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Ionospheric Response to the Acoustic Gravity Wave Singularity

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Abstract

An original model of atmospheric wave propagation from ground sources to the ionosphere in the atmosphere with a realistic high-altitude temperature profile is analyzed. Shaping of a narrow domain with elevated pressure in the resonance region where the horizontal phase wave velocity is equal to the sound velocity is examined theoretically within the framework of linearized *Eq.s*. Numerical simulations for the model profiles of atmospheric temperature and viscosity confirm analytical result for the special feature of wave fields. The formation of the narrow domain with plasma irregularities in the *D* and low *E* ionospheric layers caused by the acoustic gravity wave singularity is discussed.

Key words: atmosphere, realistic temperature profile, ionosphere, acoustic gravity waves, *D* and *E* layers.

1. INTRODUCTION

An actual problem for many applications is explaining the role of acoustic gravity waves (AGW) in the transfer of oscillating processes from the Earth's surface to the upper atmosphere. The cause for the surface sources of these waves can be earthquakes, explosions, sea waves, and other artificial and natural processes (Blanc 1985). Both infrasound and internal waves play an important role in different atmospheric phenomena (Francis 1975). Many important tasks for acoustics gravity waves in the Earth's atmosphere have been studied theoretically. A general form of wave *Eq.s* is too complex for analytical solution, even in the linear approximation. This is caused by the real inhomogeneity of the atmosphere. Altitude profiles of the atmospheric parameters are taking into account different analytical models (see, *e.g.*, Savina 1996). At present, the numerical methods of the solution of this problem are developed successfully and give a possibility of describing a propagation AGW taking into account different important factors, including the nonlinearity (see, *e.g.*, Lund and Fritts 2012).

In this paper, one of the aspects of the problem in question is examined. We carry out an analysis of AGW behavior near the resonance level, at which the condition of equality of horizontal phase wave velocity to the local value of the sound velocity is satisfied. We have shown that in the Earth's atmosphere the temperature profile is such that there is a range of wave phase velocities (or a frequency range with fixed horizontal dimensions of the source) in which the wave does not pass through a resonance domain. We have found that local disturbances of the wave pressure component are formed at the narrow resonance level.

The resonance mentioned above is the reason for ionospheric irregularities with relatively small vertical scales in the *D* and *E* layers. As will be shown in what follows, such irregularities, flattened vertically, contribute to the generation of sporadic *E* layers of the ionosphere with a large range of translucency.

Sections 2, 3, and 4 of this paper are complementary. Some analytical properties of the waves in a non-isothermal atmosphere are discussed in Section 2. Analytical conclusions on specific features of the fields of acoustic gravity waves at the resonance level are confirmed by numerical calculations, whose results are given in Section 3. Consideration of the ionospheric effects in the *D* and *E* ionospheric layers in Section 4 is based on the approximation of passive impurity and the results obtained in the foregoing sections.

2. WAVE FIELD NEAR THE RESONANCE LEVEL

The linearized system of equations of gas dynamics for the pressure perturbations $p_\sim$, the horizontal velocity $\upsilon_\sim$, and the vertical velocity $w_\sim$ is well known. We select axis z in the vertical direction and axis x in the horizontal direction. To simplify equations, it is convenient to introduce new field variables, namely: $V = (\rho/\rho_E)^{1/2}\upsilon_\sim$, $W = (\rho/\rho_E)^{1/2}w_\sim$, and $P = (\rho/\rho_E)^{1/2}p_\sim$, where ρ and ρ_E are the basic state densities in the current layer and at the ground level, respectively. Field variables are proportional to $\exp(i\omega t + ik_\perp x)$ for a monochromatic signal with frequency ω in plane atmospheric layers. Then the linearized system of equations for the wave perturbation can be reduced to the following form (Gossard and Hooke 1975):

$$\begin{aligned} &\left[-\omega^2+\omega_g^2(z)\right]W - i\frac{\omega}{\rho_E}\left[\frac{\partial}{\partial z}+\Gamma(z)\right]P = 0 \ , \\ &\left[-\omega^2+c_s^2(z)k_\perp^2\right]P - i\omega\rho_E c_s^2(z)\left[\frac{\partial}{\partial z}-\Gamma(z)\right]W = 0 \ . \end{aligned} \tag{1}$$

The perturbation of horizontal velocity can be determined as $V = (k_\perp/\omega)P$. In Eq. 1, i is the imaginary unit, $k_\perp$ is the horizontal wave number, $c_s = \sqrt{\gamma p/\rho}$ is the sound velocity, where

$$\begin{aligned} \omega_g^2 &= (\gamma-1)g^2/c_s^2 + (g/T)(\partial T/\partial z) \ , \\ \Gamma &= (2-\gamma)g/2c_s^2 - (1/2T)(\partial T/\partial z) \ , \end{aligned}$$

γ is the ratio of specific heats at constant pressure and volume, respectively, g is the acceleration due to gravity, and $T(z)$ is the basic state temperature of the atmosphere. In adiabatic approximation, we have

$$c_s(z) = 20.05 \cdot T(z)^{1/2} \ [\mathrm{m/s}] \ ,$$

where T is Kelvin temperature. It is important for the following analysis that in the steady atmosphere (Lighthill 1978) the Ekkard parameter $\Gamma > 0$ and $\omega_g^2 > 0$ where ω_g is the Brunt Väisälä frequency. According to Eq. 1, the averaged vertical energy flux $S = (1/2)(PW^* + P^*W) = \text{const}$, *i.e.*, S does not depend on altitude (Rapoport *et al.* 2004) a superscript asterisk is the sign of complex conjugation. Note that the basic-state temperature of the atmosphere $T(z)$ is altitude dependent. The dependence $\rho(z)$ is governed by the hydrostatic election $dp/dz = -\rho g$, where p is the basic state pressure. Taking into account the ideal gas law, we have the basic state density dependence in the form

$$\rho(z) = \rho_E T_E T^{-1}(z) \exp\left(-\int_0^z H^{-1}(z')dz'\right) \ ,$$

where $H(z)$ is the pressure scale height.

Some altitude domain near $z = z_*$, where

$$0 < \left[\omega - c_s(z)k_\perp\right]^2 / \omega^2 < \varepsilon$$

and ε is small in the mathematical meaning with reasonably high accuracy, $W \approx 0$ the sound velocity and the Ekkard parameter are almost constants, and system 1 is reduced to the following form:

$$\left[\frac{\partial}{\partial z} + \Gamma(z)\right] P = 0, \quad z \neq z_*, \tag{2a}$$

$$\left[-\omega^2 + c_s^2(z) k_\perp^2\right] P = 0, \quad z = z_*. \tag{2b}$$

Consider in greater detail the processes near the resonance level $z = z_*$, which are described by the equation system 2. Pressure disturbances both above and below the level $z = z_*$ are satisfied according to Eq. 2a, but not according to Eq. 2b, because $\omega \neq c_s k_\perp$, and W is infinitely small (see equation system 1), but its derivative can be finite. Exactly at the level, $z = z_*$ the conditions $W = 0$ and $dW/dz = 0$ are fulfilled. Therefore, as follows from Eq. 2b, P can have an arbitrary value and Eq. 2a is not defined at this point. The absence of disturbances of both the vertical velocity and its derivative, leads to a conclusion that above level $z = z_*$ the solutions both for W and for P are identically equal to zero. Consequently, in order to balance the pressure jump at the level in question, the finite mass should be concentrated at level $z = z_*$, which is taken into account in the solution using a delta function.

A formal solution of equation system 2 near the level $z = z_*$ can be written as follows (Savina and Bespalov 2014):

$$\begin{aligned} &P = P_* E(z_* - z)\exp\left(\Gamma_*(z_* - z)\right) + \frac{P_* c_{s*}^2}{g}\delta(z - z_*), \\ &W(z = z_*) = 0, \end{aligned} \tag{3}$$

where $E(z - z_*)$ is the Heaviside step function and $\delta(z - z_*)$ is the Dirac delta function; by a subscript asterisk we mark the values of the variables at the layer $z = z_*$. Solution 3 depends on a single constant P_*, which is determined by the boundary condition on the Earth's surface. Thus, local disturbance (Eq. 3) consists of a special feature in the altitude distribution of the pressure disturbance and characteristic structures $P(z)$ and $W(z)$ adjoining it from below. Actually, one or more resonance levels can exist in the real atmosphere for specified ω and $k_\perp$. Perturbations of the vertical velocity and pressure are absent ($W = 0$ and $P = 0$) everywhere above the first resonance level. Hence, if for the wave perturbation in the nonisothermal atmosphere at some level $z = z_*$ a condition $\omega = c_s(z_*)k_\perp$ is satisfied, then the averaged vertical energy flux is equal to zero. Above the first of such levels, wave perturbation are absent along the vertical propagation path. This effect is responsible for the formation of a waveguide channel between the Earth's surface and the resonance level for the waves whose horizontal phase velocity is equal to the local sound velocity.

Under the actual conditions, the resonance in the form of a delta function in the pressure disturbance, as well as in the horizontal velocity perturbation that is proportional to it $(V = P/c_s(z = z_*)\rho_E)$, is smeared due to the molecular viscosity and the nonlinearity. In the numerical calculations given below the molecular viscosity was taken into account.

3. FULL-WAVE CALCULATIONS FOR RESONANCE ATMOSPHERIC PERTURBATION

In this section, we determine perturbations of the pressure and vertical velocity by means of full-wave numerical calculations. We assume that on the ground level there is a monochromatic source of vertical velocity and that at the altitudes higher than 200 km level the atmosphere is isothermal. Numerical calculation of Eq. 1 made it possible to find the high-altitude distribution of the wave perturbations.

The wave fields are conveniently calculated in dimensionless variables, which we selected as follows:

$$\tilde{\omega} = \omega / \omega_{gE}\ , \quad \tilde{k}_\perp = k_\perp c_{sE} / \omega_{gE}\ , \quad \tilde{c}_s = c_s(z)/c_{sE}\ , \quad \tilde{W} = W/W_E\ , \quad \tilde{P} = P / \rho_E c_{sE}^2\ .$$

Here, subscript E indicates the value of the variable on the Earth's surface.

The temperature profile depends on many factors in the real atmosphere. We selected the altitude temperature profile following the MSIS-E-90 model (Hedin 1991). For example, we chose one temperature profile $T(z)$ for 10 August 2012 at 15:00 UT for geographic latitude 65° and longitude 45°. We determine the analytical function for this temperature profile (Fig. 1a) and kinematic viscosity profile (Fig. 1b) (Kikoin 1976) using the model values marked by boxes and stars in Fig. 1, respectively. The temperature and viscosity profiles were approximated by polynomials of the tenth order.

We give in Fig. 2 the results of numerical simulation for the inhomogeneous plane wave with the fixed frequency $\tilde{\omega} = 0.52$ and the horizontal wave number $\tilde{k}_\perp = 0.5$. Therefore, simulations are one-dimensional, and task is two-dimensional. These results correspond to theoretical examination in Section 2. Numerical calculation is conducted in much the same way as in papers Rapoport *et al.* (2004) and Bespalov and Savina (2012).

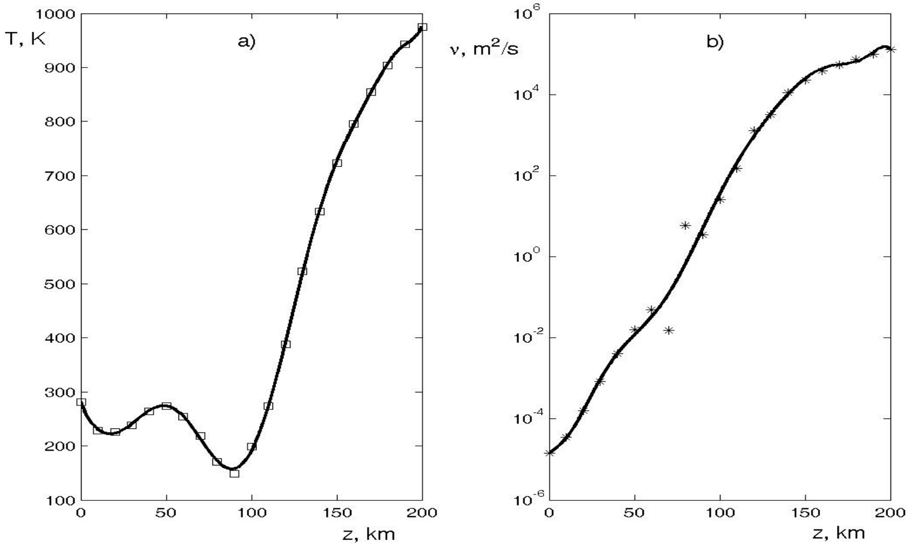

Fig. 1. Typical atmosphere temperature profile $T(z)$ as obtained from the MSIS-E-90 model marked by boxes and spline curve $T(z)$ (a), and typical altitude profile of molecular kinematic viscosity $\nu(z)$ in the Earth's atmosphere (b).

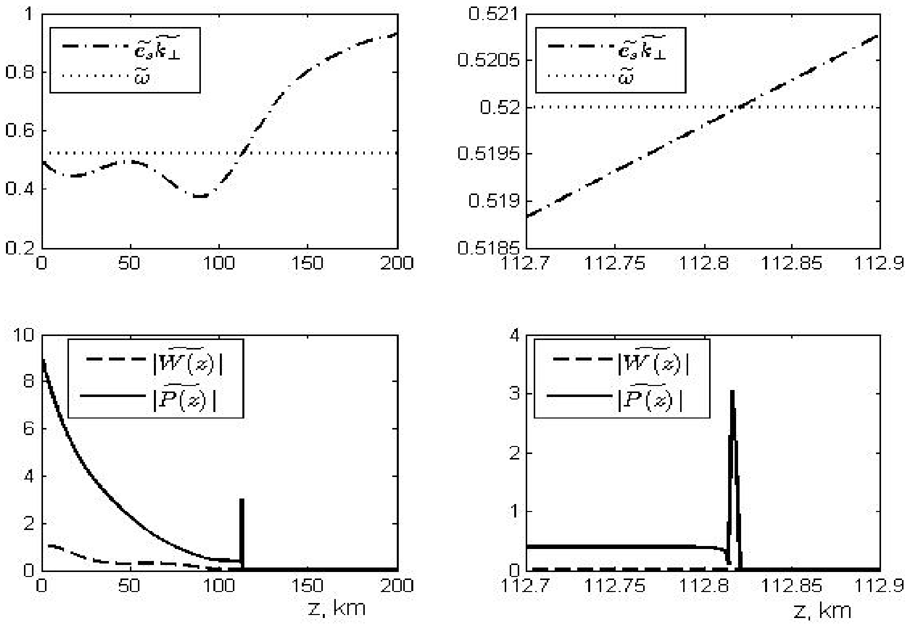

Fig. 2. Results of numerical calculations of AGW in the atmosphere (left column) and near the local disturbance (right column).

Figure 2 shows the local disturbance at an altitude near 113 km. In the upper panels, the dashed dotted curves are the altitude dependence $\tilde{c}_s(z)\tilde{k}_\perp$ and the dotted lines are the frequency $\tilde{\omega}$. In the bottom panels, the solid curves are the altitude dependence $\left|\tilde{P}(z)\right|$ and the dashed curves are the altitude dependence $\left|\tilde{W}(z)\right|$. In Figure 2 on the right bottom panel the dependencies $\left|\tilde{P}(z)\right|$ and $\left|\tilde{W}(z)\right|$ are given in greater detail for altitudes near 113 km, which correspond to the local disturbance. Calculations show that in this case the averaged vertical energy flux $\tilde{S} = 0$.

The local disturbance at the resonance level, which corresponds to solution 3, is limited by the atmospheric viscosity. Numerical calculations were carried out within the framework of the linearized system of *Eq.s*, which is correct for sufficiently small Reynolds numbers $\mathrm{Re} = \upsilon_\sim L/\nu$, where $\upsilon_\sim = \sqrt{\rho_E/\rho}\, P/c_s\rho_E$ is the disturbance of horizontal velocity, and L is a typical vertical scale of local disturbances. For conditions near the local disturbances, $\upsilon_\sim \simeq 10$ m/s, $L \simeq 20$ m, $\nu \simeq 300$ m^2/s, and $\mathrm{Re} < 1$.

4. IONOSPHERIC RESPONSE TO THE ATMOSPHERIC WAVE SINGULARITY

At the altitudes of the ionosphere, acoustic gravity waves are the reason for the occurrence of ionospheric irregularities. It was shown above that at the resonance level a singularity appears in the atmospheric gas pressure disturbance, knowing which one could estimate the horizontal velocity $\upsilon_\sim$. As numerical simulations show, resonance layers can be formed lower than the level given as an example in Fig. 2. This depends on the profile of temperature and values of $k_\perp$ and ω. The processes occurring in the neutral medium influence are passed here to the electrons and ions through collisions. At the altitudes of the D and E layers, the frequency of the acoustic gravity wave is much lower than the frequencies ν_{in} and ν_{en} of the ion and electron collisions with neutrals ($\nu_{\mathrm{in}} \approx 10^3$ s^{-1} and $\nu_{\mathrm{en}} \approx 10^4$ s^{-1} at an altitude close to 110 km). Then, neglecting the collisions between charged particles, for the velocity of drag of the electrons and ions by neutrals in the absence of external electric fields and sharp density gradients one can write (Gershman 1974)

$$\vec{u}_{\mathrm{e,i}} = \frac{\nu^2_{\mathrm{en,in}}}{\nu^2_{\mathrm{en,in}} + \omega^2_{He,\mathrm{i}}}\left\{\vec{u}_{\mathrm{n}} \mp \frac{\omega_{He,\mathrm{i}}}{\nu_{\mathrm{en,in}}}\left[\vec{u}_{\mathrm{n}}\vec{h}_0\right] + \frac{\omega^2_{He,\mathrm{i}}}{\nu^2_{\mathrm{en,in}}}\vec{h}_0\left(\vec{h}_0\vec{u}_{\mathrm{n}}\right)\right\}, \tag{4}$$

where $\vec{u}_{\mathrm{e,i,n}}$ are velocities of electrons, ions and neutrals, $\vec{h}_0$ is a unit vector along the geomagnetic field direction, and $\omega_{H\mathrm{e,i}}$ are the electron and ion gyrofrequencies values, respectively. The minus sign is chosen for the electron velocity and the plus sign is chosen for the ion velocity. The disturbance of the densities $N \approx N_{\mathrm{i}} \approx N_{\mathrm{e}}$ of the ionospheric plasma, which we assume to be quasi-neutral, can be estimated from the continuity equation:

$$\frac{\partial N}{\partial t} + \mathrm{div}\left(N\vec{u}_{\mathrm{i,e}}\right) = 0 \ . \tag{5}$$

At the altitudes of the D layer and in the lower part of the E layer, where the conditions $\nu_{\mathrm{en,in}} > \omega_{H\mathrm{e,i}}$ are fulfilled, it can be assumed that the ionospheric plasma is completely dragged by neutrals and $\vec{u}_{\mathrm{i}} \approx \vec{u}_{\mathrm{e}} \approx \vec{u}_{\mathrm{n}}$. The time τ of onset of forced distributions of electrons and ions can be estimated from the formula

$$\tau = \frac{L^2}{D} \ , \quad \text{where} \quad D = \frac{2\kappa T}{m_{\mathrm{i}}\nu_{\mathrm{in}} + m_{\mathrm{e}}\nu_{\mathrm{en}}}$$

is the diffusion coefficient, κ is Boltzmann's constant, T is the absolute temperature, and $m_{\mathrm{e,i}}$ are the electron and ion masses, respectively. A rough estimate shows that for the altitudes of the D and lower E layers, the time τ is of the order of a few seconds, which is much less than the period of the considered atmospheric gas oscillations. This means that the disturbances of the ionospheric plasma density in our case have temporal and horizontal spatial scales corresponding to the disturbance of neutral gas. Considering that in the linear approximation the disturbances of the electrons and ions densities are small and assuming that velocities of the charged particles are close to $\upsilon_{\sim}$, Eq. 5) leads to the following formula

$$N = \frac{k_\perp V}{\omega}\sqrt{\frac{\rho_E}{\rho}}N_0 = \frac{P}{c_s^2}\frac{N_0}{\sqrt{\rho\rho_E}} \ , \tag{6}$$

where N_0 is the basic state of the ionospheric plasma density. In the derivation of the Eq. 6 it was taken into account that the vertical velocity disturbances of the atmospheric gas, which are related to the effect considered in the previous sections, is absent in the resonance region, and the equality $\omega = c_s k_\perp$ is justified. Since the disturbance of ionospheric plasma density is proportional to the pressure disturbance of the neutral gas, it should be expected that thin (several ten meters) and extended (of the order of the acoustic gravity wave horizontal scale determined by the horizontal scales of the source on the Earth's surface) ionospheric irregularities with a periodically varied plasma density will be generated in the ionospheric D layer with a weak gradient of the background density in the resonance level. The impact of this resonance effect on the plasma at the altitudes of the ionospheric *E* layer, where $\nu_{in} \sim \omega_{Hi}, \nu_{en} << \omega_{He}$ (electrons are strongly magnetized) and there are conditions for the sporadic *E* layers generation, is more difficult for analysis. This is due to the fact that the electron density gradient should be taken into account and the Whitehead force, which leads to a still greater reduction of the domain thickness, should play an important role. In this case the formation of finite mass on the resonance level (Fig. 2) must lead to the formation of narrow (in vertical directive) horizontal ionospheric irregularities. Irregularities of such a type can be observed on the ionograms in the form of mildly sloping weakly diffuse sporadic layers with a large range of translucency (Fatkullin *et al.* 1985).

5. SUMMARY

Local acoustic-gravity disturbance in the nonisothermal atmosphere has been studied. According to the analytical results, the pressure wave amplitude has a local wave singularity near the layer at which the horizontal phase velocity is equal to the sound velocity. It is shown for the real altitude temperature profile in the atmosphere that near this layer the wave pressure component has a singularity, and the vertical velocity in the disturbance becomes zero.

In real conditions, many resonance layers (for different ω and $k_\perp$) can exist. The fact is that the atmospheric viscosity and nonlinearity limits the pressure wave singularity. We discussed the case when the viscosity influence on the density spike is more important. For such a situation, the vertical dimensions of singularity domain will be of the order of the mean free path of the molecules at the resonance level.

Estimates show that at altitudes of about one hundred kilometers the viscosity limits the scales to several hundreds of meters. Ionospheric plasma at these altitudes and below behaves as a passive impurity and therefore ionospheric irregularities with the same characteristic features as the neutral gas disturbances should be observed, including the case where the condition of the resonance feature generation is fulfilled. It is shown in paper Erukhimov and Savina (1980) that irregularities such a structure are the main reason for the formation of weakly diffuse sporadic *E* layers with a large range of translucence. In the framework of the model in question, the time of existence, altitude, and space scales of such layers are fundamentally dependent on the parameters of the ground-based acoustic gravity waves sources and ionospheric conditions.

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