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A JOINT NON-PARAMETRIC APPROACH TO THE DECOMPOSITION OF BOND YIELDS AND CDS SPREADS: APPLICATION OF EUROZONE MARKET DATA[3]

In this paper[4] we develop a joint non-parametric approach to the problem of the decomposition of bond yields and CDS spreads. The proposed approach is essentially an infinite-dimensional modification of the Heath-Jarrow-Morton framework and is general enough to capture even very non-trivial shapes of the yield and hazard-rate curves. The approach allows us to jointly estimate entire term structures of yields, hazard rates, and liquidity premiums, no matter what shapes they take. We apply the developed methodology to data on major Eurozone sovereign borrowers and consider the most recent period of the Eurozone debt crisis. Our data set includes instruments with maturities from 6 months to 30 years. As a result, we found several interesting interaction effects between those components in terms of term structure. Treating the bond-CDS basis as a measure of the cross-market liquidity spread, we find that cross-market liquidity evolves in a rather non-trivial and pronounced manner. As the credit quality of the reference entity deteriorates, the liquidity of the CDS market dries up, starting from longer terms.

JEL Classification: C14, G12.
Keywords: term structure; interest rates; credit risk; default intensity; liquidity premium; bond; credit default swap; risk premium.

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1. Introduction

During the last decade, significant attention has been paid to the problem of the decomposition of bond yields into constituents. This problem is not only interesting from an academic point of view, but is also important for financial engineering and risk-management. Since bond yields depend on multiple economic factors, an accurate decomposition requires fairly extensive data, realistic assumptions, interpretable measures, and advanced mathematical instruments.

The development of the credit default swap (CDS) market has extended information on market perceptions of the creditworthiness of bond issuers. The use of CDS quotes and prices helps us to refine estimates for bond credit spreads and to more accurately estimate other components of bond yields, such as the liquidity premium (see Smirnov & Tarasova, 2011). However, this requires a joint estimation framework based on joint modeling that is consistent with observable price data and that takes into account specific features of the bond and CDS markets.

The main purpose of this paper is to propose an approach for the decomposition of bond yields and CDS spreads within a joint pricing framework that is based on a reduced-form model. We also apply the original methodology to data from euro-denominated sovereign bonds and CDS and find several interesting interaction effects between the examined factors in terms of term structure, among other things.

Thus, our paper contributes to the field in several ways.

First, we develop a more sophisticated methodology for term-structure construction. It is based on a non-parametric approach that is essentially an infinite-dimensional modification of the Heath-Jarrow-Morton framework. The proposed approach is sufficiently general to capture even very non-trivial shapes of yield and hazard-rate curves.

Second, we do not refer to any bonds as default-free, since no CDS has a zero spread. In contrast, we estimate a default-free yield curve via an original procedure that takes into account traded bonds and CDSs. In particular, this feature is very important for the Eurozone bond market.

Third, since there is still no clear idea about how to formalize the individual liquidity component of bond yields and CDS spreads without imposing any limiting assumptions, we consider individual liquidity characteristics – such as bid-ask spreads – solely as a tolerance level for estimating other components of the instrument yield. Instead of analyzing individual liquidity, we study the cross-market liquidity spread, which represents extra yield or a discount on the bond over the CDS-implied yield. Cross-market liquidity may characterize the spillover effect and the liquidity commonality of the two markets.

Finally, we apply the developed methodology to the data on bonds and CDSs of different maturities, whereas most previous studies deal with the instruments of a particular term. In such papers, the results are usually obtained via a time-series analysis. Our paper is focused on term structures of yield constituents and based on a market snapshot analysis. Our data set includes instruments with maturities from 6 months to 30 years. The use of such data allows us to jointly estimate entire term structures of yields, hazard rates, and liquidity premiums, regardless of what shapes they take.

The reminder of the paper is organized as follows. Section 2 is devoted to background information: Subsection 2.1 overviews the roots of the problem and previous literature on the topic, while subsection 2.2 briefly presents a general formalization of interest rates and hazard rates within the reduced-form model class, and subsection 2.3 demonstrates the weaknesses of simple models of default-free yields and hazard rates and explains why more sophisticated methodologies, such as the one presented in this paper, are preferable. The study's methodology and data are described in Section 3: The model description, estimation procedure, and data are discussed in subsection 3.1, subsection 3.2, and subsection 3.3, respectively. Section 4 demonstrates the obtained results and section 5 concludes the paper.

2. Background and Related Literature

2.1 Components of bond yields and CDS spreads

The decomposition of bond yields is a non-trivial problem, since yields depend on a wide range of factors. In the academic and practical literature, the effects of those factors are primarily reduced to the effects of the zero-coupon yield curve and credit risk. During the previous decade significant attention was also paid to the impact of market liquidity on yields of fixed-income instruments.

Generally, a zero-coupon curve may be estimated separately from other components. If a liquid market exists for bonds that are default-free or that at least have a stable credit rating, then one may extract a zero-coupon yield curve from mid quotes and/or the trade prices of those bonds, applying any of the methods briefly discussed in next subsection. Having obtained a yield curve, one may go further and estimate the credit spread as excessive return on risky bonds over the zero-coupon curve. The liquidity component in this case is measured via the bid-ask spread around the mid-quote. Thus, the problem of decomposition may be solved consecutively by starting from construction of a yield curve.

In practice, government bonds are commonly treated as the best proxy for default-free bonds. For instance, a risk-free dollar yield curve may be constructed fairly easily, since the US Treasuries market is homogenous in terms of credit risk and liquidity. However, bonds issued by Eurozone countries substantially diverge in terms of credit risk and liquidity. Therefore, the risk-free yield and credit spreads for the Eurozone market should be estimated using a joint procedure. Since the problem is ill posed in this case, it requires some restrictive but plausible assumptions to be imposed on the term structure of credit spreads in order to provide regularization. Such methodology has been promoted by EFFAS-EBC (see Smirnov et al., 2006).

To avoid any strong assumptions on the term structure of credit spreads, one may try to extract corresponding information from credit derivatives, such as CDSs. For example, Longstaff, Mithal, and Neis (2005) considered a CDS spread as a "pure" measure of the credit component included in a bond yield. By aggregating the credit spreads implied by CDS and zero-coupon yields and comparing the resulting theoretical yields with observed yields of risky bonds, the authors found that the unexplained spread over the theoretical yield may be attributed to market liquidity. The researchers assumed that the CDS market is perfectly liquid or, less strongly, that the component of liquidity in the CDS spread is negligible. In fact, such an assumption appears to be too strong to be realistic. A recent study of Chen, Fleming, Jackson, Li, and Sarkar (2011) analyzes CDS transition data and shows that the CDS market is rather concentrated and heterogeneous in terms of market activity. The liquidity of CDS contracts also varies with tenors and the credit quality of reference entity. Therefore, the CDS market may hardly be considered to be liquid.

Thus, the liquidity component of CDS spread should be taken into account as well. There is a series of papers that incorporate CDS liquidity in their analysis. For example, Chen, Fabozzi, Sverdlove (2010) and Buhler and Trapp (2009) investigate zero-coupon yields, credit risk, and liquidity within a stochastic approach to reduced-form models. Both studies model liquidity as a stochastic process. Chen et al (2010) argue that the liquidity premium is earned by a CDS buyer and consider the CDS ask quote as price that is clean from any liquidity premium. The authors introduce liquidity in the model as an additional discount factor that reduces CDS spreads, all else remaining equal. The liquidity factor is modeled in terms of the intensity that follows the CIR process, thereby guaranteeing a mean-reversion property of the liquidity component. The results of this study show that the incorporation of a CDS liquidity component improves estimates of the CDS-implied credit component and therefore improves the estimate for the bond liquidity premium obtained using such an estimate.

In the conditions of Buhler and Trapp's experiment, the bid and ask quotes bear their own liquidity components and both are affected by a reference to bond liquidity. The authors' model is fairly general and allows for correlations between zero-coupon yield, hazard rates, and liquidity components. In contrast to the previous work of Buhler and Trapp, the model measures the intensity of liquidity factor via arithmetic Brownian motion. Such a specification simplifies calculations, but appears to be unrealistic, since the processes for Brownian motion approach infinity almost as surely as a term approaches infinity. Calibrating their model to corporate bond market prices and bid/ask quotes placed by a main CDS dealer, the authors found significant spillover effects between CDS and the bond market. They also documented a negative correlation between the credit rating of the reference entity and the liquidity of bond and CDS contracts. But the most valuable results of their paper are estimates for the contribution of credit risk, individual liquidity, and correlation between them to bond and CDS spreads. According to their estimates, 60% of bond spreads may be attributed to credit risk, 35% to liquidity, and 5% to correlation, whereas a full 90% of a CDS spread is due to credit risk, 4% to liquidity, and only 1% to correlation.

Whereas a particular specification of the liquidity process may affect results obtained via applying the model, Calice, Chen, and Williams (2011) follow a model-independent approach in measuring credit and liquidity components. In their study, a credit component is calculated separately for bonds and CDS contracts as the difference between the ratio of particular bond/CDS and German bond/CDS. The liquidity component is simply measured as a quoted bid/ask spread. The authors used a time-varying vector autoregression framework to establish the credit and liquidity spread interactions in the Eurozone market over the 2009–2010 crisis period. Their results show that the CDS market generally leads the bond market. Their study also found a significant lagged transmission from the liquidity spread of the CDS market to the credit spread in the bond market.

One more important note should be made. Previous studies mostly focused on corporate bonds, which are usually rather few for any issuer, or on a CDS of particular maturation period (5 years). The authors conducted their research within a risk-neutral pricing framework based on a popular stochastically dynamic low-parametric model[5] for the short rate and hazard rate. Those studies usually decompose yields based on instruments of particular maturity, while properties of the term structure for hazard rates and liquidity premiums are inferred from the model or totally ignored. Therefore, results in such papers are usually obtained via a time-series analysis. In

[5] E.g. Brownian motion, Vasicek or CIR models

subsection 3.1 of this paper we explain why we focus on a term structure for yield constituents and why we deal with a snapshot analysis.

Our study relates to the reduced-form modeling approach. But, in contrast, to the studies mentioned above, we use a rather general stochastic process that does not impose any critical limitation on the properties of the modeled objects. But our approach is definitely not assumption-free, at least because such a poorly investigated variable as loss-given-default (LGD) necessarily requires simplified formalization, and it may affect the accuracy of hazard-rate estimates. The next section briefly discusses the basics of reduced-form modeling and its joint application to modeling zero-coupon curves and hazard rates.

2.2 Interest Rates and Hazard Rates

It is common practice to describe interest rates in terms of spot rates and instantaneous forward rates. We denote the spot rate prevailing at time t as r_t and the zero-coupon yield curve prevailing at time t as $r_t(\cdot)$. Using continuous compounding, we may express corresponding discount functions as follows:

$$d_t(s) = \exp\left[-s\, r_t(s)\right] = \exp\left[-\int_0^s f_t(\tau)d\tau\right]$$

where $f_t(\tau)$ is the instantaneous forward rate prevailing at time t for the time $t+s$.

It is also a well-known fact that the discount function may also be expressed in terms of the risk-neutral dynamics of the spot rate $r_t = r_t(0) = f_t(0)$:

$$d_t(s) = E_t^Q\left(\exp\left[-\int_t^{t+s} r_\tau d\tau\right]\right).$$

Many different models exist for interest rate modeling[6]. Almost all these models assume that the interest rate in question is the risk-free rate – that the bonds are not subject to default risk, whereas real bonds almost always bear the issuer's default risk.

It is also common practice to introduce credit risk in the following way, very similar to the interest rate mechanics. If default is treated as an absolutely random event that is completely unpredictable, then one may focus solely on default probability distribution. The time of the default Θ is considered to be a stopping time. Let the probability of default up to time $t+s$ conditional on no default at time t be $1-Q_t(s)$, where $Q_t(s)$ is called the survival probability.

It is also well known that, under mild regularity conditions, there exist a predictable process λ_t, the "spot default rate", which is generally termed the "spot hazard rate", such that

[6] See James & Webber (2000) and (Andersen & Peterbarg (2010) for a comprehensive review.

$$Q_t(s) = E_t^Q\left(\exp\left[-\int_t^{t+s}\lambda_\tau d\tau\right]\right).$$

Continuing the analogy with interest rates, we may introduce the notions of "hazard rate to maturity" $\Lambda_t(s)$ and "instantaneous hazard rate" $\lambda_t(s)$ via

$$Q_t(s) = \exp\left[-s\Lambda_t(s)\right] = \exp\left[-\int_t^{t+s}\lambda_\tau d\tau\right].$$

Many hazard rate models also exist, mainly devised from existing interest rate models.[7] The need for the joint modeling of interest rates and credit risk has been acknowledged for some time, and since many of the hazard rate models are just interest rate models rewritten for new variables, they naturally combine together into what is called a double stochastic model.

In what follows, we show that naïve methods, such as bootstrapping, or one-factor models, such as CIR, might be inappropriate in the current circumstances.

2.3 Inappropriateness of Popular Methods

It is common practice to bootstrap the implied hazard rates from CDS spreads. While the deficiency of such an approach to estimating the term structure of interest rates has already been acknowledged for some time, the same approach is being used for the term structure of hazard rates, which might (and we show that it does) suffer from the same problems.

For example, the bootstrapped spot hazard rates for Greece on June 20, 2011, became negative at about 5 years. This is due to the very low precision of the bootstrapping method and to the very high implied-default probability, which implies a very high impact of the fitting error.

Furthermore, the most popular parametric methods of fitting fail to represent complex, but still plausible shapes of the yield and hazard-rate curves, which we discuss in more detail further below.

It should be noted that "naïve dynamic methods", such as the Nelson-Siegel dynamic, adopted by the Central Bank in Chili, or the G-Curve, adopted by MICEX in Russia (Gambarov, Shevchuk, & Balabushkin, 2004), while being sometimes capable of estimating the term structure of interest rates for illiquid markets, suffer from other serious drawbacks. They are still low-parametric methods having a limited range of shapes for the interest-rate and hazard-rate curves. Moreover, they have been proven to introduce arbitrage opportunities into model.[8] Therefore, we consider such methods to be inappropriate for our purposes, since the joint estimation framework requires arbitrage-free conditions and parametric estimation methods generally introduce model bias.

[7] Please refer to Brigo & Mercurio (2006) for the details and the analogies between interest and hazard rates.
[8] See Filipovic (1999) and Björk & Christensen (1999).

In such extreme market conditions as the Eurozone debt crisis, the application of more sophisticated approaches may provide more insightful information on the interaction between determinants of bond yields and CDS spreads and provide more accuracy for pricing sovereign debt and credit derivatives.

3 Methodology and Data

3.1 The Non-Parametric Model

Our approach to the spread decomposition problem is to specify a doubly stochastic model, like the one developed by Filipovic, Overbeck, and Schmidt (2011), but let both the spot forward rate r_t and hazard rate λ_t evolve according to an infinite-dimensional HJM-style equation, which is the infinite-dimensional extension of the doubly stochastic HJM model of study by Schönbucher (1998):

$$\begin{cases} dr_t = \left(Dr_t + \alpha_t\right)dt + \sum_{j=1}^{\infty} \sigma_t^j d\beta_t^j \\ d\lambda_t = \left(D\lambda_t + \tilde{\alpha}_t\right)dt + \sum_{j=1}^{\infty} \tilde{\sigma}_t^j d\tilde{\beta}_t^j \end{cases} \tag{1}$$

where r_t and λ_t are the infinite-dimensional state variables and β_t^j and $\tilde{\beta}_t^j$ are independent standard Wiener processes. When the processes r_t and λ_t are assumed to be independent, it reduces to a separate non-parametric estimation of spot forward rates and the hazard rate, as described for the interest rate case by Filipović (2001). The exact dynamic parameters $\alpha_t, \tilde{\alpha}_t, \sigma_t^j, \tilde{\sigma}_t^j$ are unimportant since the estimation only involves a snapshot of the market – the state variables r_t and λ_t. An infinite-dimensional model in our setting is essentially equivalent to a non-parametric estimate of the spot forward rate and hazard rates via a procedure similar to the one developed by Smirnov and Zakharov (2003) or Lapshin (2009). Yet, the underlying infinite-dimensional model is needed because it serves as a guarantee for the approach to be internally consistent and arbitrage-free for use on an everyday basis.

We now present arguments on the necessity of introducing such a complex model into consideration. The motivation behind this is threefold.

Firstly, we need a sound methodological basis for conducting analysis. Previous analyses relied on simple low-parametric dynamic models on interest rates because they worked with a time series of interest rates for specific selected terms. These models offered a sound basis for those kinds of research since they allowed for an arbitrage-free dynamic framework for the short rate (or for any other fixed-term rate). But since the work of Longstaff, Santa Clara, and

Schwartz (2001), it is generally understood that no pricing should be done using one-parametric models, including the CIR model[9]. And we are dealing with the pricing of CDS and risky bonds.

Instead of diving into the stochastic dynamics of interest and hazard rates, we concentrate on the term structure of the studied entities, namely interest and hazard rates, cross-market liquidity spreads, etc. This is possible because we restrict ourselves with one snapshot of the market at a time: The same pricing which is usually done via a stochastic model may be done using a snapshot term structure. A low-parametric model necessarily implies low-parametric and sometimes unrealistic shapes for the term-structure curves,[10] introducing a great deal of model-inflicted bias into the calculations. Since our main concern is the term structure, we need an algorithm that fits the interest rate and hazard rate and that is flexible enough and general enough so as to inflict minimal model bias and be capable of fitting most complex term-structures.

Low-parametric groups, such as CIR, Nelson-Siegel, and Svensson, are commonly known to have a very limited spectrum of possible shapes. And while it might turn out that this limited spectrum could be sufficient for our purposes, there is no way to know it in advance, so we have to first plan and conduct our experiment using a general model, within which we might show that some simpler models truly offer comparable performance in our task.

Thus, the model should be (a) non-parametric, since there is no reason to prefer one parametric specification to another, and (b) as general as possible, for the reasons described above.

Secondly, we plan to continue our studies and consider the dynamics of the studied entities, meaning that we will need a stochastic dynamics model. The above-cited works of Filipovic and Björk have shown that virtually no parametric term structure models offer this opportunity in an arbitrage-less setting. Our model cannot stem from a low-dimension dynamic short rate model, as this would assume awkward low-parametric term-structure curves and introduce a substantial model bias.

Finally, not only does this particular model possess all the needed properties – specifically, it is non-parametric (and thus general) and it has an arbitrage-free dynamic extension (Lapshin, 2010) – but it may also easily be tuned to offer any desired goodness-of-fit test if necessary.

However, we argue that it is not necessary and that the model quality should be measured not only by goodness-of-fit (which is still an important criterion), but also by the visual appeal of

[9] Nevertheless, we have reason to believe that most of the CDSs were being priced using the CIR model, at least at the time covered by the present study – a claim that should be carefully studied separately because its implications might be more serious than they seem.

[10] See previous footnote on pricing CDS with CIR.

resulting curves – since this is an important criterion used by traders. It also has an economic justification, as the continuity of market expectations requires the term structures to be smooth.[11]

So the choice of this model is really motivated not by its quantitative properties, such as a low fitting error that can be tuned to be arbitrary small, but by more qualitative criteria, such as generality and the possibility of a dynamic extension. Any such model will do and we would be delighted to perform a quantitative comparison of two such models when another such model is brought to our attention.

3.2 Model Estimation

We have estimated model (1) using data described earlier in the following way. We simultaneously search for a risk-free spot forward-rate curve $f(\tau)$ and issuer-specific spot hazard rates $h_k(\tau)$, such that CDS quotes are fitted with weights proportional to their relative liquidity, risky bond prices are fitted with weights proportional to bid-ask spreads, and fitted curves are sufficiently smooth. This may be formalized as follows:

$$\alpha \int f'(\tau)^2 d\tau + \sum_k J_1^k + \beta \sum_k \left(\int h_k'(\tau)^2 d\tau \right) + \sum_k J_2^k \to \min_{f(\cdot), h_k(\cdot)}$$

Here the first and the third terms correspond to the desired smoothness of the curves $f(\tau)$ and $h_k(\tau)$, the risk-free spot yield curve and the issuer-specific spot hazard rates curves, respectively. The second term corresponds to the fitting of the bond prices while the fourth makes the solution fit the observed CDS quotes. Hence:

$$J_1^k = \sum_T \frac{1}{ask_T^k - bid_T^k} \left[P_T^k - \left(\sum_{i=1}^{N_{k,T}} d(t_i) F_{i,T}^k Q(T_i) + \int_0^T (1 - LGD) \mathrm{d}(\tau) d\left(1 - Q_k(\tau)\right) \right) \right]^2$$

$$J_2^k = \sum_T w_T \begin{bmatrix} \int_0^T LGD\, \mathrm{d}(\tau) d\left(1 - Q_k(\tau)\right) - c_T^k \sum_{i=1}^{N} \mathrm{d}(T_i) \alpha\left(T_{i-1}, T_i\right) Q_k(T_i) + \\ + \int_0^T \mathrm{d}(\tau) \alpha\left(T_{I(\tau)}, \tau\right) d\left(1 - Q_k(\tau)\right) \end{bmatrix}^2$$

where

$$\mathrm{d}(t) = \exp\left(-\int_0^T f(\tau) d\tau \right)$$

$$Q_k(t) = \exp\left(-\int_0^T h_k(\tau) d\tau \right)$$

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[11] See Smirnov and Zakharov (2003), Lapshin (2009), and Lapshin (2010) for a more detailed discussion of the topic.

are the discount and the survival functions, respectively. P_T^K is the bond mid quote and c_T^k corresponds to the CDS composite spread provided by Reuters.

The assumptions on LGD should be discussed more closely. Loss-given-default is one of the most poorly studied variables in financial economics. For risk-neutral pricing purposes, one usually assumes LGD to be independent from other variables, such as the probability of default or the risk-free yield. In such settings there is no need to estimate the distribution of LGD, but rather its expected value under risk-neutral probability measures. Some studies documented that, in some range, CDS spreads are fairly non-sensitive to the variation of expected LGD (Houweling & Vorst, 2005). In this paper, we assume LGD to be constant and use a standard value (60%) recommended by ISDA for pricing CDSs written on senior debt. Only for Greek bonds did we assume LGD to be equal to 80%, since in the considered period Greece suffered deep financial distress and the debt write-off rate was approximately known.

It is worth recalling that in our problem settings we distinguish between liquidity of individual contract and cross-market liquidity.

As mentioned in Section 1, individual liquidity is measured by bid-ask spreads, trade frequencies, and other individual characteristics of a contract. These quantities are used as a measure of accuracy for yields and hazard rates and are included in the model as inputs. For instance, the bid and ask quotes for bonds were transformed into feasibility bands, as in Smirnov and Zakharov (2003), mentioned in subsection 2.3 and depicted by jagged curves. It was first introduced as arbitrage bounds by Jaschke (1998) and reflects a constraint, which the absence of arbitrage opportunities imposes on yield-curve estimates. It is defined as the locus of all points that might possibly belong to the zero-coupon yield curve, given the bid-ask quotes. It effectively transfers the bid-ask bounds from the price domain to the interest-rate domain. The equations governing its construction are

$$\begin{cases} r\left(t_i\right) = -\dfrac{1}{t_i}\ln \mathrm{d}_i; \\ \mathrm{d}_i \to \max, \min; \\ 1 \geq \mathrm{d}_1 \geq \ldots \geq \mathrm{d}_N \geq 0; \\ ask_k \geq \sum_{i=1}^{N} F_{i,k}\mathrm{d}_i \geq bid_k. \end{cases}$$

With regards to cross-market liquidity, we refer to assets that are traded on different but tightly related markets and the price discrepancies of which are due to differences in the market microstructure, convenience yield, and liquidity commonality. The term structure of cross-market liquidity is an output of our model and its properties are further studied.

The hazard rates may be estimated to fit both the risky bond and the CDS prices at the same time. But we found that the dependence of CDS spreads on the risk-free rate is very weak. So there is also another way to conduct the estimation. We may take only the risk-free curve, which is independent of the issuer, and ignore the estimated hazard rates. We then may use the CDS spreads again and the known risk-free rate to find the implied hazard rates for each issuer. Then we may use the known risk-free rate and the known hazard rates in order to price the risky bonds and thus estimate the risky zero-coupon yield curve. This estimate may then be compared with the actual bid-ask quotes for the same bonds and with the actual zero-coupon yield curve. We assess the significance of the deviations of theoretical yields from observed yields via the feasibility band described above.

3.3 Data

We applied the developed methodology to real data. Our data set includes daily bond and CDS information on major sovereign borrowers in the Eurozone, such as Germany, France, Italy, Spain, Portugal, and Greece, and covers the period from March 2010 to June 2011. So our sample represents both those countries considered to be financially sound and those that currently suffer financial distress.

The choice to use this kind of data is justified by the following reasons.

First, since our approach deals with an entire term structure of yield or spread, the data selection is justified by the existence of a fairly wide range of maturities for sovereign issues.

The bond data is obtained from Bloomberg™ and contains daily bid quotes, ask quotes, and last trade prices. For each country we have from 11 to 15 bond issues. Those issues are considered as benchmarks and cover maturity terms from 1 year to 30 years.

We also gather CDS spread data from Thomson Reuters 3000 Xtra. Those spreads are essentially Reuter's composite spreads, which the information agency calculates for single-name tenors from 6 month to 30 years and which are calculated from quotes posted by multiple dealers. For France and Germany, the spread data is expressed under the par spread concept, since spreads of CDSs for those counties are usually quite small (less than 100 bps). For other countries, we have data on conventional spreads, as those countries are subject to standard coupon conventions. Unfortunately, information on bid and ask quotes is unavailable to us, so we proxy the liquidity of on individual contact with the weekly trade frequencies obtained from Market Risk Transaction Activity page on the DTCC web-site.

Second, the use of daily data is motivated by the fact that the analysis we conduct is by definition point-in-time, employing only a snapshot of the market. We could use more regular data if it were readily available. The non-parametric estimation method we employ is known to successfully smooth over part of the random noise in the high-frequency data thanks to the regularization term (Smirnov & Zakharov, 2003; Lapshin, 2009). The rest of the noise cannot be dealt with from within the snapshot approach and requires a dynamic extension of the model, which is the subject of ongoing research.

Finally, before 2010, CDSs, which are crucial component of our approach, were traded under conventions and at a transparency level that is significantly different for those prevailing nowadays. The market changes[12] undertaken during 2009–2010 were implemented to improve the transparency and reliability of CDS data. It is obvious that one cannot categorically consider CDS prices before and after 2010 as homogenous. But the study of homogeneity for the time-series before and after 2010 is a topic for special research and thus is beyond the scope of this paper. We also recall that we focus on a demonstration of developed methodology rather than an investigation of crisis impact on yield components. Furthermore, in comparison with earlier data, the sample of 2010–2011 has fewer gaps in the data. Therefore we decided to limit our CDS data sample to the 2010 – 2011 period.

4 Results

Fig. 1 shows the non-parametric (magenta) estimates of the zero-coupon yield curve for France on April 7, 2011. It also depicts the yield-to-maturity versus duration points (green asterisks), which are usually closer to the zero-coupon yield curve than the yield-to-maturity versus time-to-maturity points due to the coupon effect. The feasibility band is plotted with jagged blue lines. The risk-free zero-coupon yield-curve is plotted with the dotted line and lies below the others, which is normal. The synthetic risky zero-coupon yield curve is colored red. The spread between this curve and the risk-free curve is the "pure" credit spread, as measured by the CDS. The spread between the actual curve and the synthetic risky curve presumably contains all other premiums, namely the convenience yield, the liquidity premium, etc.

On Fig. 1 we see that the credit spread is slightly less than 1% for France and shrinks to 0.3% for short maturities. We also observe that the risk-free yield, combined with the CDS-implied hazard-rate, approximates the observed yield fairly well, leaving a small spread to be considered as the liquidity spread.

[12] See Markit Reports on Big Bang Protocol (http://www.markit.com/cds/announcements/resource/cds_big_bang.pdf) and Small Bang Protocol (http://www.markit.com/cds/announcements/resource/cds_small_bang_07202009_upd.pdf).

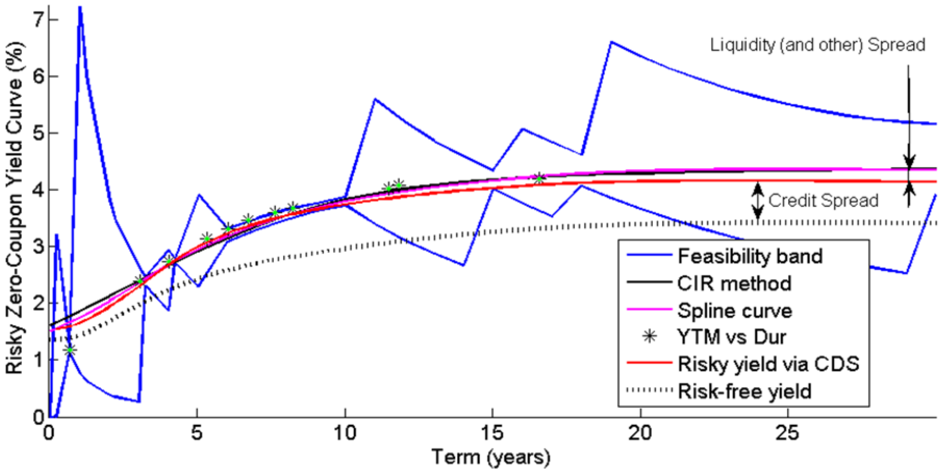

Fig. 1. Risk yield curves for France on April 7, 2011.

For countries like Portugal, we observe quite a different pattern. As Fig. 2 (a) shows, on April 14, 2011, bonds with a term to maturity of less than 17 years were traded with premium relative to the CDS market, whereas beyond this term the yields of bonds are less than the yields of the corresponding synthetic position.

Fig. 2 (b) illustrates the term structure of hazard rate for Portuguese debt. Since the hazard rate curve in essence is the density of default-time probability conditional on currently available information, the figure shows that, according to observed prices of Portuguese bonds, there are two pronounced scenarios for Portugal's debt future. Under the first scenario, Portugal will default within the next five years, while according to the second scenario the country's creditworthiness will stabilize in five to seven years. These two scenarios are weighted with according to market participant sentiments on the likelihood of each of these two outcomes.

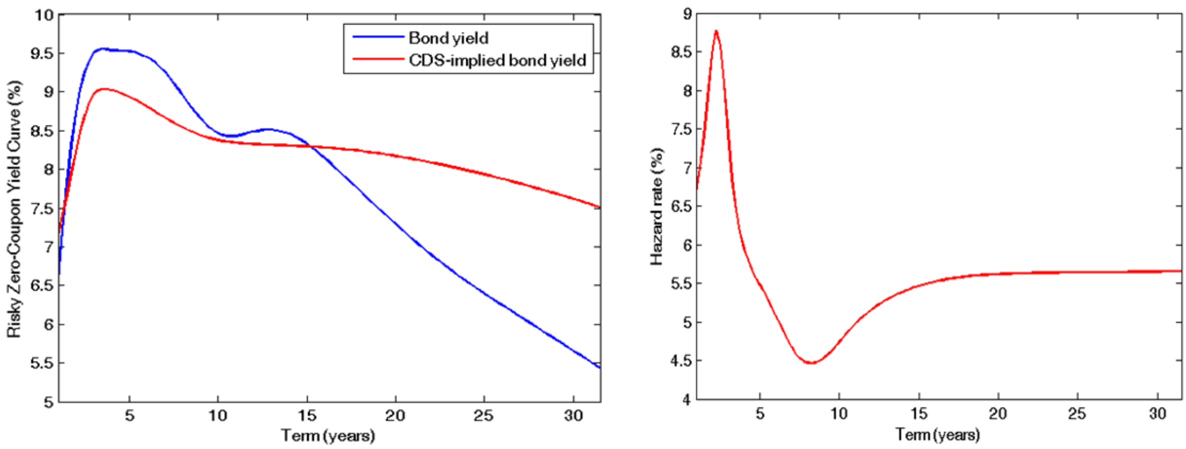

Fig. 2. (a) Risk yield curves for Portugal on April 14, 2011; (b) Hazard-rate term-structure for Portugal on April 14, 2011.

Fig. 3 illustrates the cross-market liquidity spread for Portugal on the same date. In essence, the cross-market liquidity spread curve is the difference between curves on Fig. 2 (a). As the figure demonstrates, the CDS contracts with maturities up to 10 years are merely more liquid than corresponding bonds. In contrast, the long-term bonds have a smaller liquidity premium than do CDS contracts. The yield on 30-year bonds is 2% less than that implied by CDS.

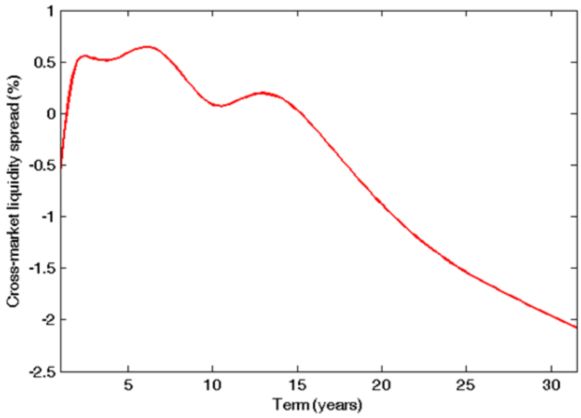

Fig. 3. Cross-market liquidity spread term structure for Portugal on April 14, 2011.

Fig. 4 shows results obtained for Greece. While the risk-free zero-coupon yield lies well below typical yields for Greece, there are other features to observe in the figure. The synthetic risk yield (red) resembles the non-parametric estimate at the short end, but lies consistently higher, with a spread of 2–3%. The feasibility band plotted on the same graph suggests that this difference is statistically significant given the observed bid-ask spreads for Greek bonds. This might mean that the CDS market overestimates the probability of Greece defaulting on a large horizon, that the bond market underestimates the same risk, or that such yield discrepancies are due to liquidity differences between the bond and CDS market.

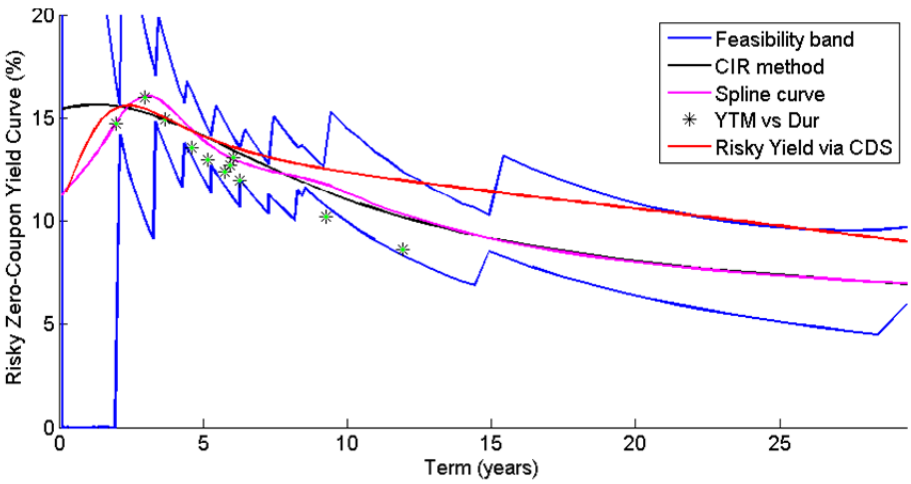

Figure 4. Risk yield curves for Greece on April 7, 2011.

Fig. 5 (a) presents the hazard-rate term-structure, which has the same shape that Portugal's has. In contrast to Portugal, Greece's hazard-rate curve has a higher peaked hump on short terms and a lower curve tail. This tells us that a negative scenario is more likely for Greece than for Portugal.

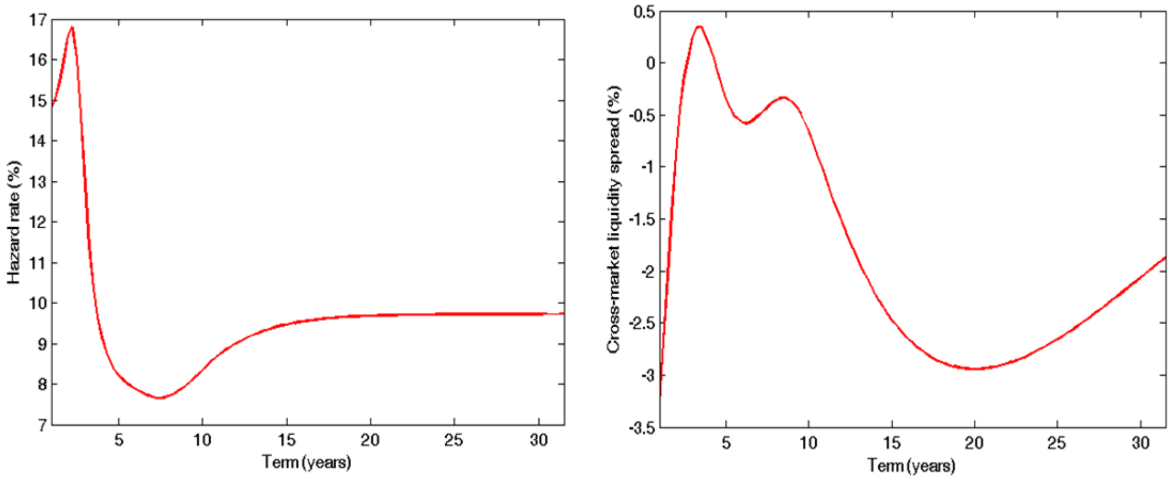

Fig. 5. (a) Hazard-rate term-structure for Greece on April 7, 2011; (b) Cross-market liquidity-spread term-structure for Greece on April 7, 2011.

We also depict the difference in yield curves, which we treat as cross-market spread (Fig. 5 (b)). The cross-market liquidity spread is negative almost everywhere and reaches -3% at a 20-year term. Furthermore, the term structure of the cross-market liquidity spread has a non-trivial shape that can hardly be interpreted in economic terms. However, comparing this figure with Fig. 1 and Fig. 3, we clearly see that the more risky bond is, the shorter the range of CDS maturities is that are as liquid as the corresponding bonds. As the credit rating of the reference entity deteriorates, the liquidity of the CDS market dries up starting from longer terms.

This effect of negative basis is also more pronounced during a continuous time period. These periods may associate with periods in which the relevant information for an unexpected decline in the creditworthiness of a reference entity comes on the market. Such a phenomenon may be due to several reasons. On the one hand, it may be caused by the active trading of speculators who try to make a profit from a "naked" long position in CDSs. On the other hand, a similar effect may be due to active hedging transactions by holders of governments bonds or holders of instruments whose values depend tightly on a sovereign's creditworthiness. Hedging may affect market prices significantly, especially for long maturities and when market depth is small. It is very hard to distinguish between these two effects without information about the exposure of market participants on both the bond and CDS market.

Indeed, the question of described phenomenon requires a special study, which is beyond the scope of this paper.

5 Conclusion

In this paper, we analyzed bond prices and CDS spread data for major Eurozone borrowers, including Germany, France, Italy, Spain, Portugal, and Greece. Our data set covers the most recent period of the debt crisis. We consider joint effects observed in the sovereign bond market and CDS market during the dataset's horizon. In order to decompose bond yields and CDS spreads into components of zero-coupon yield, credit-risk, and cross-market liquidity spread, we apply our original methodology, representing the infinite-dimensional Heath-Jarrow-Morton framework for joint modeling interest rate and credit risk.

We applied our methodology to the bond and CDS information on the aforementioned major Eurozone sovereign borrowers over the period from March 2010 to June 2011. Advanced features of the new approach have allowed us to find several interesting interaction effects between those components in terms of term structure. In particular, while the term structures of risk-free yield and hazard rates have quite regular shapes, the bond-CDS basis, which we treat as a cross-market liquidity spread, evolves in a rather non-trivial, but nevertheless pronounced manner. We show that the liquidity of the CDS market starts to dry up from longer maturities with a decline in the creditworthiness of the referenced entity. We also observe a significant liquidity squeeze in the CDS market over a continuous time span. We believe that this results from the unidirectional behavior of speculators and hedgers, and the small depth of the CDS market.

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