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MODELING POLICY RESPONSE TO GLOBAL SYSTEMICALLY IMPORTANT BANKS REGULATION³

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In this paper we elaborate a simple model that allows for the predicting of possible reactions from financial institutions to more stringent regulatory measures introduced by the Basel Committee on Banking Supervision (BCBS) in regard to global systemically important banks (G-SIBs). The context is framed by a 2011 BCBS document that proposes higher capital requirements for global systemically important banks. We attempt to analyze bank interactions in an oligopolistic market that is subject to demand constraints on loan amounts and additional loss absorbency requirements introduced by the regulator. We distinguish between the bank's announced funding cost that determines both the loan amount issued and the market interest rate, and the bank's true funding cost that has a direct impact on retained earnings. We conclude that in a two-stage game both banks will announce the highest funding cost, thus reducing the amount of loans granted (in line with the regulator's objective), but at the expense of a higher cost of borrowing established in the market. If the game is repeated, then both banks also choose lower loan amounts in the periods prior to the last one in which the declared funding cost is the lowest possible. It should be noted that the designated outcome also coincides with the findings of the Monetary Economic Department of the Basel Committee on Banking Supervision.

Keywords: Basel Committee on Banking Supervision, capital adequacy requirements, additional loss absorbency requirements, systemically important banks, game-theoretical approach

JEL Classification: C70, E58, G21

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... an ICAAP should include an analysis of the impact that the actions of a firm's competitors might have on its performance.

BIPRU⁴, 2012

1 Introduction

The world financial crisis that started in 2007-2008 and that had a dramatic impact on both the banking and real sectors has brought about serious questions concerning the regulation of financial institutions. The markets had declined for the better part of 2008, however it is commonly accepted that the financial turmoil was finally unleashed on Monday September 15th, 2008, when Henry Paulson, then the secretary of the US Treasury, announced that Lehman Brothers would be allowed to fail. Before the collapse of Lehman Brothers, the bankruptcy of any large bank was impossible to anticipate. Banks, protected by a concept of being “too big to fail”, were tempted to take on more risk in search of higher margins, thus giving rise to concerns of moral hazard. Nevertheless, the refusal of the US government to bail out the notorious investment bank revealed both that not all losses could be absorbed just by lobbying political decisions and spending taxpayers' contributions, as well as that regulation of systemic risk should be given particular attention.

Two important questions have thus arisen. Firstly, how can those institutions that are primary sources of systemic risk be determined? Secondly, what measures and policies should be applied to those institutions in order to reduce the level of systemic risk and the probability of infecting the whole financial system as a result of difficulties encountered by a particular financial institution? Hereinafter the term “systemic risk” will refer to the commonality of risk exposures of financial institutions and, hence, to the losses brought about by losses and failures of total entities. Institutions that are most likely to foster the realization of systemic risks are considered to be systemically important.

The size of a financial institution is generally considered to be a key indicator of the institution's systemic importance. However, as shown by Xin et al. (2010), it is not always a sufficient criterion.

The methodology proposed by BCBS in its July 2011 consultative document and affirmed in its November 2011 final document is an example of an indicator-based measurement approach. All the indicators are classified into five categories of cross-jurisdictional activity, size, interconnectedness, substitutability, and complexity, with each category being of equal weight (20%). Within each category, each indicator is also given an equal weight (that is, if a category includes two indicators, each will have a weight of 10% in the total score). The size, measured by bank's total exposures, has the highest weight (20%) among all indicators. Other indicators are more or less related to a bank's size; it will be reasonable to predict that larger banks exhibit more complexity compared to smaller banks, for example.

⁴ ICAAP - *Internal Capital Adequacy Assessment Process*. Source: FSA Handbook, Release 121 (January 2012): BIPRU (Prudential sourcebook for Banks, Building Societies and Investment Firms) 2: Capital: Internal capital adequacy standards, point 2.2.40G, page 17

URL : <https://fsahandbook.info/FSA/html/handbook/BIPRU/2/2>

It is important to note that this calculation methodology emphasizes the *relative* value of each indicator, since the score for each indicator is measured as a proportion in the aggregate amount summed across all banks in the sample for a given indicator. (cf. BCBS (2011B)).

The Basel Committee’s consultative document of July 2011 on global systemically important banks (G-SIB) introduces an additional loss absorbency requirement for each category of G-SIBs that are classified into five buckets depending on their relative weight in the financial system.

Table 1. Bucketing approach

Bucket	Score range	Minimum additional absorbency (common equity as a percentage of risk-weighted assets)
5 (empty)	D –	3.5%
4	C – D	2.5%
3	B – C	2.0%
2	A – B	1.5%
1	Cut-off point – A	1.0%

The Basel Committee emphasizes that those requirements are the minimal percentage that should be added to the standard Common Equity Tier 1 requirement. Basel III requires banks to hold tier 1 equity at 7% of risk-weighted assets, made up of a 4.5% minimum and a 2.5% conservation buffer.

The objective of this more stringent regulation of G-SIBs in comparison with other financial institutions is to discourage excessive risk-taking behavior by larger banks in order to reduce “the costs associated with moral hazard” (BCBS (2011B)) and limit “the cross-border negative externalities created by systematically important banks which current regulatory policies do not fully address”. In this context, it appears reasonable to analyze the possible reaction of banks to the proposed measures.

The paper is organized as follows: Section 2 describes research objectives, while Section 3 provides a review of the literature regarding the identification and regulation of SIFIs and the application of game theory modeling in economic decisions. Section 4 then develops the principal model of interaction of the two banks and Section 5 offers a conclusion.

2 Research objectives

The aim of the paper is to describe the envisaged policy response of banks to the implementation of G-SIB regulation. This means finding out the optimal strategy of the bank, considering a rise in profits from the credit expansion and increased capital requirement from G-SIB treatment due to the relatively high credit exposure. Particular attention is given to the conditions of receiving G-SIB status for relatively large banks.⁵

The game-theoretical approach is used to model the choice of one bank regarding an optimal loan to be offered when it is unaware of the other bank's decision. In the event that the simultaneous decision of the banks is to grant loans of equal amounts (either a low or high loan), the bank escapes G-SIB treatment, as in the proposed model each loan constitutes only one half of the overall market. The trade-off for the bank, or the incentive to shift to another

⁵ Though the paper uses the oligopoly framework, as it will be shown later on, a discussion on the interaction of competition and capital regulation is beyond the scope of this study. The issue deserves more focused research and is not the subject of the current paper. For an example of dealing with the issue of competition and capital regulation, please refer to Allen et al (2011).

strategy (i.e. to offer a lower loan amount when the competitor is to offer a higher loan amount), consists of benefits coming from the opportunity not to be definitely treated as a G-SIB in the next period. The proposed model shows that the benefits from following a Pareto-optimal multi-period strategy are not exceeded by the outcomes obtained from asymmetric strategy.

The paper primarily targets the policy-makers, especially those Central Banks that are currently working out approaches to dealing with the global systemically important financial institutions – in particular banks.

3 Literature review

This section provides a brief description of the existing evidence and research on the issues raised in our paper. Firstly, we review different approaches to identifying systemically important financial institutions (SIFI). Secondly, we analyze current views on SIFI regulation, including comments on the recently published BCBS consultative document. Finally, some evidence on game theory application in economic decisions is presented.

There are two broad approaches to defining SIFIs: indicator-based and model-based approaches. The first methodology, based on data from individual financial statements, is preferred by policy makers due to its simplicity and could be either purely qualitative or quantitative. The qualitative indicator-based approach is proposed in IMF/BIS/FSB (2009) and is built on a set of relevant measures that are considered to be determinants of systemic importance, such as size, substitutability, interconnectedness, complexity, and others. The quantitative indicator-based approach consists in building an overall score by weighing indicators related to systemic importance, including size, substitutability, cross-jurisdictional activity, interconnectedness, and complexity (BCBS (2011B)). An often-cited criticism of this approach is the predominance of the size metric, given that other proposed indicators are more or less correlated with it. Hence, the role of other risk factors is deemed to be underestimated. For example, Huang et al (2011), basing their study on multiple regression analysis, note that size, correlation, and probability of default are all important determinants of systemic risk contributions.

The model-based approach can be further broken down into network analysis and market-based methodology. The network analysis consists of contagion effect analysis (cf. Furfine (1999)) and centrality approach (cf. Bech et al (2010), von Peter (2007)). Aikman et al (2009) develop a RAMSI (Risk assessment model for systemic institutions) that controls for network interactions, macro-credit, and income risks and that incorporates the analysis of balance-sheet adjustments to exogenous shocks.

The BCBS consultative document (cf. BCBS (2011B)) proposes using a quantitative indicator-based approach for identifying global systemically important banks and to set higher capital charges for systemically important institutions. This document has received a number of comments from the banking sector and academia⁶, among which the most frequently cited are the following. Firstly, it is noted that BCBS does not stipulate that the size is the defining factor, but, in addition to the direct measure of size (weighting of 20%), a number of other measures are correlated to the total size. The size dimension is, therefore, given significant prominence in the overall score. As shown by Tarashev et al (2010), the measures based only on size will tend to underestimate the systemic importance of large financial institutions. Secondly, the use of relative measures is debatable due to the fact that if all banks in the sample were to reduce their systemic importance to the same extent, then their overall score

⁶ <http://www.bis.org/publ/bcbs201/cacomment.htm>

would experience no change. Thirdly, it is stated that the focus on banks only can promote the growth of a shadow banking system that is not subject to the proposed regulatory measures. Therefore, it is advised that non-bank financial institutions also be considered.

In our paper we use a game-theoretic approach to model the policy response. The application of game theory in economic decisions has been elaborated in numerous research articles. Carraro & Sgobbi (2007) investigate the application of non-cooperative bargaining theory in the analysis of negotiations. Recent developments include the use of psychological issues in game theory and modeling strategic market games. Carrillo & Palfrey (2009) and Aldashev et al (2009) have addressed behavioral game theory and decision-making procedures.

The extension of the current research might be considered within what is currently the most widely used framework of strategic market games that was first discussed in Shapley & Shubik (1969) and Shubik & Tsomocos (1990). A useful overview of the strategic market games approach is introduced in Giraud (2003), while the inclusion of cyclical endowments is analyzed in Bennie (2009).

4 Model description

For the purposes of investigating the potential behavior of financial institutions, we develop a framework of interaction for financial institutions based on game theory applications. For the sake of simplicity, we assume that the banking system is composed of only two banks. Though the concentration measures of their credit portfolios might differ, we suppose that they have equal credit ratings and, therefore, equal default probabilities.

Since the loan offer market is oligopolistic, the banks should take into account each other's behavior when making their own decisions. We use the Cournot duopoly model to find the equilibrium levels of loan amount and market interest rate. The choice of the model was conditioned by the following reasoning. Firstly, given that the banks make decisions simultaneously in our setting, we excluded Stackelberg's leader-follower model. For obvious reasons, the models for heterogeneous goods markets (Gutenberg's and Hotelling's horizontal differentiation model) were not considered given the nature of the loan market. We then have to decide whether the banks should compete on prices (Bertrand duopoly) or loan amounts (Cournot duopoly). The Bertrand model does not suit our framework since, due to price wars between the two duopolists, the equilibrium price in the model is established at a level of marginal costs that is supposed to be identical for both players. However, the objective of our research is to find the relation between the funding costs and amount of loans granted. Thus, if one of the banks has higher funding costs, then it will be forced to establish a higher loan interest rate, so that all the clients will choose the rival bank, leading to a loan amount of zero issued by the first bank and its subsequent exit from the market. This scenario seems highly improbable, even if we would intuitively tend to assume that banks would compete on loan interest rates. Thus, we suppose that the two banks will decide on the loan amount to issue given the market interest rate as in a Cournot duopoly.

Two types of interactions are presented in this paper. First, we analyze the strategies of the two banks in a two-period game. Second, we consider the case where the game is extended over a larger number of periods.

4.1 Interaction of Cournot duopolists in a two-period game

4.1.1 Key model assumptions

The game is modeled given the following assumptions:

- The rationality of all players and the rules of the game are commonly known.
- The banking system is presented by two banks that are initially treated as equally important by the regulator, but are not yet treated like G-SIBs with extra-capital requirements.
- Banks are faced with a limited demand for loans that is defined by a linear decreasing function of the market interest rate: $r_M = a - bL$ (where r_M is the market interest rate, L is the market quantity of loan demanded, a and b are demand function parameters such as $a \in R, b \in R; a > 0, b > 0$). Each period the players choose a loan amount conditioned by growth constraints and competition. The banks adopt trigger strategies: if Bank 1 issues a higher loan amount in period 1 and is consequently considered systemically important, then in the next period it is faced with a higher capital adequacy ratio $CAR+CS$ and thus higher funding cost r_A (cf. Lemma 1 for the relationship between CAR and r_A). In the next period it can grant a lower loan amount based on its higher CAR . The trigger happens if in the next period this bank chooses an announced funding cost such that it offers a lower loan amount than the other bank and stops being treated as a G-SIB. Inversely, the second bank becomes a G-SIB in the second period and in order to activate the trigger once again it has to choose an announced funding cost higher than the competitor to leave the G-SIB category.
- The game is a two-period game.
- “Penalties”, if any, are applied during the second period, depending on the outcomes of the first period, i.e. G-SIB special treatment.
- At $t=1$ the regulator observes the decisions made by the two banks at $t=0$. If the amounts are equal then no additional requirement applies. If one bank issued a larger loan than the other, then the latter is considered as systemically important by size criterion. Hence, there are three focal points: $t=0$ consists of decision-making for the first period; $t=1$ includes earning profits for the period from $t=0$ to $t=1$, and decision-making by the regulator and by the banks for the second period; $t=2$ sees earning profits for the period from $t=1$ to $t=2$.
- Objective functions: the banks seek to maximize their utility function U_i , which is the present value of total profit for two periods. The regulator needs to limit the risk exposure, i. e. the amount of issued loan in the current framework, for a relatively larger bank.

$$U_i = \frac{\pi_{1,i}}{1+\rho} + \frac{\pi_{2,i}}{(1+\rho)^2},$$

where $\pi_{j,i}$ stands for the profit realized by Bank i in period j ,

ρ denotes the discount rate (for reasons of simplicity, we assume that the banks do not have time preferences, meaning that the discount rate is set at zero).

Lemma 1. *Higher capital adequacy requirements result in higher funding costs.*

Let us denote A as the bank’s assets. The liability side is composed of two sources: external financing F , and the bank’s equity K (we denote the bank’s equity by K in line with the Basel Committee conventions). Let r_A be the funding cost, let r_F stand for the cost of external financing, and let r_K be the cost of internal financing (we make a realistic assumption that internal financing is more expensive than external financing: $r_K > r_F$).

The funding cost is determined by:

$$r_A \cdot A = r_F \cdot F + r_K \cdot K \quad (1)$$

From the accounting equation, it follows that

$$A = F + K \quad (2)$$

If the bank does not hold additional reserves, than the capital adequacy ratio is equal to:

$$CAR = \frac{K}{A} \quad (3)$$

By combining (2) and (3) into (1), and by dividing both sides of the equation by A , we have:

$$r_A = r_F \cdot (1 - CAR) + r_K \cdot CAR,$$

or in other terms,

$$r_A = r_F + (r_K - r_F) \cdot CAR.$$

Given that r_K exceeds r_F , r_A is an increasing function of CAR , so higher capital requirements lead to an increase in the funding costs.

4.1.2 Equilibrium in the Cournot duopoly in the loan market

The loan supply is provided by two banks i and j , and the **true** funding cost (equivalent to marginal costs in a classical Cournot model for manufacturing firms) are equal to c_i and c_j , respectively. The funding cost announced by each bank (and, consequently, assumed by the other bank) may be the same as its true funding cost or may differ from it, depending on the strategy pursued. We will denote the **announced** funding costs c_i^* , c_j^* for banks i and j , respectively.

The idea to separate *true* and *announced* funding costs comes from the internal capital management process, which consists of linking the capital adequacy held by the bank to its pricing decisions. Pillar II of Basel II enables banks to use metrics to allocate internal capital and consequently price the products given parameters different from the ones used to calculate overall capital adequacy (e.g. confidence level for credit risk can be used different from 99.9% for internal capital allocation purposes). This explains the difference in *true* (*external*) and *announced* (*internal*) funding costs.

It is necessary to mention that the banks being assumed to be rational might not form proper beliefs about the announced funding cost because they lack all the relevant information on the internal procedures to allocate capital and make risk-adjusted performance evaluation, for example (although some pieces of this information are disclosed in annual reports).

Concerning the modeling part, we argue the following. The banks maximize a two-period profit (as stated within the objectives part), having obtained the price and volume parameters from the one-time interaction. The outline of how one-time interaction parameters are estimated is provided below.

The banks will issue the amount of loans corresponding to their announced funding cost. Let us denote P_i and P_j as the assumed profit of banks i and j , respectively. The banks make decisions simultaneously, and take the output of the other bank as given, so that the optimization problem is the following:

$$\max_{L_i} P_i = \max_{L_i} [(r_M - c_i^*) \cdot L_i] = \max_{L_i} [(a - b(L_i + L_j) - c_i^*) \cdot L_i] \quad (4)$$

$$\max_{L_j} P_j = \max_{L_j} [(r_M - c_j^*) \cdot L_j] = \max_{L_j} [(a - b(L_i + L_j) - c_j^*) \cdot L_j] \quad (5)$$

From (1) and (2) we obtain the following best response functions:

$$L_i = \frac{a - c_i^*}{2b} - \frac{L_j}{2} \quad (6)$$

$$L_j = \frac{a - c_j^*}{2b} - \frac{L_i}{2} \quad (7)$$

From (6) and (7) we have the following symmetric Nash equilibrium:

$$L_i = \frac{a - 2c_i^* + c_j^*}{3b} \quad (8)$$

$$L_j = \frac{a - 2c_j^* + c_i^*}{3b} \quad (9)$$

The total loan supply is thus given by:

$$L = L_i + L_j = \frac{a - c_i^* - c_j^*}{3b}$$

And the market interest rate is established at the following level:

$$r_M = \frac{a + c_i^* + c_j^*}{3}$$

Consequently, the loan amounts and the market interest rates are determined by the announced funding costs c_i^* and c_j^* . However, the true retained earnings (π) are also functions of the true funding cost since they are determined by:

$$\pi_i = (r_M - c_i) \cdot L_i$$

$$\pi_j = (r_M - c_j) \cdot L_j$$

The banks' target function is the *true* retained earnings (cf. the discussion of the choice of the capital structure in section 4.1.3).

The following table summarizes the parameters of the Nash equilibrium in a Cournot's duopoly model of the loan market.

Table 2. Nash equilibrium parameters in a Cournot's duopoly model of the loan market.

	Bank i	Bank j
Loan amount (L)	$\frac{a - 2c_i^* + c_j^*}{3b}$	$\frac{a - 2c_j^* + c_i^*}{3b}$
Lending rate (r_M)	$\frac{a + c_i^* + c_j^*}{3}$	$\frac{a + c_i^* + c_j^*}{3}$
True retained earnings (π)	$\frac{(a + c_i^* + c_j^* - 3c_i)(a - 2c_i^* + c_j^*)}{9b}$	$\frac{(a + c_i^* + c_j^* - 3c_j)(a - 2c_j^* + c_i^*)}{9b}$

4.1.3 Choice of the capital structure

In our framework, the banks' target functions are retained earnings. Although a rigorous proof of a positive relationship between retained earnings and the firm's value is beyond the scope of this paper, we provide a few arguments for accumulating retained earnings for the purpose of enhancing value.

In Lemma 1 we determine that external financing is less costly than internal financing ($r_F < r_K$) due to the priority claims of debt holders. So, assuming that the costs are independent of financial leverage ($r_F = \text{const}$, $r_K = \text{const}$), the weighted average cost of capital would be lower (and, consequently, the value of the firm would be higher) if the bank were entirely financed by borrowings. However, in this case, we completely ignore financial distress costs and regulatory requirements.

The Modigliani-Miller equivalence theorem (Modigliani (1958)) about the irrelevance of the capital structure is not applicable in our case due to a number of restrictive unrealistic assumptions, such as an unlimited capacity to increase indebtedness and the absence of financial distress costs.

The trade-off theory of capital structure is based on the belief that an optimal debt and equity mix exists that permits the maximization of the firm's value by balancing the costs (financial distress costs) and benefits (tax shield) of the debt. Given that the objective of our research is not to estimate the tax benefits and probability of default concerning the use of external financing, we do not use it in our study.

The pecking order theory was first suggested by Donaldson in 1961 and was modified by Stewart C. Myers and Nicolas Majluf in 1984. Its proponents argue that the capital structure choice influences the companies' valuation due to a signaling effect. According to this theory, internal sources of financing (retained earnings / cash) are preferred to external sources of financing (debt => convertible bonds => preferred stock => common stock). The proof is based on the asymmetry of information. It is believed that managers are better informed than outside investors and prioritize their sources of financing according to the principle of least effort. Hence, internal funds are used first, then debt and then new equity (if the company prefers debt to equity it believes that its shares are undervalued, otherwise it would prefer stock issuance). According to this argument, companies would likely accumulate retained earnings in order to rely on them in case of necessity.

The pecking order theory is also appealing in our setting given that the banking industry is heavily regulated. The banks have to maintain a determined percentage of their funds in the form of equity. Taking into account the fact that the regulator imposes higher capital requirements on systemically important banks, the banks face the risk of being obliged to reinforce their equity. Issuing new shares is both costly and sends a negative signal to investors. Consequently, retained earnings, making up part of the bank's equity, provide a necessary cushion.

Besides purely regulatory reasons, there are also economic reasons. Accumulating retained earnings permit the company to have a necessary buffer to ensure a stable dividend stream that ameliorates the market perception of the company.

4.1.4 Description of the game

There are two players (two banks in our case), who make decisions simultaneously, and one regulator (equivalent to "nature" in general games).

At $t=0$ the banks make a simultaneous choice regarding the amount of loan to issue. This choice is conditioned by the capital requirements in place (set initially at $CAR = c$), demand constraints and the rival's best-response function. Since no capital surcharges apply to either bank initially, the banks can declare either their true funding cost linked to c , or a higher funding cost based on $c + \delta$ (where $\delta > 0$).

Lemma 2. *A higher announced cost of capital is associated with a lower loan amount issued.*

As shown in section 4.1.2, the equilibrium loan amounts in the Cournot model are determined by equations (8) and (9):

$$L_i = \frac{a-2c_i^*+c_j^*}{3b} \quad (8)$$

$$L_j = \frac{a-2c_j^*+c_i^*}{3b} \quad (9)$$

where c_i^* and c_j^* are the announced funding costs by banks i and j , respectively.

Let $c_i^* > c_j^*$. Then $(a - 2c_i^* + c_j^*) < (a - 2c_j^* + c_i^*)$, which implies $L_i < L_j$.

Lemma 2 provides some evidence of the banks' incentives to claim a funding cost that is higher than their true funding cost. Since a higher funding cost results in a lower loan amount (observed by the regulator), the bank's relative size will be lower and consequently will not face capital surcharges in the second period.

At the end of the first period the regulator observes the decisions made by the two players. If both banks announce the same funding cost (both declare either c or $c + \delta$) then, due to the symmetric nature of Nash equilibrium in the Cournot model both will issue the same loan amount and neither bank will be considered as systemically important.⁷ On the other hand, if bank i declares a funding cost of c while bank j declares a funding cost of $c + \delta$, then the loan amount issued by bank i will be higher than issued by bank j ($L_i > L_j$), meaning that bank i will have to abide by higher capital requirements and its true funding cost will be equal to $c + \varepsilon$ (where $\varepsilon > 0$).

In the second period, if there are no systemically important banks, then the choices available for the two banks are the same as in the first period (either c or $c + \delta$). However, if bank i is considered systemically important, then it must comply with higher capital requirements. Thus, bank i can announce either its new true funding cost $c + \varepsilon$ or even higher funding cost $c + \delta + \varepsilon$ (the role of a trigger) in order to impose a penalty on the bank j . Bank j , whose true funding cost remained unchanged at a level of c will take into account the new true funding cost of bank i . Consequently, bank j has the same choices as bank i , but, given that its true funding cost is lower, it can announce a lower funding cost $c + \varepsilon - \delta$ (we assume that $0 < \delta \leq \varepsilon$, to include any costs between c (including) and $c + \varepsilon$ (excluding) in this option).

The extensive form of the game is presented in Figure 1. Points A, B, C, D , and Q, R, S, T reflect the event where the banks are treated equally after the first period, while the other points correspond to the event where one of the banks is considered systemically important after the first period (Bank 1 for points E, F, G, H, I, J ; Bank 2 for points K, L, M, N, O, P). The dotted line on the graph represents the information constraint that Bank 2 has available. While Bank 2 makes a move, it is not aware of the decision taken by Bank 1 in the corresponding decision point. Due to the information set, Bank 1 does not know Bank's 2 moves either.

For each case, each player's payoff after two periods is calculated using the formulae in Table 2. The payoff, or utility, of a bank is calculated as the sum of present values of the true retained earnings realized in each period. In our model, we assume a zero discount rate, so the final payoff for each bank is the sum of profits from the first and the second period.

Table 3 resumes the payoffs at each point.

⁷ This assumption is quite realistic since in the proposed regulatory measures the focus is made on the *relative* size of the bank, as "the score of each bank is calculated as the amount of total exposures [loans granted, in our case] divided by the sum of total exposure for all banks in the sample" (cf. BCBS (2011), p. 7; paragraph 28).

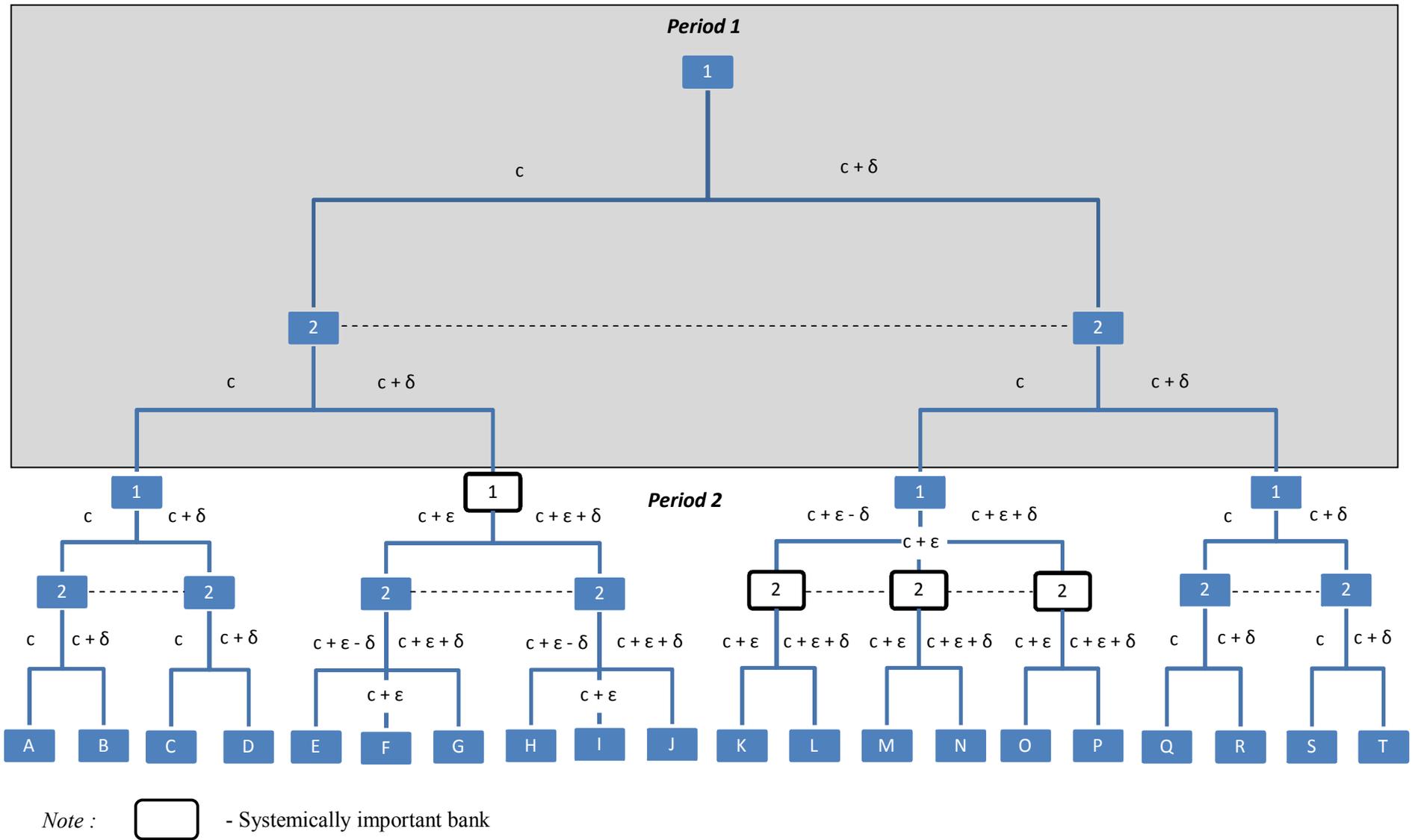


Figure 1. Extensive form of the loan-choice game

Table 3. Final payoffs in a loan-choice game of two initially equal banks with a zero discount rate

Outcome	Bank's 1 Payoff		Bank's 2 Payoff	
	Period 1	Period 2	Period 1	Period 2
A	$(a - c)^2$	$(a - c)^2$	$(a - c)^2$	$(a - c)^2$
B		$(a - c + \delta)^2$		$(a - c + \delta)(a - c - 2\delta)$
C		$(a - c + \delta)(a - c - 2\delta)$		$(a - c + \delta)^2$
D		$(a - c + 2\delta)(a - c - \delta)$		$(a - c + 2\delta)(a - c - \delta)$
E	$(a - c + \delta)^2$	$(a - c - \varepsilon - \delta)^2$	$(a - c + \delta) \cdot (a - c - 2\delta)$	$(a - c + 2\varepsilon - \delta)(a - c - \varepsilon + 2\delta)$
F		$(a - c - \varepsilon)^2$		$(a - c - 2\varepsilon)(a - c - \varepsilon)$
G		$(a - c - \varepsilon + \delta)^2$		$(a - c + 2\varepsilon + \delta)(a - c - \varepsilon - 2\delta)$
H		$(a - c - \varepsilon)(a - c - \varepsilon - 3\delta)$		$(a - c + 2\varepsilon)(a - c - \varepsilon + 3\delta)$
I		$(a - c - \varepsilon + \delta)(a - c - \varepsilon - 2\delta)$		$(a - c + 2\varepsilon + \delta)(a - c - \varepsilon + \delta)$
J		$(a - c - \varepsilon + 2\delta)(a - c - \varepsilon - \delta)$		$(a - c + 2\varepsilon + 2\delta)(a - c - \varepsilon - \delta)$
K	$(a - c + \delta) \cdot (a - c - 2\delta)$	$(a - c + 2\varepsilon - \delta)(a - c - \varepsilon + 2\delta)$	$(a - c + \delta)^2$	$(a - c - \varepsilon - \delta)^2$
L		$(a - c + 2\varepsilon)(a - c - \varepsilon + 3\delta)$		$(a - c - \varepsilon)(a - c - \varepsilon - 3\delta)$
M		$(a - c + 2\varepsilon)(a - c - \varepsilon)$		$(a - c - \varepsilon)^2$
N		$(a - c + 2\varepsilon + \delta)(a - c - \varepsilon + \delta)$		$(a - c - \varepsilon + \delta)(a - c - \varepsilon - 2\delta)$
O		$(a - c + 2\varepsilon + \delta)(a - c - \varepsilon - 2\delta)$		$(a - c - \varepsilon + \delta)^2$
P		$(a - c + 2\varepsilon + 2\delta)(a - c - \varepsilon - \delta)$		$(a - c - \varepsilon + 2\delta)(a - c - \varepsilon - \delta)$

Outcome	Bank's 1 Payoff		Bank's 2 Payoff	
	Period 1	Period 2	Period 1	Period 2
Q	$(a - c + 2\delta) \cdot$ $(a - c - \delta)$	$(a - c)^2$	$(a - c + 2\delta) \cdot$ $(a - c - \delta)$	$(a - c)^2$
R		$(a - c + \delta)^2$		$(a - c + \delta)(a - c - 2\delta)$
S		$(a - c + \delta)(a - c - 2\delta)$		$(a - c + \delta)^2$
T		$(a - c + 2\delta)(a - c - \delta)$		$(a - c + 2\delta)(a - c - \delta)$

Note: payoffs in all cells should be divided by $9b$ (where b is the absolute value of the slope of the demand curve for loans. Since we are primarily interested in comparing the payoffs across the outcomes, we can ignore this term because it plays no role in ordinal comparisons.

We impose the following intuitive condition: Each term used in calculating the retained earnings for each period is positive, considering the fact that retained earnings are the product of interest margin and loan amount issued. The loan amount cannot be negative (and, as we believe that neither bank exits the market, we introduce a stricter condition: the loan amount is positive). The interest margin should also be positive, otherwise the bank that suffered losses would not stay in the sector in the long run.

4.1.5 Nash equilibrium in a two-period game

We then need to rank those profits in order to determine which strategies the two players will pursue. Writing down the strategy space for multi-period games is space-demanding, even if the game is repeated over just two rounds. To present the game in a table form, it is necessary to draw a table $(3 \cdot 2^3) \times (3^2 \cdot 2^7)$ as the first bank has four decision-making points (out of which one is attributed with three possible choices) and the second has nine (out of which two are attributed with three possible choices).

Figure 2 presents the strategic form of a two-period game. Bank choices (*announced* funding costs) are denoted by c , $c + \delta$, $c + \varepsilon$, $c + \varepsilon + \delta$, and $c + \varepsilon - \delta$. The rows represent the decisions made by Bank 1, while the columns represent the decisions made by Bank 2. The central table shows the possible outcomes after the choice made by the two banks in the first period. The surrounding tables contain the information about the possible outcomes at the end of the second period, conditioned by the choice from the first period.

Lemma 3. *In period 2 the absence of systemically important institutions in the banking system results in a higher payoff for each bank if it declares a lower funding cost, no matter what the decision of the other bank is.*

Let us determine the dominant strategies in the matrix 2 (outcomes A , B , C , and D). If Bank 1 announces the funding cost equal to c , then the best response of Bank 2 will be to announce the funding cost c , since

$$\pi_2^A > \pi_2^B$$

We should compare $(a - c)^2$ and $(a - c + \delta)(a - c - 2\delta)$. The difference $\pi_2^A - \pi_2^B$ is equal to $\delta(a - c + 2\delta) > 0$ (given that $(a - c) > 0$ and $\delta > 0$).

If Bank 1 announces a funding cost equal to $c + \delta$, then the best response of Bank 2 will be to announce the funding cost c since

$$\pi_2^C > \pi_2^D$$

We should also compare $(a - c + \delta)^2$ and $(a - c + 2\delta)(a - c - \delta)$. The difference $\pi_2^C - \pi_2^D$ is equal to $\delta(a - c + 3\delta) > 0$ (given that $(a - c) > 0$ and $\delta > 0$).

From the previous relations, it follows that Bank 2 will choose c independently of the choice of Bank 1.

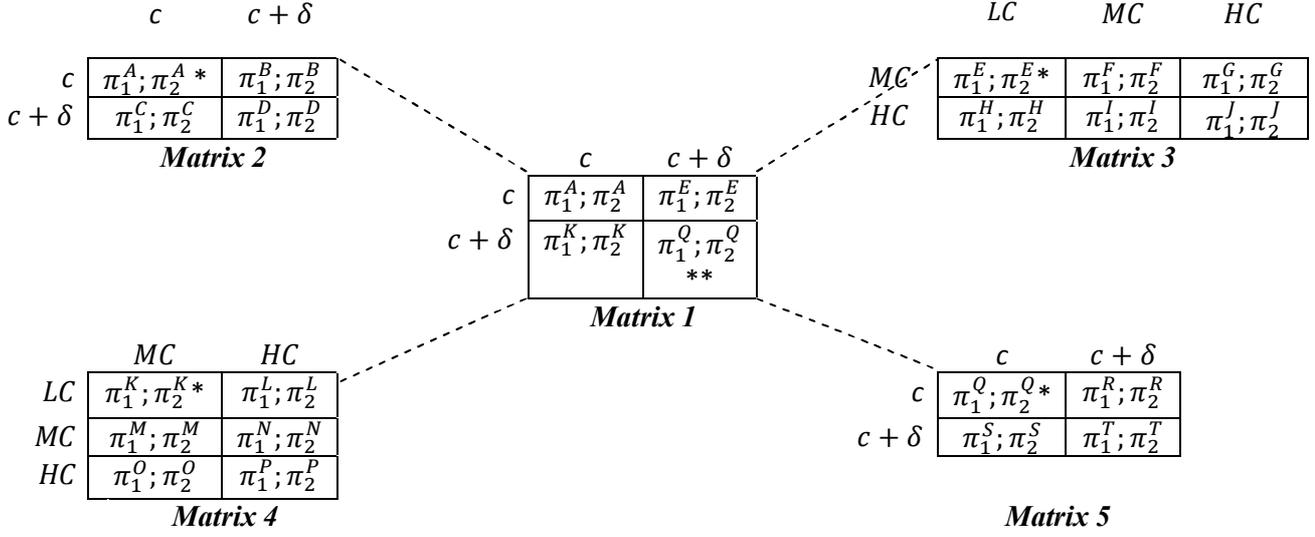
Symmetrically, if Bank 2 announces a funding cost equal to c , then the best response of Bank 1 will be to announce the funding cost c , since

$$\pi_1^A > \pi_1^C$$

Due to the symmetric nature of the banks' payoffs, the comparison is the same as for π_2^A vs. π_2^B .

If Bank 2 announces a funding cost equal to $c + \delta$, then the best response of Bank 2 will be to announce a funding cost c , since

Figure 2. Strategic form of a loan-choice game of two initially equal banks in a two-period game



Note : This figure presents the strategic form of a two-period repeated game between two banks of initially equal sizes. If at $t=0$ one bank decides to announce higher funding costs ($c + \delta$), and thus issue a lower loan than the other, then at $t=1$ the former is treated as systemically important by the regulator and should adhere to tougher capital requirements in the second period.

The banks' choices (*announced* funding costs) are denoted by c , $c + \delta$, LC , MC , and HC , where LC , MC , and HC stand for $c + \varepsilon$, $c + \varepsilon + \delta$, $c + \varepsilon - \delta$, respectively. The rows represent the decisions made by Bank 1, while the columns represent the decisions made by Bank 2. By convention, the first number stands for Bank's 1 payoff, and the second number denotes Bank's 2 payoff.

The central table shows the possible outcomes after the choice made by the two banks in the first period. The surrounding tables contain the information about the possible outcomes at the end of the second period, conditioned by the choice from the first period.

The Nash equilibrium in sub-games is denoted ($*$), while ($**$) stands for the Nash equilibrium in the game.

$$\pi_1^B > \pi_1^D$$

Due to the symmetric nature of the banks' payoffs, the comparison is the same as for π_2^C vs. π_2^D .

From the previous relations, it follows that Bank 1 will choose c independently of the choice of Bank 2.

The dominant strategies in matrix 5 (outcomes Q , R , S , and T) are the same as in matrix 2 due to the symmetric nature of the payoffs in period 2 for matrices 2 and 5.

Lemma 4. *In period 2 if Bank i is considered as systemically important, then each bank has a higher payoff if it declares the lowest possible funding cost, no matter what the decision of the other bank is.*

Let us now determine the dominant strategies in the matrix 3 (outcomes E , F , G , H , I , and J). Bank 1 is systemically important and has to comply with higher capital adequacy requirements. If Bank 1 announces a funding cost equal to $c + \varepsilon$, then the best response of Bank 2 will be to announce a funding cost $c + \varepsilon - \delta$, since

$$(\pi_2^E > \pi_2^F) \cup (\pi_2^E > \pi_2^G)$$

The difference $\pi_2^E - \pi_2^F$ is equal to $\delta(a - c + 5\varepsilon - \delta) > 0$ (given that $(a - c - \delta) > 0$ to assure positive loan amounts in outcomes D and T).

The difference $\pi_2^E - \pi_2^G$ is equal to $2\delta(a - c + \varepsilon) > 0$ (given that $(a - c) > 0, \delta > 0, \varepsilon > 0$).

If Bank 1 announces a funding cost equal to $c + \varepsilon + \delta$, then the best response of Bank 2 will be to announce the funding cost $c + \varepsilon - \delta$, since

$$(\pi_2^H > \pi_2^I) \cup (\pi_2^H > \pi_2^J)$$

The difference $\pi_2^H - \pi_2^I$ is equal to $\delta(a - c + 5\varepsilon + 3\delta) > 0$ (given that $(a - c) > 0, \delta > 0, \varepsilon > 0$).

The difference $\pi_2^H - \pi_2^J$ is equal to $\delta(3a - 3c + 5\varepsilon - \delta) > 0$ (given that $(a - c - \delta) > 0$ to assure positive loan amounts in the outcomes D and T ; $\varepsilon > 0$).

From the previous relations, it follows that Bank 2 will choose $c + \varepsilon - \delta$ independently of the choice of Bank 1.

If Bank 2 announces a funding cost equal to $c + \varepsilon - \delta$, then the best response of Bank 1 will be to announce the funding cost $c + \varepsilon$, since

$$\pi_1^E > \pi_1^H$$

The difference $\pi_1^E - \pi_1^H$ is equal to $[\delta(a - c) + \varepsilon(\varepsilon - \delta) + \delta^2] > 0$ (given that $(a - c) > 0, \varepsilon > \delta$).

If Bank 2 chooses $c + \varepsilon$, the best response of Bank 1 is $c + \varepsilon$, since

$$\pi_1^F > \pi_1^I$$

The difference $\pi_1^F - \pi_1^I$ is equal to $\delta(a + c - \varepsilon + 2\delta) > 0$, since $(a - c - \varepsilon + \delta) > 0$ (to assure positive loan amounts in the second period in outcomes G and I).

If Bank 2 chooses $c + \varepsilon + \delta$, the best response of Bank 1 is $c + \varepsilon$, since

$$\pi_1^G > \pi_1^J$$

The difference $\pi_1^G - \pi_1^J$ is equal to $\delta(a - c + 3\delta) > 0$, since $(a - c) > 0$.

From the previous relations, it follows that Bank 1 will choose “ $c + \varepsilon$ ” independently of the choice of Bank 2.

The dominant strategies in matrix 4 (outcomes K, L, M, N, O , and P) are the same as in the matrix 3 due to the symmetric nature of the payoffs for period 2 in matrices 3 and 4.

Theorem 1. *In period 2, a strategy of announcing the lowest possible funding cost is strictly dominant for each bank.*

For the proof see Lemmas 1 and 2.

Theorem 2. *In period 1, a strategy of announcing the highest funding cost is strictly dominant for each bank.*

If Bank 1 announces a funding cost equal to c , then the best response of Bank 2 will be to announce the funding cost $c + \delta$, since

$$\pi_2^A < \pi_2^E$$

The difference $\pi_2^A - \pi_2^E$ is equal to $\delta(4\delta - 5\varepsilon) - \varepsilon(a - c - 2\varepsilon) < 0$ since $\delta(4\delta - 5\varepsilon) < 0$ (given that δ and $\delta < \varepsilon$) and $\varepsilon(a - c - 2\varepsilon) > 0$ (given that $(a - c - \varepsilon - \delta) > 0$ to assure positive loan amounts in outcomes J and P and it should hold $\forall \delta$, including $\delta_{max} = \varepsilon$).

If Bank 1 announces a funding cost equal to $c + \delta$, then the best response of Bank 2 will be to announce a funding cost $c + \delta$ since

$$\pi_2^K < \pi_2^Q$$

The difference $\pi_2^K - \pi_2^Q$ is equal to $-\delta(a - c - 4\delta) - \varepsilon(\varepsilon - 2a + 2c + 2\delta) < 0$, since $-\delta(a - c - 4\delta) < 0$ (given that $(a - c - \varepsilon - 3\delta) > 0$ to assure positive loan amounts in outcomes H and L and it should hold $\forall \delta$, including $\delta_{max} = \varepsilon$); and $\varepsilon - 2a + 2c + 2\delta < 0$ (given that $(a - c - \varepsilon - \delta) > 0$ to assure positive loan amounts in outcomes J and P).

From the previous relations, it follows that Bank 2 will choose $c + \delta$ independently of the choice of Bank 2.

Symmetrically, if Bank 2 announces a funding cost equal to c , then the best response of Bank 1 will be to announce a funding cost $c + \delta$, since

$$\pi_1^A < \pi_1^K$$

The comparison of π_1^A and π_1^K is the same as for π_2^A vs. π_2^E

If Bank 2 chooses $c + \delta$, the best response of Bank 1 is $c + \delta$:

$$\pi_1^E < \pi_1^Q$$

The comparison of π_1^E and π_1^Q is the same as for π_2^K vs. π_2^Q .

From the previous relations, it follows that Bank 1 will choose $c + \delta$ independently of the choice of Bank 1.

4.2 Interaction of Cournot duopolists in a multi-period repeated game

This section provides some considerations on the possible extension of the game. Let us suppose that after the second period the banks repeat the game (with additional loss absorbcency requirements in place). The same choices are available, meaning that in the absence of systemically important institutions the banks can choose between c and $c + \delta$. If the bank is systemically important it can choose between $c + \varepsilon$ and $c + \varepsilon + \delta$, while the other bank has the options of $c + \varepsilon - \delta$, $c + \varepsilon$, and $c + \varepsilon + \delta$.

Let us analyze what happens in the third period. The optimal strategies in the third period are defined according to Theorem 1, such that for each decision point the sum of the payoffs in periods 1 and 2 is the same and the ranking of the payoffs in period 3 is the same as in the last period of the two-stage game. Table 4 summarizes these points (the first strategy is the response of Bank 1, the second is the response of Bank 2).

Table 4. Optimal strategies in period 3.

Outcome	Systemically important	Optimal strategy in period 3
A, D, F, J, M, P, Q, T	No	$(c; c)$
B, G, K, L, N, R	Bank 1	$(c + \varepsilon; c + \varepsilon - \delta)$
C, E, H, I, O, S	Bank 2	$(c + \varepsilon - \delta; c + \varepsilon)$

We then proceed to determine optimal strategies in period 2 (see Figure 2, matrices 2, 3, 4, and 5). We show (cf. Appendix) that if the game is not terminated in period 2 then the best response of each bank will be to announce the highest possible funding cost. In other words, the optimal strategy in matrices 2 and 5 would be $(c + \delta; c + \delta)$ (i. e. the Nash equilibrium is at the points $(\pi_1^D; \pi_2^D)$ and $(\pi_1^T; \pi_2^T)$, respectively); in matrices 3 and 4 (Figure 2) - $(c + \varepsilon + \delta; c + \varepsilon + \delta)$ (i. e. the Nash equilibrium is at the points $(\pi_1^J; \pi_2^J)$ and $(\pi_1^P; \pi_2^P)$, respectively).

In period 1 the Nash equilibrium establishes in $(c + \delta; c + \delta)$ (cf. Appendix), meaning that the banks will announce the highest possible funding costs.

5 Conclusions

The simple model elaborated in this paper is the first attempt to analyze the possible reactions of the banking system to the Basel Committee's recently published consultative document on the introduction of additional loss absorbency requirements for systemically important banks. The decision-maker should first and foremost be assured that the proposed measures will be efficient in terms of the desired outcome.

In this paper we develop a game-theoretical framework where the banking system is represented by two banks of either equal or different sizes. Based on the decisions made by the banks in the first period, the regulator determines whether any of the banks can be considered as systemically important depending on the relative size of the issued loan and, therefore, whether more stringent capital requirements should be imposed on it in the second period.

The banks face a loan-demand constraint. The Cournot model is used to construct the response functions and to determine both the loan amounts granted and the market interest rate. The retained earnings (the objective function) are calculated as the interest margin (difference between the market interest rate and the true funding cost) times the loan amount issued.

In the case introducing capital surcharges to systemically important banks, the regulator's objective is to limit the risk-taking by large banks. We have shown that in oligopolistic markets this measure effectively leads to a lower loan amount issued by all players, which results in a higher market interest rate paid by borrowers. Thus, reducing the total amount of funding available in the economy and increasing the cost of borrowing for clients together attain the regulator's objective. It is important to stress that these findings fully coincide with the ones announced by the Monetary Economic Department of the Basel Committee on October 11, 2011 (BCBS (2011A)). As the report's executive summary states, "The costs of the G-SIB proposals stem from the adverse impact on economic activity, especially investment, of banks' actions to increase interest rate spreads and cut lending in order to build up their capital buffers."

Thus, as the research findings suggest, the optimal (dominating) policy response of the banks for implementing G-SIB regulation is to choose symmetric strategies limiting the total loan amount offered and increasing the cost of borrowing.

It is not reasonable to control for the banks' reputation in the framework described above, since it is not possible to prove whether the quantity of loans granted was the result of a conscious choice or whether it was determined by the economic environment.

For simplicity reasons, the current study does not focus on time patterns in demand for loans. The model would be more comprehensive if periods of higher and lower demand for loans were considered. The procyclical effects in the banking sector are beyond the scope of this study, but might be an interesting issue for further research.

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Appendix

a. Ranking payoffs in matrices 2 and 5 in a three-stage game

Given that the payoffs in period 1 are the same, it is sufficient to compare the sum of the payoffs in periods 2 and 3 (cf. Figure 1 for the list of outcomes).

Outcome	Bank 1		Bank 2	
	Period 2	Period 3	Period 2	Period 3
A	$(a - c)^2$	$(a - c)^2$	$(a - c)^2$	$(a - c)^2$
B	$(a - c + \delta)^2$	$(a - c - \varepsilon - \delta)^2$	$(a - c + \delta) \cdot (a - c - 2\delta)$	$(a - c + 2\varepsilon - \delta) \cdot (a - c - \varepsilon + 2\delta)$
C	$(a - c + \delta) \cdot (a - c - 2\delta)$	$(a - c + 2\varepsilon - \delta) \cdot (a - c - \varepsilon + 2\delta)$	$(a - c + \delta)^2$	$(a - c - \varepsilon - \delta)^2$
D	$(a - c + 2\delta) \cdot (a - c - \delta)$	$(a - c)^2$	$(a - c + 2\delta) \cdot (a - c - \delta)$	$(a - c)^2$

The ranking of the payoffs in matrix 2 is the following:

$$\pi_2^A < \pi_2^B \text{ (respective payoffs are the same as } \pi_2^A \text{ vs. } \pi_2^E \text{ in a two-stage game).}$$

$$\pi_2^C < \pi_2^D \text{ (respective payoffs are the same as } \pi_2^K \text{ vs. } \pi_2^Q \text{ in a two-stage game)}$$

Consequently, the dominant strategy of Bank 2 is $c + \delta$ in period 2. Due to the symmetric nature of the two banks' payoffs, the dominant strategy of Bank 1 is also $c + \delta$ in period 2, and the sub-game Nash equilibrium establishes in $(\pi_1^D; \pi_2^D)$.

Matrix 5 is symmetric to matrix 2, and the dominant strategies of the players are the same, such that the sub-game Nash equilibrium establishes in $(\pi_1^T; \pi_2^T)$.

b. Ranking payoffs in matrices 3 and 4 in a three-stage game

Given that the payoffs in period 1 are the same, it is sufficient to compare the sum of the payoffs in periods 2 and 3 (cf. Figure 1 for the list of outcomes).

Out- come	Bank 1		Bank 2	
	Period 2	Period 3	Period 2	Period 3
E	$(a - c - \varepsilon - \delta)^2$	$(a - c + 2\varepsilon - \delta) \cdot (a - c - \varepsilon + 2\delta)$	$(a - c + 2\varepsilon - \delta) \cdot (a - c - \varepsilon + 2\delta)$	$(a - c - \varepsilon - \delta)^2$
F	$(a - c - \varepsilon)^2$	$(a - c)^2$	$(a - c - 2\varepsilon) \cdot (a - c)$	$(a - c)^2$

Out- come	Bank 1		Bank 2	
	Period 2	Period 3	Period 2	Period 3
			$(a - c - \varepsilon)$	
G	$(a - c - \varepsilon + \delta)^2$	$(a - c - \varepsilon - \delta)^2$	$(a - c + 2\varepsilon + \delta) \cdot$ $(a - c - \varepsilon - 2\delta)$	$(a - c + 2\varepsilon - \delta)$ \cdot $(a - c - \varepsilon + 2\delta)$
H	$(a - c - \varepsilon) \cdot$ $(a - c - \varepsilon - 3\delta)$	$(a - c + 2\varepsilon - \delta) \cdot$ $(a - c - \varepsilon + 2\delta)$	$(a - c + 2\varepsilon) \cdot$ $(a - c - \varepsilon + 3\delta)$	$(a - c - \varepsilon - \delta)^2$
I	$(a - c - \varepsilon + \delta) \cdot$ $(a - c - \varepsilon - 2\delta)$	$(a - c + 2\varepsilon - \delta) \cdot$ $(a - c - \varepsilon + 2\delta)$	$(a - c + 2\varepsilon + \delta) \cdot$ $(a - c - \varepsilon + \delta)$	$(a - c - \varepsilon - \delta)^2$
J	$(a - c - \varepsilon + 2\delta) \cdot$ $(a - c - \varepsilon - \delta)$	$(a - c)^2$	$(a - c + 2\varepsilon + 2\delta)$ \cdot $(a - c - \varepsilon - \delta)$	$(a - c)^2$

The ranking of the payoffs in matrix 2 is the following:

$\pi_2^E < \pi_2^F$, since $\pi_2^E - \pi_2^F = -(\delta + 2\varepsilon)(a - c - 4\delta) - 5\delta^2 - 3\varepsilon^2 - \delta\varepsilon < 0$ (given that $\delta > 0, \varepsilon > 0$; and $a - c - 4\delta > 0$ due to the fact that $a - c - 3\delta - \varepsilon > 0$ to assure positive loan amounts in outcomes H and L in a two-stage game, and $\delta < \varepsilon$).

$\pi_2^F < \pi_2^G$, since $\pi_2^F - \pi_2^G = -4\varepsilon(a - c - 3\varepsilon) + 4\delta^2 + 6\varepsilon^2 < 0$ (the maximum is attained when $\delta = \varepsilon$; in this case the expression is equal to $-4\varepsilon(a - c - 3\varepsilon) - 2\varepsilon^2 < 0$).

Consequently, if Bank 1 announces $c + \varepsilon$, the best response of Bank 2 is to announce $c + \varepsilon + \delta$.

$\pi_2^H > \pi_2^I$ (respective payoffs are the same as π_2^H vs. π_2^I in a two-stage game).

$\pi_2^H < \pi_2^J$, since $\pi_2^H - \pi_2^J = -2\varepsilon(a - c) + 12\delta\varepsilon + 3\delta^2 + \varepsilon^2 < 0$ (the maximum is attained when $\delta = \varepsilon$; in this case the expression is equal to $-2\varepsilon(a - c - 8\varepsilon)$ and is negative if the regulator imposes relatively low capital surcharges in comparison with normal capital adequacy requirements).

Consequently, if Bank 1 announces $c + \varepsilon + \delta$, the best response of Bank 2 is to announce $c + \varepsilon + \delta$.

$\pi_1^E > \pi_1^H$ (respective payoffs are the same as π_1^E vs. π_1^H in a two-stage game).

$\pi_1^F < \pi_1^I$, since $\pi_1^F - \pi_1^I = -\varepsilon(a - c - 4\varepsilon) - \delta(4\delta - 6\varepsilon) < 0$.

$\pi_1^G < \pi_1^J$, since $\pi_1^G - \pi_1^J = -\varepsilon(a - c - \varepsilon) - (\varepsilon + \delta)(a - c - 3\delta - \varepsilon) + \delta(\delta - 3\varepsilon) - \varepsilon^2 < 0$.

Consequently, if Bank 2 announces $c + \varepsilon - \delta$, the best response of Bank 1 is to announce $c + \varepsilon$; if Bank 2 announces $c + \varepsilon$ or $c + \varepsilon + \delta$, the best response of Bank 1 is to announce $c + \varepsilon + \delta$.

The sub-game Nash equilibrium is thus obtained in $(\pi_1^J; \pi_2^J)$. Matrix 4 is symmetric to matrix 3, and the best responses of the players are the same, such that the sub-game Nash equilibrium establishes in $(\pi_1^P; \pi_2^P)$.

c. Ranking payoffs in matrix 1 in a three-stage game

The Nash sub-game equilibria in matrix 1 for a three-stage game is as follows:

		<i>Bank 2</i>	
		c	$c + \delta$
<i>Bank 1</i>	c	$\pi_1^D; \pi_2^D$	$\pi_1^J; \pi_2^J$
	$c + \delta$	$\pi_1^P; \pi_2^P$	$\pi_1^T; \pi_2^T$

$$\pi_2^D \text{ vs. } \pi_2^J$$

$$(a - c)^2 + (a - c + 2\delta)(a - c - \delta) + (a - c)^2$$

vs.

$$(a - c + \delta)(a - c - 2\delta) + (a - c + 2\varepsilon + 2\delta)(a - c - \varepsilon - \delta) + (a - c)^2$$

$$\pi_2^D - \pi_2^J = (\delta - \varepsilon)(a - c) + 2(\varepsilon + \delta)^2$$

The sign depends on the parameter values.

$$\pi_2^P \text{ vs. } \pi_2^T$$

$$(a - c + \delta)^2 + (a - c - \varepsilon + 2\delta)(a - c - \varepsilon - \delta) + (a - c)^2$$

vs.

$$2(a - c + 2\delta)(a - c - \delta) + (a - c)^2$$

$$\pi_2^P - \pi_2^T = (\delta - 2\varepsilon)(a - c - \varepsilon) - \varepsilon^2 + \delta^2 < 0$$

$$\pi_1^D \text{ vs. } \pi_1^P \Rightarrow \text{has the same ranking as } \pi_2^D \text{ vs. } \pi_2^J$$

$$\pi_1^J \text{ vs. } \pi_1^T \Rightarrow \text{has the same ranking as } \pi_2^P \text{ vs. } \pi_2^T$$

Thus the Nash equilibrium establishes in $(c + \delta; c + \delta)$, such that in period 1 both banks choose the highest funding cost.

The Nash equilibrium $(c; c)$ is achievable only under certain conditions.

Similar analysis can be extended to a number of periods greater than three, though the proofs would be more demanding of space.

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