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A magnetic fluid bridge between coaxial cylinders with a line conductor

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Abstract

The behavior (break-up and creation) of a magnetic fluid bridge between two coaxial cylinders in a magnetic field of a line conductor for any values of wetting angles of a magnetic fluid is investigated.

Keywords: Magnetic fluid; Line conductor

1. Introduction

A surface shape of a magnetic fluid (MF) changes spasmodically in a magnetic field of a line conductor [1, 2, 3]. In [1] a MF drop on a line conductor was considered. In [2, 3] the behavior of a MF between two coaxial cylinders (there is a line conductor in the bulk of the inner cylinder) was investigated when MF wets cylinders (wetting angles are no more than $\pi/2$). In the present paper the behavior of a MF bridge between two coaxial cylinders is investigated for any values of wetting angles.

2. Surface shape of MF near a line conductor

Consider a MF bridge (V_m is the volume of a MF) between two coaxial cylinders with circular cross section and the radii R_c and r_0, $R_c > r_0$, Fig.1. A line conductor with current I is along the axis of cylinders. Let the MF be a heavy, incompressible fluid. The MF is immersed in a liquid with the same density. The surface of a MF is $z = h(r)$, $r^2 = x^2 + y^2$ and the magnetic field of a conductor $H = 2I / (cr)$ is not deformed. The magnetization of a MF is described by the Langevin's law: $M(\xi) = M_S L(\xi)$, $L(\xi) = cth(\xi) - 1/\xi$, $\xi = mH/(kT)$, $m = M_S/n$. The surface of a MF near the conductor in the non-dimensional form is [3] $(P^*(r^*) = \ln(sh(\xi_0 H^*)/\xi_0 H^*))$

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$$h^*(r^*) = \int \frac{G}{(1-G^2)^{1/2}} dr^*, \qquad G(r^*) = \frac{C}{r^*} + Br^* - \frac{P_1}{r^*} \int_1^{r^*} r^* P^* \, dr^*. \qquad (1)$$

Here $r^* = r/r_0$, $h^* = h/r_0$, $H^* = H/H_0$, $H_0 = 2I/(cr_0)$, $\xi_0 = mH_0/(kT) = 2mI/(cr_0kT)$, $P_1 = nkTr_0/\sigma$, $R^*_c = R_c/r_0$. Later the signs"*" are omitted and parameters are considered as non-dimensional. If the MF bridge exists the values of constants B and C are determined from boundary conditions: $G(r = 1) = -cos(\theta_1)$, $G(r = Rc) = -cos(\theta_2)$ (here θ_1, θ_2 are wetting angles) $C = -cos(\theta_1) - B$,

$$B = (P_1 \int_1^{R_C} rPdr + \cos(\theta_1) - \cos(\theta_2)R_c)/(R_c^2 - 1).$$

In Fig.1 the MF is represented in case of non-wetting a) before break-up and b) after break-up.

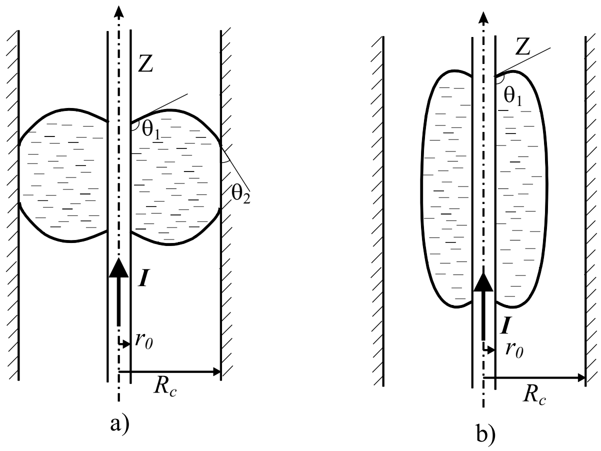

Figure 1: The MF in case of non-wetting a) before break-up and b) after break-up

3. Break-up of the MF bridge between two cylinders

A critical value of the Langevin's parameter ξ_c exists when $G = -1$, $dG/dr = 0$. For $\xi_0 > \xi_c$ the MF bridge between cylinders can not exist. When $cos\ \theta_2 = -cos\ \theta_1$, the value ξ_c depends on P_1, R_c, $cos\ \theta_1$. The calculation $\xi_c = \xi_c(\cos\theta_1)$ and other calculations were done numerically for $P_1 = 5.65$ ($r_0 = 0.1$ cm, $T = 300°$ K, $n = 0.95*10^{17}$ cm^{-3}, $\sigma = 70$ din/cm) and different values of R_c (Fig.2). It may be seen that ξ_c depends on $cos\ \theta_1$ non-monotonically and the single maximum of ξ_c exists for $\theta_1 > \pi/2$ (the case of non-wetting). For $\xi_0 = \xi_{0b}$, $\xi_{0b} < \xi_c$ the bridge break-up occurs in the moment of the touch of upper and lower surfaces at some point $r = r_1$ and when the MF volume V_m is equal to a minimal MF volume between cylinders V_0, $V_m = V_0(\xi_{0b})$ (this equality is used for the determination of ξ_{0b}). The dependences $V_0 = V_0(\xi_0)$ in case of non-wetting are monotonic (Fig.3). The dependence $V_0 = V_0(\xi_0)$ in case of wetting is non-monotonic (Fig.4). It means that in case of wetting the bridge break-up and the formation of MF volumes V_1 and V_2 on inner and outer cylinders may occur for both the increase and the decrease of current, when $V_m < V_0(0)$. In case of non-wetting the bridge break-up (the formation of a drop on the conductor) may occur only if the current increases.

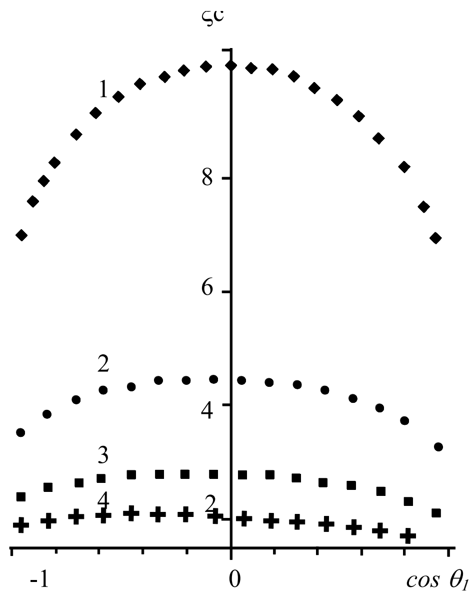

Figure 2: The value of ξ_c versus $cos\ \theta_I$ for 1 – R_c=1.5, 2 – R_c=2, 3 – R_c=3, 4 – R_c=6

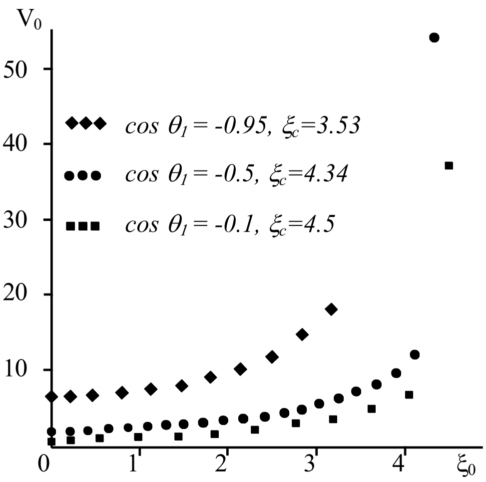

Figure 3: The value of V_0 versus ξ_0 for $cos\ \theta_I < 0$, R_c=2

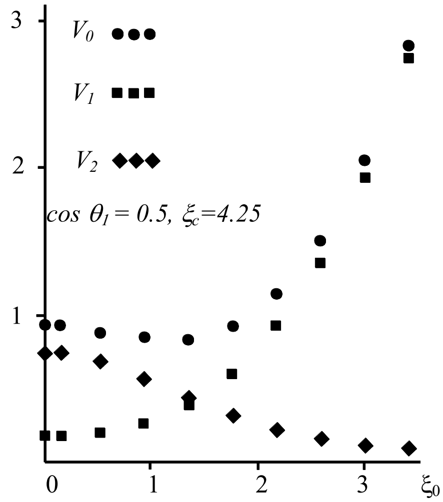

Figure 4: Values of V_0, V_1, V_2 versus ξ_0 for $cos\ \theta_1 > 0$, R_c=2

4. MF drop on a line conductor

After the bridge break-up for some ξ_{0b}, $\xi_{0b} < \xi_c$ a MF drop appears on a line conductor with the current I_b of the radius r_0 (Fig.1 b). In case of wetting the drop volume V_d is equal to $V_1\ (\xi_{0b})$ and in case of non-wetting the drop volume V_d is equal to V_m. The drop shape is described by the formula (1). The constant C is determined as $C = -cos\theta_1 - B$ from the condition $G(r = 1) = -cos\theta_1$. The dependence $B = B\ (r_d)$ (r_d is the thickness of the drop) may be obtained from the equation $G\ (r_d) = -1$. The thickness of the drop r_d is not given. But we know the value of the drop volume V_d. The dependences $V = V\ (r_d)$ for different ξ_0 (here V is the volume of some drop) in case of non-wetting ($cos\theta_1 = -0.5$) were done numerically and are shown in Fig.5. It is shown that there are three critical values of ξ_0: ξ_{01}, ξ_{02}, ξ_{03}, when the dependence $V\ (r_d)$ strongly changes. If $\xi_0<\xi_{01}$ the dependence $V = V\ (r_d)$ is a monotonous one (lines 1, 2 in Fig.5). If $\xi_{01}<\xi_0<\xi_{02}$ the dependence has a maximum and a minimum, and some values of the volume V correspond to three values of the drop thickness r_d (lines 3, 4 in Fig.5). If $\xi_{02}<\xi_0<\xi_{03}$ the dependence has two branches, some values of the volume V correspond to three values of the drop thickness r_d (lines 5, 6 in Fig.5). If $\xi_0>\xi_{03}$ the dependence $V = V\ (r_d)$ is a monotonous one (lines 7, 8 in Fig.5). So for the big enough drops and for $\xi_0>\xi_{01}$ spasmodic changes of the drop radius and hysteresis phenomena can be observed for cyclic increase and decrease of current.

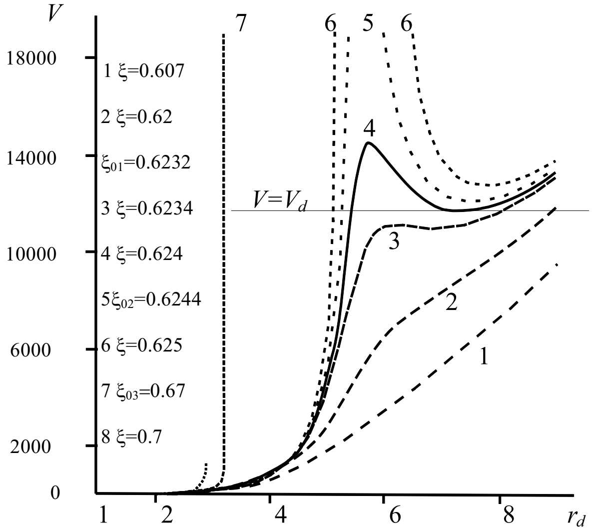

Figure 5: The dependences $V = V(r_d)$ ($\cos\theta_1 = -0.5$, $P_1 = 42.3$, $r_0 = 0.065$ cm, $T = 300°$ K, $\sigma = 12$ din/cm)

5. Formation of the MF bridge between two cylinders in case of non-wetting

Let the current increases from zero to I_b ($\xi_0 = \xi_{0b}$) for which the break-up of the MF bridge with volume V_m occurs. In case of non-wetting the MF drop of the volume V_m appears on a line conductor after the break-up. We can solve the problem about formation of the MF bridge after its break-up when the current decreases from I_b to zero using the dependence $V = V(r_d, \xi_0)$, Fig.5. If the inequality $R_c > r_d\ (I = 0)$ is valid the MF bridge can not appear. The value of $r_d\ (I = 0)$ is the solution of the equation: $V_m = V(r_d, 0)$. We can find geometrically the value of $r_d\ (I = 0)$: $r_d\ (I = 0)$ is an abscissa of the point of intersection of lines $V = V_m$ and $V = V(r_d, 0)$. The minimal volume $V^{(min)}{}_m$ for which the MF bridge appears is equal to $V^{(min)}{}_m = V(R_c, 0)$. If the inequality $R_c > r_d\ (I = 0)$ is not valid the MF bridge between two cylinders can appear for some current I_f, $0 < I_f < I_b$. The value of $\xi_{0f} = \xi_0(I_f)$ is the solution of the equation $V_m = V(R_c, \xi_{0f})$. This value can be found geometrically: ξ_{0f} is the Langevin's parameter for which a line $V = V(r_d, \xi_{0f})$ goes across the point with coordinates (R_c, V_m) on the plane (r_d, V).

6. Conclusion

It is shown that a break-up of the MF bridge between cylinders is for $\xi_{0b} < \xi_c$ when the current increases from zero to I_b. The volumes of the MF which appear on cylinders, ξ_c and ξ_{0b} are calculated numerically. The problem about a MF drop on a line conductor is solved for arbitrary wetting angles. The dependences of the volume of some drop on the thickness of the drop $V = V(r_d)$ for different ξ_0 are calculated. It is shown that there are three critical values of the parameter ξ_0 when the dependence $V(r_d)$ strongly changes. The least critical value ξ_{01} is found for the first time. The dependence $V = V(r_d)$ is necessary for the solution of the drop problem and for the solution of the problem of formation of the MF bridge between two cylinders after its break-up when the current decreases from I_b to zero. It is shown that the MF bridge can appear if $V_m \geq V^{(min)}{}_m$ or can not appear if $V_m < V^{(min)}{}_m$, when the current decreases from I_b to zero. The minimal volume $V^{(min)}{}_m$ of the MF for which the MF bridge appears is found for different wetting angles.

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