NATIONAL RESEARCH UNIVERSITY
HIGHER SCHOOL OF ECONOMICS

Dmitry Dagaev and Vladimir Yu. Rudyak

SEEDING THE UEFA CHAMPIONS LEAGUE PARTICIPANTS: EVALUATION OF THE REFORM

BASIC RESEARCH PROGRAM
WORKING PAPERS

SERIES: ECONOMICS
WP BRP 129/EC/2016

***SERIES**: ECONOMICS*

Dmitry Dagaev[1] *and* *Vladimir Yu. Rudyak*[2]

SEEDING THE UEFA CHAMPIONS LEAGUE PARTICIPANTS: EVALUATION OF THE REFORM

We evaluate the sporting effects of the seeding system reform in the major football club tournament — the Champions League — organized by the Union of European Football Associations (UEFA). In the UEFA Champions League, before the 2015-16 season, the teams were seeded in the group stage with respect to their ratings. Starting from the 2015-16 season, national champions of the Top-7 countries are seeded in the first pot, whereas other teams are seeded by their rating as before. We propose a probabilistic model for predicting the score of a single match in UEFA tournaments as well as the whole UEFA season. This model uses clubs' ratings as inputs. Applying Monte-Carlo simulations, we show that the expected rating of the UEFA Champions League winner, as well as the sum of the finalists' ratings, slightly decreased after the reform. At the same time, the difference in the finalists' ratings, which is a measure of competitive balance, increased. The UEFA Europa League became stronger and less balanced. We check the robustness of the results by introducing local fluctuations in the clubs ratings. Also, we study which national associations took advantage of the reform. For seeding rules before and after the reform, we estimate the transition matrix (p_{ij}), where p_{ij} is the probability of i-th strongest national association moving to j-th position after the season. The effect of reform on a single national association measured by the change in probability to increase or decrease the association's UEFA rank is not more than 3%.

Keywords: tournament; design; seeding; competitive balance; UEFA Champions League; Monte-Carlo simulations

JEL Classification: Z20, C44

[1]National Research University Higher School of Economics, Associate Professor. Dmitry Dagaev was supported within the framework of the Academic Fund Program at the National Research University Higher School of Economics (HSE) in 2014-2015 (grant 14-01-0007) and within the framework of a subsidy granted to the HSE by the Government of the Russian Federation for implementing the Global Competitiveness Program. E-mail: ddagaev@gmail.com.

[2]Lomonosov Moscow State University, Research Fellow. E-mail: vurdizm@gmail.com

1 Introduction

In September, 2013, the Union of European Football Associations (UEFA), a European football governing body, decided to change the format of European football club tournaments starting from the 2015-16 season. One of the most important changes was the establishing a new seeding system for the UEFA Champions League. The short-term effect of the new seeding policy was that the quantitative characteristics of single matches, as well as whole tournament characteristics, changed. At the club level, some teams will benefit from the reform as they will face weaker opponents during the group stage. At the national association level, some countries will have an increased probability of moving up in the rankings, thus, increasing the number of representatives that they field in international UEFA tournaments. Our research aims to evaluate the expected sporting effects of the reform using computer simulation techniques.

Each season, UEFA organizes two major international club competitions: the UEFA Champions League, the most prestigious tournament, and the UEFA Europa League, the second most prestigious. All 54 national associations – UEFA members – receive a certain number of places in these tournaments, based on UEFA's ranking of the respective national associations. Each national association then determines the teams that will enter the international European competitions based on domestic tournament results – standings from the domestic championship and domestic cup.[1] Clubs enter the Champions League and the Europa league at different stages of the tournament depending on both the country's position in the UEFA ranking of national associations and the club's own achievements in domestic competitions. A full list for the 2015-16 season is provided in the Appendix (see Table 1).

The European Cups have the following structure. Both the Champions League and the Europa League are made up of four qualifying rounds, followed by a group stage and a concluding play-off tournament. In every qualifying round and the play-off round, each team faces a single opponent. They play two games – a home and an away match (the only exception is the final which is played at a pre-determined neutral stadium). The winner is determined on the aggregate score. In the Europa League, the winner of these knock-out matches qualifies for the next round and the loser goes home. The same is true for the first two qualifying rounds and the play-off round of the Champions League. However, losers in

[1]Several national associations hold more tournaments. For example, there are two cups in England: the Football Association Challenge Cup and the Football League Cup. Winners of both tournaments automatically get a place in the UEFA Europa League.

the 3^{rd} and 4^{th} Champions League qualifying rounds are not completely eliminated: these teams enter the Europa League 4^{th} qualifying round, and group stage, respectively.

In both tournaments, the group stage is made up of several groups of 4 teams (8 groups in the Champions League, 12 groups in the Europa League). In each group, teams play a double round-robin tournament. That is, each team faces each other 2 times – a home and an away match. A win counts for 3 points, a draw – 1 point, a loss – 0 points. After all matches are completed, the teams are ranked based on the total number of points from the 6 group matches. The first two teams in the group advance to the play-off round. The fourth team is eliminated. In the Champions League, the third-place team in the group drops down to the Europa league. In the Europa League, the third-place team in the group is eliminated.

Sorting teams into groups or knock-out matches may be done in many different ways. However, the quality of the participating teams varies greatly. Hence, two different draws are not equal. The probability of a team advancing further depends significantly on its particular opponents. Without additional protections, the second best team may be eliminated by the best one in the very first round. This is likely an undesirable event for tournament organizers, since spectators prefer watching stronger teams. Hart, Hutton, and Sharot (1975) found that stadium attendance positively correlates with a visiting team's strength. Hence, early elimination of a favourite may reduce overall spectator interest in a tournament. Tournament organizers are interested in a draw procedure that protects stronger teams from being eliminated early. Forrest and Simmons (2002) tested whether match attendance depends on outcome uncertainty. They found that, all else equal, matches that are thought to be intense attract more spectators.

UEFA exploits coefficients[2] that reflect the strength of European clubs and national associations. The are two different coefficient rankings: the UEFA club ranking and the UEFA national association ranking. Club coefficients are used by UEFA for seeding (and, thus, protection) purposes, whereas national association coefficients determine the number of representatives from each association in European club tournaments.

UEFA national association coefficients are calculated in the following way. All clubs participating in the Champions League and Europa League collect points for the national association that they represent. In the qualifying rounds, each win adds 1 point to the association's total, a draw – 0.5 points, a loss – 0 points. Starting from the group stage, these values increase to 2, 1, and 0, respectively. After the season, the total number of

[2]See also the unofficial, but very comprehensive UEFA coefficient website compiled by Bert Kassies `http://kassiesa.home.xs4all.nl/bert/uefa/calc.html`.

points is divided by the number of participants from this association. The resulting average is the association's yearly coefficient. It is calculated for the last 5 seasons; the sum is the association's 5-year coefficient.

The UEFA club coefficient is calculated in a similar way based on the club's performance in European cups during the last 5 seasons, as well as on the coefficient of the association the club belongs to. Technically, these calculations are more complicated. Additional details are relegated to the Appendix.

In both tournaments, the group stage draw is an important event that determines the contest flow for months. At the draw, UEFA team coefficients are used to protect stronger teams and avoid heterogeneity between groups. Before the 2015-16 season, at the beginning of the draw, participants were split into 4 pots with respect to their coefficients. The only exception was for the title-holder who was guaranteed to be seeded into the first pot, irrespective of their rating. After that, groups were formed by picking 1 team from each of the 4 pots. This procedure guarantees that each group contains a very strong top-8 team and a relatively weak bottom-8 team.

Starting from the 2015-16 season, UEFA changed the seeding procedure. Namely, the first pot comprises the title-holder and the champions of the top-7 countries (or the champions of the top-8 countries, if the title-holder is a champion from one of the top-7 countries). Since a national champion may have a relatively low UEFA coefficient, the changes in the seeding procedure lead to less homogeneous groups. In this paper, we evaluate how the changes affect competitive balance, quality, and other key metrics.

The problem of optimally sorting objects into groups has long been a topic of academic interest. The literature includes both the theoretical studies of sorting properties and applications to various industries. A comprehensive introduction to the mathematical theory of sorting may be found in (Andrews , 1998). A review of operational research classification and sorting methods is provided in (Zopounidis and Doumpos , 2010). These techniques may be applied to solve such real-life problems as diagnosing based on the set of symptoms (Stefanowski and Slowiński , 1997), routing the packages at the post (McWilliams, Stanfield, and Geiger , 2005), or choosing the sample for sociological interview (see, for example, (Zopounidis and Doumpos , 2010) for more details).

To choose between different partitions, an optimization criterion (or criteria) must be selected. The simplest problem is to maximize the objective function, which is defined explicitly on the set of all partitions. In this case in the tournament seeding context we call the optimized value a tournament success measure. Since a finite number of objects may be split into a finite number of groups in a finite number of ways, one may theoretically evaluate the objective function for each partition and choose the one that maximizes the objective function. In practice, the total number of partitions may be very large, which makes a full search hard (or even impossible). In such situations, one needs to establish properties that significantly shrink the number of alternatives.

Searching for the optimal seeding in a knock-out tournament with 4 participants, given pairwise winning probabilities, Horen and Riezman (1985) consider 3 objective functions:

1) the probability of the strongest team winning the tournament;

2) the probability of the two strongest teams meeting in the final;

3) the expected rating of the winner.

They show that a seeding in which the strongest team faces the weakest team in the semifinal, while the two other teams play each other in the other semifinal, is a solution to each of the three maximization problems. This, however, is not necessarily the case for a knock-out tournament with 8 participants. Vu (2010) models tournament revenue with a function that includes game parameters, such as competitive balance and quality. The competitive balance (CB) of a game is the difference in the opponents' strengths always taken with a minus sign (thus, a closer opponent rating corresponds to a more intense game with a higher CB value). The quality (Q) of a game is the sum of the opponents' ratings. As discussed earlier, spectators are more interested in games with greater competitive balance. At the same time, spectators prefer higher quality games. Aggregating the revenue over all tournament games, we reach the organizers' optimization problem. Simulating parameters of the model for each seeding, Vu found the frequency at which this seeding is optimal. In this paper, we will use competitive balance and quality as the main objectives.

More complicated sorting problems arise when the objective function is given implicitly. In contest theory, contest organizers are interested in maximizing the overall effort levels exerted by participants in the Nash equilibrium. Following a Tullock contest model (Tullock , 1980), the contestants independently choose the level of costly efforts. The probability of a win equals the share of the participant's efforts in overall efforts exerted by the players. Baik (2008) considers a contest between N groups of players, $N \geq 2$. Similarly, the probability of a group winning equals the share of the group's efforts. If a group wins a contest, each member of the group receives a prize valued at 1. Baik shows that if the cost function is linear on efforts, then, in each group, only the player with the lowest marginal cost of efforts exerts positive efforts. It follows from this fact that effort-maximizing organizers must assign top N players to N different groups; other player assignments do not matter. In the same between-groups contest, Ryvkin (2011) considers general convex cost functions. He shows that under reasonable assumptions, the optimal partition of the contestants into groups either minimizes or maximizes the contestants' strength variance between groups. Note that this problem is different from the one faced by UEFA, because there is no competition between groups in either the Champions League or Europa League.

The rest of the paper is organized as follows. In Section 2 we describe our simulation models: a model for simulating the outcome of a single match (subsection 2.1), and a model for simulating the whole UEFA season (subsection 2.2). In Section 3 we present the results of the comparison of the two seeding systems (subsection 3.1) and test robustness (subsection 3.2). Section 4 concludes.

2 The model

In this section, we model the Champions League and Europa League tournaments. First, we propose a simulation model for a single game in both competitions. Second, we simulate the whole tournament.

2.1 Simulation of a single game score

An adequate model is key for making a good prediction about the outcome of a single game between two football teams. After (Maher , 1982), the most popular approach assumes that each team i is assigned two parameters a_i and d_i that characterize the team's attack and defence strengths, respectively. Then, the number of goals scored by the home side i against the away team j is expressed as a random variable, with a Poisson distribution, and parameters a_i and d_j (and vice-versa with a correction for the home advantage). Attack and defence parameters may be estimated with a maximum likelihood approach taking into account previous matches results. Maher model generalizations are applied to various tournaments and datasets. Dixon and Coles (1997) introduced dynamic attack and defence parameters which vary throughout the season into the model. This novelty allows the model to capture changes in a team's condition. Also, the authors developed a successful betting strategy based on their model. Another dynamic bivariate Poisson model for predicting the results of English Premier League matches is presented in (Koopman and Lit , 2015). Koning et al. (2003) adopted the model for major international tournaments, including the World Cup. To deal with the 'golden goal' rule, the authors introduced scoring intensity.

While these models have demonstrated good performance, there is an important issue one has to deal with when trying to replicate the model for UEFA club competitions. Each season, hundreds of teams participate in the Champions League and Europa League. Most of them exit the tournament after only several games. Therefore, for most teams, there is simply not enough data to estimate their attack and defence strength. It is not possible to use domestic competition results, because the level of national leagues and cups varies dramatically across Europe. Thus, we use UEFA team coefficients instead of attack and defence parameters to predict the number of goals scored by teams. UEFA coefficients are determined for each team participating in European cups and reflect team achievements on the international level over the last five years.

2.1.1 Poisson model

We have analysed the results of all 688 games from the UEFA Champions League and Europa League 2014-15 editions. The goal distributions (Fig. 1) show the difference in statistics for home and away goals scored. Poisson fitting of the data (solid lines on Fig. 1) indicates the strong potential of the Poisson distribution-based stochastic process as a model to predict the score.

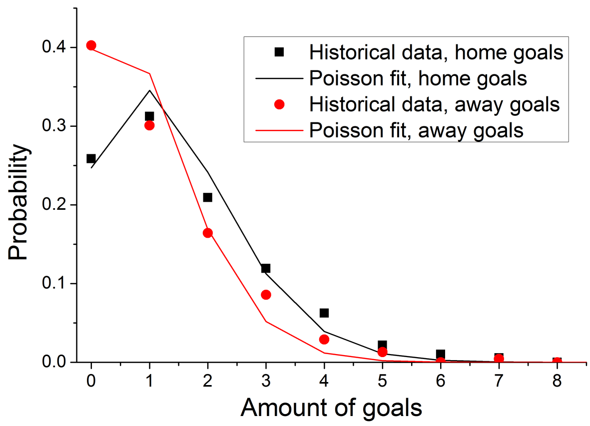

Figure 1: Distributions of goals in 688 CL and EL games from the 2014-2015 UEFA season. Solid lines show fitting with Poisson distribution ($\lambda = 1.40 \pm 0.04$, $R^2 = 0.98$ for home goals; $\lambda = 0.92 \pm 0.04$, $R^2 = 0.97$ for away goals).

Thus, we suggest the Poisson model for the score of a single game between clubs H (home) and A (away). Denote by n_h and n_a the number of goals scored by H and A, and by λ_h and λ_a the mean number of goals scored by H and A, respectively, in a match between them. Suppose that n_h and n_a follow Poisson distributions with parameters λ_h and λ_a, respectively, and the corresponding probability mass functions $P_h(n_h)$ and $P_a(n_a)$ are given as follows:

$$P_h(n_h) = \frac{\lambda_h^{n_h}\exp(-\lambda_h)}{n_h!}$$
$$P_a(n_a) = \frac{\lambda_a^{n_a}\exp(-\lambda_a)}{n_a!}. \tag{1}$$

To choose the model for λ_h and λ_a, we take into account two empirical observations. First, the number of goals scored in European international club matches correlates with the difference in the opponents' UEFA coefficients. Second, the home side has an advantage. Therefore, we consider the following parametric family of models for λ_h and λ_a:

$$\lambda_h = \alpha_h + \beta_h \cdot \exp\left(\frac{R_h - \chi_h R_a}{\gamma_h}\right)$$
$$\lambda_a = \alpha_a + \beta_a \cdot \exp\left(\frac{R_a - \chi_a R_h}{\gamma_h}\right), \tag{2}$$

where R_h and R_a stand for home and away club ratings, respectively, and α_j, β_j, γ_j, χ_j, $j \in \{a, h\}$ are constants determining the model. Now, equations (1) can be rewritten as

$$P_h(n_h) = P_h(n_h, \alpha_h, \beta_h, \gamma_h, \chi_h) = \frac{\lambda_h^{n_h}\exp(-\lambda_h)}{n_h!}$$
$$P_a(n_a) = P_a(n_a, \alpha_a, \beta_a, \gamma_a, \chi_a) = \frac{\lambda_a^{n_a}\exp(-\lambda_a)}{n_a!}. \tag{3}$$

Let $\bar{\upsilon} = (\alpha_h, \beta_h, \gamma_h, \chi_h, \alpha_a, \beta_a, \gamma_a, \chi_a)$.

2.1.2 MLE

We estimate 8 parameters of the model with a Maximum Likelihood Estimation (MLE) approach based on 2014-15 UEFA season data. The dataset contains the results of each of $M = 688$ matches in the Champions League and Europa League, as well as the UEFA coefficients of the opponents. Within the MLE, model parameters $\bar{\upsilon}$ are varied to maximize the objective function

$$P_{total}(\bar{\upsilon}) = \prod_{m=1}^{M} P_m(\bar{\upsilon}), \tag{4}$$

where $P_m(\bar{\upsilon})$ is the estimated probability of the observed score of the match number m (see (3)):

$$P_m(\bar{\upsilon}) = P_h(n_h, \alpha_h, \beta_h, \gamma_h, \chi_h) \cdot P_a(n_a, \alpha_a, \beta_a, \gamma_a, \chi_a). \tag{5}$$

We use Monte-Carlo simulated annealing (Kirkpatrick, Gelatt and Vecchi , 1983) with Metropolis criterion (Beichl and Sullivan , 2000) to find MLE-estimations. The starting values of $\bar{\upsilon}$ are random. On i-th Monte-Carlo step, the new set of parameters $\bar{\upsilon}_{i+1}$ is tested. The new set of parameters $\bar{\upsilon}_{i+1}$ is generated by adding a random normally distributed fluctuation to the previous set $\bar{\upsilon}_i$:

$$\bar{\upsilon}_{i+1} = \bar{\upsilon}_i + \sigma T_{MC} \Delta \bar{\upsilon},$$

where T_{MC} is Metropolis 'temperature' (it regulates the degree of allowed fluctuations in the system), and σ is a coefficient, the value of which is chosen to obtain a mean acceptance ratio equal to 0.5. The step acceptance probability is equal to $p_{MC} = \exp\left(\frac{P_{total}(\bar{\upsilon}_{i+1}) - P_{total}(\bar{\upsilon}_i)}{T_{MC}}\right)$. During optimization, we decreased T_{MC} from 10^6 to 10^{-5} in 10^6 steps, which corresponds to a random walk in the parameter space at the beginning of optimization and negligibly small parameter fluctuations in the end.

Since scores follow a Poisson distribution, λ_h and λ_a must always be positive; parameters $\bar{\upsilon}$ that lead to negative λ_h or λ_a are considered invalid. During Monte-Carlo optimization, we used only valid sets of parameters $\bar{\upsilon}$. New fluctuations (and, thus, new sets of parameter values) were generated at each step until a valid set was obtained.

Based on the scores of UEFA 2014-15 matches in both tournaments and corresponding participant coefficients, we obtained the following parameter estimations: $\alpha_h = -0.135$, $\beta_h = 1.562$, $\gamma_h = 132.4$, $\chi_h = 0.977$, $\alpha_a = -0.199$, $\beta_a = 1.264$, $\gamma_a = 163.0$, and $\chi_a = 1.006$. These values lead to an average value for home and away teams equal to 1.51 and 1.10, respectively, which is close to the real-life average values 1.65 and 1.01.

2.2 Simulation of UEFA season outcome

According to the above-mentioned model, each match is a random process. The UEFA season comprises hundreds of games. Thus, any particular outcome of the season is a coincidence of many random variable realisations. In such complex systems, even for very simple events, it is difficult to find exact probabilities. To find the empirical probability distributions for these events, one may accumulate statistics from multiple independent runs. This technique, called Monte-Carlo simulations, can be treated as averaging multiple measured values. The more Monte-Carlo trials one completes, the less errors of averaged values will be obtained (typically, the error $S \sim \sqrt{K}$, in which K is the number of trials).

We estimated how errors in tournament success measures (expected rating of the winner, expected competitive balance in the final, expected quality of the final, and others) depend on the number of runs in simulations[3]. We produced simulations with 10^3 to 10^7 runs, 10 times each. For each particular number of runs, we calculated unbiased sample variance of the average values of the tournament success measures. Then, we converted it to relative errors (see Fig. 2). At first glance, the relative error drops down quickly, and even 10^3–10^4 runs are enough to estimate results. However, our primary goal was to estimate the difference between the two seeding policies that produce relatively similar data and, thus, high errors. Therefore, we produced our primary simulations and robustness check simulations with 10^6 and 10^7 independent runs per simulation. The errors in the values of differences between the average values for new and old seeding procedures are the following: 0.008 for the winner rating, 0.01 for the competitive balance in the final, 0.015 for quality in the final, and $3 \cdot 10^{-4}$ for the probability of the top team winning the Champions League.

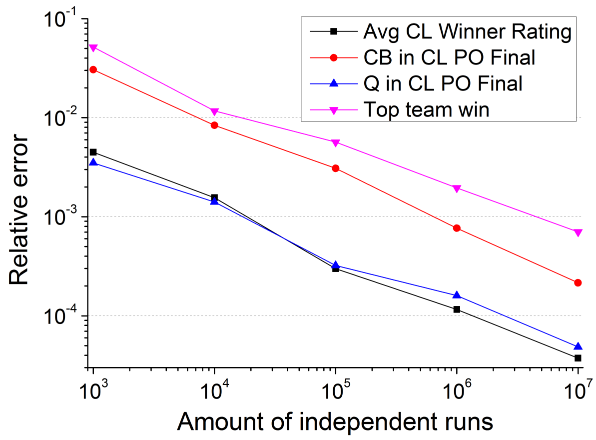

Figure 2: Dependence of relative errors for various tournament success measures on amount of independent Monte-Carlo runs.

3 Results and discussion

3.1 Main results

In the literature (see, for example, Horen and Riezman, 1985; Scarf, Yousof and Bilbao, 2009), the following quantities are often regarded as tournament success measures: the

[3]The stochastic part of the observational error is meant. The k^{th} simulation run is treated as a single measurement x_k of a certain tournament success measure. As we estimate average values $\bar{x}$ of this tournament success measure across all possible outcomes, the resulting error of the average value is an estimation of the feasible difference between the measured average and the true average, calculated as an unbiased sample variance: $S(\bar{x}) = \frac{1}{K-1}\sum_{k=1}^{K}(x_k - \bar{x})^2$. The relative error is $s(\bar{x}) = S(\bar{x}) \cdot \bar{x}^{-1}$.

expected rating of the winner, the expected quality of the final (the sum of the finalists' ratings), and the expected competitive balance of the final (the difference between the finalists' ratings).

Figures 3–5 illustrate the changes with respect to these parameters after the adoption of the new seeding system. The expected rating of the winner decreased slightly after the reform from 157.2 to 157.1 (-0.1%). The expected quality of the final decreased from 300.5 to 299.6 (-0.3%). This decrease is due to the lower probability of top clubs reaching the final stages of the tournament. The intuition behind this is rather strict: under the new seeding system, there is a higher probability of drawing two very strong teams into one group. Thus, the probability that one or both of them will fail to qualify for the knock-out stage increases. At the same time, the expected competitive balance of the final worsened from -30.0 to -30.7, which means a less competitive match by approximately 2.4%. The probability of a final between two low-ranked teams increased. However, this difference is much less than the increase in the probability of a final between a low-ranked and a high-ranked team.

The changes in the Champions League seeding system indirectly affected the Europa League. The expected rating of the Europa League winner increased from 95.4 to 96.9, and the expected quality of the final increased from 179.7 to 182.1 (+1.5% and +1.3%, respectively). As for expected competitive balance in the final, it fell from -26.5 to -27.1 making the Europa League final less competitive by 2.2%. All in all, after the reform, both tournaments acquired less competitive finals, and the Europa League winners became closer in strength to the Champions League winners.

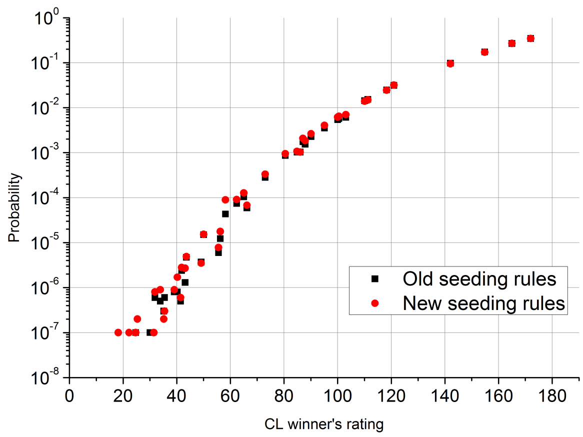

Figure 3: CL winner expected rating.

Another issue is ranking dynamics in European football. At the national association level, one may think of ranking stability as a matrix (p_{ij}), where p_{ij} is the probability of a country ranked i moving to position j after the season $(i, j = 1, \ldots, 54)$. For example, the

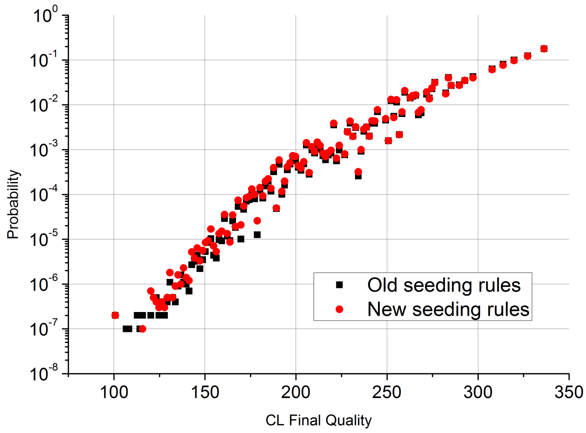

Figure 4: Sum of CL finalists expected rating.

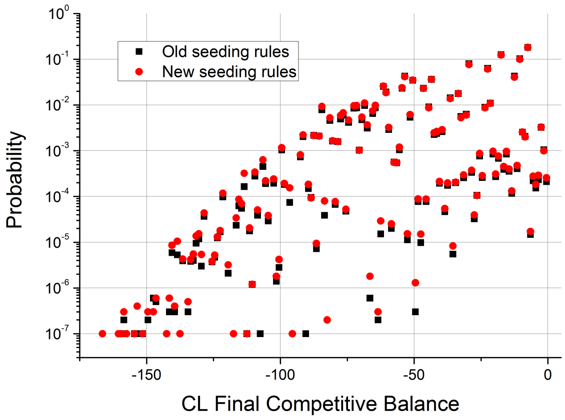

Figure 5: Competitive balance in the CL final.

unit matrix corresponds to absolutely stable systems, in which rankings after the next season are predetermined. Both excessive stability and instability lead to economic ineffectiveness. Based on 2015-16 season inputs, we calculated the ranking stability matrices (p_{ij}) for the current and previous systems, and study their common properties and differences. Figure 6 shows the effect of the reform on the countries' positions. The overall influence of the seeding changes on a country's ranking may be estimated to be at most 3%. The new rules favoured Italy and Netherlands, because they got one first-pot representative (Juventus and PSV, respectively), instead of none. As a result, Italy reduced by almost 3% the probability of dropping down to fifth place, and the Netherlands reduced by nearly 2% the probability of dropping down to ninth place. Other changes are close to zero.

3.2 Testing the model's robustness

The results presented above were obtained based on certain club ratings. We need to separate the role of the tournament structure itself from dependency on the initial dataset. To deal

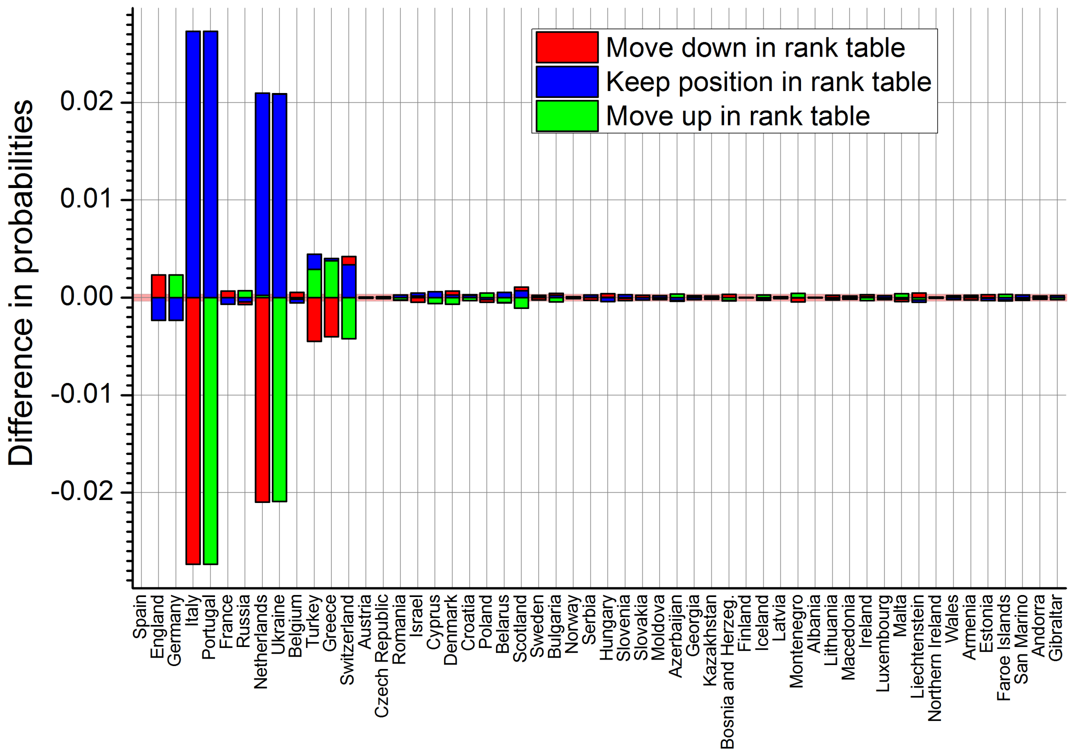

Figure 6: The difference in probabilities (the new system minus the old system) to move up or move down in the ranking. The absolute error of the probability difference values is $2.5 \cdot 10^{-4}$, shown with a light red stripe.

with this point, we developed a robustness test procedure. We varied the clubs' initial ratings, while keeping clubs' and countries' initial rankings fixed. We produced the same simulations with the old and new seeding procedures, using randomly modified input ratings (called hereinafter "setup"). We examined the stability of the resulting shifts in metrics between the two seeding procedures over the amplitude of the clubs' rating fluctuations.

3.2.1 Algorithm for a random rating setup generation

One of the simplest and most natural ways to apply random fluctuations to club ratings is to add a random normally distributed perturbation to each club's rating. However, the ratings of clubs participating in the UEFA Champions League and Europa League are very close, so adding random fluctuations may lead to a ranking change. Thus, we used only such fluctuations in the clubs' ratings that preserved rankings. The algorithm utilizes a user-defined amplitude of fluctuations A and produces a random setup in the following steps:

1. Clubs are sorted according to their true rating $R = (R_1, ..., R_C)$, where R_c is the coefficient of the c-th strongest club, $c = 1, ..., C$.

2. For each club n, except the one with the lowest rating, the difference to the next club that has a lower rating is calculated as: $\Delta R_c = R_c - R_{c+1}$.

3. The new rating is generated by the equations:

$$R_C^* = R_C \cdot (1.00 + \xi A),$$

$$R_c^* = R_{c+1}^* + \Delta R_c \cdot (1.00 + \xi A), \quad c = C - 1, ..., 1,$$

where ξ is a (0,1)-normally distributed random variable. For each club, ξ value is re-generated until $(1.00 + \xi A)$ is positive.

4. After the new rating R^* is generated for each club, the ratings are normalized to the initial sum of the ratings:

$$\tilde{R}_c^* = R_c^* \frac{\sum R_c^*}{\sum R_c}, \quad c = 1, ..., C.$$

The rankings of the clubs remain the same in this procedure.

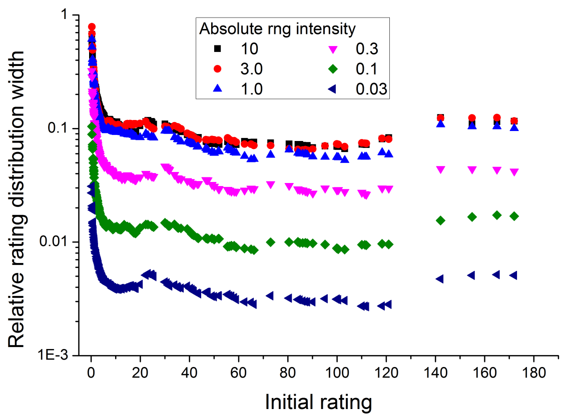

Figure 7: Dependence of the resulting dispersion of club ratings after applying fluctuations of varying strengths (from 0.03 to 10) on initial club ratings. Dispersions are saturated in fluctuation strength, ranging from 3 to 10.

We have varied fluctuation amplitude A from 0.03 to 10, and generated 100 random setups per value (also called "setup samplings", or "samplings"). We calculated the dispersion of the normalized disturbed rating $\tilde{R}^*$ for each club in each sample. The dependency of the relative dispersions on A is plotted in Fig. 7. There are two important observations. First, the relative dispersion in each sampling remains at nearly the same level for all clubs, except for the clubs with the lowest rating. The increased relative dispersion in the lowest rating region appears due to the rapid increase in $\frac{\Delta R}{R}$ ratio because R is close to zero. Second, one can see the saturation of dispersion values at $A \approx 3$. Thus the conditions of simultaneously (1) maintain club rankings and (2) keep constant average rating limit the maximum possible fluctuation of the rating. It allows us to test robustness at the whole range of fluctuation.

3.2.2 Analysis of the robustness

Now, we have a set of setup samplings for various fluctuation amplitudes A. For each A, the comparison of two seeding procedures is made. The case $A = 0$ corresponds to the basic comparison (see subsection 3.1). To illustrate the persistence of this result, we compare the outcomes under the two seeding mechanisms with increased fluctuation amplitude values A. We produced the simulations for every setup of every sampling. For each setup, we calculated the changes in the expected rating of the winner (R^{new}, R^{old}), the expected quality of the final (Q^{new}, Q^{old}) and the expected competitive balance of the final (CB^{new}, CB^{old}) for both the new and old seeding procedures:

$$\begin{aligned}
\Delta R(A) &= R^{new}(A) - R^{old}(A), \\
\Delta Q(A) &= Q^{new}(A) - Q^{old}(A), \\
\Delta CB(A) &= CB^{new}(A) - CB^{old}(A).
\end{aligned}$$

For each value of A, we calculated empirical distribution functions of $\Delta R(A)$, $\Delta Q(A)$, $\Delta CB(A)$, using 100 setups from the corresponding setup sampling. The obtained distributions are shown on Fig. 8–10. All three empirical distributions are spreading wider with increasing fluctuation amplitude A, and saturate at $A = 10$. In all cases, the probabilities of a sign inversion of the changes $\Delta R(A)$, $\Delta Q(A)$, $\Delta CB(A)$ are negligible. Moreover, the distribution averaged values (shown on inserts of Fig. 8–10) are conservative across the full range of A. Therefore, the robustness test demonstrates that the shift in metrics described in Sec. 3.1 is a steady consequence of changes in the seeding system, and not an effect of a temporary local distribution of the club's positions and power.

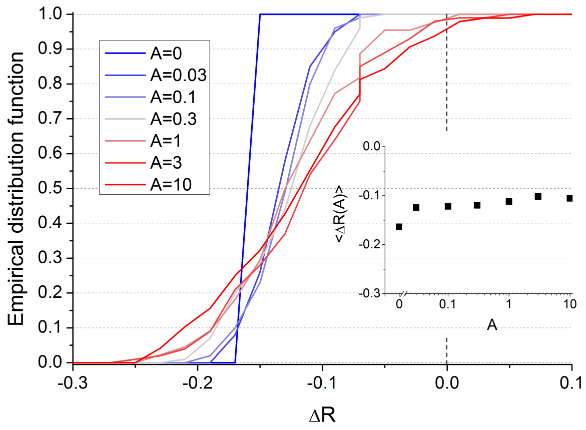

Figure 8: Empirical distribution functions of the winner rating difference between new and old seeding rules for various fluctuation strengths. While larger fluctuations lead to wider distributions of ΔR, the mean value remains approximately the same (see insert).

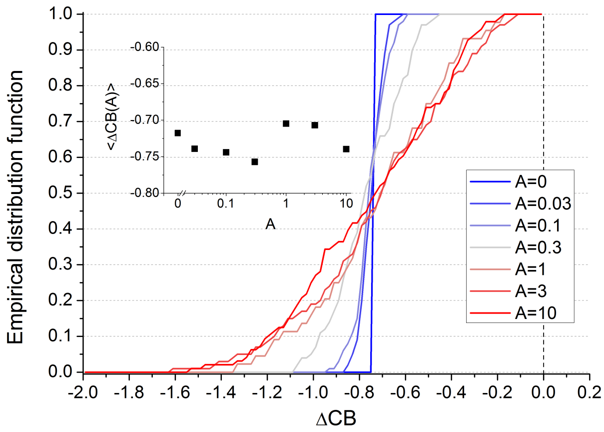

Figure 9: Empirical distribution functions of CB in the CL Final difference between the new and old seeding rules for various fluctuation strengths. While larger fluctuations lead to wider distributions of ΔCB, the mean value remains approximately the same (see insert).

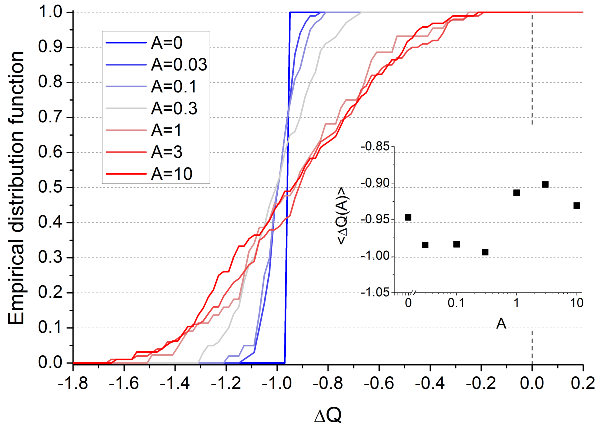

Figure 10: Empirical distribution functions of Q in the CL Final difference between new and old seeding rules for various fluctuation strengths. While larger fluctuations lead to wider distributions of ΔQ, the mean value remains approximately the same (see insert).

4 Conclusions

We estimated the changes in competitiveness in the UEFA Champions League and Europa League after the new seeding system was introduced in the Champions League in the 2015-16 season. We found the ex-ante expected values of main tournament success measures and compared the results under seeding rules before and after the reform. Our simulations show that in the UEFA Champions League the new seeding policy resulted in a decrease in tournament quality with respect to all main measures. Whereas the decrease in the expected rating of the winner and the expected quality of the final is only marginal, the expected competitive balance in the final worsened by approximately 2.4%. At the same time, in the Europa League, the expected rating of the winner and the expected quality of the final increased by nearly 1.5% due to an increase in the probability of higher-ranked teams being eliminated from the Champions League after the group stage. Expected competitive balance in the Europa League final worsened by approximately 2.2%. These results do not imply that the reform was a failure. First, our estimations were made only for one set of the inputs, namely, for the set of the teams participating in the 2015-16 UEFA season. These results were supported by consideration of inputs that are close to the initial dataset. Other datasets are expected to provide the results directed in the same way, however, absolute values of the changes may appear very different. Second, the goal of the reform could be primarily commercial. Nevertheless, the governing body should not neglect the immediate sporting effects when changing the tournament format. Evaluation of other tournament format changes made by UEFA, including the major changes in qualification system for the UEFA Europa League as well as the changes in Access list to European cups, is a possible direction for future research.

Acknowledgments

Dmitry Dagaev was supported within the framework of the Academic Fund Program at the National Research University Higher School of Economics (HSE) in 2014-2015 (grant 14-01-0007) and within the framework of a subsidy granted to the HSE by the Government of the Russian Federation for implementing the Global Competitiveness Program.

The authors are grateful to the Supercomputing Center of Lomonosov Moscow State University: calculations were performed using supercomputer "Lomonosov".

References

Andrews GE (1998) The theory of partitions. Cambridge University Press.

Baik KH (2007) Contests with group-specific public-good prizes. *Social Choice and Welfare*, **30**, 103–117.

Beichl I, and Sullivan F (2000) The Metropolis algorithm. *Computing in Science & Engineering*, **2**(1), 65–69.

Dixon MJ, and Coles SG (1997) Modelling association football scores and inefficiencies in the football betting market. *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, **46**(2), 265–280.

Forrest D, Simmons R (2002) Outcome uncertainty and attendance demand in sport: the case of English soccer, *Journal of the Royal Statistical Society: Series D (The Statistician)*, **51**, Part 2. 229–241.

Hart RA, Hutton J, and Sharot T (1975) A Statistical Analysis of Association Football Attendances, *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, **24**(1), 17–27.

Horen J and Riezman R (1985) Comparing draws for single elimination tournaments. *Operations Research* **33**(2), 249–262.

Kirkpatrick S, Gelatt CD, and Vecchi MP (1983) Optimization by Simulated Annealing. *Science* **220**(4598), 671–680.

Koning RH, Koolhaas M, Renes G, and Ridder G (2003) A simulation model for football championships. *European Journal of Operational Research*, **148**, 268–276.

Koopman SJ and Lit R (2015) A dynamic bivariate Poisson model for analysing and forecasting match results in the English Premier League. *Journal of the Royal Statistical Society. Series A: Statistics in Society*, **178**(1), 167–186.

Maher MJ (1982) Modelling association football scores. *Statistica Neerlandica*, **36**(3), 109–118.

McWilliams DL, Stanfield PM, and Geiger CD (2005) The parcel hub scheduling problem: A simulation-based solution approach. *Computers & Industrial Engineering*, **49**(3), 393–412.

Ryvkin D (2011) The optimal sorting of players in contests between groups. *Games and Economic Behavior*, **73**(2), 564–572.

Scarf PA, Yusof MM, Bilbao M (2009) A numerical study of design for sporting contests. *European Journal of Operational Research* **198**(1), 190–198.

Stefanowski J and Slowiński K (1997) Rough set theory and rule induction techniques for discovery of attribute dependencies in medical information systems, in: Principles of data mining and knowledge discovery, Springer Berlin Heidelberg, 36–46.

Tullock G (1980) Efficient rent-seeking, in: J. Buchanan et al. (Eds.), Towards a Theory of the Rent-Seeking Society, Texas A&M University Press, College Station, 97–112.

Vu TD (2010). Knockout tournament design: a computational approach. PhD dissertation, Stanford University, Department of Computer Science.

Zopounidis C and Doumpos M (2002) Multicriteria classification and sorting methods: A literature review. *European Journal of Operational Research*, **138**(2), 229–246.

5 Appendix

Appendix contains supplementary information on the UEFA tournaments format and rankings.

5.1 National association coefficient

All clubs participating in the Champions League and Europa League collect points for the national association they represent. In the qualifying rounds, each win adds 1 point to the association's total, a draw – 0.5 points, a loss – 0 points. Starting from the group stage, these numbers are 2, 1 and 0, respectively. After the season, the total number of points is divided by the number of participants from this association. The resulting average is the association's yearly coefficient. It is calculated over the last 5 seasons; the sum is an association's 5-year coefficient.

5.2 Club coefficient

A UEFA team coefficient consists of two summands: number of points gained by this team in the Champions League and Europa League for the last 5 years and 20% of the coefficient of the association this club belongs to. Starting from the group stage, a win adds 2 points to the team's coefficient, a draw – 1 point, a loss – 0 points. Bonus points are awarded for qualification into the latter stages of the tournaments. In the Champions League, it's 4 points for reaching the group stage, plus 5 points for the Last-16 stage, plus 1 point for the quarterfinals, plus 1 point for the semifinals, plus 1 point for the final. In the Europa League, it's 2 points for reaching the group stage, plus 1 point for the quarterfinals, plus 1 point for the semifinals, plus 1 point for the final. As an exception from all other bonus points, 2 points for reaching the group stage of the Europa League are partly awarded only if a team fails to obtain more than 2 points. More precisely, if a team gets x normal points during the group stage, it gets $\max(x, 2)$ points instead. Thus, a total of 2 points is a guaranteed minimum number for the teams that reached the group stage of the Europa League. The participants that fail to reach the group stage get points on a different basis. Namely, a team that leaves the Champions League from the 1^{st} qualifying round gets a fixed amount of 0.5 points, from the 2^{nd} qualifying round – 1 point. A team that finishes its participation in the Europa League in the 1^{st} qualifying round gets 0.25 points, in the 2^{nd} qualifying round – 0.5 points, in the 3^{rd} qualifying round – 1 point, in the 4^{th} qualifying round – 1.5 points.

5.3 Access list

Below we use the following abbreviations:

- GS – group stage;
- NCQ3(NCQ4) – non-champions qualifying round 3 (4);
- CQ3(CQ4) – champions qualifying round 3 (4);
- CH – national champion;
- CW – national cup winner;
- Nx – x-placed team in the national championship;
- FP – National Fair-Play ranking leader.

Table 1 determines the number of clubs from each of the national associations and clubs' starting points in the Champions League and Europa League for the 2015-16 season[4].

[4]See official rules at http://www.uefa.com/memberassociations/uefarankings/index.html (Reterieved December 26, 2015) or UEFA European Cup Football site by Bert Kassies http://kassiesa.home.xs4all.nl/bert/uefa/access2015.html (Reterieved December 26, 2015).

	Association	GS	NCQ4	NCQ3	CQ4	CQ3	Q2	Q1		GS	Q4	Q3	Q2	Q1
1	Spain	CH,N2,N3	N4							CW,N5		N6		
2	England	CH,N2,N3	N4							CW,N5		N6		FP
3	Germany	CH,N2,N3	N4							CW,N5		N6		
4	Italy	CH,N2	N3							CW,N4		N5		
5	Portugal	CH,N2	N3							CW		N4,N5		
6	France	CH,N2		N3						CW		N4,N5		
7	Russia	CH		N2						CW		N3,N4		
8	Netherlands	CH		N2						CW		N3,N4		FP
9	Ukraine	CH		N2						CW		N3,N4		
10	Belgium	CH		N2						CW		N3	N4	
11	Turkey	CH		N2						CW		N3	N4	
12	Greece	CH		N2						CW		N3	N4	
13	Switzerland			N2		CH				CW		CW,N3	N4	
14	Austria			N2		CH						CW,N3	N4	
15	Czech Republic			N2		CH						CW,N3	N4	
16	Romania						CH					CW	N2	N3
17	Israel						CH					CW	N2	N3
18	Cyprus						CH					CW	CW	N2,N3
19	Denmark						CH						CW	N2,N3
20	Croatia						CH						CW	N2,N3
21	Poland						CH						CW	N2,N3
22	Belarus						CH						CW	N2,N3
23	Scotland						CH						CW	N2,N3
24	Sweden						CH						CW	CW,N2,N3
25	Bulgaria						CH						CW	CW,N2,N3
26	Norway						CH							CW,N2,N3
27	Serbia						CH							CW,N2,N3
28	Hungary						CH							CW,N2,N3
29	Slovenia						CH							CW,N2,N3
30	Slovakia						CH							CW,N2,N3

Table 1. Access list to European cups in 2015-16 (Part 1)

	Association	GS	NCQ4	NCQ3	CQ4	CQ3	Q2	Q1		GS	Q4	Q3	Q2	Q1
31	Moldova						CH							CW,N2,N3
32	Azerbaijan						CH							CW,N2,N3
33	Georgia						CH							CW,N2,N3
34	Kazakhstan						CH							CW,N2,N3
35	Bosnia and Herzegovina						CH							CW,N2,N3
36	Finland						CH							CW,N2,N3
37	Iceland						CH							CW,N2,N3
38	Latvia						CH							CW,N2,N3
39	Montenegro						CH							CW,N2,N3
40	Albania						CH							CW,N2,N3
41	Lithuania						CH							CW,N2,N3
42	Macedonia						CH							CW,N2,N3
43	Ireland						CH							CW,N2,N3,FP
44	Luxembourg						CH							CW,N2,N3
45	Malta						CH							CW,N2,N3
46	Liechtenstein													CW
47	Northern Ireland							CH						CW,N2,N3
48	Wales							CH						CW,N2,N3
49	Armenia							CH						CW,N2,N3
50	Estonia							CH						CW,N2,N3
51	Faroe Islands							CH						CW,N2,N3
52	San Marino							CH						CW,N2
53	Andorra							CH						CW,N2
54	Gibraltar							CH						CW

Table 1. Access list to European cups in 2015-16 (Part 2)

Corresponding author:

Dmitry Dagaev

National Research University Higher School of Economics

Associate Professor, Department of Higher Mathematics

E-mail: ddagaev@gmail.com

Any opinions or claims contained in this Working Paper do not necessarily reflect the views of HSE.