

SHORT CATALOG OF PLANE TEN-EDGE TREES

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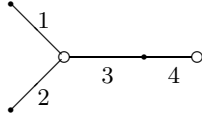
ABSTRACT. A type is the set all pairwise nonisotopic plane binary trees with the same passport. A type is called decomposable, if it is a union of several Galois orbits. In this work we present the list of all passports of plane binary trees with ten edges and the list of all Galois orbits.

1. INTRODUCTION

1.1. Rotation group. A plane tree is a tree imbedded in the plane. Two trees are considered the same if there exists an isotopy that maps one tree into another. A plane tree possesses a binary structure — a coloring of its vertices into two colors: black and white (adjacent vertices have different colors).

Two operators acts on edges of a plane binary tree: α — the counterclockwise rotation around white vertices, β — the counterclockwise rotation around black vertices. If some enumeration of edges is given, then operators α and β define permutations a and b in S_n , where n is the number of edges. Subgroup $\langle a, b \rangle \subset S_n$ generated by permutations a and b is the *rotation group* of the tree [1]. The rotation group is defined up to conjugation.

Example 1.1. For the tree



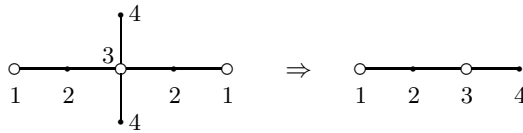
$a = (1, 2, 3)$, $b = (3, 4)$ and $\langle a, b \rangle = S(4)$.

A map of a plane binary tree T_1 with set of vertices V_1 and set of edges E_1 to a plane binary tree T_2 with set of vertices V_2 and set of edges E_2 is a map from V_1 to V_2 such that

- white vertices are mapped into white and black — into black;
- adjacent vertices remain adjacent under the mapping (this condition defines the map from E_1 to E_2);
- the map commutes with operators α and β .

If $|E_1| > |E_2|$, then tree T_1 is called *reduced* to tree T_2 . We will call a tree *non-reducible*, if it cannot be mapped into a smaller tree.

Example 1.2. Example of a map.



Here the vertex with number i , $i = 1, 2, 3, 4$, is mapped to the vertex with the same number. Thus, the left six-edge tree is reduced to 3-chain.

Remark 1. A tree is reducible, if the set of its edges can be partitioned into a union of pairwise disjoint classes in a way such that, if two edges e_1 and e_2 belong to one class, then edges $\alpha(e_1)$ and $\alpha(e_2)$ belong to one class and edges $\beta(e_1)$ and $\beta(e_2)$ also belong to one class.

Remark 2. Automorphisms of a plane binary tree are rotations around its center of symmetry (if it exists). Hence, the group of automorphisms is cyclic. Let $\text{Aut}(T)$ be the group of automorphisms of a tree T and $|\text{Aut}(T)|$ be the order this group. If the group $\text{Aut}(T)$ is nontrivial of order k , then T will be called k -symmetric.

1.2. The Goulden-Jackson formula [2].

Definition 1.1. The *passport* of a plane binary tree is the list of valencies of its white vertices plus the list of valencies of its black vertices. The set of all trees with the same passport is called a *type* and is denoted as

$$\Xi = \langle a_1, \dots, a_m \mid b_1, \dots, b_n \rangle,$$

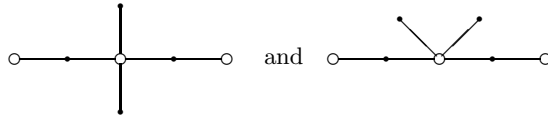
where m is the number of white vertices, n is the number of black vertices, a_1, \dots, a_m is the list of white valencies, written in the nonincreasing order, and b_1, \dots, b_n is the list of black valencies (also written in the nonincreasing order). Thus the tree from Example 1.1 belongs to the type $\langle 3, 1 \mid 2, 1, 1 \rangle$.

Theorem (the Goulden-Jackson formula). Let Ξ be a type and $w(\Xi)$ be a weighted sum $w(\Xi) = \sum_{T \in \Xi} (1/|\text{Aut}(T)|)$. Then

$$w(\Xi) = \frac{(m-1)!(n-1)!}{\prod_i k_i! \prod_j l_j!}.$$

Here m is the number of white vertices, n is the number of black vertices, k_i , $i = 1, 2, \dots$, is the number of white vertices of valency i and l_j , $j = 1, 2, \dots$, is the number of black vertices of valency j .

Example 1.3. The Goulden-Jackson formula for the type $\langle 4, 1, 1 \mid 2, 2, 1, 1 \rangle$ gives the number $3/2$. Indeed, two trees belong to this type



The left tree is 2-symmetric and the right is non-symmetric.

1.3. Galois orbits.

Definition 1.2. A polynomial $p \in \mathbb{C}[z]$ is called a *generalized Chebyshev polynomial* or *Shabat polynomial*, if it has exactly two finite critical values: 0 and 1. The set $p^{-1}[0, 1]$ is a plane binary tree: white vertices are inverse images of $\{0\}$ and black vertices of $\{1\}$.

For each plane binary tree T there exists a Shabat polynomial p with algebraic coefficients such, that the tree $p^{-1}[0, 1]$ is isotopic to T . In what follows all our Shabat polynomials will have algebraic coefficients.

Polynomial p is unique up to linear change of variable z . Let $S(t)$ be the set of all Shabat polynomials that correspond to a tree T , i.e. $p \in S(T) \Rightarrow p^{-1}[0, 1] = T$. Each $p \in S(T)$ is defined over some algebraic field and let K be the minimal of such fields. K will be called the *definition field* of the tree T . If the definition field is \mathbb{Q} , then the tree will be called *rational*.

Let p be a Shabat polynomial with algebraic coefficients and γ be an elements of absolute Galois group. The action of γ on p is the action on its coefficients:

$$\begin{aligned} q = \gamma(p) &= \gamma(a_n z^n + a_{n-1} z^{n-1} + \dots + a_1 z + a_0) = \\ &= \gamma(a_n) z^n + \gamma(a_{n-1}) z^{n-1} + \dots + \gamma(a_1) z + \gamma(a_0). \end{aligned}$$

Then q is a Shabat polynomial and trees $p^{-1}[0, 1]$ and $q^{-1}[0, 1]$ belong to the same type.

The action of Galois group can be defined on trees. Indeed, if Shabat polynomials p_1 and p_2 define equal trees, then $p_2(z) = p_1(cz + d)$, where c and d are algebraic numbers. Then

$$\gamma(p_2(z)) = \gamma(p_1)(\gamma(c)z + \gamma(d)),$$

i.e. Shabat polynomials $\gamma(p_1)$ and $\gamma(p_2)$ also define equal trees. Thus, a type is a union of disjoint Galois orbits and the degree of definition field of a given tree is equal to the cardinality of its Galois orbit.

Definition 1.3. A type Ξ is called *non-decomposable*, if all its trees are in one Galois orbit. Otherwise, a type is called *decomposable*.

The order of rotation group is a Galois invariant [1]. Thus, if a type contains two trees with different rotation groups, then it is decomposable. Analogously, a type is decomposable if it contains symmetric trees and non-symmetric trees. Also a type is decomposable, if it contains two trees such, that Shabat polynomial of the first is a degree of Shabat polynomial and Shabat polynomial of the second is not.

A decomposable type is called *trivially decomposable*, if its partition into orbits can be explained by above-mentioned combinatorial reasons. Otherwise, a type is called *non-trivially decomposable*. Examples of non-trivial decomposition are known only for trees of diameter four.

Example 1.4. One of the first discovered example of non-trivial decomposition is the "Leila flower" [3, 4]:

$$\Xi = \langle 5, \underbrace{1, \dots, 1}_{15} \mid 6, 5, 4, 3, 2 \rangle.$$

A tree in this type is completely defined by cyclic order of black vertices under the counterclockwise going around of white vertex of valency 5. If the circuit begins with the vertex of valency 6, then a tree is defined by permutation of numbers 2, 3, 4, 5. Here even permutations correspond to trees in one orbit and odd — in another.

2. SETTING OF THE PROBLEM

N	passport	$w(\Xi)$	N	passport	$w(\Xi)$
1	$\langle 10 1, 1, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	1/10	2	$\langle 9, 1 2, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	1
3	$\langle 8, 2 2, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	1	4	$\langle 8, 1, 1 3, 1, 1, 1, 1, 1, 1, 1 \rangle$	1
5	$\langle 8, 1, 1 2, 2, 1, 1, 1, 1, 1, 1, 1 \rangle$	7/2	6	$\langle 7, 3 2, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	1
7	$\langle 7, 2, 1 3, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	2	8	$\langle 7, 2, 1 2, 2, 1, 1, 1, 1, 1, 1, 1 \rangle$	7
9	$\langle 7, 1, 1, 1 4, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	1	10	$\langle 7, 1, 1, 1 3, 2, 1, 1, 1, 1, 1, 1, 1 \rangle$	6
11	$\langle 7, 1, 1, 1 2, 2, 2, 1, 1, 1, 1, 1, 1 \rangle$	5	12	$\langle 6, 4 2, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	1
13	$\langle 6, 3, 1 3, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	2	14	$\langle 6, 3, 1 2, 2, 1, 1, 1, 1, 1, 1, 1 \rangle$	7
15	$\langle 6, 2, 2 3, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	1	16	$\langle 6, 2, 2 2, 2, 1, 1, 1, 1, 1, 1, 1 \rangle$	7/2
17	$\langle 6, 2, 1, 1 4, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	3	18	$\langle 6, 2, 1, 1 3, 2, 1, 1, 1, 1, 1, 1, 1 \rangle$	18
19	$\langle 6, 2, 1, 1 2, 2, 2, 1, 1, 1, 1, 1, 1 \rangle$	15	20	$\langle 6, 1, 1, 1, 1 5, 1, 1, 1, 1, 1, 1 \rangle$	1
21	$\langle 6, 1, 1, 1, 1 4, 2, 1, 1, 1, 1, 1, 1 \rangle$	5	22	$\langle 6, 1, 1, 1, 1 3, 3, 1, 1, 1, 1, 1, 1 \rangle$	5/2
23	$\langle 6, 1, 1, 1, 1 3, 2, 2, 1, 1, 1, 1, 1 \rangle$	10	24	$\langle 6, 1, 1, 1, 1 2, 2, 2, 2, 1, 1, 1, 1 \rangle$	5/2
25	$\langle 5, 5 2, 1, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	1/2	26	$\langle 5, 4, 1 3, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	2
27	$\langle 5, 4, 1 2, 2, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	7	28	$\langle 5, 3, 2 3, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	2
29	$\langle 5, 3, 2 2, 2, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	7	30	$\langle 5, 3, 1, 1 4, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	3
31	$\langle 5, 3, 1, 1 3, 2, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	18	32	$\langle 5, 3, 1, 1 2, 2, 2, 1, 1, 1, 1, 1, 1 \rangle$	15
33	$\langle 5, 2, 2, 1 4, 1, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	3	34	$\langle 5, 2, 2, 1 3, 2, 1, 1, 1, 1, 1, 1, 1 \rangle$	18
35	$\langle 5, 2, 2, 1 2, 2, 2, 1, 1, 1, 1, 1, 1, 1 \rangle$	15	36	$\langle 5, 2, 1, 1, 1 5, 1, 1, 1, 1, 1, 1, 1 \rangle$	4
37	$\langle 5, 2, 1, 1, 1 4, 2, 1, 1, 1, 1, 1, 1, 1 \rangle$	20	38	$\langle 5, 2, 1, 1, 1 3, 3, 1, 1, 1, 1, 1, 1 \rangle$	10
39	$\langle 5, 2, 1, 1, 1 3, 2, 2, 1, 1, 1, 1, 1, 1 \rangle$	40	40	$\langle 5, 2, 1, 1, 1 2, 2, 2, 2, 1, 1, 1, 1 \rangle$	10
41	$\langle 5, 1, 1, 1, 1, 1 4, 3, 1, 1, 1, 1, 1 \rangle$	4	42	$\langle 5, 1, 1, 1, 1, 1 4, 2, 2, 1, 1, 1, 1 \rangle$	6
43	$\langle 5, 1, 1, 1, 1, 1 3, 3, 2, 1, 1, 1, 1 \rangle$	6	44	$\langle 5, 1, 1, 1, 1, 1 3, 2, 2, 2, 1, 1, 1 \rangle$	4
45	$\langle 5, 1, 1, 1, 1, 1 2, 2, 2, 2, 2, 1, 1, 1 \rangle$	1/5	46	$\langle 4, 4, 2 3, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	1
47	$\langle 4, 4, 2 2, 2, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	7/2	48	$\langle 4, 4, 1, 1 4, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	3/2
49	$\langle 4, 4, 1, 1 3, 2, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	9	50	$\langle 4, 4, 1, 1 2, 2, 2, 1, 1, 1, 1, 1, 1 \rangle$	15/2
51	$\langle 4, 3, 3 3, 1, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	1	52	$\langle 4, 3, 3 2, 2, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	7/2
53	$\langle 4, 3, 2, 1 4, 1, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	6	54	$\langle 4, 3, 2, 1 3, 2, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	36
55	$\langle 4, 3, 2, 1 2, 2, 2, 1, 1, 1, 1, 1, 1, 1 \rangle$	30	56	$\langle 4, 3, 1, 1, 1 4, 2, 1, 1, 1, 1, 1, 1 \rangle$	20
57	$\langle 4, 3, 1, 1, 1 3, 3, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	10	58	$\langle 4, 3, 1, 1, 1 3, 2, 2, 1, 1, 1, 1, 1, 1 \rangle$	40
59	$\langle 4, 3, 1, 1, 1 2, 2, 2, 2, 1, 1, 1, 1, 1, 1 \rangle$	10	60	$\langle 4, 2, 2, 2 4, 1, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	1
61	$\langle 4, 2, 2, 2 3, 2, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	6	62	$\langle 4, 2, 2, 2 2, 2, 2, 1, 1, 1, 1, 1, 1, 1 \rangle$	5
63	$\langle 4, 2, 2, 1, 1 4, 2, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	30	64	$\langle 4, 2, 2, 1, 1 3, 3, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	15
65	$\langle 4, 2, 2, 1, 1 3, 2, 2, 1, 1, 1, 1, 1, 1, 1 \rangle$	60	66	$\langle 4, 2, 2, 1, 1 2, 2, 2, 2, 1, 1, 1, 1, 1, 1 \rangle$	15
67	$\langle 4, 2, 1, 1, 1, 1 3, 3, 2, 1, 1, 1, 1, 1, 1, 1 \rangle$	30	68	$\langle 4, 2, 1, 1, 1, 1 3, 2, 2, 2, 1, 1, 1, 1, 1, 1 \rangle$	20
69	$\langle 4, 2, 1, 1, 1, 1 2, 2, 2, 2, 2, 1, 1, 1, 1, 1 \rangle$	1	70	$\langle 4, 1, 1, 1, 1, 1, 1 3, 3, 3, 1, 1, 1, 1, 1, 1, 1 \rangle$	1
71	$\langle 4, 1, 1, 1, 1, 1, 1 3, 3, 2, 2, 1, 1, 1, 1, 1, 1 \rangle$	3/2	72	$\langle 3, 3, 3, 1 3, 2, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	6
73	$\langle 3, 3, 3, 1 2, 2, 2, 1, 1, 1, 1, 1, 1, 1 \rangle$	5	74	$\langle 3, 3, 2, 2 3, 2, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	9
75	$\langle 3, 3, 2, 2 2, 2, 2, 1, 1, 1, 1, 1, 1, 1 \rangle$	15/2	76	$\langle 3, 3, 2, 1, 1 3, 3, 1, 1, 1, 1, 1, 1, 1, 1 \rangle$	15
77	$\langle 3, 3, 2, 1, 1 3, 2, 2, 1, 1, 1, 1, 1, 1, 1 \rangle$	60	78	$\langle 3, 3, 2, 1, 1 2, 2, 2, 2, 1, 1, 1, 1, 1, 1 \rangle$	15
79	$\langle 3, 3, 1, 1, 1, 1 3, 2, 2, 2, 1, 1, 1, 1, 1, 1 \rangle$	10	80	$\langle 3, 3, 1, 1, 1, 1 2, 2, 2, 2, 2, 1, 1, 1, 1, 1 \rangle$	1/2
81	$\langle 3, 2, 2, 2, 1 3, 2, 2, 1, 1, 1, 1, 1, 1, 1 \rangle$	40	82	$\langle 3, 2, 2, 2, 1 2, 2, 2, 2, 1, 1, 1, 1, 1, 1 \rangle$	10
83	$\langle 3, 2, 2, 1, 1, 1 2, 2, 2, 2, 2, 1, 1, 1, 1, 1 \rangle$	2	84	$\langle 2, 2, 2, 2, 2 2, 2, 2, 2, 1, 1, 1, 1, 1, 1 \rangle$	1/2

In table above all types of ten-edge trees are enumerated. The lists of white and black valencies are partitions of number 10. The list of white valencies in each type

is lexicographically higher, than the list of black (as the number of vertices is 11, then these two lists cannot be equal). For each type we compute the corresponding weighted sum with the use of Goulden-Jackson formula. This weighted sum is equal to the type cardinality, if there is no symmetric trees in the type. Otherwise, it is less.

Remark 3. In what follows we will write "Galois n -orbit" instead of "Galois orbit of cardinality n ".

Remark 4. Shabat polynomials and definition fields of nine-edge trees are described in catalog [6] and with number of edges ≤ 8 — in catalog [5].

Remark 5. In what follows critical values of Shabat polynomial will be 0 and some nonzero rational number.

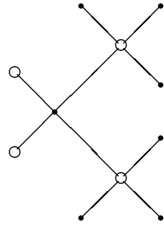
Remark 6. In what follows we will give only schematic pictures of trees and will not try to present their true forms (see [5, 7]).

3. ORBITS

3.1. Types 1, 25, 45, 80 and 84. Each of them contains only one tree — a symmetric tree with symmetry of the order 10, 2, 5, 2 and 2, respectively.

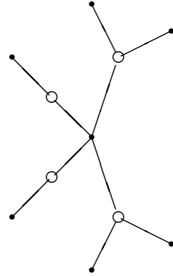
3.2. Types 2 — 4, 6 — 15, 17 — 21, 23, 26 — 32, 34 — 42, 44, 46, 49, 51, 53 — 60, 62, 63, 65, 67 — 70, 72 — 74, 77, 79, 81 and 82. All these types are non-decomposable and do not contain symmetric trees.

3.3. Types 5, 22, 24, 48, 52 and 71. Each of them contains one 2-symmetric tree (Galois 1-orbit). Non-symmetric trees in each of these types constitute Galois 3-orbit, 2-orbit, 2-orbit, 1-orbit, 3-orbit and 1-orbit, respectively. Hence, non-symmetric trees in types 48 and 71 are rational.



$$\left(x^2 + \frac{100}{27}\right)^4 \left(x^2 + \frac{100}{27}x + \frac{100}{27}\right)$$

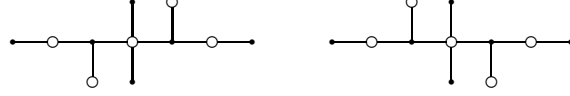
The rational non-symmetric tree in the type 48 and its Shabat polynomial.



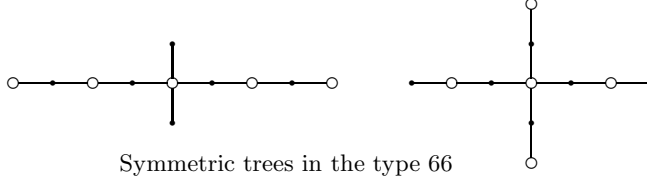
$$\left(x^2 + \frac{6075}{5476}\right)^3 \left(x^2 + \frac{2025}{592}x + \frac{2025}{592}\right)^2$$

The rational non-symmetric tree in the type 71 and its Shabat polynomial.

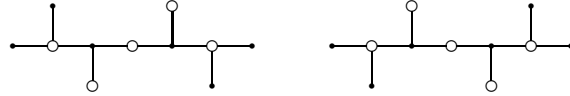
3.4. Types 64, 66, 76 and 78. Each of them contains two 2-symmetric trees and 14 non-symmetric. Symmetric trees constitute Galois 2-orbit and non-symmetric — 14-orbit.



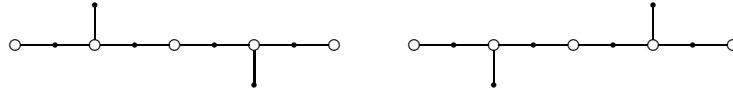
Symmetric trees in the type 64



Symmetric trees in the type 66

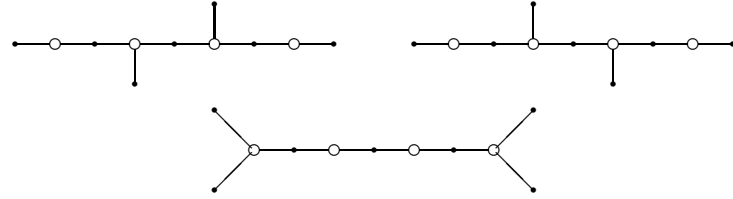


Symmetric trees in the type 76

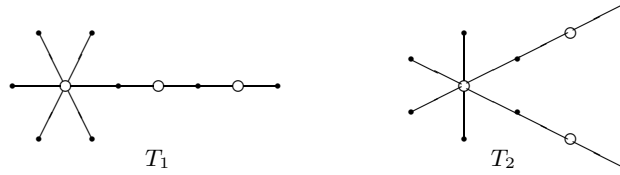


Symmetric trees in the type 78

3.5. Type 75. It contains three 2-symmetric trees (Galois 3-orbit) and six non-symmetric (Galois 6-orbit). Schemas of symmetric trees are presented below.

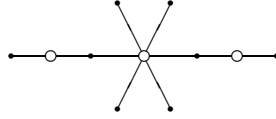


3.6. Type 16. It contains four trees. Two of them — T_1 and T_2



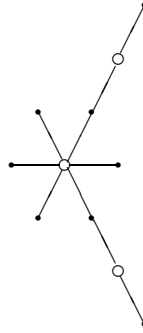
constitute 2-orbit. They can be reduced to 2-chain and their rotation group has order 14400.

2-symmetric tree T_3

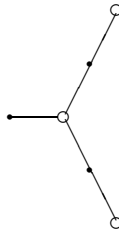


constitute 1-orbit. The order of its rotation group is 240.

Tree T_4



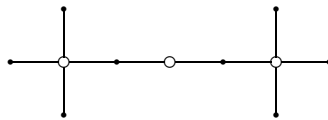
is rational and constitute 1-orbit. Its Shabat polynomial $x^6(x^2 - 2x + 32/5)^2$ is the square of Shabat polynomial $x^3(x^2 - 2x + 32/5)$ that corresponds to the tree



In "squaring a tree", inverse images of segment $[-1, 0]$ become new edges.

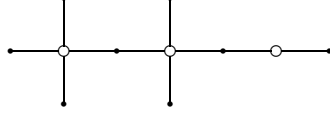
The tree T_4 can be reduced to 2-chain. The order of its rotation group is 7200.

3.7. Type 47. Here everything is more or less the same, as in the previous type 16. There are also 4 trees. Two of them belong to the Galois 2-orbit. They can be reduced to 2-chain and their rotation group has order 14400. One tree is 2-symmetric

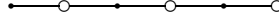


Its rotation group has order 240.

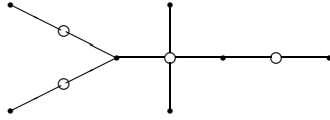
The fourth tree



is rational and can be reduced to 2-chain. The order of its rotation group is 200. Its Shabat polynomial $(x^2 - 1/5)^4(x - 1)^2$ is the square of Shabat polynomial $(x^2 - 1/5)^2(x - 1)$ that corresponds to 5-chain



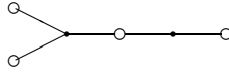
3.8. **Type 61.** It contains 6 trees. Each of them can be reduced to 2-chain. The rotation group of each tree has order 28800. The type is decomposable: 5 trees constitute Galois 5-orbit and the sixth tree



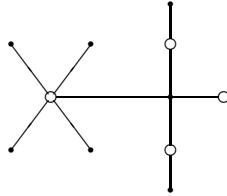
is rational. Its Shabat polynomial

$$x^4 \left(x^3 + \frac{5}{9}x^2 - \frac{5}{81}x - \frac{5}{81} \right)^2$$

is the square of Shabat polynomial that corresponds to the tree



3.9. **Type 33.** It contains 3 trees. The rotation group of each of them is S_{10} . Two trees belong to Galois 2-orbit and the tree

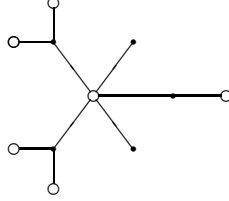


is rational with Shabat polynomial

$$x^5 \left(x^2 - \frac{27}{16} + \frac{27}{32} \right)^2 (x - 1).$$

Trees in this type are trees of diameter 4. The black vertex of valency 4 is the center and a cyclic order of white vertices under counterclockwise going around of the center defines the tree. This type is an example of non-trivial decomposition.

3.10. **Type 43.** It contains 6 trees. The rotation group of each of them is S_{10} . Five trees constitute a Galois 5-orbit and the sixth tree

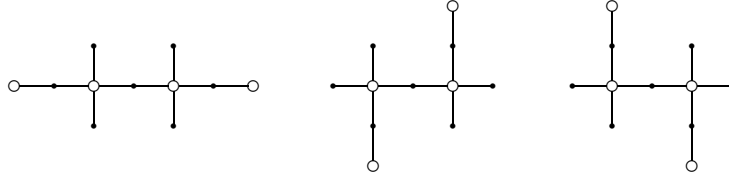


is rational with Shabat polynomial

$$\int x^4(x-1)(x^2+2x+2)^2 dx.$$

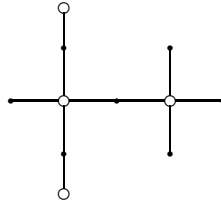
Trees in this type are trees of diameter 4. The white vertex of valency 5 is the center and a cyclic order of black vertices under counterclockwise going around of the center defines the tree. This type also is an example of non-trivial decomposition.

3.11. **Type 50.** It contains 9 trees. Three of them are 2-symmetric



and they constitute Galois 3-orbit. The rotation group of each of them has order 3840.

One tree



is rational with Shabat polynomial

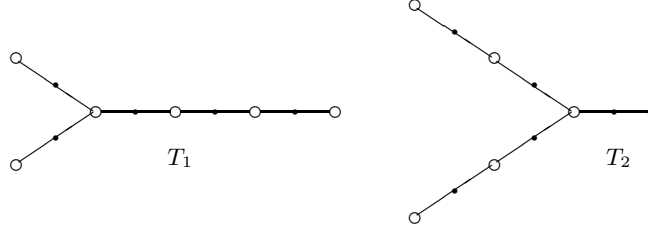
$$(x^2 - 500/441)^4(x^2 + 500x/189 + 500/189).$$

Its rotation group has order 1440 and it belong to Galois 1-orbit.

Remark 7. This tree is, so called, "special" tree, i.e. a tree with a primitive rotation group with order less, than $n!/2$ (n is the number of edges) [8, 1].

Remaining 5 trees constitute Galois 5-orbit. Their rotation group is S_{10} .

3.12. **Type 83.** It contains 2 trees:



Both are rational with Shabat polynomials

$$(3x + 20)^3(3x - 10)^2(x - 10)^2(3x^3 + 20x^2 - 400x - 4000)$$

for tree T_1 and

$$(60x - 7)^3(1200x^2 + 280x + 343)^2(100x^2 + 35x + 49)(15x - 7)$$

for tree T_2 . Their rotation groups have orders 14400 and 7200, respectively.

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