety of statistical issues. Indeed, a review of over 4,000 articles published in management journals indicates that almost 10% of these articles used at least some form of power transformation in the analysis. Specifically, over 14% of the articles published in *Strategic Management Journal* during the 2000s reported a transformation of at least one variable. Unfortunately, the first-order variable and its higher-order polynomials are usually highly correlated, resulting in a wide range of multicollinearity problems. However, the majority of the studies analyzed articles fell short of ideal in addressing this issue. A review of the top journals publishing organizational research indicates that several critical issues were ignored during the implementation of these transformations. Specifically, researchers did not typically: (1) provide an explanation for their decision to use a specific transformation (e.g., z-score, Legendre, polynomial); (2) did not test the effects of their transformation procedures on the focal variable, and (3) report the results of their analyses both before and after the transformation to assess the effects of their correction procedure. Therefore, the purpose of this manuscript is threefold. First, we provide a review of 324 articles published in the organizational sciences that describe current practices in research using variable transformations. Second, we conduct several Monte Carlo simulations examining the effects of the four types of transformations that were most commonly reported in the literature. Finally, the results of these simulations are used to help develop a set of recommended best practices for researchers. We conclude with a discussion of implications for editors, reviewers, and researchers in the organizational studies field.

Keywords: polynomial transformations; Legendre; strategic management research; statistics; Monte Carlo simulation; skew; kurtosis; type II error; r-square; standardized; mean center.
JEL Classification: C46.

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[4] This study was carried out within "The National Research University Higher School of Economics' Academic Fund Program in 2012-2013, research grant No. 11-01-0192".

Polynomial transformations (square, cube, and higher-order transformations of a variable) remain a popular method for testing the curvilinearity of relationships in the organizational sciences. Out of over 4,000 articles published in the top six management journals over the past 10 years, 324 used a polynomial transformation in the analysis. The transformation was found to be used on a large variety of variables and in multitude of statistical techniques, including multiple linear and logistic regression, factor analysis, and analysis of covariance structures. Most commonly transformed variables included age (e.g., Morris et al., 2000), density (e.g., Gomez-Mejia et al., 2007), growth (e.g., Von Nordenflycht, 2007), ownership (e.g., Mayer & Whittington, 2003), and percentage variables (e.g., Connelly et al., 2010), with strategy and entrepreneurship studies accounting for a vast majority of studies with curvilinear relationships (83%). Unfortunately, the first order variable and its higher-order polynomials are usually very highly correlated (in the magnitude of .9 and above). Therefore, using these variables simultaneously in a model results in a wide range of multicollinearity problems well-known by researchers, and amply discussed in the statistical literature (e.g., Farrar & Glauber, 1967). The issues that are of most serious concern are the sensitivity of parameter estimates to model specification and sample coverage, high variance of estimates, and increased chance of a Type II error.

Polynomial collinearity requires special treatment by researchers, since the most commonly recommended solution – removing one of the correlated variables (Farrar & Glauber, 1967) – cannot be used when testing for curvilinearity, because the focal variable and its polynomial transforms have to remain in the model. Other methods of dealing with collinearity include ridge regression, principal components regression, shrunk estimates, and partial least squares (for a comprehensive review, see Wold et al., 1984). These methods, for the purpose of testing curvilinearity of a relationship, may be simply ineffective as they would not partial out the contribution of the monomial and each of the polynomial terms. This leaves researchers with two options: to use a transformation on a variable of interest and its polynomials to improve precision of estimates, or leave the issue unattended. Literature review revealed that currently, 81% of the published studies chose the latter approach, ignoring the collinearity between polynomials altogether. Doing so leaves the precision of the estimates, and claims of the relative impact of each individual variable reported by these studies questionable.

In the small number of studies that use transformations to combat multicollinearity, the methods used are often inappropriate. For example, about 90% of the reviewed stidies that did perform transformation, used mean-centering, shown by Echambadi & Hess (2007) to be ineffective for collinearity reduction. This issue has to be at least partially due to the fact that currently, a multitude of methods exist for dealing with collinearity (e.g., Aiken & West, 1991; Cohen & Cohen, 1983; Cohen et al., 2003; Neter et al., 1990). However, contributing to the confusion may be the fact that, to the best of our knowledge, none of the studies provide a comprehensive review of all the transformations, compare the methods to each other, and offer specific recommendations for use in organizational and behavioral research. Therefore, such review is warranted.

Another potential issue of concern that was found during literature review was inadequate reporting practices, especially assumptions of the statistical tests and full correlation matrices. In this study, only 3.4% of reviewed studies have reported the check for data normality. This issue is widespread in all areas of social sciences: as Osborne et al. (2001) indicate, few articles report having tested assumptions of the statistical tests they rely on for drawing their conclusions. This is probably due to the fact that some of the tests' requirements are “robust” to violations, and researchers often ignore the tests' pre-requisites (Osborne & Waters, 2002). However, normality is not one of the requirements that is robust to violations (Osborne & Waters, 2002). As distributions depart from normality, statistical inference become less robust (Bradley, 1982), especially under conditions of multicollinearity (Vasu, 1979).

Lack of reporting of full correlation matrices is a problem that has been addressed in management literature in the past. A study by Boyd et al. (2005) indicated that 71.4% of the studies they've reviewed included correlation matrices; one-third of those, or 25% of the total, included all the variables in the matrix. Consistent with the findings of Boyd et al. (2005), which mentioned that polynomial variables were regularly missing from correlation matrices, current literature review found that only 17.6% of all studies included polynomial terms in the correlation matrices. This is a disturbing trend, since replicability of studies diminishes substantially when readers do not have access to full correlation matrices. In addition, the impact of polynomial collinearity on the results of the study cannot be fully evaluated when these correlations are not reported.

As it is apparent that current methodological practices involving polynomial transformation are somewhat deficient methodogically, and exemplar journals must maintain a high standard of methodological rigor (Hoetker, 2007), the purpose of the present study is three-fold. First, we summarize transformation methods currently used to address polynomial multicollinearity; second, compare performance of these techniques in a variety of methodological settings using Monte Carlo simulation, and finally, provide best practices of addressing polynomial collinearity.

Polynomial Transformations

Literature offers several transformations for reducing collinearity (e.g., Shacham & Brauner, 1997). In addition to commonly accepted methods, this paper also presents other transformations from statistical literature to offer researchers the broadest spectrum of transformations for their specific needs.

Mean-centering. A number of researchers, such as Aiken and West (1991) and Jaccard et al. (1990), recommend using mean-centering for alleviating collinearity concerns. Mean-centered is achieved by subtracting the mean of the variable from each observation: $x_i - \bar{x}$. This is one of the most common methods of addressing collinearity in the social sciences, but as Echambadi & Hess (2007) show, it does not alleviate multicollinearity in moderated regression. Echambadi & Hess (2007) show in detail the mathematics behind their findings, so they are omitted here. But mean-centering is included in the Monte Carlo simulation of this study to demonstrate that the same logic applies to collinearity induced by polynomial transformations of the variables, and that mean-centering does nothing for collinearity reduction.

Mean-centering and rescaling. Also known as standardizing, this technique for reducing collinearity was proposed by several researchers, and specifically for polynomial collinearity by Kim (1999). This transformation (with result notated as *z*) is accomplished by first subtracting the mean of the variable from each observation, and dividing the result by the standard deviation: $Z_x = \frac{x_i - \bar{x}}{\sigma_x}$. As Kim (1999) shows, the collinearity between polynomial terms is substantially reduced, while correlations of polynomial terms with other variables are unaffected. However, Kim (1999) does not indicate whether this transformation performs consistently under various methodological settings – an issue that is addressed in the current study.

A substantial limitation of this transformation is that it cannot be used for analysis of covariance structures models. As a result of the transformation, all variables have a mean of zero and standard deviation of one, deeming them unusable by methods such as structural equation modeling (Cudeck, 1989).

The transformation $\boldsymbol{W_i = \frac{(x_i - x_{min})}{(x_{max} - x_{min})}}$, for the purpose of polynomial collinearity reduction, was analyzed by Shacham and Brauner (1997), and described in detail by Wagner (1973). Transformed values range from 0 to 1, making resulting parameter estimates easy to compare and interpret. While both studies show that *Wi* transformation results in substantial reduction in polynomial collinearity, they do not indicate, again, whether this transformation performs consistently under different settings.

The transformation $\boldsymbol{Z_i = \frac{2x_i - x_{max} - x_{min}}{x_{max} - x_{min}}}$ was first introduced by Seber (1977) and recommended for polynomial collinearity reduction by Shacham & Brauner (1997). The distribution of the transformed variable ranges from -1 to 1, again making the resulting parameter estimates easy to interpret. While this transformation was determined by Shacham & Brauner (1997) to be the most effective for polynomial collinearity reduction (compared to *Wi* and dividing the value by the variable's max), the study did not test performance of this transformation under different settings.

Orthogonal polynomials have been recommended as a very effective method for collinearity reduction (Seber, 1977). There is a wide variety of techniques for generating orthogonal polynomials. For example, ORTHOG command in STATA generates a list of orthogonal variables using a modified Gram-Schmidt orthogonalization algorithm (Golub & Van Loan, 1989). One of the criticisms of this approach is interpetability of the regression coefficients (Yang et al., 2006), yet it is supposedly one of the most effective methods for reducing collinearity between polynomials (Shacham & Brauner, 1997). For this study, we are using Legendre polynomials - a method that generates orthogonal polynomials independent of the original data distribution. The method itself, as well as interpretation of the resulting coefficients, is presented in detail in Appendix 1.

Identifying Critical Issues: A Review of the Literature

To identify the critical issues to consider when performing variable transformations, we performed a review of the extant organization research literature. Specifically, we conducted a manual search of six top management journals (*Academy of Management Journal, Administrative Science Quarterly, Journal of Management, Organization Science, Personnel Psychology*, and *Strategic Management Journal*) from the years 2000 to 2010 to identify articles reporting at least one variable transformation. We selected these specific journals because previous research (e.g., Podsakoff et al., 2005) has indicated that these outlets were consistently ranked as having strong impact and reputation in the management field. The time period was selected because we wanted to review the more current, up-to-date trends in the use of variable transformations. Total number of the reviewed articles in all journals over this time period was 4,472.

The Method section of every article was manually reviewed to determine whether any of the variables have undergone a polynomial transformation. If at least one variable was transformed by creating a square, cube, fourth degree, or other degree polynomial, and both the monomial and polynomial variables were used in the analysis simultaneously, the article was selected for further analysis. Out of all articles reviewed, 324 (7.2%) contained at least one variable that met the above criteria, and comprised the final dataset. For each article, we have recorded article orientation (macro or micro), sample size, R-square of the regression models (where applicable), number of variables transformed (with actual variables listed for each article), and polynomial degree utilized (square, cube, fourth, or other). Article characteristics based on the above criteria are presented in Table 1.

Each article with polynomial transformation was further evaluated on the following criteria: type of transformation performed on the polynomial variable for any purpose, including collinearity reduction (none, standard z, orthogonal, other with specification), whether normality checks were performed on the data before and/or other transformation, and whether the correlations between the first and higher order degree polynomials were reported. Average correlations between polynomials before and after transformations, if reported, were recorded. We also took a note of any special methods of dealing with collinearity that were discussed in the article, even if collinearity issues addressed were not specifically aimed at polynomial collinearity. Table 2 reports a summary of the codes for each variable on the above criteria.

As indicated in Table 2, almost 97% of the studies did not mention whether the normality check was performed. To the extent that lack of normality may affect results of some of the statistical procedures, such as ordinary least squares regression, whenever the data is not normal, the resulting reported coefficients become suspect. Further, less than 20% (17.6%) reported all of the variables, including polynomials, in the correlation matrix – a finding consistent with that of Boyd et al. (2005). On some occasions, polynomial transformations were not included in the correlation matrices (or matrices were not provided altogether), but the study did mention high collinearity between polynomials, and indicated the steps taken to reduce it. Therefore, the number of studies reporting some kind of adjustment for collinearity is slightly higher, 19.1%. Out of all methods for collinearity reduction, mean-centering was by far the most frequently used method (in almost 40% of all transformations, or 7.4% of the total sample), despite the fact that it was previously shown to have no effect on collinearity reduction.

We have also noted the average correlations between the polynomials for those studies in which they were reported. Before adjustments for multicollinearity, average correlations between polynomials were over .9 (.94), and after various transformations - .73. It is worth noting that the apparent reduction in collinearity after transformation was due to only a few observations that were very low in magnitude. Out of 22 studies that report adjustments for collinearity, over half (12) showed polynomial correlations over .9 in magnitude, while only five had correlations lower than .5. Without these five studies, where collinearity reduction was achieved by the means of some type of orthogonal transformation (three articles) or standardization (two articles), the average correlation between polynomials was just slightly lower than that of the non-transformed group, with a magnitude of .86.

Method

In order to understand how much of a problem current practices are regarding reports of variable transformations, we conducted several Monte Carlo simulations (Lewis & Orav, 1989). Monte Carlo simulation is an alternative to analytical mathematics, which evaluates the behavior of a statistic using random samples from known populations of simulated data (Mooney, 1997). These analyses are often used to examine the effect of various factors on statistical procedures when actual organizational data is very difficult to obtain or does not exist. In our case, we were specifically interested in the effect that each of the transformation types had on several statistical criteria (Type II error, collinearity reduction, skewness reduction) given various sample conditions (effect size, sample size, and amount of skew) in samples with high collinearity between a variable and its polynomials created to test the significance of curvilinearity in the model. The most common types of polynomials (square, cube, and fourth degree) were tested, because other types of transformation (square root or higher power polynomails) are relatively rare (for examples, see e.g. Wiegelt et al., 2009; Kim et al., 2009).

As is well known and described in statistical literature, high multicollinearity results in a large number of problems (Farrar & Glauber, 1967). When using polynomial transformation for the purpose of testing curvilinearity of the relationships, inflated Type II error rates become, arguably, the most serious problem. This is because under conditions of high multicollinearity, individual effects of monomial variable and its respective polynomials on the dependent variable may not be separable. Therefore, the main outcome of the Monte Carlo simulation presented here is the effect of collinearity on the Type II error rate and resulting power of the test (for a review of Type II error rate vs. the power of the test, see, e.g., Cohen, 1992). Additional outcomes of interest are the percentage of collinearity reduction and reduction in skewness achieved by each of the reviewed transformations.

The study was testing the null hypothesis $\boldsymbol{H_0}$***:*** $\boldsymbol{\beta = 0}$ against an alternative hypothesis, $\boldsymbol{H_a}$***: at least one*** $\boldsymbol{\beta}$ ***is not equal to zero***. Without loss of generality, β_1 (relating y and x, the monomial variable) was pre-specified to be significantly different from zero. Relationships between y and x^2, x^3, and x^4 may or may not have been significant, which would not have affected the outcome of the hypotheses test. Using the recommended power level of .80 (Cohen, 1992), and predetermined effect sizes ranging from 0.05 to 0.95, we have calculated that the minimum size of a sample necessary to detect the smallest effect was 242; largest – 18. To test for the effect of the sample size, taking into account the variability of sample sizes from the reviewed literature, and for the reasons of convenience, we have opted to create samples of 25, 50, 100, and 250 observations for each effect size.

Data

Populations of data were generated using @RISK simulation software (Palisade Corporation, 2010); each population had 10,000 observations. To test for effect sizes, correlations between the two variables of interest (x and y) were set to vary from 0.1 to 0.9 in increments of 0.1. In addition, one effect sizes (0.05) was added to the study. To assure that the strength of the relationship was at least as high as specified in advance, we used the simulation methodology for generating bivariate polulations of Lewis & Orav (1989). The procedure is as follows (Lewis & Orav, 1989: 301):

1. Generate independent $N(0,1)$ variables X_1 and X_2
2. If ρ=0, set $X = X_1$ and $Y = X_2$
3. Else, set $X = X_1$ and $Y = \rho X_1 + (1- \rho^2)^{1/2} X_2$

This methodology is conservative, and in most cases, resulting correlations between *x* and *y* become higher than specified, but it assures a reduced chance of Type II error due to data variations.

Simulation Factors

Type of transformation. We examined five popular types of polynomial transformations in our analyses: mean-centering, standardized (z-score), W_i, Z_i, and Legendre. A summary of explanations and formulas for these transformations are described above and provided in the Appendix.

Amount of skew. As was recommended in the methodological literature (Lewis & Orav, 1989), the amount of skew present in the relationship was manipulated to be either heavily skewed, moderately skewed, lightly skewed, or none (a normal distribution), corresponding to the skewness coefficients of approximately 3, 2, 1, and 0. As was mentioned above, normality is one of the statistical assumptions not robust to violations. Previous literature (e.g., Vasu, 1979) has also shown that effects of multicollinearity may become more severe in conditions of increased non-normality. Therefore, to test performance of each transformation under conditions of non-normality, we generated three populations of different non-normality levels. Without loss of generality, we varied skewness for each effect size without adjusting kurtosis, as skewed distributions present sufficient conditions of non-normality. This is because the higher the absolute value of the skewness, the higher the kurtosis (Mooney, 1997), so it was sufficient to control for only one of these values.

We followed recommendations of Lewis & Orav (1989), and generated non-normal populations using gamma distributions. For lightly skewed distributions, we used [Gamma(5,1)]; for moderately skewed distributions – [Gamma(1,1)], and heavily skewed distributions – [Gamma(0.5,1)] (Lewis & Orav, 1989: 220). Resulting populations were lightly, moderately, and severely skewed, corresponding to skewness index of approximately 1, 2, and 3. The fourth population for each effect size was normally distributed, with both skewness and kurtosis pre-specified to be zero. It was necessary to control for normality of kurtosis in the last population because while there is a direct positive relationship between increasing skewness and kurtosis, the reverse is not always true (Mooney, 1997) – that is, it is possible to have bell-shaped distributions with high kurtosis number.

Simulation procedure

The simulation program was written in Visual Basic using VB.NET, with Microsoft Excel 2010 as an output interface. The program performed the following procedure: using a random number generator, a sample of n = 25, 50, 100, and 250 pairs of *x* and *y* observations (following the bivariate variable generation procedure described above) was drawn from the pre-determined population of a certain effect size. Then, for each sample, additional variables were generated in by creating a square, cube, and fourth-order polynomial transformation of the x-variable. Next, each sample was taken through a series of transformations: mean-centering, standardization, W_i, Z_i, and Legendre orthogonal transformation. Resulting samples were then subjected to multiple regression procedure, regressing x, x^2, x^3, and x^4 on y. Regression results (parameter estimates, standard errors, p-values, R-square, F-value), as well as sample skewness and correlations between polynomials before and after transformations were recorded, and procedure was repeated again. While there is no general agreement on the number of simulation runs that is adequate for each combination of parameters, Mooney (1997) recommended adjusting the number of runs to the desired acceptable rate of Type I error (alpha), calculated as $1/\sqrt{n}$, where n is the number of runs. In this study, for each effect and sample size, the procedure was repeated 5,000 times, resulting in an alpha level of approximately 0.01. Results were then processed to calculate the actual Type II error rates for each combination of sample size, effect size, and transformation type.

Outcome variables

Reduction in collinearity. We first assessed the actual reduction in collinearity achieved by applying the specified transformations to the polynomial variables. As was discussed above, various studies have shown the effectiveness of each transformation in isolated extent, but it was important to compare their performance relative to each other. We have calculated percentage reduction in collinearity by comparing the absolute value of correlation coefficient between the monomial variable and its corresponding polynomials for each transformation, against the original correlation on untransformed variables. In some cases, it was necessary to take the absolute value of the correlation coefficient, because some transformations (such as orthogonal) have changed the direction of correlation from positive to negative in some cases, thus resulting in erroneous calculation of over 100% reduction in collinearity.

Reduction in Type II error. Next, we assessed the percentage reduction in Type II error. Type II error refers to the likelihood of wrongly failing to reject a false null hypotheses. The Type II error probability was calculated following the procedure of Duval & Groeneveld (1987) and Mooney (1997). Because the null hypothesis was known to be false in all of the populations with effect size (correlation between *x* and *y*) higher than zero, the percentage reduction in Type II error was calculated as the proportion of trials for which the false null was (incorrectly) not rejected – that is, for trials where the regression coefficient for the relationship between *x* and *y* was not significant at a 0.05 level. As was specified above, we have only looked at the Type II error rate on the relationship between *y* and *x* variables, as this was the only relationship pre-set in advance to be significant for each effect size higher than zero. Any relationships between *y* and x^2, x^3, and x^4, even if significant, were ignored from calculations. This was because these relationships were not specified to be significant *a priori*, and could have been erroneous as a result of a Type I error.

Reduction in skewness. Because we varied the normality of the pseudopopulations, and applied transformations have changed the mathematical relationships between the variables, it was reasonable to expect at least some changes in the shapes of the distributions. We have checked the skewness achieved as a result of each transformation, and compared it to the original skewness number for the same sample before the transformation to see if there was any difference. Results were then aggregated for each sample as a simple average.

Results

Before we evaluated the effect of transformations on each of the outcome variables, we have compared sample means for all samples with different effect size, skeweness size, and normality, to see if it was possible to aggregate some of the results by one of the parameters. We used Bonferroni correction to control for the familywise inflation of Type I error. We have determined that mean differences in skewness and collinearity were significant on at least the 0.0001 level for the level of skewness and the sample size, but none of the sample mean differences have reached even the traditional significance level of 0.05 for the effect size. However, for reduction in Type II error, mean differences were significant on at least the 0.0001 level for all three simulation factors – the level of skewness, the sample size, and the effect size. Therefore, for analysis of skewness reduction and collinearity reduction, simulation results were aggregated on an effect size level, but were analyzed separately for each sample size and level of normality. For Type II error reduction, results were analyzed separately for each sample size, effect size, and normality level.

Collinearity reduction. Results of collinearity reduction between polynomials, achieved by different transformations, are presented in Table 3. We have observed that despite the fact that there are significant mean differences for collinearity reduction for the sample size and normality, the general pattern of collinearity reduction was approximately identical for all sample sizes and all transformations except the *Zi*. This general pattern is presented in Figure 1. Results indicate that for heavily skewed distributions, none of the transformations, with exception of *Zi*, provide adequate reduction of collinearity. As sample sizes increase, so does improvement in the performance of transformations, with *Legendre transformation* offering the highest percentage reduction for all sample sizes and skewness levels. The pattern of the *Zi* transformation is somewhat different – it varies and does not to appear to depend significantly on sample size. Therefore, if collinearity reduction (not the Type II error reduction) is the only goal of the transformation, than we recommend using *Zi* transformations for smaller sample sizes, followed by *Legendre* transformation and standardization.

Reduction in Type II error. Complete results of Type II error reduction, achieved by different transformations, are presented in graphical form in Figures 2-5. In each figure, four graphs correspond to four different sample sizes; vertical axes show percentage reduction in collinearity, and horizontal axes – effect sizes.

Results indicate that the pattern of reduction in Type II error is actually quite different than the pattern of reduction in collinearity. First, it depends not only on skewness level, but also on sample size. Second, it appears that for smaller effect sizes (that is, when collinearity between polynomials is rather small) and small samples, none of the methods are substantially effective. Noticeable reduction in collinearity for smaller sample sizes starts when collinearity reaches 0.3-0.5. However, it's worth noting that such small collinearity effect size is probably has no consequences on the results of the analysis, anyway, and transformations with such effect sizes may not even be warranted. Overall, *Legendre* transformations and standardization perform by far superior compared to other three methods, achieving, in some cases, almost 100% reduction in Type II error.

Reduction in skewness. Even though much more effective methods are available for skewness reduction, we have found that some of the transformations analyzed here also improve the data by reducing non-normality. However, only the *Zi* transformation, for all sample and effect sizes, showed consistent and reliable results for skewness reduction. As was mentioned earlier, if reduction in collinearity is the ultimate goal of the transformation, then *Zi* becomes the transformation of choice as it also reduces skewness of the sample somewhat.

Summary and Recommendations

Results of the Monte Carlo simulation, discussed above, can be summarized as follows. With respect to individual performance in collinearity reduction, mathematical *Zi* provides the highest absolute reduction in collinearity, working better for smaller samples and heavy skewness; orthogonal polynomials and standardization work better for larger samples; mean-centering provides zero reduction in collinearity, and reduction achieved by *Wi* is negligible. With respect to individual performance in Type II Error reduction, orthogonal polynomials provide the highest absolute reduction in the rate of Type II error, closely followed by the standardization. Reduction achieved by other methods depended on the level of non-normality and sample size; mean-centering provided zero reduction in Type II error.

As a result, we recommend the following. First, for all analysis methods involving polynomial variable transformations, we recommend checking for normality and reduction in non-normality of the sample, whenever possible (log, square root, other transformation methods). To alleviate problems with individual transformation performance, we recommend increasing sample size, whenever possible, as percentage reduction in Type II error was higher with all methods for larger sample sizes.

Second, the type of transformation used should also depend on the purpose for which transformations are implemented. If polynomial variables are used as controls (such as firm size and firm size squared to control for non-homogeneity in the size of the companies used for analysis), then individual effects of a variable and its polynomial are not relevant; they are only relevant as a block. As a result, when individual parameter estimates are not relevant, we recommend to use orthogonal polynomials for all samples and skewness levels if using analysis of covariance models. The method is complex, but achieves the best results. Standardization is acceptable to reduce complexity of the analysis, especially if using OLS or other non-covariance structure methods, as it cannot be used in, for example, structural equation modeling techniques which require variables to have non-unity variance.

When polynomial variables are used as individual independent variables, so polynomial transformations are used for hypothesized curvilinearity effects, we recommend to use mathematical Zi transformations for extremely small sample sizes as this is the only method that achieves notable reduction in both collinearity and type II error; use orthogonal polynomials for smaller samples or higher levels of non-normality, or with covariance structure models, and use standardization for medium- to large sample sizes with non-covariance-structure methods.

Conclusion

The purpose of this study was to evaluate the transformation methods currently used to address polynomial multicollinearity, compare performance of these techniques in a variety of methodological settings using Monte Carlo simulation, and provide best practices of addressing polynomial collinearity. As we have shown in the paper, current state of management research leaves much to be desired in terms of methodological rigor with respect to polynomial transformations. We have shown that existence of polynomial collinearity presents a substantial problem not only in terms of unreliable relationship coefficients, but also in terms of an increased chance of Type II error – that is, inability to find the effect when it is in fact present. We have also shown that not all transformations perform equally in all settings of sample sizes and collinearity effects between polynomials, and provided specific recommendations for their use depending on the setting.

One of the major limitations of this study is our inability to demonstrate that existence of polynomial colliearity between variables also increases the chance of Type I error – the more dangerous type that finds the presence of an effect where there is actually none. As Vasu (1979) has demonstrated in her working paper, polynomial collinearity under conditions of non-normality can as much as triple the chance of type I error. However, much like Vasu (1979), though we have observed an increase in type I error in our simulations, we are unable to decisively prove this claim, as we can never guarantee that the underlying population of data has an exactly zero effect size between a variable and its polynomial transformation. We have attempted to generate many populations with effect size of zero, but none of them, upon testing, resulted in an exactly zero effect size, though some were as small as 0.001. Even with that small of an effect size, there is a potential relationship if a sample is large enough, and as a result, we cannot claim that the population has a true effect size of zero between polynomials, and any increase in Type I error are not due to the true, however minimal, correlation between the variables. We hope that future studies are able to find a way to decisively demonstrate the damaging effects of polynomial collinearity, including increased chance of a Type I error, but in the meantime, we hope we have convinced researchers to use polynomial transformations in their studies more widely.

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Table 1
Summary of Organizational Studies Reporting Variable Transformations

Journal	No. of studies w/ polynomial transformations	Domain		Range of sample size		Mean no. of variables transformed	Polynomial degree			
		Macro	Micro	Low	High		Square	Cube	Fourth	Other
Academy of Management Journal (AMJ)	118	76.3%	23.7%			1.59	100.0%	3.4%	0.8%	0.8%
Administrative Science Quarterly (ASQ)	41	87.8%	12.2%			1.51	100.0%	0.0%	0.0%	0.0%
Journal of Management (JOM)	9	66.7%	33.3%			2.11	100.0%	11.1%	0.0%	0.0%
Organizational Science (OS)	33	75.8%	24.2%			1.73	100.0%	0.0%	0.0%	3.0%
Personnel Psychology (PP)	11	9.1%	90.9%			1.36	100.0%	0.0%	0.0%	0.0%
Strategic Management Journal (SMJ)	112	100.0%	0.0%			1.61	99.1%	4.5%	0.0%	0.9%
Total	324	83.3%	16.7%			1.61	99.7%	3.1%	0.3%	0.9%

Table 2
Summary of Methodological Approaches to Polynomial Collinearity

	Type of transformation performed to correct for multicollinearity					Normality check performed			Articles reporting correlations between polynomials	Average reported Correlations	
Journal	**None**	**Mean-centering**	**Standard Z**	**Orthogonal**	**Other**	**None**	**Before**	**After**		**Without transforms**	**With transforms**
AMJ	77.1%	5.1%	16.9%	0.8%	0.8%	96.6%	0.8%	4.2%	17.8%	0.94	0.78
ASQ	82.9%	7.3%	0.0%	7.3%	2.4%	100.0%	0.0%	0.0%	26.8%	0.96	0.51
JOM	77.8%	22.2%	0.0%	0.0%	0.0%	100.0%	0.0%	0.0%	0.0%	N/A	N/A
OS	63.6%	24.2%	6.1%	0.0%	6.1%	87.9%	0.0%	12.1%	21.2%	0.94	0.70
PP	90.9%	0.0%	9.1%	0.0%	0.0%	81.8%	0.0%	18.2%	9.1%	0.94	N/A
SMJ	88.4%	4.5%	6.3%	0.0%	1.8%	100.0%	0.0%	0.0%	15.2%	0.93	0.81
Total	80.9%	7.4%	9.3%	1.2%	1.9%	96.9%	0.3%	3.4%	17.6%	0.94	0.73

Table 3
Collinearity Reduction Achieved by Transformations (from the Monte Carlo Simulations)

Sample size / Normality	Correlation before transformation	Mean-centering Raw	Mean-centering % Reduction	Standardization Raw	Standardization % Reduction	Wi Raw	Wi % Reduction	Zi Raw	Zi % Reduction	Legendre Raw	Legendre % Reduction
		Achieved collinearity reduction with transformation									
n = 25											
Heavily-Skewed	0.93	0.93	0.0%	0.90	3.9%	0.93	0.0%	0.02	98.3%	0.89	4.8%
Medium-Skewed	0.93	0.93	0.0%	0.86	8.0%	0.93	0.1%	0.07	92.1%	0.85	8.8%
Lightly Skewed	0.95	0.95	0.0%	0.68	28.8%	0.93	2.3%	0.21	77.6%	0.67	29.5%
Normal	0.98	0.98	0.0%	0.30	69.3%	0.93	4.9%	0.31	68.6%	0.28	71.1%
n = 50											
Heavily-Skewed	0.93	0.93	0.0%	0.88	5.4%	0.94	0.4%	-0.04	95.9%	0.86	8.2%
Medium-Skewed	0.93	0.93	0.0%	0.84	9.4%	0.93	0.3%	0.00	99.7%	0.82	11.5%
Lightly Skewed	0.95	0.95	0.0%	0.67	28.7%	0.93	1.6%	0.13	85.8%	0.64	32.3%
Normal	0.98	0.98	0.0%	0.30	69.7%	0.94	4.2%	0.30	69.2%	0.28	71.4%
n = 100											
Heavily-Skewed	0.89	0.89	0.0%	0.86	3.4%	0.89	0.0%	-0.17	80.8%	0.86	4.2%
Medium-Skewed	0.89	0.89	0.0%	0.84	6.3%	0.89	0.0%	-0.15	83.6%	0.83	7.4%
Lightly Skewed	0.93	0.93	0.0%	0.73	21.5%	0.91	2.0%	0.03	97.3%	0.72	22.3%
Normal	0.97	0.97	0.0%	0.29	70.4%	0.93	4.2%	0.29	69.8%	0.27	72.2%
n = 250											
Heavily-Skewed	0.89	0.89	0.0%	0.86	3.0%	0.89	-0.5%	-0.24	72.6%	0.85	3.9%
Medium-Skewed	0.89	0.89	0.0%	0.83	6.0%	0.89	-0.6%	-0.23	73.9%	0.82	7.1%
Lightly Skewed	0.92	0.92	0.0%	0.72	21.9%	0.91	1.2%	-0.10	89.1%	0.71	22.7%
Normal	0.97	0.97	0.0%	0.29	70.5%	0.94	3.3%	0.29	70.3%	0.27	72.3%

Figure 1
Collinearity Reduction Achieved by Transformations

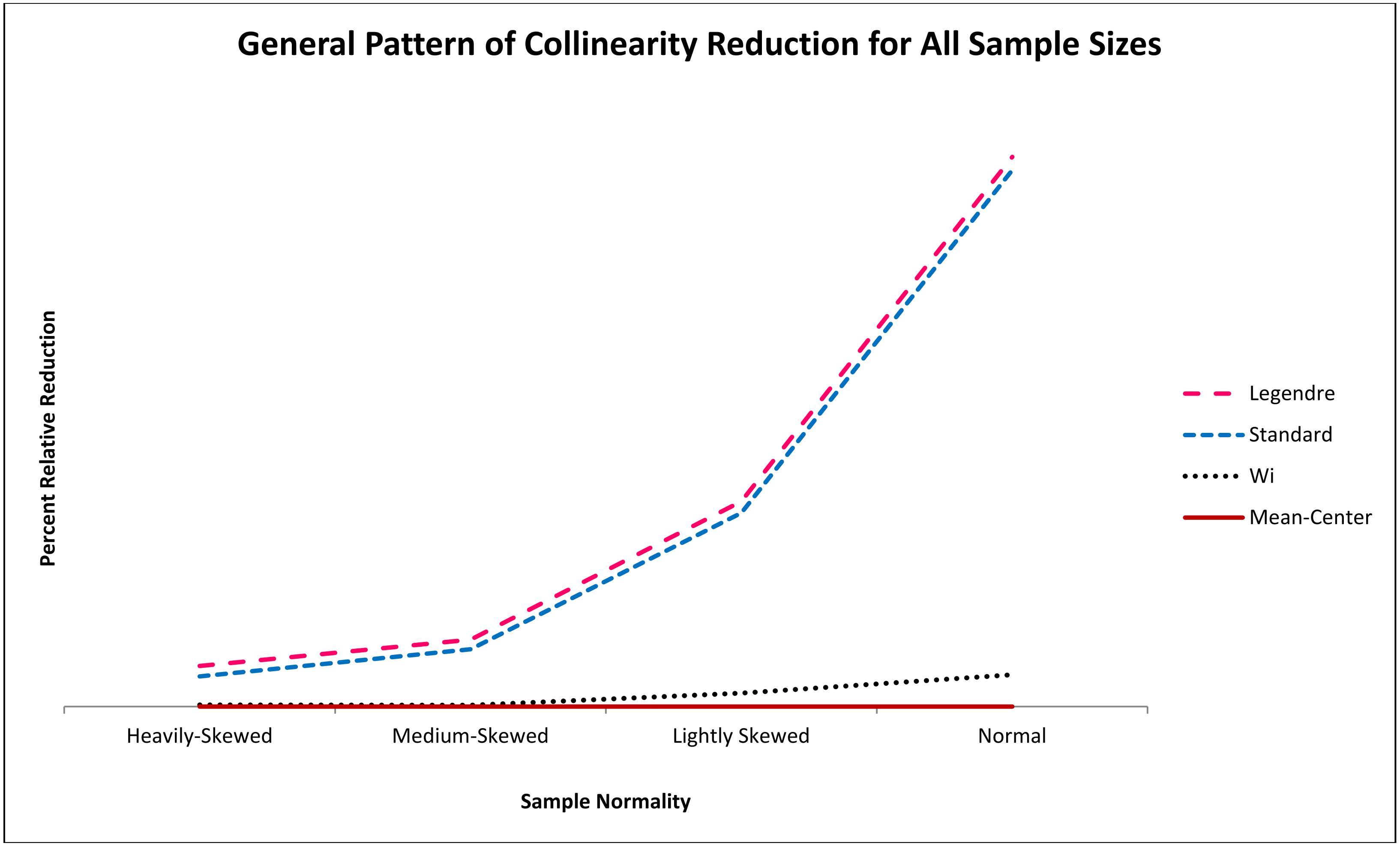


Figure 2. Type II Error Reduction Achieved by Transformation for Heavily Skewed Distributions

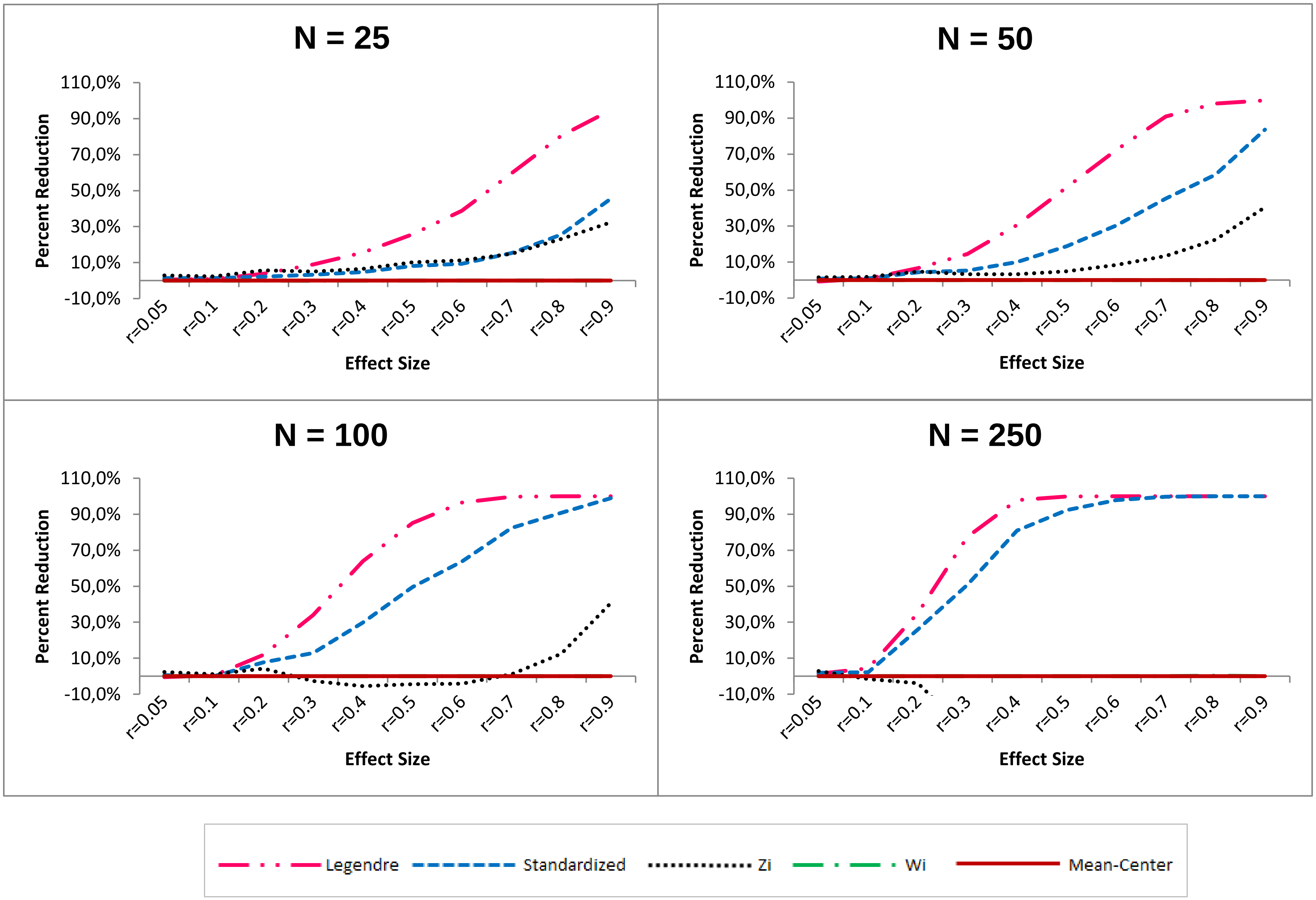


Figure 3. Type II Error Reduction Achieved by Transformation for Moderately Skewed Distributions

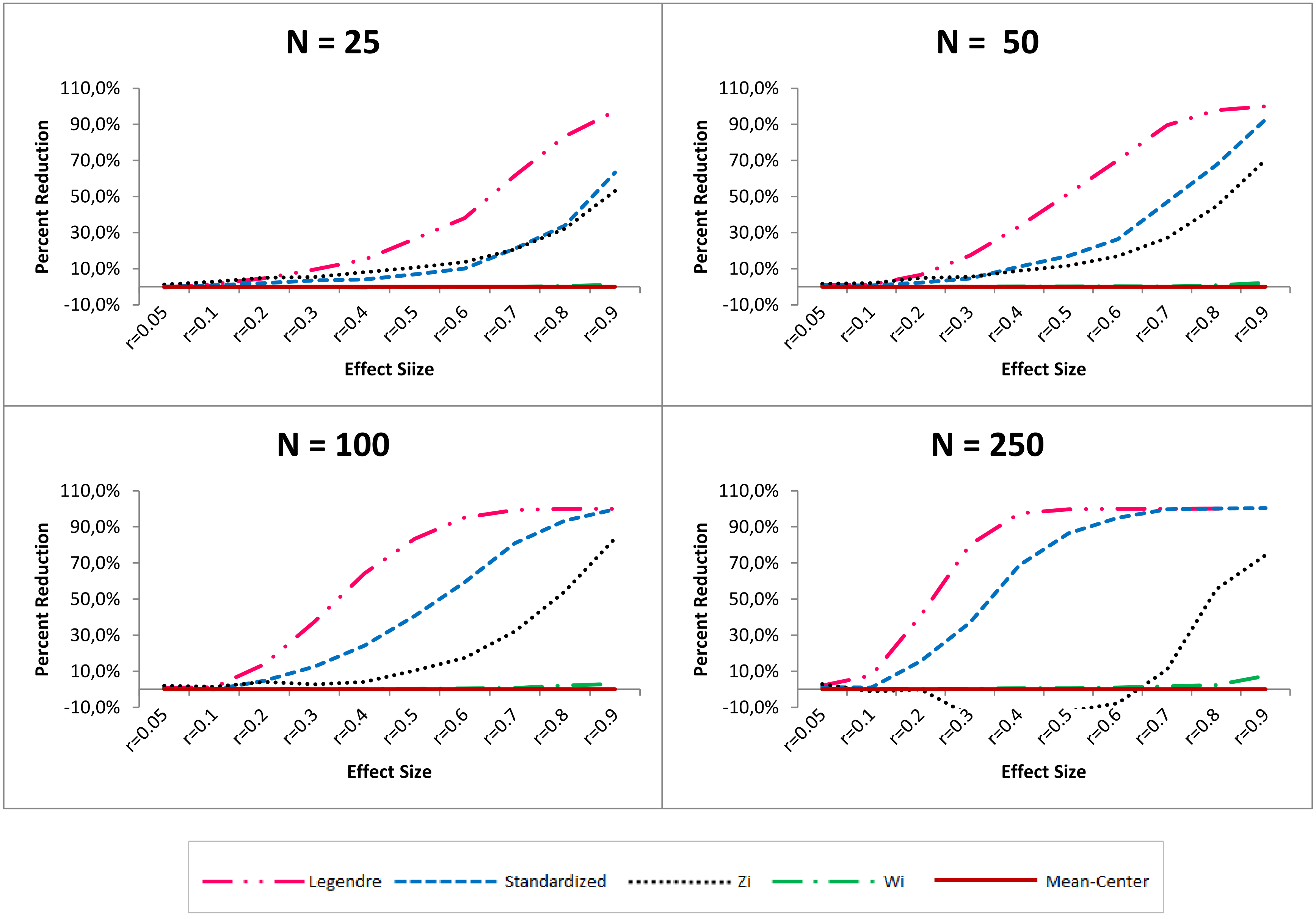


Figure 4. Type II Error Reduction Achieved by Transformation for Lightly Skewed Distributions

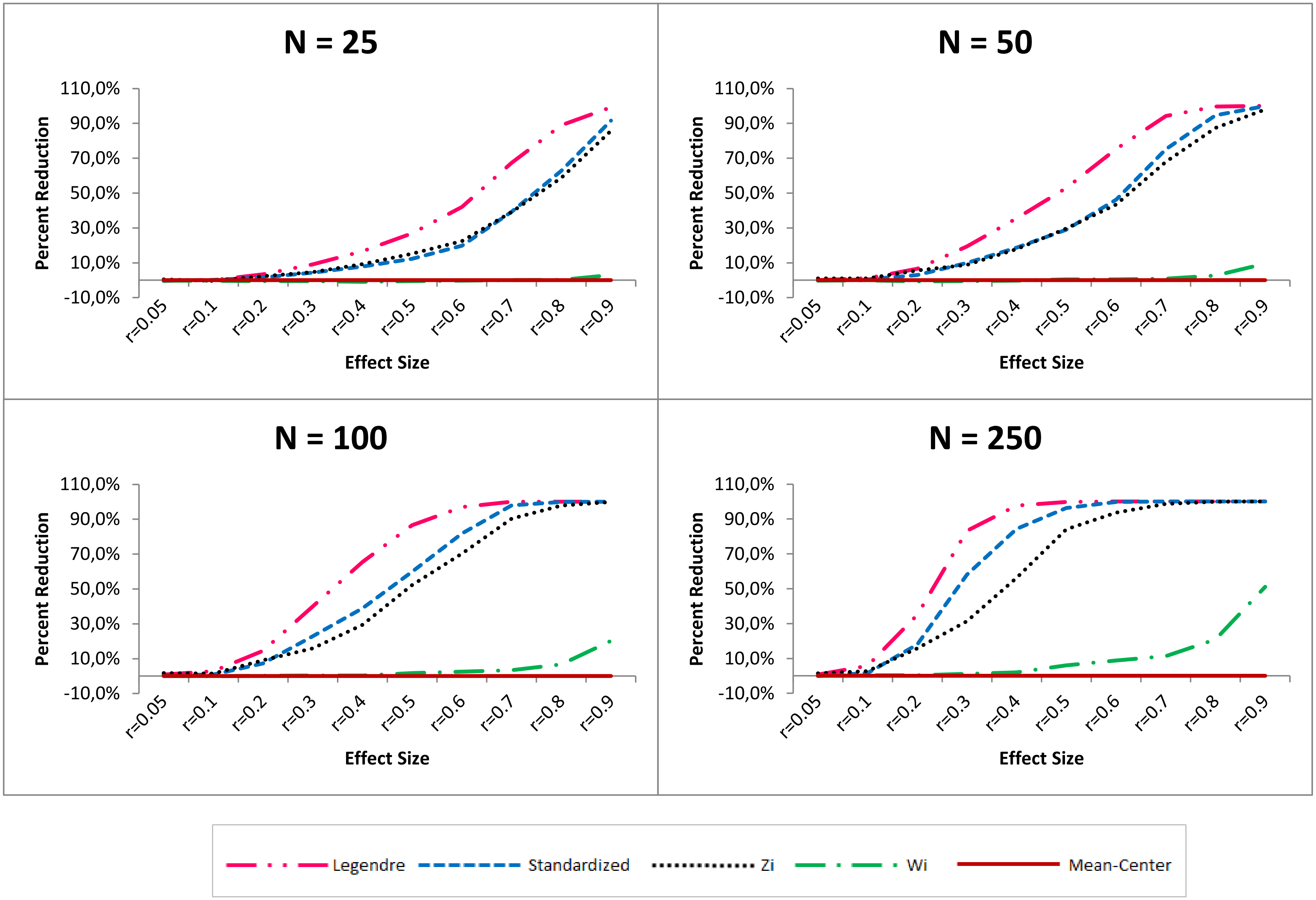
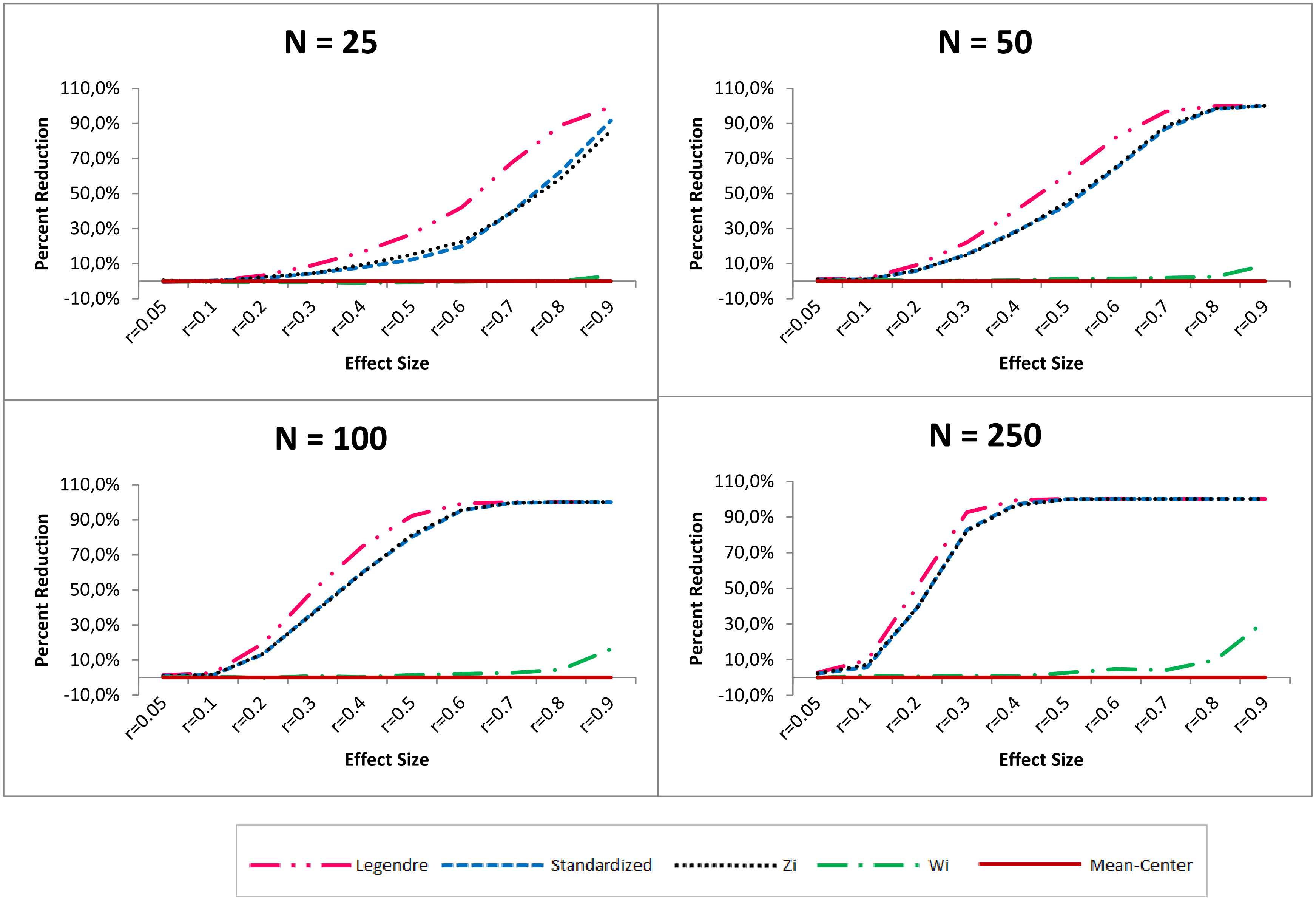


Figure 5. Type II Error Reduction Achieved by Transformation for Normal Distributions



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