Freight Car Routing Problem

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At present time in most countries for freight transportation interaction of several companies is necessary. First company is freight, it has got railway cars and directly executes customer orders for the transportation of good by providing cars. Second company is railway, it provides transportation of the cars under special rules. We consider the freight car routing problem.

We know initial car distribution, orders, profits for delivering and tariffs for standings and for empty transfers. Most orders we can accept or reject. We should find subset of profitable orders and optimal logistic for empty cars. We have sets of stations, car models, good types, orders and sources (station can be source of cars). For all stations, car models and good types we know 2 functions: price and duration of empty transfer. Price depends on the last good type. We have also planning horizon.

For the formulation of integer programming problem we'll use integer variables for empty transfers, standings and order transfers, binary variable for orders (that indicates delivered or not). Problem can be formulated as problem of finding several flows with maximal cost in oriented graph without cycles and additional constraints. Every flow corresponds to movement the cars of the same model. Objective function maximizes profit, that equals difference between profit for delivered orders and charges for standings and empty transfers:

$$\max \qquad \sum p_{q,t-r_q} z_{qt}^{w} \quad - \quad \sum c \cdot y_{itw}^{\alpha c} \quad - \quad \sum M(i,j,k,w) x_{ijtw}^{c'c''k} \qquad (1)$$

Constraints (2) - (3) give minimal and maximal number of cars for delivering orders:

$$\sum_{w \in W_q} \sum_{t=r_q}^{r_q+\Delta_q} z_{qt}^{w} \leq v_q^{\max} u_q \qquad (2)$$

$$\sum_{w \in W_q} \sum_{t=r_q}^{r_q+\Delta_q} z_{qt}^w \geq v_q^{\min} u_q \tag{3}$$

Constraints (4) - (5) are balance equations for cars transferred for the order and freed after order:

$$y_{itw}^{1c} = y_{i,t-1,w}^{1c} + \sum x_{j,i,t-D(j,i,k,w),w}^{c'ck} - \sum z_{qt}^w, \tag{4}$$

$$y_{itw}^{2ck} = y_{i,t-1,w}^{2ck} - \sum x_{ijtw}^{cc'k} + \sum z_{q,t-d_q}^w + \sum \vec{v}_s, \tag{5}$$

Then we reformulate the problem in terms of routes. We use integer variables only for numbers of cars in routes. Then we solve its linear relaxation by column generation.

This approach gave good results for tests from freight company. In several cases working time was similar and in some cases it was significantly reduced.

The authors were supported by the Russian Foundation for Basic Research (project no. 11-08-13121).

References

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