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ENDOGENOUS RENT-SEEKING SUCCESS FUNCTIONS: A MECHANISM DESIGN APPROACH

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We describe optimal rent-seeking success functions (RSSFs), which maximize expected revenues of an administrator who allocates under informational asymmetry a source of rent among competing bidders. Optimal mechanism design produces RSSFs similar or identical to those widely used in the literature, thus offering a solid microeconomic foundation for such functional forms. Various properties and extensions of optimal RSSFs are analyzed.

Key words: Rent seeking, asymmetric information, Bayesian mechanism design

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1. Introduction

Tullock (1980) introduced input-output approach in rent-seeking analysis in the form of the context success function (CSF) that describes outcomes of contest where several agents bid resources to secure a source of rent. If $s_i, i = 1, \dots, n$ are participating agents' outlays in a rent-seeking contest, then obtained payoffs are given by CSFs $z_i(s_{1,}, \dots, s_n), i = 1, \dots, n$. Under the standard definition contests are of "winner takes all" nature, in which case CSFs describe expected gains of participants; with additional assumption of risk-neutrality they are probabilities of winning multiplied by the valuation of the prize. Sometimes the notion of contest is extended to include situations, common in rent seeking, where prizes are divisible (Hillman, Riley, 1989; Corchon, Dahm, 2008; Fey, 2008), and success functions characterize shares of the prize obtained by contenders. Both versions permit the same description when agents are risk-neutral[1], but to explicitly allow prize divisibility we will hereafter refer to the above model as *rent-seeking success function* (RSSF), borrowing the terminology from Hirshleifer (1989).

While it is natural to assume some general properties of RSSFs (e.g. they should monotonically increase in an agent's own rent-seeking outlays and monotonically decrease in outlays of other contenders), their exact forms are far from obvious. Since Tullock's seminal work, a simple fractional model

$$z_i(s_1, \dots, s_n) = V \frac{s_i}{\sum_{j=1}^{n} s_j} \tag{1}$$

where V is the value of the prize, is widely used, or its immediate logit extension

$$z_i(s_1, \dots, s_n) = V \frac{\xi(s_i)}{\sum_{j=1}^{n} \xi(s_j)} \tag{2}$$

with some monotonically increasing function ξ. Plausibility and analytical tractability were the main appeals of these forms, which explain their popularity in literature, but these and other RSSFs obviously require more solid and rigorous foundations.

Two broad approaches were proposed to address this problem. The first is *axiomatic*, whereby a particular set of axioms provide necessary and sufficient conditions for a given class of RSSFs. Scaperdas (1996) presented such axioms for logit RSSFs (2); the centrepiece of his characterization is an appropriately formulated independence of irrelevant alternatives condition. Polishchuk and Savvateev (2004) noted that if a RSSF is of the form $z_i(s_{1,}, \dots, s_n) = \Phi(s_i, \sum_{j \neq i} s_j)$, then such RSSF is identical to (1).

[1] With risk-aversion (Hillman, Katz , 1984) this is no longer the case.

The second approach supplies *micro-foundations* for RSSFs by deducing particular functional forms of rent-seeking outcomes from assumptions about rent-seeking mechanisms, institutions and information available to participating agents. An example is the well-known model of the commons (Dasgupta, Heal, 1979), when the source of rent is in public domain and aggregate payoff which depends on the total investments by participating agents is shared among them in proportion to their outlays:

$$z_i(s_1, \dots, s_n) = F\left(\sum_{j=1}^{n} s_j\right) \frac{s_i}{\sum_{j=1}^{n} s_j}; \qquad (3)$$

this model[2] is a straightforward generalization of (1). Another set of examples is given by auction-type contests when the prize goes to the highest bidder; additional randomness assumptions, e.g. that agents' outlays are augmented by random shocks (Hillman, Riley, 1989; Jia, 2008), or when agents are uncertain about how their bids are valuated (Corchon, Dahm, 2008), produce RSSFs similar to (1), (2).

In the above examples rules of the rent-seeking game are set exogenously and are not themselves a decision variable. In many instances however a source of rent is controlled by an administrator (e.g. government official) who has her own priorities and preferences over the outcome of rent seeking and resources invested by contenders. In such cases the administrator could manipulate rent-seeking rules to achieve a more preferable outcome, and these rules thus become *endogenous.* This leads to the optimal contest design approach in rent seeking (Dasgupta, Nti, 1998; Epstein, Nitzan, 2006, 2007), which yields particular types of RSSFs that under given institutional constraints best suit the administrator.

A natural setup for implementation of the optimal rent-seeking design is *informational asymmetry* between the administrator and participants, when individual characteristics (types) of the latter are not directly observable by the administrator. Indeed, if the administrator has full information, she can simply identify the first-best outcome (rent allocation and participants' outlays), subject to appropriate participation constraints, and present agents with "take it or leave it" offers that would implement such outcome. A more sophisticated approach involving RSSFs where agents make their bids in anticipation of other rent-seekers' bidding strategies, would be superfluous, if not inferior, in such case[3]. Optimal mechanism design can still generate particular RSSFs under full information, assuming that the administrator *has* to use RSSF-based allocation mechanisms and is furthermore restricted to certain classes of such functions

[2] For this model applications in a conventional rent-seeking setup see e.g. Grossman (1994).

[3] Rent-seeking equilibria based on commonly used RSSFs usually leave agents above their reservation utility levels, despite of partial dissipation of rent; this is an indication that such equilibria are *not* first-best outcomes for the rent administrator (assuming that the administrator's sole concern is revenue collection and that she does not care about agents' welfare – for a more general formulation see Epstein, Nitzan, (2006)).

(e.g. the class of logit functions (2) where ξ is concave, as in Dasgupta, Nti (1998); see also Corchon (2007)). Such assumptions could reflect e.g. institutional restrictions on rent allocation mechanism such as requirements of competitive bidding and collusion-proofness.

However, if the administrator does not have full information about agents' types, top-down take-it-or-leave-it-type contracts could no longer be optimal and not even, for that matter, feasible. In such case bidding is not a constraint imposed upon the administrator, but her instrument of choice, since bids, apart from their immediate material value, are also signals that reveal valuable information about agents' types. Optimal RSSFs that describe administrator's best response to such signals thus become fully endogenous.

Below we derive optimal RSSFs through Bayesian implementation, on the assumption that agents' types – their valuations[4] of the resource allocated by the administrator which are known only to agents themselves – are randomly drawn from a given distribution which is a common knowledge. It is shown that no matter what mechanism the administrator uses to communicate with the agents, as long as it involves trading the source of rent for agents' payments to the administrator, the best of all such mechanisms can always be represented through some RSSFs; therefore Bayesian implementation endogenizes the very model of rent seeking based on RSSFs. Furthermore the proposed approach leads to a class of RSSF which are generalizations of Tullock's logit functional forms (2), and the latter forms obtain if and only if agents have Cobb-Douglas utilities, irrespective of distribution of their types.

To get further insight into the class of RSSFs obtained through Bayesian mechanism design, we study asymptotic behaviour of such functions when the number of participating agents grows to infinity. Such analysis reveals increasing returns to scale in rent seeking as suggested in Murphy, Shleifer, Vishny (1993), and possible exclusion from rent seeking of agents with low valuation of the prize – a phenomenon observed under different assumptions by Hillman and Riley (1989). Finally, as an extension of the base model, we derive optimal RSSFs when the administrator can invest a portion of agents' contributions to augment the allocated resource – in such case, still assuming Cobb-Douglas utilities, optimal RSSFs combine features of forms (2) and (3).

The rest of the paper is organized as follows. In Section 2 a Bayesian mechanism design problem leading to optimal RSSFs is presented. This problem is solved in Section 3, producing a class of endogenous RSSFs which are optimal for given

[4] There could be other kinds of private information, e.g. costs of rent-seeking outlays (efforts) to agents, as in Fey, (2008); however such case could be re-formulated in terms of unobservable valuations.

preferences of participating agents and distributions of their types. Properties of the derived RSSFs are analyzed in Section 4, including conditions under which such RSSFs can be represented in the logit form proposed by Tullock. Section 5 investigates asymptotic properties of optimal RSSFs for large numbers of participating agents. In section 6 the analysis is extended on rent-seeking contests when the source rent is of variable size and can be enhanced by investing some of the payments collected from rent-seekers. Section 7 concludes.

2. Rent-seeking and Bayesian mechanism design

Consider a model where the administrator allocates one unit of resource which is a source of rent for n agents, each with a quasi-linear utility function $f(z_i, w_i) - s_i$, $i = 1,..,n$; here z_i is the quantity of resource obtained from the administrator, s_i – rent-seeking outlay, and w_i – agent's type which is his private information. We assume positive and diminishing marginal returns to the allocated resource: $f_z > 0, f_{zz} < 0$; utility increasing in agent's type: $f_w > 0$; and a single-crossing property $f_{wz} > 0$. One way to interpret agents' types is to view them as endowments of another resource, complementary to the one allocated by the administrator, in which case types become indicators of agents' wealth, and f is a two-input production function. In what will follow we use a multiplicative specification $f(z, w) = w\varphi(z)$, where $\varphi(\cdot)$ is monotonically increasing, smooth, concave and satisfies the Inada conditions $\varphi(0) = 0, \lim_{z\to\infty} \varphi'(z) = 0, \lim_{z\to 0} \varphi'(z) = \infty$. However, most of our results also hold for a general constant returns to scale two-input production function.

Agents' types are randomly and independently drawn from a distribution with cumulative function $G(w)$ and density $g(w), w \in [\underline{w}, \overline{w}]$, $0 \leq \underline{w} < \overline{w} \leq \infty$; this distribution is common knowledge to all parties involved. The function $\rho(w) \equiv w - \frac{1-G(w)}{g(w)}$ (marginal revenue, or valuation, as it is known in the auction theory – see e.g. Klemperer (1999)) is assumed monotonically increasing – a condition which is satisfied for most commonly used distributions, including those with increasing hazard rate $\frac{g(w)}{1-G(w)}$. Both the administrator and agents are risk-neutral.

Informational asymmetry prompts the administrator to communicate with agents prior to allocating the resource. Such communication is based on a mechanism $\mathcal{M} = (M_1,..., M_n; a(\cdot))$, conventionally defined as a collection of strategy sets from which agents select their messages $m_i \in M_i$, and an allocation function $a(m_1, ..., m_n)$ which describes administrator's decision in response to received messages. In mechanisms considered below an allocation comprises a set of payments $s_1, \dots, s_n$ of the agents to the administrator, and a division $\sum_{i=1}^{n} z_i \leq 1$ of the unit stock of resource among the agents:

$$a(m_1, \dots, m_n) = (s_1(m_1, \dots, m_n), \dots, s_n(m_1, \dots, m_n), \\ z_1(m_1, \dots, m_n), \dots, z_n(m_1, \dots, m_n)). \qquad (4)$$

This mechanism works as follows: once all agents have communicated to the administrator their messages $m_i, i = 1, \dots, n$, agent i is required to make the payment $s_i(m_1, \dots, m_n)$ to the administrator and receives in exchange the amount $z_i(m_1, \dots, m_n)$ of the allocated resource. Notice that there are no a priory restrictions on the content of messages (or, what is the same, on information sets); in particular, no communication is also an option with a constant allocation function.

Rent-seeking success functions form a sub-set of such mechanisms, whereby messages are payments $s_i \geq 0$ offered to the administrator, and the allocation is the $2n$-tuple $\langle s_1, \dots, s_n;\ z_1(s_1, \dots, s_n), \dots, z_n(s_1, \dots, s_n)\rangle$ of rent-seeking outlays and outcomes. It will be shown in the next section that the administrator can restrict her choice of the best mechanism to this subset.

We assume Bayesian mechanism implementation, in which case agents' strategies form a Bayes-Nash equilibrium – they are functions $m_i(w_i),\ i = 1, \dots, n$ of agents' types such that for every type w_i $m_i(w_i)$ maximizes agent i's expected utility, conditional on other agents playing strategies $m_j(w_j),\ j \neq i$:

$$E_{-w_i}[w_i \varphi(z_i(m_i(w_i), m_{-i}(w_{-i}))) - s_i(m_i(w_i), m_{-i}(w_{-i}))] \geq \\ E_{-w_i}[w_i \varphi(z_i(m'_i, m_{-i}(w_{-i}))) - s_i(m'_i, m_{-i}(w_{-i}))], i = 1, \dots, n, \qquad (5)$$

for all feasible messages $m'_i \in M_i$.[5]

Since participation in rent-seeking game is voluntary, the administrator also needs to ensure that equilibrium outcomes leave agents (non-strictly) above their reservation utility levels which in the present context equal zero:

$$E_{-w_i}[w_i \varphi(z_i(m_i(w_i), m_{-i}(w_{-i}))) - s_i(m_i(w_i), m_{-i}(w_{-i}))] \geq 0, \\ i = 1, \dots, n. \qquad (6)$$

Now the optimal rent-seeking mechanism design problem can be stated as maximization of the expected gross payoff collected by the administrator from the agents

[5] We use the notation $"-i"$ as a conventional shortcut for "all variables other than i". For more on Bayes-Nash equilibria in rent-seeking games with asymmetric information see Malueg, Yates (2004), Fey (2008).

$$\max E \sum_{i=1}^{n} s_i\big(m_i(w_i), m_{-i}(w_{-i})\big) \tag{7}$$

over all mechanisms (4) subject to conditions (5), (6) and the resource constraint

$$\sum_{i=1}^{n} z_i\big(m_i(w_i), m_{-i}(w_{-i})\big) \leq 1. \tag{8}$$

3. Optimal rent-seeking success functions

The problem of optimal mechanism design is considerably simplified when mechanisms are *direct*, i.e. agents' messages are announcements (truthful or otherwise) of their types. In the present context a direct mechanism includes strategy sets $M_i = [\underline{w}, \overline{w}]$ and functions $\tilde{s}_i$ and $\tilde{z}_i$, defined over $\underbrace{[\underline{w}, \overline{w}] \times \ldots \times [\underline{w}, \overline{w}]}_{n \text{ times}}$, $i = 1, \ldots, n$, such that once agents' types are reported as w'_i, the mechanism requires agent $i = 1, \ldots, n$ to make the payment $\tilde{s}_i(w'_1, \ldots, w'_n)$ to the administrator against obtaining from her $\tilde{z}_i(w'_1, \ldots, w'_n)$ units of the allocated resource. Direct mechanism is *incentive compatible* if correct reporting by agents of their types constitutes a Bayes-Nash equilibrium, i.e.

$$E_{-w_i}[w_i \varphi(\tilde{z}_i(w_i, w_{-i})) - \tilde{s}_i(w_i, w_{-i})] \geq \tag{5'}$$
$$E_{-w_i}[w_i \varphi(\tilde{z}_i(w'_i, w_{-i})) - \tilde{s}_i(w'_i, w_{-i})], \forall\, w'_i \in [\underline{w}, \overline{w}],\ i = 1, \ldots, n.$$

According to the revelation principle (Myerson 1981), if functions $m_1(\cdot), \ldots, m_n(\cdot)$ form a Bayes-Nash equilibrium for a mechanism (4), then the functions

$$\tilde{s}_i(w'_1, \ldots, w'_n) = s_i\big(m_1(w'_1), \ldots, m_n(w'_n)\big), \tag{9}$$
$$\tilde{z}_i(w'_1, \ldots, w'_n) = z_i\big(m_1(w'_1), \ldots, m_n(w'_n)\big),$$

$i = 1, \ldots, n$, represent a direct incentive-compatible mechanism such that for any combination of agents' types $w_1, \ldots, w_n$ the two mechanisms yield the same allocation. Therefore the choice of optimal mechanisms can be confined to direct mechanisms $\tilde{z}_i(\cdot), \tilde{s}_i(\cdot)$, and the administrator's problem set forth in the previous section can be re-stated as follows:

$$\max E \sum_{i=1}^{n} \tilde{s}_i(w_i, w_{-i}) \qquad (7')$$

subject to the resource constraint

$$\sum_{i=1}^{n} \tilde{z}_i(w_i, w_{-i}) \leq 1, \qquad (8')$$

the incentive compatibility constraints (5'), and participation constraints

$$E_{-w_i}[w_i \varphi(\tilde{z}_i(w_i, w_{-i})) - \tilde{s}_i(w_i, w_{-i})] \geq 0, \qquad i = 1, \dots, n. \qquad (6')$$

We will now demonstrate that the optimal solution of this problem (which delivers the best results over all conceivable mechanisms (4)) can be implemented by appropriately chosen RSSFs. To this end, first notice that in a direct incentive-compatible mechanism $\tilde{s}_i(\cdot), \tilde{z}_i(\cdot)$ satisfying participation constraints (6') transfer functions $\tilde{s}_i(w_i, w_{-i})$ can be replaced by $\tilde{s}_i(w_i) \equiv E_{-w_i}\tilde{s}_i(w_i, w_{-i})$ (for simplicity we keep the same notation for such reduced single-variable form) – the new mechanism remains incentive-compatible, also meets participation constraints, and yields the same value to the maximand (7'). Therefore without loss of generality $\tilde{s}_i(\cdot)$ can be assumed depending on w_i alone; this assumption is kept through the rest of the paper.

Next, tools of the optimal auction theory (Myerson, 1981; Maskin, Riley, 1989; Klemperer, 1999) are used to solve the problem (5')-(8').

Proposition 1 *Optimal direct mechanism which solves the problem (5')-(8') is as follows*:

$$\tilde{z}_i(w_i, w_{-i}) = F\left(\frac{[\rho(w_i)]_+}{A_F([\rho(w_1)]_+, \dots, [\rho(w_n)]_+)}\right), \quad i = 1, \dots, n, \qquad (10)$$

where $\rho(w)$ *is the marginal revenue function for distribution* G*;* $[x]_+ \equiv max(x, 0)$*;* $F(t) \equiv (\varphi')^{-1}(1/t)$*; and symmetric function* $A_F(x_1, \dots, x_n)$ *is uniquely determined for all* $x_1 \geq 0, \dots, x_n \geq 0$, $\sum_{i=1}^{n} x_i > 0$ *by the following equation:*

$$\sum_{i=1}^{n} F\left(\frac{x_i}{A_F(x_1, \dots, x_n)}\right) = 1 \qquad (11)$$

(if $\rho(w_i) \leq 0$ *for all* i*, then all* $\tilde{z}_i$ *are equal zero); and*

$$\tilde{s}_i(w_i) = \tilde{s}(w_i) \equiv w_i \bar{\varphi}(w_i) - \int_{\underline{w}}^{w_i} \bar{\varphi}(s) ds, \quad (12)$$

where

$$\bar{\varphi}(w_i) \equiv E_{w_{-i}} \varphi\big(\tilde{z}_i(w_i, w_{-i})\big). \quad (13)$$

Proofs of this and subsequent propositions can be found in the Appendix.

Finally, a set of endogenous RSSFs which solve the optimal mechanism design problem (without an a priory requirement that such mechanism is RSSF-based) obtains from the above direct mechanism. To this end, one has to eliminate agents' types w_i from (10), (12). Recall that the marginal revenue function ρ monotonically increases in type, and therefore there exists $w^0 \in [\underline{w}, \overline{w})$ such that $\rho(w_i) > 0$ for all $w_i \in [\underline{w}, \overline{w}]$, $w_i > w^0$, and $\rho(w_i) < 0$ for all $w_i \in [\underline{w}, \overline{w}], w_i < w^0$. Notice further that for all $w_i < w^0$ agent i obtains no resource from the administrator and hence due to (12), (13) makes no contribution, whereas for $w_i > w^0$ both amounts are positive. It is shown in the Appendix that over the range of $[w^0, \overline{w}]$ the function $\tilde{s}(\cdot)$ monotonically increases, and therefore the mechanism (10)-(13) indeed yields RSSFs which solve the problem (5)-(8).

Proposition 2 *The function $\tilde{s}(\cdot)$ monotonically increases for $s \in [\underline{s}, \overline{s}]$, where $\underline{s} = \tilde{s}(w^0)$, $\overline{s} = \tilde{s}(\overline{w})$, and optimal RSSFs solving the problem (5)-(8) are defined over $s_i \in [\underline{s}, \overline{s}], i = 1, \dots, n$ as follows[6]:*

$$z_i(s_i, s_{-i}) = F\left(\frac{\rho\big(\tilde{s}^{-1}(s_i)\big)}{A_F\Big(\rho\big(\tilde{s}^{-1}(s_1)\big), \dots, \rho\big(\tilde{s}^{-1}(s_n)\big)\Big)}\right), \quad i = 1, \dots, n. \quad (14)$$

4. Properties of optimal rent-seeking success functions

Optimal RSSFs (14) can be represented as

$$z_i(s_i, s_{-i}) = F\left(\frac{\eta(s_i)}{A_F(\eta(s_1), \dots, \eta(s_n))}\right), i = 1, \dots, n, \quad (15)$$

[6] These functions can be extrapolated beyond the "equilibrium range" $[\underline{s}, \overline{s}]$ by letting $z_i(s_i, s_{-i}) = 0$ for $s_i < \underline{s}$ and $z_i(s_i, s_{-i}) = z_i(\overline{s}, s_{-i})$ for $s_i > \overline{s}$.

where $\eta(s) \equiv \rho(\tilde{s}^{-1}(s))$ is a monotonically increasing function; note that the function F is also monotonically increasing and $F(0) = 0$. Generally RSSFs (14) are not of Tullock's logit form (2), although they share with that form some common properties. Thus, both classes of RSSFs – (2) and (15) – conform to the basic intuition of rent-seeking technologies – rent-seeking outcome for a given agent increases in his own outlay s_i and decreases in outlays of all other agents; furthermore, such outcome is determined by a ratio of an appropriate valuation (monotone transformation) of the agent's outlay $\eta(s_i)$ to an aggregate (average) of such valuations of outlays of all agents.

Proposition 3 *The following statements hold:*

(i) *The function $A_F(x_1, \dots, x_n)$ is monotonically increasing in its arguments.*

(ii) *The function $t_n A_F$, where $nF(t_n) = 1$, is a generalized average*[7] *of $x_1, \dots, x_n$ in that it is symmetric and such that $\min x_i \le t_n A_F(x_1, \dots, x_n) \le \max x_i$; in particular $t_n A_F(x, \dots, x) = x$.*

(iii) *The function $z_i(s_i, s_{-i})$ monotonically increases in $s_i \in [\underline{s}, \overline{s}]$ and monotonically decreases in $s_j \in [\underline{s}, \overline{s}]$ for all $j \ne i$.*

For logit RSSFs (2) $F(t) = t, A_F(x_1, \dots, x_n) = \sum_{i=1}^{n} x_i$, and $t_n = 1/n$, and therefore $t_n A_F(x_1, \dots, x_n) = \sum_{i=1}^{n} x_i/n$ is the conventional average of $x_1, \dots, x_n$. Functions (15) can be reduced to the logit form if the utility function is of Cobb-Douglas type: $\varphi(z) = \alpha^{-1} z^{\alpha}$, $\alpha \in (0,1)$. In such case $F(t) = t^{1/1-\alpha}$, and RSSFs (15) take form (2) with $V = 1$ and the following
monotonically increasing functions

$$\xi(s) = \eta(s)^{1/1-\alpha} = [\rho(\tilde{s}^{-1}(s))]^{1/1-\alpha}. \qquad (16)$$

It turns out that Cobb-Douglas utility is not just sufficient, but also necessary for logit representation of RSSFs (15).

Proposition 4 *Rent-seeking success functions (15) admit logit representation (2) if and only if $\varphi(z) = Cz^{\alpha}$, $\alpha \in (0,1), C > 0$.*

Resource allocations achieved through optimal RSSFs (14) as a rule are not ex post socially efficient (the only exception is the Pareto distribution $G(w) = 1 -$

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[7] For a similar but more restrictive concept of generalized averages see Kolmogorov (1985).

$(\underline{w}/w)^k, k > 1, \underline{w} > 0, \overline{w} = \infty$, when $\rho(w)/w = \text{const}$)[8]. Efficiency losses are the toll of the informational asymmetry[9]; such losses are especially severe for "low" types w_i, and in the case $a(\underline{w}) < 0$ (or, what is the same, $\underline{w} < w^0 < \overline{w}$) take the extreme form of complete exclusion of agents in the $[\underline{w}, w^0]$ range from the resource allocation process, whereas social efficiency requires allocation of positive amounts of the resource to all agents with $w_i > 0$. If agents' types are treated, as in Section 2, as endowments of a complementary production input, such exclusion could be interpreted as *informational discrimination* of poorer agents, which are "too small" to be of interest for the resource administrator and would restrict her ability to extract revenue from wealthier rent-seekers[10]. This observation sheds new light on the causes of entry barriers that owners of small assets face: in addition to political economy/public choice explanations (Djankov et al., 2002; Polishchuk, 2008) and inequality of stakes arguments (Hillman, Riley, 1989), such discrimination could also have informational rationales.

5. Limiting case: a continuous model

Additional insight into properties of endogenous RSSFs can be gained by considering the limiting case of an "atomless" model which approximates rent seeking with a large number of participants.

Suppose that rent-seekers form a unit continuum of agents with the distribution $G(w)$ of their types, and the resource administrator allocates one unit of resource across this continuum by implementing a direct mechanism $\tilde{s}_\infty(\cdot), \tilde{z}_\infty(\cdot)$, so that an agent that reveals his type as w gets $\tilde{z}_\infty(w)$ units of resource against a contribution of $\tilde{s}_\infty(w)$. This mechanism is incentive-compatible iff

$$w\varphi(\tilde{z}_\infty(w)) - \tilde{s}_\infty(w) \geq w\varphi(\tilde{z}_\infty(w')) - \tilde{s}_\infty(w'), \forall w, w' \in [\underline{w}, \overline{w}], \qquad (17)$$

and the participation constraint takes form

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[8] Note however that if the resource is non-divisible, optimal auctions with symmetric bidders and monotonically increasing marginal valuation function ρ always deliver efficient outcomes (Klemperer, 1999).

[9] Social losses and rent dissipation due to informational asymmetry in rent-seeking contest were observed in a different setting in Hillman, Riley (1989).

[10] Similarly a price-discriminating monopolist could elect under informational asymmetry not to cater to lower wealth/valuation segment of the market in order to enhance the yield of the more lucrative part.

$$w\varphi\big(\tilde{z}_\infty(w)\big) - \tilde{s}_\infty(w) \geq 0,\ \forall w \in [\underline{w}, \overline{w}]. \tag{18}$$

The optimal mechanism maximizes the administrator's aggregate revenues $\int_{\underline{w}}^{\overline{w}} \tilde{s}_\infty(w) g(w) dw$ subject to constraints (17), (18) and the resource constraint $\int_{\underline{w}}^{\overline{w}} \tilde{z}_\infty(w) g(w) dw \leq 1$ and is as follows (Tonis, 1998):

$$\tilde{z}_\infty(w) = F\left(\frac{[\rho(w)]_+}{A_\infty}\right),\ \tilde{s}_\infty(w) = w\varphi(\tilde{z}_\infty(w)) - \int_{\underline{w}}^{w} \varphi(\tilde{z}_\infty(t)) dt, \tag{19}$$

where $A = A_\infty$ is the unique solution of the equation

$$\int_{\underline{w}}^{\overline{w}} F\left(\frac{[\rho(w)]_+}{A}\right) g(w) dw = 1 \tag{20}$$

(it is assumed through the end of this section that one has $\int_{\underline{w}}^{\overline{w}} F\left(\frac{[\rho(w)]_+}{A}\right) g(w) dw < \infty, \forall A > 0$). The function $\tilde{z}_\infty(w)$, and hence $\tilde{s}_\infty(w)$, are monotonically increasing for $w \in [\underline{w}, w^0]$, and

$$z_\infty(s) \equiv \tilde{z}_\infty\big(\tilde{s}_\infty^{-1}(s)\big),\ s \in [\underline{s}_\infty, \overline{s}_\infty], \tag{21}$$

with $\underline{s}_\infty = \tilde{s}_\infty(\underline{w}), \overline{s}_\infty = \tilde{s}_\infty(\overline{w})$, is a rent-seeking success function, which in the present case depends only on an agent's own contribution. We will now show that this function approximates optimal RSSFs (14) when the number of participating agents is large.

To this end, suppose that n agents with types $w_1, \dots, w_n$ are randomly and independently drawn from the distribution $G(w)$ to obtain a discrete approximation of the said distribution, so that each agents carries a weight $1/n$. This means that if z_i and s_i are resp. the resource allocated to agent i and his payment, then the resource constraint takes form $\sum_{i=1}^{n} \frac{1}{n} z_i \leq 1$, and similarly the resource administrator's revenue equals $\sum_{i=1}^{n} \frac{1}{n} s_i$. The optimal RSSF-based mechanism for such sample $z_i^{(n)}(s_i, s_{-i})$ with $s_i \in [\underline{s}^{(n)}, \overline{s}^{(n)}]$ is described above (superscript n stands for the size of the sample) with the only modification that $A_F^{(n)}$ now satisfies the following equation:

$$\sum_{i=1}^{n} \frac{1}{n} F\left(\frac{x_i}{A_F^{(n)}(x_1, \dots, x_n)}\right) = 1 \tag{11'}$$

Functions (21) approximate RSSFs (14), (11') "on the average" in the following sense: when contributions of all agents but i are fixed at their equilibrium levels $s_j = \tilde{s}^{(n)}(w_j), j = 1, \dots, n; j \neq i$, one obtains a parametric family of single-variable rent-seeking success functions $z_i^{(n)}(s_i|w_{-i}) \equiv z_i^{(n)}(s_i, \tilde{s}_{-i}^{(n)}(w_{-i}))$, and according to the following proposition, for a given outlay s_i the expected value of such functions over other agents' types approaches $z_\infty(s_i)$ for large n. We establish such convergence in the next two propositions under an additional technical assumption $a(\underline{w}) > 0$.

Proposition 5 *Domains of RSSFs $z_i^{(n)}(s_i, s_{-i})$ approximate those of $z_\infty(s)$: $\lim_{n\to\infty} \underline{s}^{(n)} = \underline{s}_\infty$, $\lim_{n\to\infty} \overline{s}^{(n)} = \overline{s}_\infty$, and*

$$\lim_{n\to\infty} E_{-w_i} z_i^{(n)}(s_i|w_{-i}) = z_\infty(s_i), \forall s_i \in (\underline{s}, \overline{s})^{11}. \tag{22}$$

For Cobb-Douglas utilities, when according to Proposition 4 optimal RSSFs allow a logit representation

$$z_i^{(n)}(s_1, \dots, s_n) = \frac{\xi^{(n)}(s_i)}{\frac{1}{n}\sum_{j=1}^{n} \xi^{(n)}(s_j)} , \tag{23}$$

where functions $\xi^{(n)}(\cdot)$ are calculated according to (16) (with $\tilde{s}(w)$ is replaced by $\tilde{s}^{(n)}(w)$), Proposition 5 can be re-stated as convergence of $\xi^{(n)}$to a constant multiple of $z_\infty(\cdot)$.

Proposition 6 *If $\varphi(z) = Cz^\alpha$, $\alpha \in (0,1)$, and*

$$\xi^{(n)}(s_i) = \left[\rho\left((\tilde{s}^{(n)})^{-1}(s)\right)\right]^{1/1-\alpha},$$

one has

$$\lim_{n\to\infty} \xi^{(n)}(s_i) = \xi_\infty(s_i) \equiv \left[\rho\left(\tilde{s}_\infty^{-1}(s_i)\right)\right]^{1/1-\alpha} = z_\infty(s_i) \int_{\underline{w}}^{\overline{w}} [\rho(w)]^{1/1-\alpha} g(w)dw,\ \forall s_i \in (\underline{s}, \overline{s}). \tag{24}$$

[11] It is assumed that n is large enough to have $s_i \in [\underline{s}^{(n)}, \overline{s}^{(n)}]$.

Properties of the limit RSSF z_∞ can now be extended in the above described sense on the optimal RSSFs $z_i^{(n)}$, when the number of agents is sufficiently large. One such property is increasing returns to scale[12] which holds under a mild additional assumption.

Proposition 7 *If the ratio $\rho(w)/w$ monotonically non-decreases*[13]*, then the limiting RSSF $z_\infty(s_i)$ is convex.*

According to the above proposition, when agents are sufficiently numerous, those among them with higher valuation of the source of rent (larger endowments of a complementary input) obtain the resource allocated by the administrator on increasingly better terms[14] (whereas, as it was noted above, agents at the bottom of the type distribution could even opt out of rent seeking altogether). Such discrimination[15] leads to re-distribution of the allocated resource (in comparison with the socially optimal competitive benchmark when the resource is sold at the market-clearing price) from “low” to “high” types to which optimal RSSFs give a scale advantage.

Consider as an example uniform distribution of w on the [2,3] range and agents’ utility functions with $\varphi(z) = 2\sqrt{z}$. In this case the limiting function $\xi_\infty(s_i) = 12 + 4.16\, s_i - 8\sqrt{(2 + 1.04 s_i)}$, defined over the [1.92, 6.73] range; the graph of this function and convergence to it of $\xi^{(n)}(s_i)$ are shown on Fig. 1.

Another noteworthy example can be obtained by combining the Pareto distribution $G(w) = 1 - (\underline{w}/w)^{\,k}$ $(\underline{w} \le w < \infty)$ and Cobb-Douglas utility $\varphi(z) = \alpha^{-1} z^{\alpha}$ with $\alpha \in (0,1)$, $k(1-\alpha) > 1$. In this case the limit of the functions $\xi^{(n)}(s_i)$ entering optimal RSSFs (2) for finite n is as follows:

$$\xi_\infty(s_i) = C(s_i/\underline{s} \; - (1-\alpha)),$$

where $\underline{s} = \underline{w}(1 - \frac{1}{k(1-\alpha)})^{\alpha}/\alpha$, and $C = \frac{1}{\alpha}[\underline{w}\left(1 - \frac{1}{k}\right)]^{1/1-\alpha}$. Here the limiting function ξ_∞ is linear in agents’ outlays, and therefore for large n the optimal RSSFs can be approximated by Tullock’s initial fractional model (1).

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[12] Increasing returns to scale in rent-seeking activities was observed in a different context in Murphy, Shleifer, Vishny (1993). More generally on the role of economy of scale in rent-seeking see Tullock (1980).

[13] This condition holds if e.g. the hazard rate $\frac{g(w)}{1-G(w)}$ of the distribution $G(w)$ monotonically increases.

[14] Such feature commonly occurs in optimal contracts due to the single-crossing property.

[15] For interpretation of optimal auctions as monopolistic price discrimination see (Bulow, Roberts, 1989).

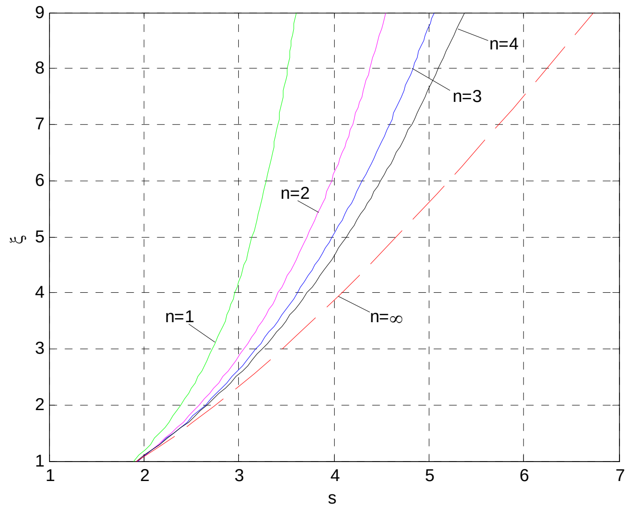

Fig. 1. Convergence of rent-seeking success functions

6. An extension: variable resource

It was assumed so far that the stock of resource allocated by the administrator is fixed; however in many applications it can be expanded at some additional cost to the administrator. To explore such situations, in this section the administrator has the option to partially invest payments collected from rent-seeker to augment the allocated resource; this will lead to a yet another class of RSSFs.

Namely, let the administrator have access to a resource-production technology with monotonically increasing and convex production function $\mathcal{F}(s)$. If the administrator invests in this technology an amount s_0 from her total receipts and keeps the balance $\sum_{i=1}^{n} s_i - s_0$, then she will have $\mathcal{F}(s_0)$ units of the resource available for allocation to rent-seekers. To obtain endogenous RSSFs in this setting, the procedure presented in Section 4 is still applicable, with the following modification: direct mechanisms now include, in addition to functions $\tilde{s}_i$ and $\tilde{z}_i$, a yet another function $\tilde{s}_0(w_1, \dots w_n)$, which together satisfy the constraints

$$\sum_{i=1}^{n} \tilde{z}_i(w_i, w_{-i}) \le \mathcal{F}(\tilde{s}_0(w_1, \dots w_n));\ \tilde{s}_0(w_1, \dots w_n) \le \sum_{i=1}^{n} \tilde{s}_i(w_i). \qquad (25)$$

The optimal direct mechanism maximizes the expected payoff of the administrator

$$\max E\,[\sum_{i=1}^{n} \tilde{s}_i(w_i) - \tilde{s}_0(w_1, \dots w_n)] \tag{26}$$

subject to constraints (5'), (6') and (25). The rest of the procedure remains the same, and its outcome, assuming again agents' Cobb-Douglas utilities $\varphi(z) = \alpha^{-1} z^{\alpha}$, is as follows.

Proposition 8 *Optimal RSSFs admit representation*

$$z_i(s_i, s_{-i}) = H(\sum_{j=1}^{n} \xi(s_j)) \ \frac{\xi(s_i)}{\sum_{j=1}^{n} \xi(s_j)} \tag{27}$$

with some monotonically increasing functions ξ *and* H. *Here* $H(t) \equiv \mathcal{F}(\Psi(t))$, *where* $\Psi(\cdot)$ *is an inverse function to* $\mathcal{F}(s)/[\mathcal{F}'(s)]^{1/1-\alpha}$; *functions* $\xi(s)$ *are calculated according to (12), (13) and (16) with the underlying direct mechanism*

$$\tilde{z}_i(w_i, w_{-i}) = H(\sum_{j=1}^{n} \rho(w_j)^{1/1-\alpha}) \ \frac{\rho(w_i)^{1/1-\alpha}}{\sum_{j=1}^{n} \rho(w_j)^{1/1-\alpha}}. \tag{28}$$

Endogenous RSSFs combine features of the functional forms (2) and (3); e.g. for a Cobb-Douglas resource production technology $\mathcal{F}(s) = s^{\beta}, 0 < \beta < 1,$ one obtains $H(t) = Ct^{\beta - \alpha\beta/1-\alpha\beta}$, for some $C > 0$. Notice that according to (27) the rent-seeking contest acquires features of public good provision, since rent-seekers' contributions, driven by individual self-interest, also increase the total supply of resource (for more on rent-seeking and public goods see Congleton, Hillman, Konrad (2008)).

Discretion of the administrator who is a net revenue maximizer over how much to invest in resource production entails additional efficiency losses, on top of those in resource allocation (see Section 4), since the equilibrium investment falls short of the ex post social optimum. Indeed, it follows from (28) that $\tilde{s}_0(w_1, \dots w_n) = \Psi(\sum_{j=1}^{n} \rho(w_j)^{1/1-\alpha})$, whereas it is easy to verify that the first-best investment s^* is as follows: $s^*(w_1, \dots w_n) = \Psi(\sum_{j=1}^{n} {w_j}^{1/1-\alpha})$, and since Ψ monotonically increases (as an inverse to a monotonically increasing function), and $\rho(w) < w,\ \forall w < \overline{w}$, one has $\tilde{s}_0(w_1, \dots w_n) < s^*(w_1, \dots w_n),$ unless all agents are of the highest possible type. Such efficiency losses are due to the administrator's inability to fully appropriate the resource rent which is partly shared with rent-seeking agents – full rent appropriation is precluded by informational asymmetry[16].

7. Concluding remarks

The paper contributes to the strand of public choice literature where rules of rent-seeking contests are not assumed upfront but instead are endogenous to some plausible behavioural, institutional, informational etc. assumptions. Here such assumptions include informational asymmetry and revenue maximization by the rent administrator. It is argued that this is a natural setup for rent-seeking contests, since bidding is essential to deal with informational asymmetry that restricts the revenue-collection ability of the administrator. Indeed, in such case the rent seeking success function model is endogenous to the above informational and behavioural assumptions, and yields RSSFs which are similar (and under additional assumptions identical) to those commonly used in the rent-seeking studies.

Analysis of endogenous RSSFs sheds light on a number of distributional issues of public choice and political economy, such as discrimination of small stake holders and increasing returns in rent seeking. It reveals origins of efficiency losses in rent seeking, including the failure to achieve socially optimal investments into rent-generating resources.

The above analysis can be extended in several ways, to reflect variations in the setups of rent-seeking and auction theory (Klemperer, 1999; Corchon, 2007; Congleton, Hillman, Konrad, 2008). Such extensions include, but are not limited to, bidders' asymmetry; risk-aversion; collective rent seeking and possibility of collusion; more complex preferences of the administrator, combining private and public interest; entry costs; etc., and are left to future research.

[16] Similarly in (McGuire, Olson, 1996) the autocrat under-invests in her tax base (in comparison to the social optimum) due to deadweight losses of taxation.

Appendix

Proof of Proposition 1 The problem (5')-(8') is solved by using tools of the mechanism design/optimal auction theory (Myerson, 1981; for the case of divisible prize see also Maskin and Riley, 1989). Consider agents' expected net equilibrium payoffs

$$\pi_i(w_i) \equiv w_i\, E_{w_{-i}}\varphi\big(\tilde{z}_i(w_i, w_{-i})\big) - \tilde{s}_i(w_i), \qquad i = 1, \dots, n. \tag{A.1}$$

Assuming interior w_i and differentiability, the necessary condition for incentive compatibility (5') is

$$w_i \bar{\varphi}_i{}'(w_i) = \tilde{s}_i{}'(w_i), \tag{A.2}$$

where

$$\overline{\varphi}(w_i) \equiv E_{w_{-i}}\varphi\big(\tilde{z}_i(w_i, w_{-i})\big), \tag{A.3}$$

or equivalently,

$$\pi_i'(w_i) \equiv \bar{\varphi}_i(w_i), \tag{A.4}$$

for all $i = 1, \dots, n$. Furthermore, incentive compatibility constraints are satisfied if and only if equalities (A.4) hold and functions $\bar{\varphi}_i(w_i)$ are monotonically non-decreasing.

According to (A.4), functions π_i are non-decreasing, and therefore once the participation constraint (6') is satisfied for the lowest type $\underline{w}$, it holds for all other types. In the optimum $\pi_i(0) = 0$, so that $\pi_i(w_i) = \int_{\underline{w}}^{w_i} \overline{\varphi}_i(s)ds$ and hence

$$\tilde{s}_i(w_i) = w_i \bar{\varphi}_i(w_i) - \int_{\underline{w}}^{w_i} \bar{\varphi}(s)ds. \tag{A.5}$$

Substituting (A.5) into the administrator's objective function, one has

$$\int_{\underline{w}}^{\overline{w}} \tilde{s}_i(w_i) g(w_i) dw_i =$$

$$\int_{\underline{w}}^{\overline{w}} w_i\, \overline{\varphi}_i(w_i) g(w_i) dw_i - \int_{\underline{w}}^{\overline{w}} \int_{\underline{w}}^{w_i} \overline{\varphi}_i\, (t)dt\; g(w_i)dw_i =$$

$$= \int_{\underline{w}}^{\overline{w}} [w_i - \frac{1 - G(w_i)}{g(w_i)}]\, \bar{\varphi}_i(w_i)\; g(w_i) dw_i.$$

Administrator's gross payoff can thus be represented as

$$\int_{\underline{w}}^{\overline{w}} \dots \int_{\underline{w}}^{\overline{w}} [\sum_{i=1}^{n} \rho(w_i)\varphi(\tilde{z}_i(w_i, w_{-i}))] g(w_1) \dots g(w_n) dw_1 \dots dw_n,$$

and, ignoring for a moment constraints (5'), (6'), functions $\tilde{z}_i(\cdot)$ can be found from the following problems:

$$\max[\sum_{i=1}^{n} \rho(w_i)\varphi(\tilde{z}_i(w_i, w_{-i})], \qquad \text{(A.6)}$$
$$\sum_{i=1}^{n} \tilde{z}_i(w_i, w_{-i}) \le 1, \tilde{z}_j(w_j, w_{-j}) \ge 0, j = 1, \dots, n,$$

for any $w_j \in [\underline{w}, \overline{w}], j = 1, \dots, n.$ This is a standard resource allocation problem, and given the neo-classical properties of φ, its solution is as follows:

$\rho(w_i)\varphi'(\tilde{z}_i(w_i, w_{-i})) = \lambda(w_1, \dots, w_n),$ for all $i = 1, \dots, n$ such that $a(w_i) > 0$;

$\tilde{z}_i(w_i, w_{-i}) = 0$, for all $i = 1, \dots, n$ such that $\rho(w_i) \le 0$.

Solving for $\lambda(w_1, \dots, w_n)$ from the budget constraint $\sum_{i=1}^{n} \tilde{z}_i(w_i, w_{-i}) \le 1$, one obtains (10); equation (11) indeed has a unique solution, since F is monotonically increasing and $F(0) = 0, F(t) \to \infty$ with $t \to \infty$. The mechanism is made complete by combining $\tilde{z}_i$ with agents' contribution functions $\tilde{s}_i$ derived according to (A3), (A5); notice that solution (10) is symmetric and hence the subscript i in $\overline{\varphi}_i$ can be dropped.

To verify optimality, notice that if $\mu_i > 0$ and at least for some $j \neq i$ $\mu_j > 0$, then in the optimal solution of the problem

$$\max \sum_{k=1}^{n} \mu_k \varphi(z_k), \quad \sum_{k=1}^{n} z \le 1, \quad z_l \ge 0, l = 1, \dots, n, \qquad \text{(A.7)}$$

x_i monotonically increases in μ_i.[17] Therefore $\tilde{z}_i(w_i, w_{-i})$ monotonically increases in w_i over the range $w_i \in [w^0, \overline{w}]$ if at least some other $w_j > w^0$, and monotonically non-decreases (being equal to zero) otherwise. This means that the expected value $\overline{\varphi}(w_i) \equiv E_{w_{-i}}\varphi(\tilde{z}_i(w_i, w_{-i}))$ monotonically increases in $w_i \in [w^0, \overline{w}]$, since $w_j > w^0$ with positive probability. Therefore the obtained mechanism indeed maximizes (7') subject to (5'), (6'), and (8'): participation constraint is met by definition, whereas incentive compatibility follows from (A.4) and monotonicity of $\overline{\varphi}$.

[17] Re-write (A.7) as $\max \mu_i\varphi(z_i) + \Phi(z_i)$, $0 \le z_i \le 1$, where $\Phi(t) \equiv \max \sum_{k \neq i} \mu_k \varphi(z_k)$, $\sum_{k \neq i} z_k \le 1 - t$, $z_l \ge 0, l \neq i$.

Proof of Proposition 2 Monotonicty of $\bar{\varphi}$ implies that the function $\tilde{s}(w_i)$ monotonically increases over the same range$[w^0, \overline{w}]$; indeed if $x < y, x, y \in [w^0, \overline{w}]$, then $\tilde{s}(y)$-$\tilde{s}(x) = (y-x)\bar{\varphi}(y) + x\big(\bar{\varphi}(y) - \bar{\varphi}(x)\big) - \int_x^y \bar{\varphi}(t)dt > x\big(\bar{\varphi}(y) - \bar{\varphi}(x)\big) > 0.$ This allows to invert $\tilde{s}$ and define RSSFs (14). These RSSFs deliver (as a Bayes-Nash equilibrium with agents' strategies $s_i(w_i) = \tilde{s}(w_i),\ i = 1, \dots, n$) the same outcomes as the optimal direct mechanism (10), (12), and participation constraint (6) follows from (6'), Q.E.D.

Proof of Proposition 3 Symmetry and monotonicity of A_F follow immediately from its definition. Since F is monotonically increasing, one has

$$1 = \sum_{i=1}^{n} F\left(\frac{x_i}{A_F(x_1, \dots, x_n)}\right) \le nF(\frac{\max x_i}{A_F(x_1, \dots, x_n)}),$$

and therefore $\frac{\max x_i}{A_F(x_1,\dots,x_n)} \ge t_n$. Monotonicity of A_F implies that $F\left(\frac{x_i}{A_F(x_1,\dots,x_n)}\right)$ monotonically decreases in x_j for $j \neq i$, and due to the constraint

$$\sum_{i=1}^{n} F\left(\frac{x_i}{A_F(x_1,\dots,x_n)}\right) = 1$$

,monotonically increases in x_i . Hence $\tilde{z}_i(w_i, w_{-i})$ monotonically increases in w_i over the range $w_i \in [w^0, \overline{w}]$ (which has been already established in the proof of Proposition 1) and monotonically decreases in $w_j,\ j \neq i$ over the same range. To complete the proof, notice that the function $\eta(s) \equiv \rho(\tilde{s}^{-1}(s))$ monotonically increases, since the marginal valuation function $\rho(\cdot)$ increases by assumption, and $\tilde{s}(\cdot)$ – due to Proposition 2.

Proof of Proposition 4 Only the second part of the proposition needs to be verified. Let

$$F\left(\frac{x_i}{A_F(x_1, \dots, x_n)}\right) = \frac{\zeta(x_i)}{\sum_{j=1}^{n} \zeta(x_j)}, i = 1, \dots, n \qquad \text{(A.8)}$$

for some monotonically increasing function $\zeta(\cdot)$. Denote $y_i = \zeta(x_i)$ and suppose first that $n = 2$, in which case (A.8) yields

$$\frac{\zeta^{-1}(y_1)}{\zeta^{-1}(y_2)} = \frac{F^{-1}(\frac{y_1}{y_1 + y_2})}{F^{-1}(\frac{y_2}{y_1 + y_2})}.$$

Let $\frac{y_1}{y_2} \equiv t$, so that

$$\frac{\zeta^{-1}(ty_2)}{\zeta^{-1}(y_2)} = \frac{F^{-1}(t/(t+1))}{F^{-1}(1/(t+1))},$$

and hence $\zeta^{-1}(ty_2) = \zeta^{-1}(y_2)H(t)$ for some function H. This leads to the functional equation $\zeta^{-1}(xy) = \zeta^{-1}(x)\,\zeta^{-1}(y)$, which implies $\zeta(x) = x^c$ for some $c > 0$ (Acz´el, Dhombres, 1989). The case $n > 2$ is treated similarly by choosing $y_k, k > 2$ such that $\sum_{k=3}^{n} y_k/y_2 = \text{const}$.

Proof of Proposition 5 Fix w_i and treat w_j, $j \neq i$ as independent random variables. According to the law of large numbers (Feller, 1968), for every given $A > 0$ the random variable $\sum_{k=1}^{n} \frac{1}{n} F\left(\frac{\rho(w_k)}{A}\right)$ converges in probability to $EF\left(\frac{\rho(w)}{A}\right) = \int_{\underline{w}}^{\overline{w}} F\left(\frac{\rho(w)}{A}\right) g(w)dw$. This implies that $A_F^{(n)}(\rho(w_1), \dots, \rho(w_n))$ converges in probability to A_∞ (recall that F monotonically increases), and hence $\tilde{z}_i^{(n)}(w_i, w_{-i}) = F\left(\frac{\rho(w_i)}{A_F^{(n)}(\rho(w_1),\dots,\rho(w_n))}\right)$ converges in probability to $F\left(\frac{\rho(w_i)}{A_\infty}\right) = \tilde{z}_\infty(w_i)$. Notice that $A_F^{(n)}(\rho(w_1), \dots, \rho(w_n)) \leq A_{\max}$ for all n, $w_1, \dots, w_n$, where $F\left(\frac{\rho(\overline{w})}{A_{\max}}\right) = 1$, and so random variables $\tilde{z}_i^{(n)}(w_i, w_{-i})$ are bounded from above by $F\left(\frac{\rho(w_i)}{A_{\max}}\right)$; therefore convergence of these variables in probability implies convergence of their expected values, so that

$$\lim_{n\to\infty} E_{-w_i} \tilde{z}_i^{(n)}(w_i, w_{-i}) = \tilde{z}_\infty(w_i). \qquad \text{(A.9)}$$

By the same token $\overline{\varphi}^{(n)}(w_i) \equiv E_{w_{-i}} \varphi\left(\tilde{z}_i^{(n)}(w_i, w_{-i})\right) \to \varphi(\tilde{z}_\infty(w_i))$, $n \to \infty$, and hence

$$\lim_{n\to\infty} \tilde{s}^{(n)}(w_i) = \tilde{s}_\infty(w_i). \qquad \text{(A.10)}$$

Letting in (A.10) w_i equal $\underline{w}$ and $\overline{w}$, one obtains resp. $\lim_{n\to\infty} \underline{s}^{(n)} = \underline{s}_\infty$, $\lim_{n\to\infty} \overline{s}^{(n)} = \overline{s}_\infty$. The functions $\tilde{s}^{(n)}(w_i)$ are monotonically increasing, and therefore due to (A.10) the inverses of these functions converge to $\tilde{s}_\infty{}^{-1}(\cdot)$. This fact in combination with (A.9) and the observation that functions $E_{-w_i}\tilde{z}_i^{(n)}(\cdot, w_{-i})$ are also monotonically increasing, leads to (22).

Proof of Proposition 6 It was shown in the proof of Proposition 5 that functions $(\tilde{s}^{(n)})^{-1}(\cdot)$ converge to $\tilde{s}_\infty^{-1}(\cdot)$, Q.E.D.

Proof of Proposition 7 The first-order version of the incentive compatibility condition (17) implies that $\frac{dz_\infty}{ds} = \frac{d\tilde{z}_\infty}{dw} / \frac{d\tilde{s}_\infty}{dw} = \frac{1}{w\varphi'(\tilde{z}_\infty(w))}$. One also has $a(w)\varphi'\big(\tilde{z}_\infty(w)\big) = A_\infty$, and hence $\frac{dz_\infty}{ds} = \frac{a(w)}{A_\infty w}$ and thus non-decreases in w. Finally, $\tilde{s}_\infty(w)$ monotonically increases in w, and hence $\frac{dz_\infty}{ds}$ monotonically non-decreases in s, Q.E.D.

Proof of Proposition 8 As in the proof of Proposition 1, optimal direct mechanism design boils down to the following problem similar to (A.6):

$$\max\ [\sum_{i=1}^{n} \rho(w_i)\varphi(\tilde{z}_i(w_i, w_{-i})) - \tilde{s}_0(w_1, \dots, w_n)] \qquad \text{(A.11)}$$

$$\sum_{i=1}^{n} \tilde{z}_i(w_i, w_{-i}) \ \le \mathcal{F}\big(s_0(w_1, \dots, w_n)\big), \tilde{z}_j\big(w_j, w_{-j}\big) \ge 0, j = 1, \dots, n.$$

Assuming an interior optimum, one has

$$\tilde{z}_i(w_i, w_{-i}) = \rho(w_i)^{1/1-\alpha}(\mathcal{F}'(\tilde{s}_0(w_1, \dots w_n))^{1/1-\alpha}, \qquad \text{(A.12)}$$

and due to the budget constraint of the problem (A.11),

$$\mathcal{F}(\tilde{s}_0(w_1, \dots w_n) = (\mathcal{F}'(\tilde{s}_0(w_1, \dots w_n))^{1/1-\alpha} \textstyle\sum_{i=1}^{n} \rho(w_i)^{1/1-\alpha}. \qquad \text{(A.13)}$$

(A.12) and (A.13) yield (28). Similarly to the proof of Proposition 1 it can be shown that here too $\tilde{z}_i(w_i, w_{-i})$ monotonically increases in $w_i \in [w^0, \overline{w}]$ if at least some other $w_j > w^0$, and monotonically non-decreases (being equal to zero) otherwise, and therefore allocation (28) is indeed a part of optimal direct mechanism. Arguments similar to those presented in the proof of Proposition 2 complete the proof of Proposition 8.

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Полищук Л., Тонис А. Конструирование механизмов борьбы за ренту: оптимальный выбор «функций успеха»: Препринт WP10/2009/05. — М.: Издательский дом Государственного университета – Высшей школы экономики, 2009. — 28 с. (на англ. яз.).

В анализе борьбы за ренту широко применяются введенные Гордоном Таллоком «функции успеха», которые ставят результаты борьбы за ренту данного участника в зависимость от его собственных усилий (затрат), а также аналогичных затрат конкурентов. Нередко форма таких функций постулируется из «правдоподобных соображений»; в настоящей работе предполагается, что правила борьбы за ренту целенаправленно выбираются в условиях информационной асимметрии администратором источника ренты. Функции успеха в таком случае оказываются эндогенными, образуя оптимальный с точки зрения администратора механизм распределения ренты. Получено описание оптимальных функций успеха и анализируются их свойства; в частности, формулируются условия, при которых эти функции принимают распространенные в литературе формы. Исследуются перераспределительные последствия и потери эффективности, возникающие при реализации такого рода механизмов.

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Зав. редакцией оперативного выпуска *А.В. Заиченко*
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