Income Distribution, Market Structure, and Individual Welfare*

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December 2009

Abstract

This paper explores how income distribution affects market structure, prices, and economic well-being of different consumer groups. I consider a general equilibrium model of monopolistic competition with free entry, heterogenous firms and consumers that share identical but non-homothetic preferences. The results in the paper suggest that poverty reduction might be of a greater importance than lowering income inequality, as lower income inequality does not necessarily lead to welfare gains of the poor. In particular, I show that higher income inequality may benefit the poor via a trickle-down effect operating through the entry of firms into the market.

*I am grateful to Kala Krishna and Andrés Rodríguez-Clare for their invaluable guidance and constant encouragement. I also would like to thank Pol Antràs, Edward Green, Sergey Lychagin, James Tybout, Kei-Mu Yi, conference participants at the Spring 2007 Midwest Economic Theory meetings and 2007 North American Summer Meeting of the Econometric Society for helpful comments and discussions. All remaining errors are mine.

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1 Introduction

What are the possible consequences of income redistribution for market structure, consumption allocation, and welfare? As Atkinson and Bourguignon (2000) argue, "it is difficult to think of economic issues without distributive consequences and it is equally difficult to imagine distributive problems without some allocational dimension". There is a large empirical and theoretical literature that relates income distribution and inequality to a number of social and economic outcomes.[1] Alesina and Rodrik (1994) show that a rise in income inequality has a negative impact on economic growth (see also Persson and Tabellini (1994)). Waldmann (1992) argues that the level of inequality is positively correlated with infant mortality. Glaeser *et al.* (2003) find that high inequality can negatively affect social and economic progress through the subversion of institutions in the economy. This paper provides another insight into the interaction between income distribution and economic outcomes, which has not been explored extensively. In particular, I show that higher income inequality may benefit the poor via a trickle-down effect operating through entry. This finding suggests that since lower income inequality does not necessarily lead to poverty reduction, focusing on poverty reduction might be more important than focusing on income inequality.

I consider a general equilibrium model of monopolistic competition with heterogenous firms and consumers. I assume that all consumers share identical but *non-homothetic* preferences and differ in their incomes.[2] Nonhomotheticity of the preferences and income heterogeneity imply that changes in prices may affect different groups differently. Furthermore, the presence of market power induces variable markups across firms, which are in turn affected by the income distribution. As a result, changes in the income distribution may have different consequences for different groups of agents.

I adopt the preference structure from Murphy *et al.* (1989) and Matsuyama (2000). The basic idea is that goods are indivisible and consumers buy at most one unit of each good. This implies that given the prices, goods are arranged so that consumers can be considered as moving down some list in choosing what to buy. For instance, in developing countries, consumers first buy food, then clothing, then move up the chain of durables from kerosene stoves to refrigerators, to

[1] See Atkinson and Bourguignon (2000) for literature review.

[2] In the paper, the income distribution is exogenous. I deliberately leave out of the scope the framework with the endogenous income distribution, as my main goal is to understand the effects of changes in income distribution not in some other parameters.

cars. Hence, the consumer utility can only be increased by the consumption of a greater number of goods. Moreover, consumers with higher income purchase the same set of goods as consumers with lower income plus some others.

This structure of consumer preferences has enough flexibility to be applied as to the whole economy as to a certain industry where goods differ in quality. On the one hand, each good can be interpreted as a distinct good sold in the market. In this case, the structure describes the whole economy. On the other hand, we might think that firms sell not distinct goods but some characteristics of a good produced in a certain industry. For instance, consider a car industry. Each good can be treated as some characteristic of a car. The poor purchase main characteristics associated with a car, while the rich buy the same characteristics as the poor plus some additional luxury characteristics. That is, both groups of consumers buy the same good but of different quality.

I assume that goods differ in terms of the valuations consumers attach to them. By the valuation of a good, I mean the utility delivered to consumers from the consumption of one unit of this good. That is, there are goods that are more essential in consumption (necessities) and goods that are less essential (luxuries). There is free entry in the market. To enter the market, ex ante identical firms have to make costly investments that are sunk. Once firms enter, they learn about the valuations attached to their goods. The only source of firm ex-post heterogeneity is the difference in the valuations placed on their goods.[3] Depending on the valuations drawn, firms choose whether to stay or to leave the market. Firms that decide to stay engage in price competition with the other firms. This leads to the endogenous distribution of markups, which is affected not only by the market size, but also by the income distribution in the economy. Hence, the model incorporates two key features: imperfect competition and non-homothetic preferences, which allow us to analyze the effects of changes in income distribution on equilibrium prices, market structure, and welfare of different groups.

The paper focuses on the case with two types of consumers: rich and poor.[4] Depending on the valuations of their goods, firms are endogenously divided into three groups in the equilibrium. Firms with high valuations choose to serve all consumers. Firms with medium valuations decide to sell only to the rich. Finally, firms with low valuations leave the market. In the paper, I

[3] To simplify the analysis, I assume that marginal costs of production are identical across all firms.

[4] Though the model can be extended to the case with many consumer types, the analysis in this case is quite complicated. See Appendix B for details.

analyze the effects of changes in the income and the fraction of the rich on the welfare of poor consumers.

A rise in the income level of the rich has two effects. First, it leads to redistribution of firms across the groups. Second, more firms enter into the market, which in turn results in tougher competition. The former effect is negative for the poor, while the latter is positive. I show that the second effect prevails and as a result, the poor gain from a rise in the income of the rich. This is reminiscent of the trickle-down effect in Aghion and Bolton (1997), who show that in the presence of imperfect capital markets, the accumulation of wealth by the rich may be good for the poor. The effect of additional firm entry on welfare arises in models with homothetic preferences as well. For instance, in Melitz (2003) higher income of a certain group would result in more entry, tougher competition, and, thereby, higher welfare of all consumers. However, there is a difference. In the present paper, if the mass of firms is unchanged (the short run), then there is only a negative impact on the poor resulting in welfare losses. While in models with homothetic preferences, if the mass of firms is fixed, higher income of certain consumers does not affect welfare of the others. In addition, in the present model, higher income of the rich raises the markups of firms selling only to the rich and decreases the markups of firms serving all consumers, while in traditional models, we observe the same or no impact on firms' markups.

Another interesting question is to compare welfare of the poor in economies with different fractions of the rich. What is better for the poor: tiny minority or vast majority of the rich? Specifically, keeping the consumer incomes fixed, I consider a rise in the fraction of the rich and explore its implications for the poor. As before, there are two effects. First, some firms that served all consumers find it more profitable to sell only to the rich. This redistribution decreases welfare of the poor. Second, the larger fraction of the rich results in more firms entering the market. This in turn leads to tougher competition and, therefore, increases welfare of poor consumers. I find that if the fraction of the rich is sufficiently small, then the positive effect prevails and as a result, the poor benefit from a higher fraction of the rich. While if the fraction of the rich is sufficiently high, the opposite happens. Hence, we might expect that welfare of the poor has an inverted U shape as a function of the fraction of the rich.

There is a common feature of both comparative statics considered above. An increase in the personal income as well as in the fraction of the rich raises the aggregate income in the economy. In the view of policy implications, I explore the effects of changes in the income distribution keeping the aggregate income constant. To capture a pure redistribution effect, I consider a rise in the personal income of the rich together with a decrease in the fraction of the rich keeping aggregate income fixed. I show that these changes in the income distribution result in higher welfare of the poor. That is, higher income inequality may benefit the poor. The effect is based on the entry of firms in the market. In particular, I show that the changes considered result in more entry and, therefore, tougher competition in the market. This in turn reduces the prices of the necessities increasing welfare of the poor.

The related literature in this area can be divided into three strands. First, there are papers that consider monopolistic competition models assuming homothetic or quasi-linear preferences (see for instance Melitz (2003) and Melitz and Ottaviano (2005)). However, in traditional models of monopolistic competition, income distribution plays no role. If preferences are identical and homothetic, it is well understood that the distribution of income does not affect equilibrium: only aggregate income matters. When preferences are quasi-linear, the presence of a numeraire good eliminates the impact of income distribution on equilibrium outcomes. Hence, in those models, any price changes have the same impact on all consumers regardless of whether consumers have identical incomes or not.

The second group of papers explores the implications of non-homothetic preferences in a perfectly competitive environment. For instance, Matsuyama (2000) develops a Ricardian model of trade with non-homothetic preferences to analyze the interaction between income distribution and trade patterns.[5] Meanwhile, in a perfectly competitive environment, differences in the equilibrium prices are fully determined by firm technological differences (in our case, by differences in the valuations of goods) and firm profits are equal to zero. Therefore, there is no room for the effects of income distribution on the entry of firms and output prices.

Finally, the third group of papers deals with both monopolistic competition and non-homothetic preferences. Markusen (1986) extends the Krugman type model of trade with monopolistic competition by adding non-homothetic demand. He examines the role of income per capita in interindustry and intra-industry trade. Mitra and Trindade (2005) also consider a model of monopolistic competition with non-homothetic preferences. However, they directly assume the functional relationship between consumer income and its share spent on a certain type of goods, without deriving the dependence from the consumer maximization problem. Ramezzana (2000)

[5] See also Flam and Helpman (1987) and Stokey (1991).

uses the similar preference structure to explore how similarities in per capita incomes affect trade volumes between countries. In his paper, consumers are identical within a country. As a result, the effects of income inequality are assumed away.

Foellmi and Zweimueller (2004) develop a general equilibrium model with non-homothetic preferences and *fixed* mass of *identical* firms. They show that depending on the parameters in the model, higher income inequality has either no impact on firm markups or increases them. In contrast, the findings in my paper suggest that this is not necessarily the case. In fact, higher income inequality affects the prices set by firms differently. Due to free entry, higher income inequality may raise markups for firms that sell their goods only to the rich and reduce markups for firms that sell their goods to all consumers. Foellmi and Zweimueller (2006) examine a dynamic variation of Murphy *et al.* (1989). Assuming learning by R&D, they focus their analysis on the link between possible growth and inequality. In my paper, I address different questions. I do not consider the learning by R&D spillover and explore the impact of income distribution on the level of competition, markups, and individual welfare.

The rest of the paper is organized as follows. Section 2 introduces basic concepts of the model. Section 3 focuses on the case with two types of consumers, rich and poor, and analyzes the effects of income distribution on market structure and individual welfare. Section 4 concludes.

2 The Model

I consider a general equilibrium model of monopolistic competition with heterogenous firms and consumers. The preference structure is adopted from Murphy *et al.* (1989) and Matsuyama (2000).

2.1 Production

The timing in the model is as follows. There is free entry in the market. To enter the market, firms have to make sunk investments f_e. If a firm incurs the costs of entry, it obtains a draw b of the valuation of its good from a common distribution $G(b)$ with the support on $[0, B]$. This is meant to capture the idea that before they enter, firms do not know how well they will end up doing, as they do not know how highly consumers will value their products. I assume that $G'(b) = g(b)$ exists. The valuation b is interpreted as the utility delivered to consumers from the consumption of one unit of the good. Depending on the valuation they draw, firms choose to leave the market or to stay. Firms that decide to stay compete in price with the other firms. The only factor of production is labor. I assume that marginal costs of production are the same for all firms and equal to c, i.e., it takes c effective units of labor (which are paid a wage of unity) to produce a unit of any good.[6]

Consumers differ in the number of efficiency units of labor they are endowed with.[7] I assume that there are N types of consumers indexed by n. A consumer of type n is endowed with I_n efficiency units of labor. I choose indices so that $I_n > I_{n-1}$. Let us denote α_n as the fraction of type n consumers in the aggregate mass L. Then, the total labor supply in the economy in efficiency units is $L\sum_{i=1}^{N}\alpha_i I_i$.

2.2 Consumption

All consumers have the same non-homothetic preferences given by the utility function

$$U = \int_{\omega\in\Omega} b(\omega)x(\omega)d\omega,$$

where Ω is the set of available goods in the economy, $b(\omega)$ is the valuation of good ω, and $x(\omega) \in \{0,1\}$ is the consumption of good ω. Each consumer owns a balanced portfolio of shares of all firms. Due to free entry, the total profits of all firms are equal to zero in the equilibrium. This implies that the value of any balanced portfolio is equal to zero. Thus, all consumers have the same wealth, while their incomes vary with their productivity. To simplify the notation, I assume that consumers have equal shares of all firms. Let us denote π as the total profits of all firms in the economy. Given the prices of available goods, a type n consumer maximizes

$$\int_{\omega\in\Omega} b(\omega)x(\omega)d\omega$$

subject to the budget constraint

$$\int_{\omega\in\Omega} p(\omega)x(\omega)d\omega \leq I_n + \frac{\pi}{L},$$

[6] The assumption that marginal costs of production are the same across firms simplifies analytical derivations in the model and does not change qualitative results. In general, we can assume that marginal costs are also drawn from some common distribution.

[7] Throughout the paper, I use terms, endowments of efficiency units of labor and labor productivities, interchangeably.

where $p(\omega)$ is the price of good ω. The utility maximization problem merely involves moving down the list of products ordered by their valuation to price ratios, $\frac{b(\omega)}{p(\omega)}$, until all income is exhausted.

3 The Case with Two Types of Consumers

As the analysis of the general case with N types is quite complicated (see Appendix B for details), I focus on the case with two types of consumers: a high-income (high productivity) type and a low-income type. The productivity of the high-income type is defined by I_H, the productivity of the low-income type is I_L. Given the preferences, all goods consumed by the less productive type are also consumed by the more productive type. Thus, goods in the economy can be divided into two groups: the "common" group includes goods that are consumed by all consumers; the "exclusive" group includes goods that are only consumed by high-income consumers.

A firm producing a good ω obtains the profits of $(p(\omega) - c)Q(\omega)$, where $Q(\omega)$ is demand for good ω. If all consumers buy the good, then the demand is L. If only the rich purchase it, the demand is $\alpha_H L$, where α_H is the fraction of the high-income type. Hence, $Q(\omega) \in \{L, \alpha_H L, 0\}$ (demand is equal to zero if the price is so high that nobody wants to purchase good ω). Taking the valuation to price ratios of the other firms as given, firms choose prices to maximize their profits. The following proposition holds.

Proposition 1 *In the equilibrium, goods from the same group have the same valuation to price ratio.*

Proof. Suppose not. Then, there exists some group, in which there are at least two goods with different $\frac{b(\omega)}{p(\omega)}$ ratios. Since both goods belong to the same group, a firm producing the good with higher $\frac{b(\omega)}{p(\omega)}$ can raise its $p(\omega)$ without affecting the demand. This in turn would increase its profits and, thereby, contradicts to the equilibrium concept. ■

Define V_C as the valuation to price ratio of goods from the "common" group and V_E as valuation to price ratio of goods from the "exclusive" group. Note that V_C and V_E are endogenous and V_C is strictly greater than V_E.[8] Hence, if a firm with valuation $b(\omega)$ serves all consumers,

[8] By definition, $V_C \geq V_E$. If $V_C = V_E$, then all available goods have the same valuation to price ratios. In this case, the equilibrium concept implies that the high income consumers purchase all goods, while the poor buy

then its price is equal to $\frac{b(\omega)}{V_C}$ and its profits are given by

$$(p(\omega) - c)\,L = \left(\frac{b(\omega)}{V_C} - c\right) L,$$

while if the firm serves only the rich, its profits are given by

$$(p(\omega) - c)\alpha_H L = \left(\frac{b(\omega)}{V_E} - c\right) \alpha_H L.$$

As $V_C > V_E$, the firm chooses between selling to more people at a lower price and selling to fewer of them but at a higher price. In other words, firms choose $p(\omega) \in \{\frac{b(\omega)}{V_C}, \frac{b(\omega)}{V_E}\}$ to maximize their profits taking V_C and V_E as given. Notice that in the equilibrium, the price of good ω depends only on $b(\omega)$. Therefore, hereafter I omit the notation of ω and consider prices as a function of b.

Let b_M be the unique solution of the equation

$$\left(\frac{b}{V_C} - c\right) L = \left(\frac{b}{V_E} - c\right) \alpha_H L. \tag{1}$$

Then,[9]

$$\begin{aligned} \left(\frac{b}{V_C} - c\right) L \ &\geq \ \left(\frac{b}{V_E} - c\right) \alpha_H L \quad \text{if } \ b \geq b_M, \\ \left(\frac{b}{V_C} - c\right) L \ &< \ \left(\frac{b}{V_E} - c\right) \alpha_H L \quad \text{otherwise.} \end{aligned}$$

That is, if a firm draws $b \geq b_M$, then the firm finds it more profitable to serve both types of consumers. Otherwise, the firm chooses to serve only the rich. A firm with valuation b_M of its good is indifferent between selling to all consumers and selling only to the rich (see *Figure 1*). Hence, even in the presence of market power, products have a natural hierarchy. Consumers first purchase goods with higher b, which are more essential in consumption. This result is supportive of the common intuition that the poor spend most of their income on necessities, while the rich can afford to buy not only necessities, but also luxuries.

only some subset of the available goods (for instance, this subset can be randomly determined). This means that the expected demand for a certain good is strictly less than L. Hence, firms can increase their profits by slightly decreasing their prices and acquiring greater demand share. Therefore, if $V_C = V_E$, equilibrium does not exist.

[9] Notice that in the equilibrium, $\frac{\alpha_H}{V_E}$ is strictly less than $\frac{1}{V_C}$. Otherwise, for any $b \geq 0$, $\left(\frac{b}{V_E} - c\right) \alpha_H L$ is strictly greater than $\left(\frac{b}{V_C} - c\right) L$, implying that all firms would choose to sell only to the high-income consumers.

Figure 1: The Profit Function

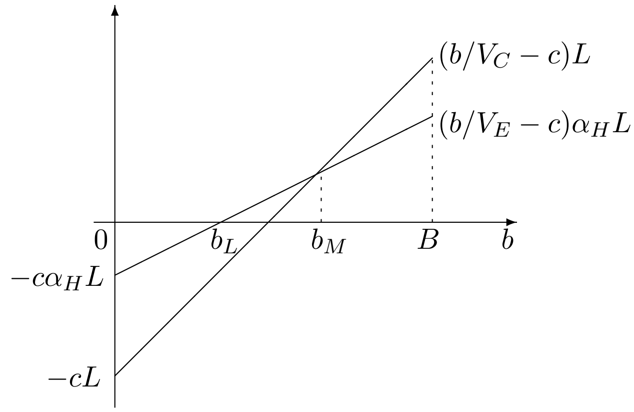
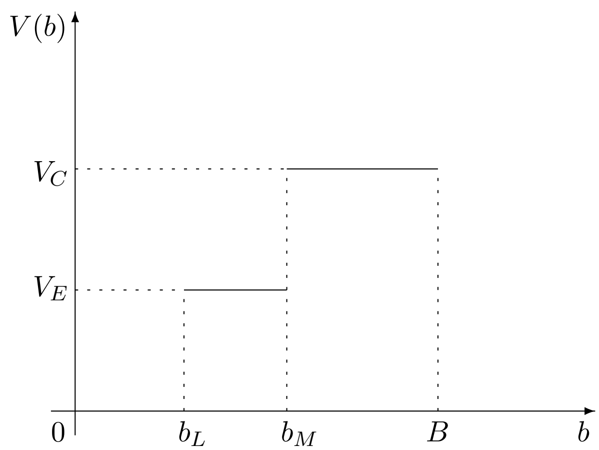


Figure 2: The Valuation to Price Function



Let us denote a function $V(b)$ equal to $\frac{b}{p(b)}$ for all b on $[0, B]$. The function $V(b)$ is depicted in *Figure 2*, where $b_L \geq 0$ is the cutoff level meaning that firms with $b < b_L$ exit because of negative potential profits.

3.1 The Equilibrium

Let us denote M_e as the mass of firms entering the market. One can think of M_e as that there are $M_e g(b)$ different firms with a certain valuation b. In the equilibrium, several conditions should be satisfied. First, as there is free entry in the market, the ex ante expected profits of firms have to be equal to zero. Second, the goods market clears. Since the poor consume only goods from the "common" group, the aggregate cost of the bundle of goods from the "common" group should be equal to the income of a poor consumer. Similarly, the aggregate cost of the bundle of all available goods in the economy should be equal to the income of a rich consumer.

Definition 1 *The equilibrium of the model is defined by the price function $p(b)$ on $b \geq b_L$, the cutoff level $b_L \geq 0$, b_M, M_e, and the valuation to price ratios V_C and V_E such that*

1) The ex ante expected profits of firms are equal to zero.

2) The goods market clears.

Next, I derive equations that satisfy the equilibrium conditions and show that there exists a unique equilibrium in the model. Let $\pi(b)$ be the variable profits of a firm with valuation b. To find the equilibrium, I express $\pi(b)$ and $p(b)$ as functions of b, b_L, b_M, and the exogenous parameters. As b_L is the cutoff level, firms with valuation b_L have zero profits. This implies that $\left(\frac{b_L}{V_E} - c\right)\alpha_H L = 0$ or $V_E = \frac{b_L}{c}$. From (1), we can express V_C as a function of b_L and b_M. As a result, the following lemma holds.

Lemma 1 *In the equilibrium,*

$$p(b) = \begin{cases} \frac{b}{V_C} = cb\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right) & \text{if } b \geq b_M, \\ \frac{b}{V_E} = cb\frac{1}{b_L} & \text{if } b \in [b_L, b_M), \end{cases}$$

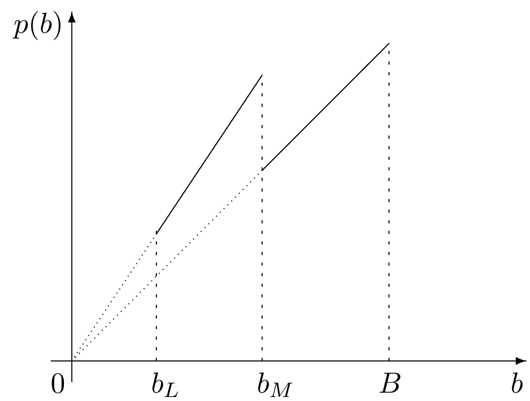
$$\pi(b) = \begin{cases} \left(cb\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right) - c\right)L & \text{if } b \geq b_M, \\ \left(cb\frac{1}{b_L} - c\right)\alpha_H L & \text{if } b \in [b_L, b_M). \end{cases}$$

Since firms with valuation b_M have the same profits from selling to all consumers as from selling only to the rich, the price function has a jump at b_M; i.e., to compensate for lower demand, firms raise their prices (see *Figure 3*). This results in the nonmonotonicity of the price function.

Due to free entry in the market, the ex ante expected profits of firms are equal to zero in the equilibrium. Using the results from *Lemma 1* and taking into account that firms with $b < b_L$ exit, I obtain

$$f_e = (G(b_M) - G(b_L))E(\pi(b)|b_L \leq b < b_M) + (1 - G(b_M))E(\pi(b)|b \geq b_M),$$

Figure 3: The Price Function



which is equivalent to

$$\frac{f_e}{cL} + 1 = \alpha_H H(b_L) + (1 - \alpha_H) H(b_M), \tag{2}$$

where $H(x) = G(x) + \frac{\int_x^B t dG(t)}{x}$. The goods market clearing condition implies that

$$\begin{cases} I_L = M_e \int_{b_M}^B p(t) dG(t) \\ I_H = M_e \int_{b_L}^B p(t) dG(t) \end{cases}. \tag{3}$$

The aggregate cost of the bundle of goods from the "common" group is equal to the income of a poor consumer, while the aggregate cost of the bundle of all available goods in the economy is equal to the income of a rich consumer. Dividing the second line in (3) by the first one and using *Lemma 1*, I obtain

$$\frac{\int_{b_L}^{b_M} t dG(t)}{\int_{b_M}^B t dG(t)} = \left(\frac{I_H}{I_L} - 1\right)\left(\alpha_H + \frac{b_L(1 - \alpha_H)}{b_M}\right).$$

Hence, given the exogenous parameters I_H, I_L, α_H, f_e, c, L, and the distribution of draws $G(\cdot)$, we can find endogenous b_M and b_L from the system of equations, which is given by

$$\begin{cases} \frac{\int_{b_L}^{b_M} t dG(t)}{\int_{b_M}^B t dG(t)} = \left(\frac{I_H}{I_L} - 1\right)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right) \\ \frac{f_e}{cL} + 1 = \alpha_H H(b_L) + (1 - \alpha_H) H(b_M) \end{cases}. \tag{4}$$

The following lemma states the existence and uniqueness of the equilibrium.

Lemma 2 *The system of equations* (4) *has a unique solution.*

Proof. In Appendix A. ■

Once b_M and b_L are found, V_C and V_E can be derived using the results in *Lemma 1.* Finally, the mass of firms can be found from (3).

3.2 Income Inequality and Welfare

Before exploring the effects of income inequality on the market structure and welfare, I examine how consumer welfare and income inequality are determined in the model.

3.2.1 Welfare

Given the preference structure, welfare of a certain consumer is equal to the sum of the valuations of goods she consumes. In this way, welfare of a poor consumer is equal to $M_e \int_{b_M}^{B} t dG(t)$. From (3), $M_e = \frac{I_L}{\int_{b_M}^{B} p(t) dG(t)}$. This implies that

$$W_p = I_L V_C. \tag{5}$$

Welfare of a poor consumer naturally rises with an increase in either her income or the valuation to price ratio of goods she consumes. Similarly, welfare of a rich consumer is given by

$$W_r = I_L V_C + (I_H - I_L) V_E. \tag{6}$$

As the rich consume the same bundle of goods as the poor plus some others, welfare of the rich is equal to welfare of the poor plus additional welfare from the consumption of the "exclusive" goods, which is in turn equal to income spent on these goods multiplied by their valuation to price ratio.

Notice that all changes in individual welfare are divided into two components: an income effect and a price effect. The price effect is determined by changes in V_C and V_E, which implicitly depend on the incomes (I_H and I_L) and the intensity of competition within the groups of goods. The income effect is explicitly determined by changes in exogenous I_L and I_H.

3.2.2 Income Inequality

As income inequality in the economy, I consider the variance of the income distribution, which is given by[10]

$$VAR = \alpha_H(1-\alpha_H)\,(I_H - I_L)^2\,. \tag{7}$$

As it can be seen, the income inequality is increasing in the income difference $I_H - I_L$ and has an inverted U shape as a function of α_H. Further, I express the variance in terms of the aggregate income per capita, the fraction of the rich, and the income of the poor. The aggregate income per capita is given by

$$AG = \alpha_H I_H + (1-\alpha_H)I_L.$$

Hence, the variance can be rewritten as follows

$$VAR = (\frac{1}{\alpha_H} - 1)\,(AG - I_L)^2\,. \tag{8}$$

The expression in (8) implies that keeping the aggregate income fixed, a rise in I_H together with a decrease in α_H raise the income inequality in the economy.

In the next sections, I examine the impact of income inequality on market structure and individual welfare. Since in the paper I focus on the effects on welfare of poor consumers, in the subsequent analysis I assume away the income effect for the poor. Specifically, I keep the income of the poor fixed and only consider changes in α_H and I_H. Recall that while changes in α_H affect consumer welfare only through the price effect, changes in I_H affect welfare through both the price and the income effects.

3.2.3 Changes in the Income of the Rich

If the rich get even richer, do the poor gain or lose? What is the impact on the prices? In this section, I consider a rise in the income of the rich I_H. From (5), higher I_H affects welfare of the poor only through changes in the prices of the "common" goods. Two opposite effects influence these prices. First, since I_H rises, some firms that used to sell their goods to all consumers find

[10] Another possible way to describe income inequality in the model is to use the Gini coefficient. However, in the case of the income distribution considered in the paper, the Gini coefficient is highly correlated with the variance. Changes in the parameters of the distribution, which increase the Gini coefficient, usually increase the variance. The exception is changes in α_H. In some cases, higher α_H decreases the Gini coefficient but increases the variance. As my main goal is to analyze the qualitative implications of changes in income distribution, without loss of generality, I consider the variance of the distribution as the measure of income inequality.

it more profitable to sell only to the rich. This reduces competition among firms serving all consumers and, therefore, raises the prices of the "common" goods. Second, higher income of the rich results in higher expected profits of firms. This in turn implies that more firms enter the market inducing tougher competition and reducing the prices. I show that the latter effect prevails over the former one. As a result, higher I_H positively affects V_C increasing welfare of the poor.

Meanwhile, a rise in I_H affects the rich through both the price and the income effects. Higher income of the rich allows firms that sell only to the rich to raise their prices. In spite of higher entry in the market, the prices of the "exclusive" goods rise and as a result, V_E falls. However, the income effect is stronger than the price effect and as a result, the rich benefit from higher I_H. Similar intuition works if we consider changes in I_L. An increase in I_L raises the prices of the "common" goods and decreases the prices of "exclusive" goods. The poor and the rich are better off (see details in Appendix A). The following proposition summarizes the findings above.

Proposition 2 *A rise in the income of the rich reduces (raises) the prices of the "common" ("exclusive") goods increasing welfare of all consumers.*

Proof. In Appendix A. ■

The intuition behind the results above may also work in traditional models with homothetic preferences. For instance, in Melitz (2003), higher income of a certain group of consumers would result in higher entry, tougher competition, and, thereby, higher welfare of all consumers. However, there are some differences. In the present model, higher income of the rich raises the markups of firms selling only to the rich and decreases the markups of firms serving all consumers. In traditional models, we would observe the same or no impact on firms' markups. Furthermore, assume for a moment that the mass of firms does not change in the model.[11] In this case, higher income of the rich raises prices of all goods and the poor are worse off. While in traditional models, if we fix the mass of firms, then higher income of one group of consumers does not affect welfare of the rest.

3.2.4 Changes in the Fraction of the Rich

This section focuses on how changes in α_H affect welfare of the poor. From (5), a rise in α_H affects the poor through the price effect. First, higher α_H raises the firm profits from serving rich

[11] To some extent, this case can be interpreted as a short run version of the model.

consumers. As a result, some firms switch from serving all consumers to serving only the rich. This reduces competition among firms selling the "common" goods and, consequently, raises the prices of the "common" goods. Second, a higher fraction of the rich results in higher ex ante expected profits and, therefore, increases entry in the market inducing tougher competition and lower prices of all goods. Remember that in the previous section, I show that the negative effect on the prices of "common" goods always dominates the positive one. In the present case, it is not necessarily true. In particular, I show that in a neighborhood around $\alpha_H = 0$, a rise in α_H increases welfare of the poor. While in a neighborhood around $\alpha_H = 1$, higher fraction of the rich decreases welfare of poor consumers. Thus, the following proposition holds.

Proposition 3 *If α_H is close to zero (close to one), a rise in α_H decreases (increases) the prices of the "common" goods increasing (decreasing) welfare of the poor.*

Proof. In Appendix A. ■

The last proposition suggests that welfare of the poor has an inverted U shape as a function of α_H.[12] However, because of mathematical difficulties arising in the analysis, I cannot strictly prove this conjecture. Instead, I make a number of numerical exercises where I consider welfare of the poor as a function of α_H. The results are supportive of the claim that welfare of the poor has an inverted U shape as a function of α_H.[13]

The fact that firms endogenously choose the type of consumers to serve makes the results regarding changes in α_H different from those that might be obtained in traditional models with homothetic preferences. In Melitz (2003), higher fraction of the rich would lead to higher welfare of the poor in the long run and has no impact in the short run (when the mass of firms is fixed). In this model, we observe an ambiguous impact of α_H on the poor in the long run and a negative impact in the short run.

[12] While a rise in α_H has an ambiguous impact on the poor, the rich always benefit from it. See details in Appendix A.

[13] Specifically, it appears that the sign of $(W_p)'_{\alpha_H}$ is the same as the sign of $K(\alpha_H) - \frac{\alpha_H}{1-\alpha_H}$, where $K(\alpha_H) > 0$ for any $\alpha_H \in [0,1]$ and $K(1) < \infty$. This implies that in the neighborhood of $\alpha_H = 0$ ($\alpha_H = 1$), $(W_p)'_{\alpha_H} > 0$ ($(W_p)'_{\alpha_H} < 0$). If $K(\alpha_H)$ is well behaved (that is, the equation $K(\alpha_H) = \frac{\alpha_H}{1-\alpha_H}$ has a unique solution), then W_p has an inverted U as a function of α_H. Unfortunately, the analysis of the behavior of $K(\alpha_H)$ on $[0,1]$ is quite complicated. We cannot exclude the possibility that the equation $K(\alpha_H) = \frac{\alpha_H}{1-\alpha_H}$ has multiple solutions. In the numerical examples I consider, I take the power distribution $G(b) = \left(\frac{b}{B}\right)^k$ with $k > 0$ as the distribution of draws. For a number of different sets of the exogenous parameters, I find the solution of $K(\alpha_H) = \frac{\alpha_H}{1-\alpha_H}$. In all cases, the solution is unique.

3.2.5 Changes in the Income and the Fraction of the Rich (Keeping the Aggregate Income Fixed)

There is a common feature for both comparative statics considered above. A rise in I_H as well as a rise in α_H increases the aggregate income in the economy. To capture a pure redistribution effect, I consider a rise in the personal income of the rich together with a decrease in the fraction of the rich keeping the aggregate income in the economy fixed.[14] I show that these changes in the income distribution result in higher entry in the market and, therefore, higher welfare of the poor.

To understand the intuition behind better, I first consider the short run implications of the changes in the income distribution. From the previous sections, we know that in the short run, higher I_H decreases welfare of the poor, while lower α_H increases it. Thus, two effects work in opposite directions. However, it appears that the impact of I_H is always stronger than that of α_H. Here the assumption that aggregate income is unchanged plays a key role.[15] This implies that in the short run, the poor are worse off from the changes in the income distribution considered.

What is about the long run? On the one hand, higher income of the rich allows firms to impose higher prices of their goods and, consequently, leads to more entry in the market. On

[14] Another exercise that holds the aggregate income unchanged is to examine changes in the incomes of the rich and the poor keeping the fraction of the rich fixed. I do not focus on this exercise, as the main goal of this paper is to explore the effects on welfare of the poor. However, if we consider a rise in I_H and a decrease in I_L holding AG constant, then we might expect that the income effect prevails over the price effect. That is, the rich gain and the poor lose. Furthermore, since the poor consume on average more valuable goods than the rich do, the changes in the incomes substitute the consumption of more valuable goods for the consumption of less valuable goods. As a result, given that $bg(b)$ is increasing in b, total welfare in the economy may decrease.

[15] The heuristic proof is as follows. Recall that welfare of a poor consumer is given by $W_p = M_e \int_{b_M}^{B} t dG(t)$. Since in the short run M_e is fixed, we only need to examine the effects on b_M. Note that b_M solves

$$\left(\frac{b}{V_C} - c\right) L = \left(\frac{b}{V_E} - c\right) \alpha_H L.$$

This implies that $b_M \left(\frac{1}{V_C} - \frac{\alpha_H}{V_E}\right) = c(1-\alpha_H)$. From the goods market clearing condition, $I_L = \frac{M_e}{V_C} \int_{b_M}^{B} t dG(t)$ and $I_H - I_L = \frac{M_e}{V_E} \int_{b_L}^{b_M} t dG(t)$. This results in

$$\frac{b_M}{M_e} \left(\frac{I_L}{\int_{b_M}^{B} t dG(t)} - \frac{\alpha_H (I_H - I_L)}{\int_{b_L}^{b_M} t dG(t)} \right) = c(1-\alpha_H).$$

In the short run, only firms that were active before may operate in the market. Therefore, in the short run, b_L is unchanged. Moreover, as aggregate income in the economy is unchanged, $\alpha_H (I_H - I_L)$ does not change as well. This implies that in $\frac{b_M}{M_e} \left(\frac{I_L}{\int_{b_M}^{B} t dG(t)} - \frac{\alpha_H (I_H - I_L)}{\int_{b_L}^{b_M} t dG(t)} \right)$, b_M changes only. As a result, a rise in $c(1-\alpha_H)$ leads to a rise in b_M decreasing W_p.

the other hand, lower fraction of the rich reduces the demand for the "exclusive" goods making ex ante expected profits lower. This results in lower entry in the market. As in the previous section, I focus on the analysis of two extreme cases: $\alpha_H \approx 0$ and $\alpha_H \approx 1$. I show that in these cases, the impact of I_H prevails over that of α_H leading to more entry in the market. Moreover, I show that the positive effect on welfare of the poor from more entry is stronger than the negative short run effect. These results are derived for the neighborhoods of $\alpha_H = 0$ and $\alpha_H = 1$ and an arbitrary distribution function $G(b)$. However, if we limit the analysis to the cases when $bg(b)$ is increasing in b, then the results hold for any $\alpha_H \in [0, 1]$.[16] The next proposition summarizes these findings.

Proposition 4 *If α_H is in the neighborhoods of $\alpha_H = 0$ and $\alpha_H = 1$ or $G(b)$ is such that $bg(b)$ is increasing in b, a rise in I_H together with a decrease in α_H (keeping AG fixed) raise welfare of the poor and the mass of firms entering the market.*

Proof. In Appendix A. ■

The assumption that $bg(b)$ is increasing in b has a strong economic interpretation. It implies that $g(b)$ does not decrease too fast; i.e., the probability of getting higher values of b does not decrease too fast with b. Moreover, in some sense, utility from the consumption of all goods with a certain valuation b is equal to $M_e bg(b)$. Hence, this assumption also guarantees that this utility increases with b.

3.3 Entry Costs, Market Size, and Welfare

In this paper, the impact of the costs of entry and market size on consumer welfare is the same as in traditional models. However, the present model implies that changes in market size or entry costs have different magnitudes for different types of consumers. In this section, I briefly describe the effects of changes in f_e and L on individual welfare and focus on the effects on the relative welfare.

An increase in the costs of entry f_e reduces the ex ante expected profits of firms. This in turn decreases the mass of firms entering the market and reduces the intensity of competition. As a result, the prices of goods from both groups rise and welfare of all consumers falls. An increase in L results in higher ex ante expected profits of firms. This leads to the higher number

[16] For instance, a family of power distributions satisfies this condition.

of firms entering the market and tougher competition. The prices of goods from both groups fall and consumers of both types are better off. Finally, any changes in f_e and L such that the ratio $\frac{f_e}{L}$ remains the same do not change the prices and individual welfare. Two opposite effects completely compensate each other (see (4)). The following proposition holds.

Proposition 5 *Larger countries and countries with lower entry costs tend to have higher individual welfare.*

Proof. In Appendix A. ■

In the next section, I examine the effect of $\frac{f_e}{L}$ on the relative welfare of the rich with respect to the poor.

3.3.1 Relative Welfare

The relative welfare of the rich with respect to the poor is given by

$$\frac{W_r}{W_p} = 1 + \frac{I_H - I_L}{I_L} \frac{V_E}{V_C}.$$

Note that welfare inequality is divided into two components: income inequality and consumption inequality. The income inequality is determined by the ratio $\frac{I_H - I_L}{I_L}$, while the consumption inequality $\frac{V_E}{V_C}$ depends on the relative prices of the "exclusive" goods with respect to the "common" goods. Hence, changes in the parameters in the model may affect either type of inequality or both. For instance, higher income of the rich raises the income inequality but decreases the consumption inequality.

The relative welfare can be rewritten as follows

$$\frac{W_r}{W_p} = 1 + \frac{I_H - I_L}{I_L} \left(\alpha_H + (1 - \alpha_H) \frac{b_L}{b_M} \right). \tag{9}$$

From (9), changes in $\frac{f_e}{L}$ affect $\frac{W_r}{W_p}$ only through the ratio $\frac{b_L}{b_M}$. From the goods market equilibrium condition

$$\frac{\int_{b_L}^{b_M} tdG(t)}{\int_{b_M}^{B} tdG(t)} - \frac{I_H - I_L}{I_L} \left(\alpha_H + (1 - \alpha_H) \frac{b_L}{b_M} \right) = 0,$$

b_M can be expressed as a function of b_L: $b_M = b_M(b_L)$. Moreover, this functional relationship does not depend on the ratio $\frac{f_e}{L}$. Hence,

$$\left(\frac{W_r}{W_p}\right)'_{\frac{f_e}{L}} = \frac{I_H - I_L}{I_L}(1-\alpha_H)\left(\frac{b_L}{b_M(b_L)}\right)'_{b_L}\frac{\partial b_L}{\partial \frac{f_e}{L}}.$$

Recall that the analysis in the previous section implies that $\frac{f_e}{L}$ is negatively correlated with the cutoff b_L. That is, $\frac{\partial b_L}{\partial \frac{f_e}{L}} < 0$. As a result, to explore the effects of $\frac{f_e}{L}$ on the relative welfare, we need to determine the sign of $\left(\frac{b_L}{b_M(b_L)}\right)'_{b_L}$. In Appendix A, I show that if $\left(\frac{b^2 g(b)}{\int_b^B t dG(t)}\right)'_b$ is greater than zero for any $b \in [0, B]$, then $\left(\frac{b_L}{b_M(b_L)}\right)'_{b_L}$ is positive. Otherwise, depending on the parameters in the model, the sign of $\left(\frac{b_L}{b_M(b_L)}\right)'_{b_L}$ might be either. The following proposition formalizes the findings above.

Proposition 6 *If* $\left(\frac{b^2 g(b)}{\int_b^B t dG(t)}\right)'_b > 0$ *for any* $b \in [0, B]$, *then the rich gain more from an increase in market size and lose more from an increase in the costs of entry than the poor.*

Proof. In Appendix A. ■

Limiting the analysis to the case when $\left(\frac{b^2 g(b)}{\int_b^B t dG(t)}\right)'_b$ is always positive, we derive that the rich lose more from an increase in $\frac{f_e}{L}$ than the poor. To understand the intuition, I separately consider two markets. The first market is the market for goods from the "common" group, while the second one is the market for the "exclusive" goods. I divide the effect of higher $\frac{f_e}{L}$ into two steps. First, given an increase in $\frac{f_e}{L}$, fewer firms enter the both markets decreasing b_L and b_M. Second, due to less competitive pressure, some firms that sold their goods only to the rich switch to selling to all consumers. This effect decreases b_M even more and in turn reduces competition in the second market allowing firms with low valuations to survive. As a result, the cutoff b_L falls even more as well. However, firms that switched from the second market to the first one have relatively high valuations of their goods compared with firms that "survived", the prices of these goods were relatively high. This implies that b_L has to fall by more than b_M to compensate for the difference in the prices. That is, higher $\frac{f_e}{L}$ decreases $\frac{b_L}{b_M}$.

4 Conclusion

In this paper, I consider a general equilibrium model of monopolistic competition with heterogenous firms and consumers. The model incorporates two key features: imperfect competition and non-homothetic preferences, which allow us to analyze the consequences of changes in income distribution on pricing, market structure and, thereby, welfare of different groups of consumers.

This framework leads to interesting theoretical results that help to understand the impact of income inequality on individual well-being. In particular, I analyze how income inequality influences welfare of the poor. I show that higher income inequality in the economy may benefit the poor via a trickle-down effect operating through entry. The model also allows us to analyze the effects of changes in market size and entry costs. An increase in market size leads to tougher competition. Therefore, markups of all firms fall and welfare of all consumers rises. Similarly, an increase in entry costs reduces the intensity of competition, raises markups, and, thereby, decreases welfare of all consumers. Furthermore, I show that the rich may gain more from an increase in market size and lose more from an increase in entry costs compare to the poor.

There are a number of plausible extensions of this model. For instance, it would be interesting to consider an open economy version of the model. In this case, the paper can be modified in two ways. First, one can explore a model of trade between two countries with different income distributions and examine how this difference affects the trade patterns. Second, it would be interesting to consider the case when income distribution is endogenous and, for instance, affected by the level of openness. I leave these issues for future work.

Appendix A

In Appendix A, I provide the proofs of the lemmas and the propositions formulated above.

Proof of Lemma 2

Consider

$$\frac{\int_{b_L}^{b_M} t dG(t)}{\int_{b_M}^{B} t dG(t)} = \left(\frac{I_H}{I_L} - 1\right)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right). \tag{10}$$

Let $b_M = F_1(b_L)$ be an implicit solution of (10). Then, it is straightforward to show that $F_1(b_L)$ is strictly increasing in b_L and $B \geq F_1(b_L) \geq b_L$. The latter implies that $F_1(B) = B$. Now, consider

$$\frac{f_e}{cL} + 1 = \alpha_H H(b_L) + (1-\alpha_H)H(b_M). \tag{11}$$

By analogy, let $b_M = F_2(b_L)$ be an implicit solution of (11). As $H(\cdot)$ is strictly decreasing, $F_2(b_L)$ is also strictly decreasing in b_L. Since $H(B) = 1$, $H(F_2(B)) = \frac{f_e}{cL(1-\alpha_H)} + 1 > 1$. This implies that $F_2(B) < B$. Let b_L^B be such that $F_2(b_L^B) = B$. Then, $H(b_L^B) = \frac{f_e}{cL\alpha_H} + 1 > 1$, i.e., $b_L^B < B$. Hence, the solution of (4) exists and is unique (see *Figure 4*).

Comparative Statics

In this section, I use a simplifying notation: $\int_x^y$ means $\int_x^y tdG(t)$.

Proof of Proposition 2

An increase in I_H shifts the curve $F_1(b_L)$ up, while the curve $F_2(b_L)$ is unchanged. As a result, b_L falls and b_M rises (see *Figure 5*). The impact on welfare of the poor is not so straightforward. Rewrite (10) and (11) as follows

$$\begin{cases} J_1 \equiv (1-\alpha_H)cLH(b_M) + \alpha_H cLH(b_L) - f_e - cL = 0 \\ J_2 \equiv I_L \int_{b_L}^{b_M} - (I_H - I_L)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)\int_{b_M}^{B} = 0 \end{cases}. \tag{12}$$

Figure 4: The Equilibrium

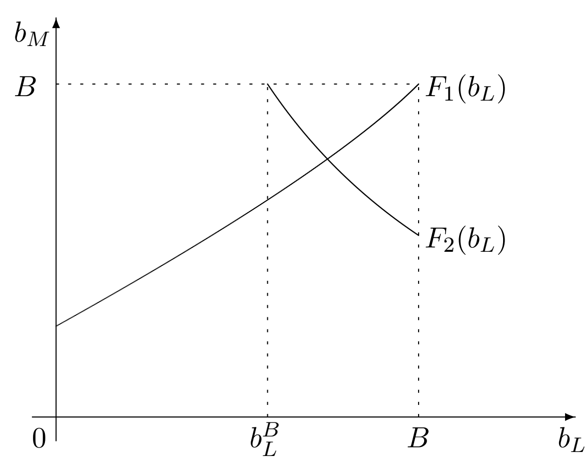
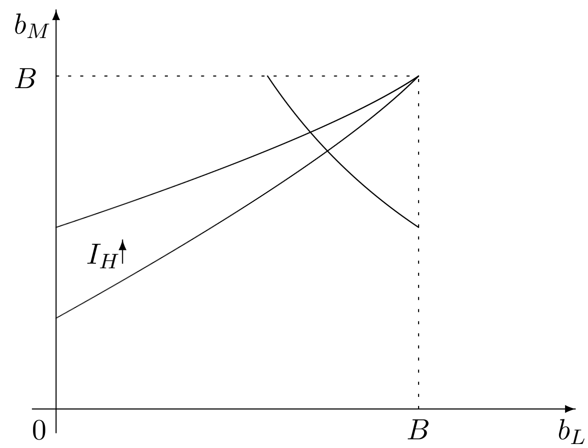


Figure 5: An Increase in I_H



Notice that equilibrium values of b_L and b_M solve (12). Using implicit differentiation, I obtain

$$\frac{\partial b_M}{\partial I_H} = \frac{\frac{\partial J_2}{\partial I_H}\frac{\partial J_1}{\partial b_L}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}} > 0 \tag{13}$$

$$\frac{\partial b_L}{\partial I_H} = \frac{-\frac{\partial J_2}{\partial I_H}\frac{\partial J_1}{\partial b_M}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}} < 0. \tag{14}$$

Consider $\frac{1}{V_C} = \frac{\alpha_H c}{b_L} + \frac{(1-\alpha_H)c}{b_M}$. We have

$$\left(\frac{\alpha_H c}{b_L} + \frac{(1-\alpha_H)c}{b_M}\right)'_{I_H} = \frac{-\alpha_H c}{(b_L)^2}\frac{\partial b_L}{\partial I_H} - \frac{(1-\alpha_H)c}{(b_M)^2}\frac{\partial b_M}{\partial I_H}.$$

From (13) and (14),

$$\frac{-\alpha_H c}{(b_L)^2}\frac{\partial b_L}{\partial I_H} - \frac{(1-\alpha_H)c}{(b_M)^2}\frac{\partial b_M}{\partial I_H} = \frac{c^2 L\alpha_H(1-\alpha_H)\frac{\partial J_2}{\partial I_H}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}\left(\frac{H'(b_M)}{(b_L)^2} - \frac{H'(b_L)}{(b_M)^2}\right).$$

Recall that $H'(x) = -\frac{\int_x^B t dG(t)}{x^2} < 0$. Then,

$$\frac{-\alpha_H c}{(b_L)^2}\frac{\partial b_L}{\partial I_H} - \frac{(1-\alpha_H)c}{(b_M)^2}\frac{\partial b_M}{\partial I_H} = \frac{c^2 L\alpha_H(1-\alpha_H)\frac{\partial J_2}{\partial I_H}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}\frac{\int_{b_L}^B - \int_{b_M}^B}{(b_L)^2(b_M)^2}.$$

Since $\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L} > 0$ and $\frac{\partial J_2}{\partial I_H} < 0$, $\left(\frac{\alpha_H c}{b_L} + \frac{(1-\alpha_H)c}{b_M}\right)'_{I_H} < 0$. Therefore, $(V_C)'_{I_H} > 0$. This implies that an increase in I_H leads to the lower prices of the "common" goods and higher welfare of the poor, which is equal to $I_L V_C$.

From the analysis above, we know that higher I_H results in lower b_L. That is, the prices of the "exclusive" goods rise. As $W_p = M_e \int_{b_M}^{B}$ and b_M increases, an increase in I_H raises M_e and, therefore, $W_r = M_e \int_{b_L}^{B}$.

Changes in I_L

Similarly, an increase in I_L shifts the curve $F_1(b_L)$ down, while the curve $F_2(b_L)$ is unchanged. Hence, b_L rises and b_M falls. To analyze the impact on consumer welfare, I use the same technique as in the previous proofs. As $W_r = M_e \int_{b_L}^{B}$ and $I_H = M_e \int_{b_L}^{B} p(t)dG(t)$, $W_r = \frac{I_H \int_{b_L}^{B}}{\int_{b_L}^{B} p(t)dG(t)}$. The sign of $(W_r)'_{I_L}$ is the same as the sign of $\left(\int_{b_L}^{B}\right)'_{I_L} \int_{b_L}^{B} p(t)dG(t) - \left(\int_{b_L}^{B} p(t)dG(t)\right)'_{I_L} \int_{b_L}^{B}$. Algebra shows that

$$\left(\int_{b_L}^{B}\right)'_{I_L} \int_{b_L}^{B} p(t)dG(t) - \left(\int_{b_L}^{B} p(t)dG(t)\right)'_{I_L} \int_{b_L}^{B} =$$

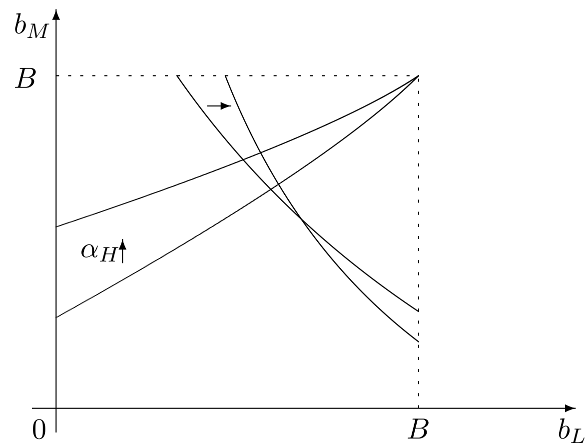
$$= \frac{c^2 L(1-\alpha_H)^2 \frac{\partial J_2}{\partial I_L}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}} \frac{\int_{b_L}^{b_M}\int_{b_L}^{B}\int_{b_M}^{B}}{(b_L)^2 (b_M)^2}$$
$$+c(1-\alpha_H)(b_M - b_L)\left\{\frac{\partial b_L}{\partial I_L}\frac{g(b_L)}{b_M}\int_{b_M}^{B} - \frac{\partial b_M}{\partial I_L}\frac{g(b_M)}{b_L}\int_{b_L}^{B}\right\}.$$

From (12), $\frac{\partial b_L}{\partial I_L} > 0$, $\frac{\partial b_M}{\partial I_L} < 0$, and $\frac{\partial J_2}{\partial I_L} > 0$. This implies that $(W_r)'_{I_L} > 0$. As $M_e = \frac{W_r}{\int_{b_L}^{B}}$, an increase in I_L raises M_e and, thereby, $W_p = M_e \int_{b_M}^{B}$.

Proof of Proposition 3

An increase in α_H shifts the curve $F_1(b_L)$ up and the curve $F_2(b_L)$ to the right around 45° degree line (see *Figure 6*). In this case, b_M rises. The impact on b_L is not so straightforward. There are two opposite effects. The upward shift of $F_1(b_L)$ decreases b_L, while the shift of the $F_2(b_L)$ increases b_L. Next, I show that $\frac{\partial b_L}{\partial \alpha_H} > 0$.

Figure 6: An Increase in α_H



From (12),

$$\begin{aligned}\frac{\partial b_M}{\partial \alpha_H} &= \frac{-\frac{\partial J_1}{\partial \alpha_H}\frac{\partial J_2}{\partial b_L} + \frac{\partial J_2}{\partial \alpha_H}\frac{\partial J_1}{\partial b_L}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}} > 0 \\ \frac{\partial b_L}{\partial \alpha_H} &= \frac{-\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial \alpha_H} + \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial \alpha_H}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}.\end{aligned}$$

To determine the sign of $\frac{\partial b_L}{\partial \alpha_H}$, I examine

$$\begin{aligned}& -\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial \alpha_H} + \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial \alpha_H} \\ = \quad & cL\left(\left(H\left(b_L\right) - H\left(b_M\right)\right)\frac{\partial J_2}{\partial b_M} - \frac{(1-\alpha_H)}{\left(b_M\right)^2}\left(I_H - I_L\right)\left(1 - \frac{b_L}{b_M}\right)\left(\int_{b_M}^{B}\right)^2\right).\end{aligned}$$

The partial derivative of J_2 with respect to b_M can be written as follows

$$\begin{aligned}\frac{\partial J_2}{\partial b_M} &= \left(I_H - I_L\right)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right) b_M g\left(b_M\right)\frac{\int_{b_L}^{B}}{\int_{b_L}^{b_M}} \\ &\quad + \left(I_H - I_L\right)\frac{b_L(1-\alpha_H)}{\left(b_M\right)^2}\int_{b_M}^{B}.\end{aligned} \tag{15}$$

Then,

$$\begin{aligned}
\frac{-\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial \alpha_H}+\frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial \alpha_H}}{cL\left(I_H-I_L\right)} &= \left(H\left(b_L\right)-H\left(b_M\right)\right)\alpha_H b_M g\left(b_M\right)\frac{\int_{b_L}^{B}}{\int_{b_L}^{b_M}} \\
&\quad+\left(H\left(b_L\right)-H\left(b_M\right)\right)b_L(1-\alpha_H)g\left(b_M\right)\frac{\int_{b_L}^{B}}{\int_{b_L}^{b_M}} \\
&\quad+\frac{(1-\alpha_H)b_L}{\left(b_M\right)^2}\int_{b_M}^{B}\left(G(b_L)-G(b_M)+\frac{\int_{b_L}^{B}-\int_{b_M}^{B}}{b_L}\right).
\end{aligned}$$

Therefore, as $b_M > b_L$ and $G(b_L)b_L - G(b_M)b_L + \int_{b_L}^{B} - \int_{b_M}^{B}$ is increasing in b_M and equal to zero when $b_M = b_L$,

$$\frac{-\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial \alpha_H}+\frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial \alpha_H}}{cL\left(I_H-I_L\right)}>0.$$

This in turn implies that $\frac{\partial b_L}{\partial \alpha_H} > 0$.

Welfare of the poor is given by $W_p = \frac{I_L}{c\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)}$. To determine the sign of $(W_p)'_{\alpha_H}$, we need to examine the sign of

$$\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}=\frac{1}{b_L}-\frac{1}{b_M}-\left(\frac{\alpha_H}{\left(b_L\right)^2}\frac{\partial b_L}{\partial \alpha_H}+\frac{(1-\alpha_H)}{\left(b_M\right)^2}\frac{\partial b_M}{\partial \alpha_H}\right).$$

The derivative of J_2 with respect to b_L can be expressed as

$$\frac{\partial J_2}{\partial b_L}=-\left(I_H-I_L\right)\left(\frac{b_L g\left(b_L\right)\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)\int_{b_M}^{B}}{\int_{b_L}^{b_M}}+\frac{(1-\alpha_H)}{b_M}\int_{b_M}^{B}\right). \tag{16}$$

Using the expressions (15) and (16), I show that

$$\begin{aligned}
\frac{\alpha_H}{\left(b_L\right)^2}\frac{\partial b_L}{\partial \alpha_H}+\frac{(1-\alpha_H)}{\left(b_M\right)^2}\frac{\partial b_M}{\partial \alpha_H} &= \frac{cL\left(I_H-I_L\right)\left(H\left(b_L\right)-H\left(b_M\right)\right)\alpha_H}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L}-\frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}} \\
&\quad+\frac{cL\left(I_H-I_L\right)H\left(b_L\right)\frac{b_L\alpha_H(1-\alpha_H)}{b_M}P_1}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L}-\frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}} \\
&\quad-\frac{cL\left(I_H-I_L\right)H\left(b_M\right)\frac{b_L\alpha_H(1-\alpha_H)}{b_M}P_1}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L}-\frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}} \\
&\quad+\frac{cL\left(I_H-I_L\right)\left(1-\frac{b_L}{b_M}\right)P_2}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L}-\frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}},
\end{aligned}$$

where

$$P_1 = \left(\frac{\alpha_H b_M g\left(b_M\right) \int_{b_L}^{B}}{\left(b_L\right)^2 \int_{b_L}^{b_M}} + \frac{(1-\alpha_H) b_L g\left(b_L\right) \int_{b_M}^{B}}{\left(b_M\right)^2 \int_{b_L}^{b_M}} + \frac{(1-\alpha_H)}{b_L \left(b_M\right)^2} \int_{b_M}^{B} \right)$$

and

$$P_2 = \frac{(1-\alpha_H)\alpha_H}{\left(b_M\right)^2 \left(b_L\right)^2} \int_{b_L}^{b_M} \int_{b_M}^{B}.$$

In addition,

$$\frac{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}{cL\left(I_H - I_L\right)} = \frac{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}} P_3 + \frac{(1-\alpha_H)\int_{b_M}^{B} t dG(t)}{\left(b_M\right)^2} P_4,$$

where $P_3 = \frac{(1-\alpha_H) b_L g(b_L) \left(\int_{b_M}^{B}\right)^2}{\left(b_M\right)^2} + \frac{\alpha_H b_M g(b_M) \left(\int_{b_L}^{B}\right)^2}{\left(b_L\right)^2}$ and $P_4 = \frac{(1-\alpha_H)\int_{b_M}^{B}}{b_M} + \frac{\alpha_H \int_{b_L}^{B}}{b_L}$. Therefore,

$$\begin{aligned}
\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} &= \frac{1}{b_L} - \frac{1}{b_M} \\
&\quad - \frac{\left(H\left(b_L\right) - H\left(b_M\right)\right)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right) P_1}{\frac{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}} P_3 + \frac{(1-\alpha_H)\int_{b_M}^{B}}{\left(b_M\right)^2} P_4} \\
&\quad + \frac{\left(1 - \frac{b_L}{b_M}\right) P_2}{\frac{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}} P_3 + \frac{(1-\alpha_H)\int_{b_M}^{B}}{\left(b_M\right)^2} P_4}.
\end{aligned}$$

After some simplifications,

$$\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} = \frac{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}{\frac{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}} P_3 + \frac{(1-\alpha_H)\int_{b_M}^{B}}{\left(b_M\right)^2} P_4} P_5,$$

where

$$\begin{aligned}
P_5 &= \frac{(1-\alpha_H)\int_{b_M}^{B}}{\left(b_M\right)^2} \left(\frac{1}{b_L} + \frac{b_L g\left(b_L\right)}{\int_{b_L}^{b_M}}\right) \left(G(b_M) - G(b_L) - \frac{\int_{b_L}^{b_M}}{b_L}\right) \\
&\quad + \frac{\alpha_H \int_{b_L}^{B}}{\left(b_L\right)^2} \frac{b_M g\left(b_M\right)}{\int_{b_L}^{b_M}} \left(G(b_M) - G(b_L) - \frac{\int_{b_L}^{b_M}}{b_M}\right).
\end{aligned}$$

Hence, the sign of $\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}$ is the same as the sign of P_5. As $b_M > b_L$, $G(b_M) -$

$G(b_L) - \frac{\int_{b_L}^{b_M}}{b_L} < 0$ and $G(b_M) - G(b_L) - \frac{\int_{b_L}^{b_M}}{b_M} > 0$. Hence, if α_H is close enough to zero then $\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} < 0$; that is, $(W_p)'_{\alpha_H} > 0$. However, if α_H is close enough to one then $\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} > 0$. This implies that $(W_p)'_{\alpha_H} < 0$. It is much more complicated to determine the sign of P_5 for all values of $\alpha_H \in [0,1]$.

The Effect of Higher α_H on the Rich

From the previous section, we know that $\frac{\partial b_L}{\partial \alpha_H} > 0$. This means that higher α_H decreases the prices of the "exclusive" goods. Welfare of the rich is given by $\frac{1}{c}\left(\frac{I_L}{\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)} + (I_H - I_L)\, b_L\right)$. This implies

$$c\,(W_r)'_{\alpha_H} = \frac{(I_H - I_L)\frac{\partial b_L}{\partial \alpha_H}\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)^2 - I_L\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}}{\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)^2}.$$

To determine the sign of $(W_r)'_{\alpha_H}$, we need to examine the sign of

$$(I_H - I_L)\frac{\partial b_L}{\partial \alpha_H}\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)^2 - I_L\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}.$$

Using the previous results,

$$\begin{aligned}
&(I_H - I_L)\frac{\partial b_L}{\partial \alpha_H}\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)^2 - I_L\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} = \\
= \quad &(I_H - I_L)\frac{\partial b_L}{\partial \alpha_H}\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)^2 - \frac{I_L\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)P_5}{\frac{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}}P_3 + \frac{(1-\alpha_H)\int_{b_M}^{B}}{(b_M)^2}P_4}.
\end{aligned}$$

After some simplifications, it appears that to prove that $(W_r)'_{\alpha_H} > 0$, it is enough to prove that

$$(I_H - I_L)\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)(H(b_L) - H(b_M)) - \frac{I_L}{b_L}\left(G(b_M) - G(b_L) - \frac{\int_{b_L}^{b_M}}{b_M}\right)$$

is greater than zero. This is in turn equivalent to

$$\frac{I_L \int_{b_L}^{B}}{b_L}\left(\frac{H\left(b_L\right)-H\left(b_M\right)}{\int_{b_M}^{B}}-\frac{1}{b_L}+\frac{1}{b_M}\right)>0.$$

For any $b_L < b_M$, $\frac{H(b_L)-H(b_M)}{\int_{b_M}^{B}}-\frac{1}{b_L}+\frac{1}{b_M}>0$ resulting in that $(W_r)'_{\alpha_H}$ is always greater than zero. Since $(W_r)'_{\alpha_H}>0$, $(b_L)'_{\alpha_H}>0$ and $W_r = M_e\int_{b_L}^{B}$; the mass of firms entering the market rises, i.e., $(M_e)'_{\alpha_H}>0$.

Proof of Proposition 4

Aggregate income per capita AG is given by $\alpha_H I_H + (1-\alpha_H) I_L$. This implies that $\alpha_H\left(I_H - I_L\right) = AG - I_L$. In this way, I rewrite (12) as follows

$$\begin{cases} J_1 \equiv (1-\alpha_H)cLH\left(b_M\right)+\alpha_H cLH\left(b_L\right)-f_e-cL=0 \\ J_2 \equiv I_L\int_{b_L}^{b_M}-\left(AG-I_L\right)\left(1+\frac{b_L(1-\alpha_H)}{\alpha_H b_M}\right)\int_{b_M}^{B}=0 \end{cases}. \tag{17}$$

Hence, it is necessary to explore the impact of a decrease in α_H on welfare of the poor given new equilibrium equations (17). Using the same technique as in the proof of *Proposition 3*, I obtain

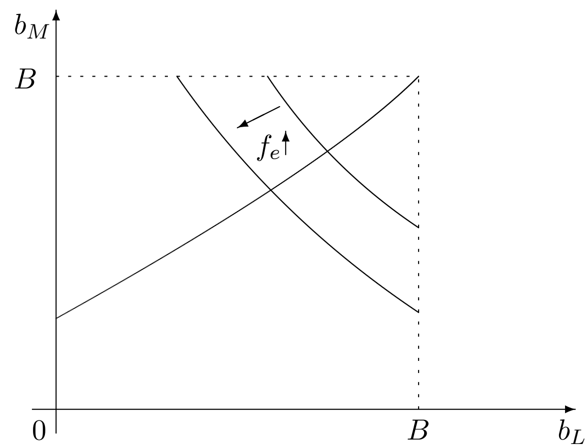
$$\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}=\frac{\left(\frac{\alpha_H}{b_L}+\frac{b_L(1-\alpha_H)}{b_M}\right)\left(\frac{(1-\alpha_H)\int_{b_M}^{B}}{b_M^2}\left(G(b_M)-G(b_L)\right)+P_6\right)}{\frac{\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}tdG(t)}P_3+\frac{(1-\alpha_H)\int_{b_M}^{B}}{(b_M)^2}P_4},$$

where

$$\begin{aligned} P_6 \quad = \quad & \frac{(1-\alpha_H)\int_{b_M}^{B}}{b_M^2}\frac{b_L^2 g\left(b_L\right)}{\int_{b_L}^{b_M}}\left(G(b_M)-G(b_L)-\frac{\int_{b_L}^{b_M}}{b_L}\right) \\ & +\frac{\alpha_H\int_{b_L}^{B}}{b_L}\frac{b_M g\left(b_M\right)}{\int_{b_L}^{b_M}}\left(G(b_M)-G(b_L)-\frac{\int_{b_L}^{b_M}}{b_M}\right). \end{aligned}$$

If α_H is close to one then $P_6>0$ and $\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}>0$. That is, welfare of the poor rises with a decrease in α_H. This result is also supported by the fact that given sufficiently high α_H, both an increase in I_H and a decrease in α_H have a positive impact on welfare of the poor. Consider $\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}$ when α_H is close to zero. From (17), $\lim_{\alpha_H\to 0} b_L(\alpha_H)=0$ and $\lim_{\alpha_H\to 0}\frac{b_L(\alpha_H)}{\alpha_H}$ is a positive constant. As for any density function $g(\cdot)$, $\lim_{x\to 0} xg(x)=0$;

Figure 7: An Increase in f_e



$\lim_{\alpha_H \to 0} P_6 > 0$. This implies that $\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H=0} > 0$.

Finally, it can be shown that if $bg(b)$ is increasing in b, then for any $\alpha_H \in [0,1]$,

$$\frac{(1-\alpha_H)\int_{b_M}^{B}}{b_M^2}\left(G(b_M) - G(b_L)\right) + P_6 > 0.$$

Proof of Proposition 5

A rise in f_e shifts the curve $F_2(b_L)$ to the left, while the curve $F_1(b_L)$ is unchanged. As a result, b_L and b_M fall (see *Figure 7)*. Since $W_p = \frac{I_L}{\frac{\alpha_H c}{b_L} + \frac{(1-\alpha_H)c}{b_M}}$ and $W_r = W_p + \frac{I_H - I_L}{c} b_L$, W_p and W_r decrease. M_e, which is equal to $\frac{W_p}{\int_{b_M}^{B}}$, falls as well. In the same way, a rise in L raises M_e, W_p, and W_r. Finally, any changes in f_e and L such that $\frac{f_e}{L}$ remains unchanged do not affect $F_2(b_L)$ and $F_1(b_L)$.

Proof of Proposition 6

I need to show that given $\left(\frac{b^2 g(b)}{\int_b^B}\right)'_b > 0$, $\left(\frac{b_L}{b_M(b_L)}\right)_{b_L} > 0$ where $b_M(b_L)$ is an implicit solution of

$$\int_{b_L}^{b_M} - \left(\frac{I_H}{I_L} - 1\right)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)\int_{b_M}^{B} = 0.$$

Notice that the sign of$\left(\frac{b_L}{b_M(b_L)}\right)_{b_L}$ is the same as the sign of $b_M - \frac{\partial b_M}{\partial b_L} b_L$. Algebra shows that

$$\begin{aligned}
b_M - \frac{\partial b_M}{\partial b_L} b_L &> 0 \iff \\
\frac{b_L g(b_L) + \frac{\int_{b_L}^{b_M}}{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)} \frac{(1-\alpha_H)}{b_M}}{\frac{\int_{b_L}^{B}}{\int_{b_M}^{B}} b_M g(b_M) + \frac{b_L(1-\alpha_H)}{(b_M)^2} \frac{\int_{b_L}^{b_M}}{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}} \frac{b_L}{b_M} &< 1 \iff \\
\frac{(b_L)^2 g(b_L)}{\int_{b_L}^{B}} &< \frac{(b_M)^2 g(b_M)}{\int_{b_M}^{B}}.
\end{aligned}$$

This finishes the proof.

Appendix B

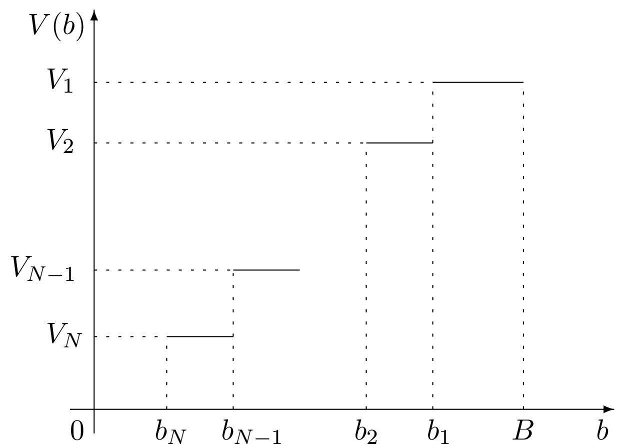
In this section, I consider a general case of the model with N consumer types.

The General Case with N Consumer Types

To complete the model, I consider the general case with N types of consumers. I show the existence and uniqueness of the equilibrium and discuss some issues related to the case when the distribution of efficiency units of labor among consumers is continuous.

In the general case, consumers differ in the number of efficiency units of labor they are endowed with. A consumer of type n is endowed with I_n efficiency units of labor. I choose indices so that $I_n > I_{n-1}$. Here α_n is the fraction of consumers of type n in the aggregate mass L of consumers. The equilibrium in the general model is similar to the equilibrium in the simple case considered before. All goods that are consumed by a certain type of consumers are also consumed by more productive consumers. Thus, goods in the economy are divided into N groups. Goods belong to group $k = 1..N$ if they are only consumed by consumers whose type is greater or equal to k. In the equilibrium, goods from the same group have the same valuation to price ratio. Let V_k be the valuation to price ratio of goods from group k. Then, in the equilibrium, $V(b)$ looks as in *Figure 8,* where b_k is such that firms with b_k are indifferent between selling to consumers with types greater or equal to k and selling to consumers with types greater or equal to $k+1$. For instance, firms with b_1 are indifferent between selling to all consumers and selling to everyone except the poorest. Firms with $b < b_N$ leave the market.

Figure 8: The Valuation to Price Function: A General Model



Without loss of generality, I assume that firms with b_k choose to sell to consumers with types greater or equal to k. As before, let M_e be the mass of firms that enter the market and draw valuation of their goods.

Definition 2 *The equilibrium of the model is defined by the price function $p(b)$ on $b \geq b_N$, M_e, the sequences $\{V_k\}_{k=1..N}$ and $\{b_k\}_{k=1..N}$ such that*

1) The ex ante expected profits of firms are equal to zero.

2) The goods market clears.

Let $\pi_k(b)$ and $p_k(b)$ be the profit and the price of a firm with valuation $b \in [b_k, b_{k-1})$, respectively.[17] Then, the following lemma holds.

Lemma 3 *In the equilibrium,*

$$p_k(b) = \frac{b}{V_k} = bc\frac{\sum_{i=k}^{N} \frac{\alpha_i}{b_i}}{\sum_{i=k}^{N} \alpha_i},$$

$$\pi_k(b) = cL\sum_{i=k}^{N} \frac{\alpha_i(b - b_i)}{b_i}.$$

Proof. See below. ■

[17] $b_0 = A$.

In the equilibrium, the expected profits of firms are equal to zero. This implies that

$$\begin{aligned} f_e &= \textstyle\sum_{k=1}^{N}(G(b_{k-1})-G(b_k))E(\pi_k(b)|b\in[b_k,b_{k-1})) \iff \\ \frac{f_e}{cL}+1 &= \textstyle\sum_{k=1}^{N}\alpha_k H(b_k). \end{aligned}$$

In addition, the goods market clearing condition should be satisfied. This implies that the aggregate cost of the bundle of goods from group k should be equal to income of a consumer of type k. In this way, I obtain

$$I_k = M_e\int_{b_k}^{B} p(t)dG(t) \quad k=1..N.$$

Hence, there is the system of $N+1$ equations

$$\begin{cases} I_k = M_e\int_{b_k}^{B} p(t)dG(t) \quad k=1..N \\ \frac{f_e}{cL}+1=\sum_{k=1}^{N}\alpha_k H(b_k) \end{cases} \tag{18}$$

with $N+1$ unknowns: $\{b_k\}_{k=1..N}$ and M_e.

Proposition 7 *The equilibrium in the general model always exists and is unique.*

Proof. See details below. ■

The Continuous Distribution of Efficiency Units of Labor

Assume that the distribution of consumer productivities is continuous. Notice that any continuous distribution can be approximated by the sequence of discrete distributions. Therefore, we can interpret equilibrium in the continuous model as the limit of equilibria in the discrete models. In this case, the function $V(b)$ is continuous, increasing on $[b_L^c, b_M^c)$, and flat on $[b_M^c, B]$, where $0 \geq b_L^c > b_M^c \geq B$. The parameter b_L^c represents the cutoff level: firms with $b < b_L^c$ leave the market. While b_M^c is determined by the support of the productivity distribution. Namely, goods with $b \in [b_M^c, B]$ are consumed by everybody in the equilibrium. This implies that $b_M^c < B$ if and only if the lower bound of the distribution support is strictly greater than zero; i.e., the minimum income in the economy is greater than zero.

Because of mathematical difficulties, it is hard to solve the continuous model for an arbitrary distribution of productivities. To solve the problem explicitly, I need to make a simplifying assumption about the distribution of efficiency units of labor. I assume that this distribution has a constant hazard rate. That is, I consider the family of exponential distributions on $[s, \infty)$, where $s \geq 0$ is the minimum endowment of efficiency units of labor. Since the upper bound of the support is infinity, the maximum income in the economy is also equal to infinity. This implies that the cutoff b_L^c equals to zero in the equilibrium. I show that in a neighborhood $b = 0$, the price function $p(b)$ is decreasing in b and $p(0) = \infty$. Hence, this model gives us a simple straightforward explanation of why some luxury goods with relatively low valuation (or quality) to price ratios are so expensive: the rich are ready to pay such high prices for these goods.

The Continuos Case: Exponential Distribution

I assume that there is a distribution $F(\cdot)$ on $[s, S]$ (with a density function $f(\cdot)$) of efficiency units of labor. That is, given the mass L of consumers, there are $F(x)L$ consumers with income less or equal to x. Define $V(b) = \frac{b}{p(b)}$. From the main body of the paper, $V(b)$ is increasing on $[b_L^c, b_M^c)$ and flat on $[b_M^c, B]$ (see *Section 3*). I assume that $V(b)$ is differentiable on $[b_L^c, b_M^c)$. To simplify the notation, I also assume that $L = 1$.

Consider a particular firm with valuation b. If $b \in [b_M^c, B]$ then demand for this good is equal to one and $p(b) = \frac{b}{V(b_M^c)}$. Suppose $b \in [b_L^c, b_M^c)$ and the firm sets price p of its good. Then, given $V(b)$ in the equilibrium, $s + \int_{V^{-1}\left(\frac{b}{p}\right)}^{b_M^c} M_e p(t) dG(t)$ is the total spendings on goods, which are bought before the good considered: goods that have higher valuation to price ratios. This implies that demand for this good is equal to $1 - F\left(s + \int_{V^{-1}\left(\frac{b}{p}\right)}^{b_M^c} M_e p(t) dG(t)\right)$. Hence, in the equilibrium, firms with $b \in [b_L^c, b_M^c)$ solve the following maximization problem

$$\max_p (p - c)\left(1 - F\left(s + \int_{V^{-1}\left(\frac{b}{p}\right)}^{b_M^c} M_e p(t) dG(t)\right)\right).$$

The first order condition implies that

$$\frac{1 - F\left(s + \int_{V^{-1}\left(\frac{b}{p}\right)}^{b_M^c} M_e p(t) dG(t)\right)}{f\left(s + \int_{V^{-1}\left(\frac{b}{p}\right)}^{b_M^c} M_e p(t) dG(t)\right)} = (p - c)\,\frac{bM_e p\left(V^{-1}\left(\frac{b}{p}\right)\right) g\left(V^{-1}\left(\frac{b}{p}\right)\right)}{p^2 V'\left(V^{-1}\left(\frac{b}{p}\right)\right)}.$$

This equation should be satisfied for any $b \in [b_L^c, b_M^c)$. That is, the price function $p(b)$ on $[b_L^c, b_M^c)$

solves the following differential equation

$$\frac{1-F\left(s+\int_b^{b_M^c} M_e p(t) dG(t)\right)}{f\left(s+\int_b^{b_M^c} M_e p(t) dG(t)\right)}=(p(b)-c)\frac{bM_e g(b)}{p(b)V'(b)} \tag{19}$$

where $V(b)=\frac{b}{p(b)}$. Using the solution of (19), free entry condition, and the goods market clearing condition, we can find b_L^c, b_M^c, and M_e.

In general, it is rather complicated to find the solution of (19). To simplify the problem, I assume that $F(x)=1-e^{-\alpha(x-s)}$ on $[s,\infty)$. This implies that $\frac{1-F\left(s+\int_b^{b_M^c} M_e p(t) dG(t)\right)}{f\left(s+\int_b^{b_M^c} M_e p(t) dG(t)\right)}=\frac{1}{\alpha}$. Thus, (19) is equivalent to

$$V'(b)=\alpha M_e(b-cV(b))\,g(b)\,. \tag{20}$$

As the maximum endowment of efficiency unit of labor is infinity, there is no exit and $b_L^c=0$. Using the initial condition $V(0)=0$ and (20), we have

$$\begin{aligned} V(b) &= \frac{1}{c}\left(b-e^{-\alpha M_e cG(b)}\int_0^b e^{\alpha M_e cG(t)}dt\right) \\ p(b) &= \frac{cb}{b-e^{-\alpha M_e cG(b)}\int_0^b e^{\alpha M_e cG(t)}dt}. \end{aligned}$$

From the goods market clearing condition, we obtain that $s=\frac{M_e}{V(b_M^c)}\int_{b_M^c}^B tdG(t)$. Using this equation and the free entry condition, we can find M_e and b_M^c.[18] Notice that $\lim_{b\to 0}p(b)=\infty$. This means that goods with the lowest valuations have the highest prices.

Proof of Lemma 3

Demand for goods from group k is equal to $L\sum_{i=k}^N \alpha_i$. From the definition of the sequence $\{b_k\}_{k=1..N}$, $\left(\frac{b_k}{V_k}-c\right)\sum_{i=k}^N \alpha_i=\left(\frac{b_k}{V_{k+1}}-c\right)\sum_{i=k+1}^N \alpha_i$. By induction,

$$\frac{\sum_{i=k}^N \alpha_i}{V_k}=\frac{1}{V_1}-c\sum_{i=1}^{k-1}\frac{\alpha_i}{b_i}. \tag{21}$$

[18] In the simplest case when $s=0$, $b_M^c=B$ and M_e can be found from $f_e=\int_0^B (p(b)-c)\,e^{-\alpha M_e\int_b^B p(t)dG(t)}dG(b)$.

From the definition of $\pi_k(b)$ and (21),

$$\pi_N(b) = \left(\frac{b}{V_N} - c\right)\alpha_N L = \frac{bL}{V_1} - cbL\sum_{i=1}^{N-1}\frac{\alpha_i}{b_i} - c\alpha_N L.$$

Recall that $\pi_N(b_N) = 0$. This implies that $\frac{1}{V_1} = c\sum_{i=1}^{N}\frac{\alpha_i}{b_i}$. From (21), $\frac{1}{V_k} = \frac{c\sum_{i=k}^{N}\frac{\alpha_i}{b_i}}{\sum_{i=k}^{N}\alpha_i}$ $k = 1..N$. Therefore,

$$\begin{aligned} p_k(b) &= bc\frac{\sum_{i=k}^{N}\frac{\alpha_i}{b_i}}{\sum_{i=k}^{N}\alpha_i}, \\ \pi_k(b) &= cL\sum_{i=k}^{N}\frac{\alpha_i(b-b_i)}{b_i}. \end{aligned}$$

Proof of Proposition 8

Using *Lemma 3*, the system of equations (18) can be rewritten as follows[19]

$$\begin{cases} \frac{f_e}{cL} + 1 = \sum_{k=1}^{N}\alpha_k H(b_k), \\ \frac{I_k - I_{k-1}}{cM_e} = \frac{\sum_{i=k}^{N}\frac{\alpha_i}{b_i}}{\sum_{i=k}^{N}\alpha_i}\int_{b_k}^{b_{k-1}} tdG(t) \quad k = 1..N \end{cases}. \tag{22}$$

Consider $k = N$. Then,

$$\frac{I_N - I_{N-1}}{cM_e} = \frac{1}{b_N}\int_{b_N}^{b_{N-1}} tdG(t). \tag{23}$$

Given M_e and b_{N-1}, there exists a unique solution $b_N(b_{N-1}, M_e)$ of the equation (23). The function $b_N(b_{N-1}, M_e)$ is strictly increasing in M_e and b_{N-1}. Given b_{N-1}, $\frac{M_e}{b_N(b_{N-1},M_e)} = \frac{I_N - I_{N-1}}{c\int_{b_N}^{b_{N-1}} tdG(t)}$ is strictly increasing in M_e.

Consider $k = N-1$. Then,

$$\frac{I_{N-1} - I_{N-2}}{cM_e} = \frac{\frac{\alpha_N}{b_N} + \frac{\alpha_{N-1}}{b_{N-1}}}{\alpha_N + \alpha_{N-1}}\int_{b_{N-1}}^{b_{N-2}} tdG(t). \tag{24}$$

Given M_e and b_{N-2}, there exists a unique solution $b_{N-1}(b_{N-2}, M_e)$ of the equation (24). The function $b_{N-1}(b_{N-2}, M_e)$ is strictly increasing in b_{N-2}. Since $\frac{M_e}{b_N(b_{N-1},M_e)}$ is strictly increasing in M_e, $b_{N-1}(b_{N-2}, M_e)$ is also strictly increasing in M_e. Finally, $\frac{\left(\frac{\alpha_N}{b_N} + \frac{\alpha_{N-1}}{b_{N-1}}\right)M_e}{\alpha_N + \alpha_{N-1}} = \frac{I_{N-1} - I_{N-2}}{c\int_{b_{N-1}}^{b_{N-2}} tdG(t)}$ is strictly increasing in M_e.

[19] $I_0 = 0$.

Using the backward induction, it can be proved that for any $k = 1..N$, there exists a unique solution $b_k(b_{k-1}, M_e)$ of the equation

$$\frac{I_k - I_{k-1}}{cM_e} = \frac{\sum_{i=k}^{N} \frac{\alpha_i}{b_i}}{\sum_{i=k}^{N} \alpha_i} \int_{b_k}^{b_{k-1}} tdG(t)$$

such that $b_k(b_{k-1}, M_e)$ is strictly increasing in b_{k-1} and M_e. This implies that for any M_e, there exists a unique solution $\{b_k(M_e)\}_{k=1..N}$ of the system of equations $\frac{I_k - I_{k-1}}{cM_e} = \frac{\sum_{i=k}^{N} \frac{\alpha_i}{b_i}}{\sum_{i=k}^{N} \alpha_i} \int_{b_k}^{b_{k-1}} tdG(t)$ $k = 1..N$. And for any $k = 1..N$, $b_k(M_e)$ is strictly increasing in M_e. Hence, (22) is equivalent to

$$\begin{cases} \frac{f_e}{cL} + 1 = \sum_{k=1}^{N} \alpha_k H(b_k(M_e)), \\ b_k = b_k(M_e) \qquad\qquad k = 1..N. \end{cases} \tag{25}$$

Consider $D(M_e) = \sum_{k=1}^{N} \alpha_k H(b_k(M_e))$. As $H(x)$ is a strictly decreasing function, $D(M_e)$ is strictly decreasing in M_e. If M_e is close to zero then $b_N(M_e)$ is close to zero and, thereby, $D(M_e)$ is high enough. If M_e is sufficiently high then for any $k = 1..N$, $b_k(M_e)$ is close to B and $D(M_e) \approx \sum_{k=1}^{N} \alpha_k H(B) = 1 < \frac{f_e}{cL} + 1$. This implies that there exists a unique solution M_e of (25). Therefore, there exists a unique solution of (18).

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