Novosibirsk State University
Lavrentyev Institute of Hydrodynamics

Abstracts

Russian-French Workshop

“Mathematical Hydrodynamics”

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Novosibirsk, Russia

Novosibirsk State University
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Lavrentyev Institute of Hydrodynamics SB RAS
Novosibirsk State University

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ABSTRACTS

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Double-deck structures of boundary layers in flows

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In [1] the double-deck structure of the boundary layer was discovered in the problem of flow around a plate with small periodic imperfections. The inverse powers of the Reynolds number **Re**, which determine the structure of the imperfections, are of course different from those generating the three-deck structure of the Smith–Stewartson boundary layer despite the fact that the equations in the thin near-wall layer are the same. Later it turned out that a similar two-deck structure also arises in problems of flow in tubes and channels with periodic imperfections on the walls. The parameters determining these imperfections differ from the Smith–Stewartson parameters and from the parameters of the two-deck boundary layer in the problem of flow around a plate. But the equations describing the flow in a thin near-wall boundary layer are always the same, and the specific character of the problem manifests itself in the boundary conditions.

In all these problems, as the amplitude of imperfections exceeds a critical value A^*, there arise vortices, which appear and disappear between the humps of the periodic structure. These vortices generate the flow oscillations on the "upper deck" of the double-deck structure. The equations describing these oscillations in the problems of flow in tubes and channels and in the problem of flow around a plate turned out to be significantly different. In the last case, this is a Rayleigh-type equation and the problem of its solvability reduces to an analysis of the eigenvalue problem for a one-dimensional Schröedinger operator on the half-line. It is of interest to note that the existence of a solution of this problem implies the non-uniqueness of the asymptotic solution of the problem of flow around a plate with small periodic imperfections (but, unfortunately, there is no discrete spectrum).

We assume that in all problems listed above, the flow of a viscous incompressible fluid is described by a system consisting of Navier-Stokes equations and the continuity equation ($\mathbf{U} = (u, v)$ is the velocity, p is the pressure):

$$\langle \mathbf{U}, \nabla \rangle \mathbf{U} = -\nabla p + \varepsilon^2 \Delta \mathbf{U}, \quad \langle \nabla, \mathbf{U} \rangle = 0, \tag{1}$$

where ε is a small parameter. We can assume that $\varepsilon = \mathbf{Re}^{-1/2}$, or we assume that this parameter is a small dimensionless viscosity. The boundary conditions in the problem of flow around a plate have the form ($S = \{y = \varepsilon^{4/3}\mu(x, x/\varepsilon)\}$):

$$\mathbf{U}|_S = \mathbf{0}, \ \mathbf{U}|_{y \to \pm\infty} \to (1, 0)^T, \quad \mathbf{U}|_{x \to -\infty} \to (1, 0)^T, \\ \partial_y u|_{y=0, x<0} = 0, \quad v|_{y=0, x<0} = 0. \tag{2}$$

The boundary conditions in problems of flow in a channel and in a tube have the form

$$\mathbf{U}|_S = \mathbf{0}, \tag{2'}$$

where $S = \{y = \varepsilon^{4/5}\mu(x, x/\varepsilon^{2/5}),\ y = l + \varepsilon^{4/5}\mu_1(x, x/\varepsilon^{2/5})\}$ and $S = \{y = l + \varepsilon^{4/5}\mu(x, x/\varepsilon^{2/5})\}$ for the channel and for the tube, and l is the width of the channel or the tube radius. The functions $\mu(x, \xi)$ and $\mu_1(x, \xi)$ are either 2π-periodic in the argument ξ, or they are stabilizing functions, i.e., $\partial_\xi \mu \in \mathcal{S}$ (Schwartz space) are smooth and have zero mean (i.e. $\int_0^{2\pi} \mu(x, \xi)\, d\xi = 0$).

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Expansion shock waves in regularised shallow water theory

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We identify a new type of shock wave by constructing a stationary expansion shock solution of a class of regularised shallow water equations that include the Benjamin-Bona-Mahoney (BBM) and Boussinesq equations. An expansion shock exhibits divergent characteristics, thereby contravening the classical Lax entropy condition. The persistence of the expansion shock in initial value problems is analysed and justified using matched asymptotic expansions and numerical simulations. The expansion shock's existence is traced to the presence of a non-local dispersive term in the governing equation. We establish the algebraic decay of the shock as it is gradually eroded by a simple wave on either side. More generally, we observe a robustness of the expansion shock in the presence of weak dissipation and in simulations of asymmetric initial conditions where a train of solitary waves is shed from one side of the shock.

This is joint work with Mark Hoefer and Michael Shearer [1], [2].

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