# Social Learning in Networks: Extraction of Deterministic Rules

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Abstract—In this paper, we want to introduce experimental economics to the field of data mining and vice versa. It continues related work on mining deterministic behavior rules of human subjects in data gathered from experiments. Game-theoretic predictions partially fail to work with this data. Equilibria also known as game-theoretic predictions solely succeed with experienced subjects in specific games - conditions, which are rarely given. Contemporary experimental economics offers a number of alternative models apart from game theory. In relevant literature, these models are always biased by philosophical plausibility considerations and are claimed to fit the data. An agnostic data mining approach to the problem is introduced in this paper - the philosophical plausibility considerations follow after the correlations are found. No other biases are regarded apart from determinism. The dataset of the paper "Social Learning in Networks" by Choi et al 2012 is taken for evaluation. As a result, we come up with new findings. As future work, the design of a new infrastructure is discussed.

Keywords—Experimental Economics; Machine Learning; Social Learning; Human Behavior; Data Mining; Game Mining

# I. INTRODUCTION

There are many scientific disciplines promising to predict outcomes of pugnacious, social and economical interactions of humans on the granularity level of individual decisions [1]. One of them is game theory, where people are assumed to be intelligent and autonomous, and to act pursuant to their existing preferences. It is important to underline that game theory is a mathematical discipline, whose task was never to define human preferences, but to calculate based on their definition. A preference is an order on outcomes of an interaction. One can be regarded as rational, if one always makes decisions, whose execution has referred to subjective estimation the most preferred consequences [2], [3]. The level of intelligence determines the correctness of subjective estimation. Beyond justifying own decisions, rationality is a base for predictions of other people's decisions. If the concept of rationality is satisfied, and applied mutually, and even recursively in a human interaction, then the interaction is called strategic. Game is a notion for the formal structure of a concrete strategic interaction [4]. A definition of a game consists of a number of players, their preferences, their possible actions and the information available for the actions. A payoff function can replace the preferences under assumed payoff maximization. The payoff function defines each player's outcome depending on his actions, other players' actions and random events in the environment. The game-theoretic solution of a game is a prediction about the behavior of the players also known as an equilibrium. The basis for an equilibrium is the assumption of rationality. Deviating from an equilibrium is outside of rationality, because it does not maximize the payoff according to the formal definition. There are games, which have no equilibria. At least one mixed strategies equilibrium is guaranteed in finite games [5].

In common language, the notion of game is used for board games or video games. In game-theoretic literature, it is extended to all social, economical and pugnacious interactions among humans. A war can be simplified as a board game. Some board games were even developed to train people, like Prussian army war game 'Kriegspiel Chess' [6] for their officers. We like it to train in order to perform better in games [7]. In most cases, common human behavior in games deviates from game-theoretic predictions [8], [9]. One can say without any doubt that if a human player is trained in a concrete game, he will perform close to equilibrium. But, a chess master is not necessarily a good poker player and vice versa. On the other side, a game-theorist can find a way to compute an equilibrium for a game, but it does not make a successful player out of him. There are many games we can play; for most of them, we are not trained. That is why it is more important to investigate our behavior while playing general games than playing a concrete game on expert level. Conducting experiments for gathering data of human game playing is called experimental economics.

Although general human preferences are a subject of philosophical discussions [10], game theory assumes that they can be captured as required for modeling rationality. Regarding people as rational agents is disputed at least in psychology, where even a scientifically accessible argumentation exposes the existence of stable and consistent human preferences as a myth [11]. The problems of human rationality can not be explained by bounded cognitive abilities only. '... people argue that it is worth spending billions of pounds to improve the safety of the rail system. However, the same people habitually travel by car rather than by train, even though traveling by car is approximately 30 times more dangerous than by train!'[12, p.527–530] Since the last six decades nevertheless, the common scientific standards for econometric experiments are that subjects' preferences over outcomes



can be insured by paying differing amounts of money [13]. However, insuring preferences by money is criticized by the term 'Homo Economicus' as well. It is even logically obvious that researchers, who claim that monetary preferences win over other preferences, can not be trusted as scientists – they do not care about anything but money.

The ability of modeling other people's rationality and reasoning as well corresponds with the psychological term 'Theory of Mind' [14], which lacks almost only in the cases of autism. For experimental economics, subjects as well as researchers, who both are supposed to be non-autistic people, may fail in modeling of others' minds anyway. In Wason task at least, subjects' reasoning does not match the researchers' one [15]. Human rationality is not restricted to capability for science-grade logical reasoning - rational people may use no logic at all [16]. However, people also make serious mistakes in the calculus of probabilities [17]. Even in mixed strategy games, where random behavior is of a huge advantage, the required sequence of random decisions can not be properly generated by people [18]. Due to bounded cognitive abilities, every human 'random' decision depends on previous ones and is predictable in this way. In ultimatum games [9, S. 43ff], the former economists' misconception of human preferences is revealed – people's minds value fairness additionally to personal enrichment. Our minds originated from the time, when private property had not been invented and social values like fairness were essential for survival.

From the view point of data miners fascinated by human behavior, the sizes of datasets originated from social networks predominate the ones from experimental economics by orders of magnitude [9]. Nevertheless, analyzing data from experimental economics has the same importance for understanding human psychology as studying Escherichia for understanding human physiology. Data from experimental economics has the advantage of originating from simple and controlled human interactions.

In experimental economics, the models are first constructed by philosophical plausibility considerations and then are claimed to fit the data. In this work, we reverse the order of common research in experimental economics. We follow the slogan 'existence precedes essence' – the philosophical plausibility considerations follow after the correlations and regularities are found. For these needs, we analyze the dataset of the paper "Social Learning in Networks" by Choi et al 2012 [19]. The only assumption about human behavior is its determinism.

The next section summarizes related work on data mining approaches and economical models. Then, the experiment setup and the gathered data are introduced. Before extracting rules of behavior, we explain the reasons for the assumption of determinism. The results and their interpretations follow afterwards. A suggestion for more efficient research on human behavior is made in future work. Summary and discussion conclude this paper.

# II. RELATED WORK

A similar approach is already explored on two datasets – a zero-sum game of mixed strategies and an ultimatum game [20]. For both datasets, extracted deterministic regularities outperformed state-of-art models. It was shown that some

regularities can be easily verbalized, what underlines their plausibility.

A very comprehensive gathering of works in experimental psychology and economics on human behavior in general games can be found in [21]. Quantal response equilibrium became popular as a model for deviations from equilibria [22]. It is a parametrized shift between mixed strategies equilibrium and an equal distribution. The basic idea for quantal response equilibrium is the concept of trembling hand – people make mistakes with certain probability. Quantal response equilibrium was used to model the dataset for our work [19]. Quantal response models could achieve significant p-value. Unfortunately, the Akaike information criterion [23] was not calculated to judge the trade-off between fit quality and model complexity.

### III. SOCIAL LEARNING IN NETWORKS

The paper [19], whose underlying dataset we use, addresses the problem of social learning. Social learning is described as the process of acquiring knowledge by observation of other players' actions. The experimentators created a basic scenario to gather relevant data of human behavior. The game of this scenario requires three players. Players have to choose between two actions -1 or 1. The knowledge, which has to be acquired by social learning is the state of environment, which is either -1 or 1. At the beginning of the game, every player might get one single private signal -1 or 1, which is equal to the state of environment at probability  $\frac{2}{3}$  and is the opposite at probability of  $\frac{1}{3}$ . There are three information levels – 'low', 'high' and 'full', which denote the probabilities of providing a signal as  $\frac{1}{3}$ ,  $\frac{2}{3}$  and 1. A round of this game lasts for 6 turns. In every turn, each player simultaneously chooses an action, which should reveal the true state of the environment. From the 6 turns one is randomly chosen and if a player guessed it right in this turn, a constant payoff of \$2 is paid. Nothing is paid otherwise.

After first and all further turns, a player might be able to observe another player's action. There are three types of networks – 'complete', 'circle' and 'star', which determine eligibility for observation. Fig.1 shows these three types. Arrows point from the observed to the observer. There are no more signals sent than the one in the beginning of a round. Every round, the state of the environment is chosen independently. Undergrad students at New York University were recruited as subjects for this experiment. All subjects were not previously trained on this game, but were carefully instructed about its rules and structure.

In order to calculate all equilibria of this game, one needs to construct its extensive form – a tree of depth 10 and branching factor up to 8. Since this game includes hidden and simultaneous actions, it is a game of imperfect information. It can be solved using the game-theoretic package GAMBIT [24]. The resulting equilibria are quite intuitive and are drafted hereafter. If a player makes a random decision or chooses always –1 or 1, he gets in average  $\$\frac{1}{2}*2=1$  per round. If a signal is available and is copied as his decision, he gets in average  $\$\frac{2}{3}*2=1.33$  per round. In the complete network under full information, players can raise their average payoff up to \$1.46 per round, if they, additionally to following own signal in the first turn, follow the median of the last turns

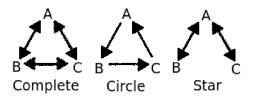


Fig. 1. Network types with players A, B and C. Arrows point from the observed to the observer.

TABLE I. Number of subjects' groups for 9 game configurations.

Information/	Low	High	Full
Network type			
Complete	6	5	6
Circle	5	6	6
Star	6	6	6

decisions in subsequent turns. If a player can observe only one of other players, he either follows his own signal or, in the case of absent signal, follows the observed player's decision. If a player did not get a signal, but can observe two other players, he follows their decisions, if these are identical.

The described solution is valid for subjects ignorant of fellows' payoffs, what rarely happens with humans. There are numerous works like [25], which underline the claim that other players' payoff matters. In [26], it is even shown that the difference between payoffs is more important than the absolute value. Therefore, subjects might not only concentrate on their own payoff, but be driven by pugnacity or graciousness.

### IV. DATASET

Table I shows the  $3 \times 3 = 9$  game configurations with available data. From 18 participating subjects, 6 groups of 3 members are created. Every group plays 15 rounds with 6 turns for almost every game configuration. This makes 3\*15\*6=270 human decisions from one group playing one of the game configurations. Summing the numbers from table I and multiplying them by 270 results the overall number of single human decision in this dataset, which is 14040 samples. In a round, a player makes sequence of 6 decisions. A signal is provided in 1593 decision sequences and absent in 747 decision sequences. The decision sequences are missing only in rounds of 'high' or 'low' information.

### V. ASSUMPTION OF DETERMINISM

Modeling human behavior outside of game playing with human subjects should not be confused with prediction algorithms of artificial players. Quite the contrary, artificial players can manipulate the predictability of human subjects by own behavior. For instance, an artificial player, which always throws 'stone' in roshambo, would success at predicting a human opponent always throwing 'paper' in reaction. Otherwise, if an artificial player maximizes its payoff based on opponent modeling, it would face a change in human behavior and have to deal with it. This case is more complex than a spectator prediction model for an 'only-humans' interaction. This work is restricted on modeling behavior

TABLE II. Decision making in the first turn with provided signal.

Signal/	-1	1
Decision		
-1	757	51
1	42	743

without participating.

Human behavior can be modeled as either deterministic or non-deterministic. Although human subjects fail at generating truly random sequences as demanded by mixed strategies equilibrium, non-deterministic models are especially used in case of artificial players in order to handle uncertainties.

'Specifically, people are poor at being random and poor at learning optimal move probabilities because they are instead trying to detect and exploit sequential dependencies. ... After all, even if people don't process game information in the manner suggested by the game theory player model, it may still be the case that across time and across individuals, human game playing can legitimately be viewed as (pseudo) randomly emitting moves according to certain probabilities.' [27] In the addressed case of spectator prediction models, non-deterministic view can be regarded as too shallow, because deterministic models allow much more exact predictions. Non-deterministic models are only useful in cases, where a proper clarification of uncertainties is either impossible or costly. To remind, deterministic models should not be considered to obligatory have a formal logic shape.

### VI. RESULTS OF RULE EXTRACTION

In the first turn of every round for all network types (2340 samples), players have no history to consult for their decisions. The inputs, which could impact their choice, are the round number, the network type, own position in the network, the information level and the signal provided. Correlation coefficients to the decision are calculated for these inputs. The highest correlation of 0.88329 with the decision results for the signal if such is provided (1593 samples). The second highest correlation of 0.03703 results for the network type and is insignificant. Therefore, the decision in the first turn has only the provided signal as a possible cause.

In the first turn, the rational choice is to copy the signal as own decision, if players are only interested in their own payoff. But the data proves that subjects deviate from the given signal in 5.8% of cases. Table II displays the distributions of decisions in the first turn of every round for all network types. The Cohen's Kappa [28] between signal and decision is 0.883 – subjects deviate almost symmetrically from the egoistic rationality. A possible assumption could be that subjects commit a fallacy and try to predict the randomly chosen current state from previous rounds. Indeed, in the samples for the first rounds only and with provided signals (106 samples), the deviation from rational choice is slightly lower at 3.8%. Since p-value according to the Fischer's exact test [29] is 0.5174 between the first rounds and the rest rounds, the statistical significance of this difference can not be claimed due to insufficient data. For the next results, the deviation of 5.8% and the Kappa of 0.8832 are taken as the upper bound for model correctness. Another interpretation of the deviation from

TABLE III. DECISION MAKING IN THE FIRST TURN WITHOUT PROVIDED SIGNAL (NUMBER OF -1/ALL).

Information/	low	high	sum
Network type			
complete	109/176(62%)	58/84(69%)	167/260(64%)
circle	77/146(53%)	45/86(52%)	122/232(53%)
star	123/184(67%)	39/71(55%)	162/255(64%)
sum	309/506(61%)	142/241(59%)	451/747(60%)

TABLE IV. Decision making in the second turn; complete and full information (270 samples).

Signal			-1	1
Last turns decisions' median/	-1	1		
Decision				
-1	145	11	131	25
1	23	91	11	103

egoistic rationality by provided signal in the first turn could be an attempt to sabotage other players' payoffs as driven by pugnacity.

For the cases, where neither the signal nor the history is provided (747 samples), players prefer to choose -1 in 60% of cases. For the network type 'circle' (232 samples), the proportion of -1 is 53% and significantly defers (pvalue is 0.0046) from the rest of network types, which stay around 64%. In contrast, there is no significant difference between 'low' and 'high' information. Table III summarizes the results - there is no 'complete' information included, because 'complete' means that the signal is always provided. For the first round (50 samples), the proportion of -1 is 78% and defers significantly (p-value is less than 0.0001) from the rest. We can assume that players prefer -1 rather than a random choice in the case they don't have any information. This preference might be interpreted as an erroneous attempt to establish cooperation by communicating missing knowledge. It can also be interpreted as human aversion to random decision making.

Since the data shows that a player's decision in the first turn is equal to the signal at probability of 94.2%, it can be claimed that if this deviation intends sabotage, then it is successful. Even in the rounds with complete network and full information (270 samples), the probability of the first turns decisions' median being equal to the actual state drops from 74% to 68% due to this deviation from egoistic rationality. And the 68% are insignificantly different to 67% probability of signal being uncompromised. That means that a player has no incentive to switch from following the signal to observing other players, even if they are all observable and supplied with signals. In fact, the signal has a correlation of 0.7414 with second turns decisions and the last turns median has almost the same correlation of 0.7352. Table IV shows the confusion matrices of the decision in the second turn with most correlated inputs. If the signal and the last turns median are equal ( 208 samples), players don't follow them at probability of only 1.9%.

Fig.2 shows the correlations of three different input types with the decision in dependence from turn number. Although it is futile for the players to follow other players' decision rather than the own signal, the correlation to the signal drops after the first turn. On the other side, the correlation to the observed players' decisions grows significantly. The

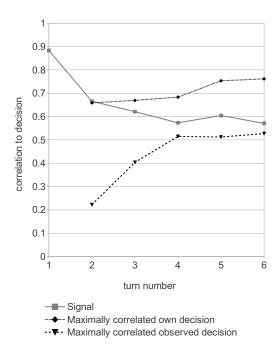
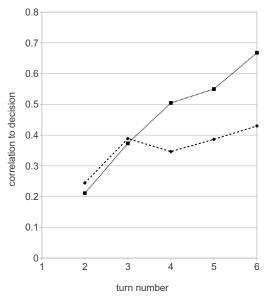


Fig. 2. Signal is provided (1593 samples); correlations between inputs and the decision. Own previous decision, which is maximally correlated to the actual decision, is the last one except in the 6th turn, where it is next-to-last. From the observed decisions, the last one has always the highest correlation.

deviation from egoistic rationality has the consequence that the correlation between state and the signal is in all turns higher than the correlation to the players' decisions, which are supplied by a signal (Fig.4). Especially in the second turn, the correlation with actual state is significantly lower, what a portion of signal supplied players are successfully sabotaged by observed decisions.

Fig.3 shows significantly high correlation of subjects' decisions with observed decisions in the case of absent signal. Following observed decisions causes a significant correlation with the actual state after the first turn (Fig.4). From the other side, the correlation to the own decision in the first turn is also high, although this first turns decision does not have any reasonable basis without a signal.

Fig.5 and 6 shows the results of rule extraction using JRip algorithm [30]. This algorithm creates disjunctive sets of logical rules, which can be easily verbalized. Cross validation is a well known procedure to estimate model's generalization - the model's performance on unseen data. The quality of a fit is less meaningful. Generalization correctness is always significantly higher than the null hypothesis except for the first turn without provided signal. The null hypothesis is a permanent choice of the majority class. Therefore, its correctness equals the frequency of this class. The same procedure that applies to estimation of correctness is applied to Kappa statics. The generalization errors are significantly higher for players without provided signal especially in the beginning of a round. This can be explained by hidden inputs, which determines the behavior without provided signal. Nevertheless, we can claim that generalizing deterministic rules deliver a good model for



- ---- Maximally correlated own decision
- --- Maximally correlated observed decision

Fig. 3. Signal is not provided (747 samples); correlations between inputs and the decision. Own previous decision, which is maximally correlated to the actual decision, is the last one except the turns after 3rd, where it is next-to-last. From the observed decision, the last one has always the highest correlation.

TABLE V. The set of derived rules for the 5th turn and available signal.

$[(Own \ decision \ in \ turn \ 3 = -1)$	695 samples,
$\land (Own\ decision\ in\ turn\ 4 = -1) \rightarrow -1]$	6.3% error
V	
[(Own decision in turn $3 = -1$ )	28 samples,
$\land (Signal = -1) \land (Player = B) \rightarrow -1]$	21% error
V	
[(Own decision in turn $3 = -1$ ) $\land$ (1st observed in turn $4 = -1$ )	15 samples,
$\land (GType = star) \rightarrow -1]$	27% error
V	
[(Own decision in turn $4 = -1$ ) $\land$ (Own decision in turn $2 = 1$ )	5 samples,
$\land (Player = C) \land (Round \le 7) \land (Observed \ in \ turn \ 2 = -1) \rightarrow -1]$	0% error
V	
[(Observed in turn $3 = -1$ ) $\land$ (Own decision in turn $3 = -1$ )	6 samples,
$\land (Round \le 7) \land (Round \ge 5) \land (Signal = -1) \rightarrow -1]$	0% error
V	
[(Own decision in turn $4 = -1$ ) $\land$ (Observed in turn $3 = -1$ )	25 samples,
$\land (Player = B) \rightarrow -1$	40% error
V	
[(Own decision in turn $4 = -1$ ) $\land$ (Own decision in turn $1 = -1$ )	12 samples,
$\land (Player = C) \rightarrow -1]$	8.3% error
V	
$[\rightarrow 1]$	807 samples,
	8.8% error

human behavior in this game.

Finally, we list a set of derived rules for the 5th turn and available signal. Every rule has a number of samples, which satisfy its conditions, and a error on the data V.

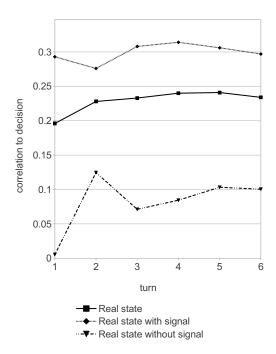


Fig. 4. Correlation of the real state to the decision in general (2340 samples), with and without provided signal. In comparison, correlation between signal and real state is 0.347. Under 0.058, correlations are insignificant at 95%.

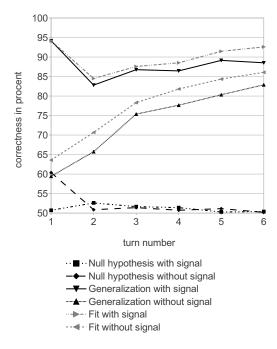


Fig. 5. Generalization and fit correctness for rule extraction. The null hypothesis choosing the majority decision mostly results correctness under 53%.

## VII. FUTURE WORK

During the work on this paper, we confronted the time consuming requesting, selection and reformatting of data. Unfortunately, there is

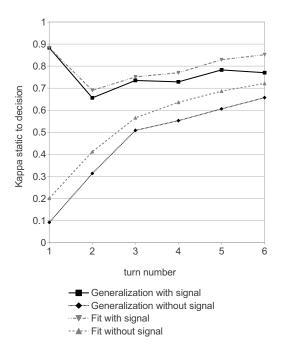


Fig. 6. Generalization and fit Kappa for rule extraction.

no online portal, where most of the datasets are offered in a common format. This is an issue, which we will address in the future. Like in the field of bioinformatics, common formats are an important part of an interdisciplinary research infrastructure and are needed to accelerate the progress [31].

As for methodological aspects of Machine Learning in the context of Experimental Economics, we would like to use the advanced pattern mining techniques for economic game data analyses. For example, in papers [32], [33] was made an attempt to use sequential patterns and similarity dependencies on pattern structures for video game players' behaviour analysis, in particular sequential attribute dependencies might be a tool of choice. We will try to apply sequential pattern mining in a supervised task, where the outcome of a game (or a turn) is a target attribute [34], [35] to see which patterns better generalize the user behaviour. These experiments are able not only to broad the tools of experimental economics, but also help to reveal potentially new knowledge of human behaviour in games based on sequential pattern description.

# VIII. CONCLUSION

In this paper, data analysis revealed a strong hint of pugnacious behavior in a social learning scenario. This finding forces to rethink assumption of egoistic preferences. Second, derived deterministic rules generalize human behavior and significantly outperform null hypothesis hereby. And finally, we suggest an interdisciplinary infrastructure to introduce more efficiency to the research on the combined field of experimental economics and machine learning.

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