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ON THE SOCIAL EFFICIENCY IN MONOPOLISTIC COMPETITITIOIN MODELS

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Abstract

We consider standard monopolistic competition models with aggregate consumer's preferences defined by two well-known classes of utility functions — the Kimball utility function and the variable elasticity of substitution utility function. It is known that market equilibruim is efficient only for the special case when utility function has a constant elasticity of substitution, but we find that in both cases a special tax on firms' output may be introduced such that market equilibrium becomes efficient.

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1 Introduction

It is a common knowledge that monopolistic competition usually leads to the inefficieny, i.e. market equilibrium differs from the social welfare state. This fact is in line with economic intuition — using their monopoly power firms charge higher prices than the ones which lead to efficient equilibrium. But one special case of efficiency in monopolistic competition model is well known — it is the case when the aggregate consumer's preferences are defined by the constant elasticity of substition (CES) utility function introduced in Dixit and Stiglitz (1977). This fact makes us think that efficiency in monopolistic competition model is rare but possible thing, so we find it meaningful to take a closer look at this subject. We consider two well-known generalizations of the CES function — the first one is the class of implicitly defined utility functions introduced in Kimball (1995) and the second one is the variable elasticity of substitution (VES) utility function introduced in Zhelobodko et al. (2012). It was proven in Dhingra and Morrow (2012) that CES is the only function in the class of variable elasticity

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of substitution utility functions which leads to the efficient equilibrium, and we establish the similar result for the Kimball class of utility functions. These results look disappointing, but we find a way to "fix" the inefficiency. More precisely, our main result is that for any utility function from the VES or Kimball classes it is possible to introduce the special tax on firms such that market equilirium becomes efficient. One can say that this idea is in the spirit of the Second Welfare Theorem, but on the side of firms, not consumers.

Kimball utility function was proposed in the article Kimball (1995), which became one of a cornerstones in both Real Business Cycle and New Keynesian literature. Kimball's paper presents "Neomonetarist" model combining standard Real Business Cycle principles and sticky prices a la Calvo (1983). This approach, although not entirely new, stimulated a great deal of research, which resulted in a number of more realistic and accurate models with helpful insights in the certain aspects of economy. Examples include New Neoclassical Synthesis model of US economy in Smets and Wouters (2007) and its modification with learning mechanism instead of rational expectations in Slobodyan and Wouters (2012), dynamic general-equilibrium model of US and Canadian economies aimed to explain exchange rate dynamics in Bouakez (2005), DSGE model with incomplete exchange rate pass-through to trade prices in Gust et al. (2009), New Keynesian model with labor market frictions in Riggi and Tancioni (2010) and Sala et al. (2008), model of unemployment in Givens (2011), the model of large devaluations in open economy in Burstein et al. (2007), monetary business cycle model with investment gestation lags and habits in consumption in Edge et al. (2007), model of endogenous currency choice in Gopinath et al. (2010), DSGE model with Taylor-type contracting in goods and labor markets in de Walque et al. (2006), New Keynesian model of inflation dynamics in Sbordone (2007). Kimball agregator is also used in the model of new economic geography in Barde (2008) and monopolistic competition model with cost price change in Klenow and Willis (2007).

Our point of interest in this paper is the utility function introduced by Kimball. Its basic principle is flexibility — it is defined via an arbitrary function and may generate any form of demand curve each firm faces. For example it seems plausible from the economic point of view to assume that firms face price elasticity of demand which in increasing in firm's relative price, so it is easier for firm to lose customers after increasing the price than to gain them after decreasing it. Another notable feature of Kimball agregator is that it is homogeneous of degree one and allows to introduce a price aggregator the same way as for constant elasticity of substitution utility function. It is also worth noting that the CES utility function is a special case of Kimball agregator.

Another generalization of CES utility function is a variable elasticity of substitution utility function introduced in Zhelobodko et al. (2012) and is motivated by the same idea of overcoming the lack of flexibility of CES utility function, most importantly the independence of firms' prices and markups from market size and the independence of firms' size from the number of consumers. Utility functions from the VES class were analyzed extensively in Dhingra and Morrow (2012).

In the next section we introduce a standard monopolistic competition models a la Dixit and Stiglitz (1977) or Melitz (2003) with Kimball and VES utility functions and derive the formulae for taxes which make market equilibria efficient.

2 Models

2.1 Kimball utility function

Consider the economy with aggregate consumer and finite but sufficiently high number n of monopolistically competititve firms. Denote the level of production of i-th firm by x_i. Consumer's utility function Y is defined by the following relation:

$$\sum_{i=1}^{n} G(y_i) = 1, y_i = \frac{x_i}{Y},$$

where $G(1) = 1, G'(\xi) > 0, G''(\xi) < 0$ for any $\xi \geq 0$.

Denote the total size of the labor force in the economy by L and the common wage by w. Each firm faces vabriable costs αx_i, fixed costs f, both measured in the units of labor, and also pays taxes $wT(y_i)$. Note that T may be defined as the function of x_i, but the formulae will be slightly more complicated. Obviously, since all firms face the same levels of costs, x_i and y_i are also the same for all firms, so we will use x and y without subscripts where it doesn't create confusion.

The problem of the social planner, who solves the utility maximization problem under technological constraints, has the following form

$$\begin{cases} Y \to \max_{y,n}, \\ (\alpha y Y + f)n = L, \\ nG(y) = 1. \end{cases}$$

After some simple calculations we get the following equation defining socially optimal level of y:

$$G(y) - yG'(y) = \frac{f}{L}. \tag{1}$$

Denote the solution of this equation by y_1^w, and the optimal number of firms may be calculated as $n_1^w = \frac{1}{G(y_1^w)}$.

Now consider the market equilibrium. Consumer solves the problem of minimization of costs of her consumption basket:

$$\begin{cases} Y \sum_{i=1}^{n} p_i y_i \to \min_{p_i} \\ \sum_{i=1}^{n} G(y_i) = 1. \end{cases}$$

Forming a Lagrangean and calculating its partial derivative with respect to p_i, we get (ignoring subscriptors)

$$p = \frac{\lambda}{Y} G'(y), \tag{2}$$

where λ is a Lagrange multiplier. Define firm's profit in the following way:

$$w\pi = px - (\alpha x + f)w - wT(y) = \lambda y G'(y) - \alpha w y Y - fw - wT(y).$$

Maximizing π with respect to y and assuming that the number of firms is high enough so the individual firm can't influence λ, we get the following condition:

$$\frac{\lambda}{w}\left(yG''(y) + g'(y)\right) - \alpha Y - T'(y) = 0. \tag{3}$$

As in Dixit and Stiglitz (1977) and Melitz (2003) we demand firm's profit to be zero:

$$\lambda y G'(y) - \alpha y Y - f - wT(y) = 0. \tag{4}$$

Consumer's income consists of wage, firms' profit (which equals to zero) and taxes and is totally spent on consumption:

$$npyY = wL + nwT(y). \tag{5}$$

Together with relation $nG(y) = 1$, equations (2) – (5) form the system of equations which define optimal consumption y_1^m, price and the number of firms n_1^m. Eliminating p, Y, λ, we get the following equation:

$$\frac{1}{L}yG'(y)T'(y) - \frac{1}{L}(yG'(y))'T(y) = yG''(y)G(y) + \frac{f}{L}G'(y). \tag{6}$$

As we can see, for the given function G values of y in (1) and (6) are the functions of one parameter f/L. We want $y_1^w = y_1^m$ defined by (1) and (6) to be the same (as it can easily be seen, in this case the number of firms is also the same), and in order to guarantee this, we can express f/L from (1) as a function of y and substitute it to (6).

$$\frac{1}{L}yG'(y)T'(y) - \frac{1}{L}(yG'(y))'T(y) = yG''(y)G(y) + G(y)G'(y) - yG'(y)^2. \tag{7}$$

Solve it as a differential equation on T:

$$\frac{\frac{1}{L}yG'(y)T'(y) - \frac{1}{L}(yG'(y))'T(y)}{((yG'(y))')^2} = \frac{yG''(y)G(y) + G(y)G'(y) - yG'(y)^2}{((yG'(y))')^2}.$$
$$\left(\frac{T(y)}{LyG'(y)}\right)' = \frac{G(y)}{y^2G'(y)} + \frac{G(y)G''(y)}{yG'^2(y)} - \frac{1}{y}.$$
$$\frac{T(y)}{LyG'(y)} = -\int_0^y \left(G(z)\left(\frac{1}{zG'(z)}\right)' - \frac{1}{z}\right)dz + C = -\frac{G(y)}{yG'(y)} + C.$$

Hence,

$$T(y) = (CyG'(y) - G(y))L \tag{8}$$

defines the tax rate which makes the market equilibrium efficient for any C. Note that $T(y)$ may be negative and thus be a subsidy. It is also worth noting that if we demand $T(y)$ to be zero, we get a differential equation on $G(y)$, and its solution is $G(y) = ay^\rho$, which corresponds to the case of the CES utility function. This proves that the only utility function in the Kimball class such that the market equilibrium is efficient without taxes is the CES function.

Every C leads to the efficient equilibrium, but perhaps it is possible to find the "most reasonable" C. We provide some guesses about this subject. First, for the case of the CES function $G(y) = y^a$, there is no need for taxes, so we can assume $T(y) = 0$. Then $C = 1/a = 1/G'(1)$. Second, in the case of monopoly it is reasonable to assume that there is also no need for taxes since there is no need for money redistribution. In this case $0 = T(1) = L(CG'(1) - 1)$ and again $C = 1/G'(1)$. These considerations make us think that this value of C is, in some sense, "better" than the others.

2.2 VES utility function

In this subsection we derive the similar formula of tax for the class of VES utility functions:

$$U = \sum_{i=1}^{n} u(x_i),$$

where u is thrice continuously differentiable, strictly increasing and strictly concave on $(0, \infty)$.

Using the fact that outputs of all firms are the same, $x_i = x$, the problem of the social planner is

$$\begin{cases} nu(x) \to \max_{x,n}, \\ (\alpha x + f)n = L, \end{cases}$$

thus the optimal level of x may be found from the following equation:

$$u'(x)(\alpha x + f) - \alpha u(x) = 0. \tag{9}$$

Denote its solution by x_2^w, and the optimal number of firms is then $n_2^w = \frac{L}{\alpha x_2^w + f}$.

In the market equilibrium consumer solves the following problem:

$$\begin{cases} \sum_{i=1}^{n} u(x_i) \to \max_{x_i} \\ \sum_{i=1}^{n} p_i x_i \le wL + \sum_{i=1}^{n} w\pi(x_i) + \sum_{i=1}^{n} wT(x_i), \end{cases}$$

and the i-th firm solves the profit maximization problem:

$$w\pi_i = p_i x_i - (\alpha x_i + f)w - wT_i(x) \to \max. \tag{10}$$

The Lagrangean for the consumer's utility maximization problem is

$$L = \sum_{i=1}^{n} u(x_i) = \lambda \left(wL + \sum_{i=1}^{n} T(x_i) - \sum_{i=1}^{n} p_i x_i \right).$$

We assume that while making her consumption choice, consumer ignores the fact that the income she recieves as taxes from firms depend on her choice, so firm's price (ignoring the subscripts) equals to

$$p = \frac{u'(x)}{\lambda}. \tag{11}$$

Similarly to the previous subsection, we impose the condition of firms' profits to be zero. Substituting (11) into (10), we get

$$\pi = \left(\frac{u'(x)}{\lambda w} - \alpha \right) x - f - T(x).$$

Again, we assume that in equilibrium firms' profit is zero. Hence, the Lagrange multiplier equals to

$$\lambda = \frac{xu'(x)}{w(\alpha x + f + T(x))}. \tag{12}$$

Substituting this expression to the firm's profit maximization condition, after some calculations we get the differential equation on $T(x)$:

$$\frac{1}{\alpha} xu'(x)T'(x) - \frac{1}{\alpha}(xu'(x))'T(x) = x^2 u''(x) + \frac{f}{\alpha}\left(u'(x) + xu''(x)\right). \tag{13}$$

Substituting the expression for f/α from (9) to (13), we get

$$\left(\frac{T}{\alpha x u'(x)}\right)' = \frac{u(x)}{x^2 u'^2(x)} + \frac{u(x)u''(x)}{xu'^3(x)} - \frac{1}{xu'(x)}.$$
$$\left(\frac{T}{\alpha x u'(x)}\right) = C - \frac{u(x)}{xu'^2(x)} - \int_0^x \frac{u(z)u''(z)}{xu'^3(z)} dx.$$

Hence,

$$T(x) = \alpha \left[Cxu'(x) - \frac{u(x)}{u'(x)} - xu'(x) \int_0^x \frac{u(z)u''(z)}{zu'^3(z)} dz \right]. \tag{14}$$

This choice of tax rate guarantees that firms' output is the same in the social welfare state and in the market equilibrium: $x_2^w = x_2^m$. In order to verify that number of firms is also the same, consider the consumer's budget constraint: $n_2^m px = wL + n_2^m wT$. Substituting (11) and (12), we get $n_2^m = \frac{L}{\alpha x_2^m + f} = \frac{L}{\alpha x_2^w + f} = n_2^w$. So the number of firms is also the same and hence the market equilibrium is efficient.

To find the “most reasonable” C, again use the intuition that in the CES case there is no need for taxes. Substitute $u(x) = x^a$ in (14). We get $0 = T(x) = Cax^a$, hence $C = 0$.

3 Conclusion

The presented approach allows to “fix” the inefficient equilibrium in the certain classes of monopolistic competition models. Further research way involve analysis of another classes of utility functions and finding another ways to make market equilibria efficient.

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