

Physical meaning and consequences of the loop infrared divergences in global de Sitter space

E. T. Akhmedov

*Institute for Theoretical and Experimental Physics, B. Chermushkinskaya 25, 117218 Moscow, Russia;**Moscow Institute of Physics and Technology, Institutskii per 9, 141700 Dolgoprudny, Russia;**and Mathematical Faculty of the National Research University Higher School of Economics, Vavilova 7, 117312 Moscow, Russia*

(Received 21 September 2012; published 22 February 2013)

Following Krotov and Polyakov [Nucl. Phys. **B849**, 410 (2011)], we show that in global de Sitter space its isometry is broken by the loop IR divergences for any invariant vacuum state of the massive scalars. We derive a kinetic equation in global de Sitter space that follows from the Dyson-Schwinger equation of the Schwinger-Keldysh diagrammatic technique in the IR limit and allows us to understand the physical meaning and consequences of the loop IR divergences. In many respects, the isometry breaking in global de Sitter space is similar to the one in the contracting Poincaré patch of de Sitter space. Hence, as a warm-up exercise we study the kinetic equation and properties of its solutions in the expanding and contracting Poincaré patches of de Sitter space. Quite unexpectedly, we find that under some initial conditions there is an explosive production of massive particles in the expanding Poincaré patch.

DOI: [10.1103/PhysRevD.87.044049](https://doi.org/10.1103/PhysRevD.87.044049)

PACS numbers: 04.62.+v

I. INTRODUCTION

The de Sitter (dS) space isometry is crucial for its stability. Obviously, if it is not broken then the cosmological constant cannot be secularly screened. There are states of the massive scalar field theories in which they respect dS isometry at tree level [1,2]; they are referred to as α vacua. But in the loops, the situation becomes more complicated. The one-loop calculation in the expanding Poincaré patch (EPP) of dS space shows that the isometry is broken for all α vacua except for the Bunch-Davies (BD) one [3] (see also the Appendix of Ref. [4]).

However, we find it rather unphysical to address the question of the stability of the system under such conditions when all its symmetries are respected exactly. We propose to consider excitations above the highly symmetric state and to trace where they evolve in the future infinity. For the finite particle density on top of any α vacuum (including the BD one), dS isometry is broken even at tree level [3,4]. The obvious question to ask at this point is whether this density decays or rapidly increases with time. Rephrasing this, we would like to understand if the system has the dS invariant ground state.

The standard answer to this question is that the density will decay due to the rapid expansion of the space. But the situation happens to be more complicated and actually quite counterintuitive. We show in this paper that the proper quantity to study is the number density per comoving volume: It is crucially important and may be counterintuitive that the particle creation and decay rates are defined at the nonlinear level via the density per comoving volume rather than per the physical one. (Unlike the physical volume, which inflates with time, comoving volume remains constant.) Note that the number density per comoving volume of the spatially homogeneous dust remains constant throughout the history of dS space independently

of whether the space is expanding or contracting. But such a density will grow unboundedly if one will turn on only the process of particle production by the gravitational field. That is, in particular, the source of the loop IR divergences and large IR contributions, which are under study in this paper. Of course for the backreaction problem it is important to know how fast is such a growth in comparison with the expansion of the space, especially when the loss processes are also taken into account (on top of the production ones).¹ The main goal of this paper is to address this problem.

In global dS space the situation is more intricate. There is a one-loop IR divergence even for massive scalars [5]. It reveals itself via the impossibility to take the moment of turning on self-interactions t_0 to past infinity. We will see that t_0 plays the role of IR cutoff. Thus, unlike the situation with the BD initial state in EPP, in global dS space, isometry is broken in the loops for all possible vacua (including the Euclidean one). Furthermore, the presence of such IR divergences means that the growth of particle density wins against the expansion of the space-time.

However, it remains unclear whether or not one can take $t_0 \rightarrow -\infty$ after the summation of all loop leading IR contributions. We show that this problem is related to the one of the presence of a stationary state in dS QFT. If such a state does exist, then t_0 can be taken past infinity and the dS

¹Note that turning on only a single process of the particle kinetics rather than all of them may cause an instability; i.e., it may cause a divergence in the collision integral (see Ref. [3] for the detailed discussion). However, as we show, in many cases these instabilities do disappear when one takes into account all kinetic processes together. I.e., the collision integral becomes well defined. But then, in turn, new instabilities of different kinds, which cause an explosion of the solution of the kinetic equation (but not of the collision integral), may appear under some conditions.

invariance is restored. Otherwise, the dS isometry is not restored at higher loops, and the backreaction due to the particle production cannot be neglected. It was explained in Refs. [4,6] that to approach such a problem one has to solve the kinetic equation, which is the tool for the summation of the leading IR contributions in all loops. Below we derive this equation in global dS space.

Although in EPP there are no IR divergences, one has large IR contributions there. As a result, one can also derive the kinetic equation in EPP for the finite initial particle density on top of the BD vacuum. In Refs. [4,6] the problem of the existence of the stationary state in EPP was addressed with the use of the corresponding kinetic equation. The conclusion was that it could exist under the appropriate initial conditions.

In the contracting Poincaré patch (CPP), the situation happens to be similar to the global dS space. There, unlike EPP, we also encounter loop IR divergences. In the main body of this paper, we explain the reason for such a difference between EPP and CPP. We derive the kinetic equation in CPP. Such equations in EPP and CPP can be simply mapped to each other because we are dealing with the spatially homogeneous states and study the density per comoving volume. As a result, there is a quite simple map between the solutions of the kinetic equations in EPP and CPP. In particular, we find that there is the stationary solution even in the CPP, which is related to the above-mentioned stationary solution in EPP. Its presence means that under the appropriate initial conditions and in the circumstances of the perfect spatial homogeneity, dS isometry can be restored after the summation of all leading loop contributions, even in CPP.

But one of the main messages of this paper is that there is also a solution of the kinetic equation (both in CPP and EPP) which describes explosion within a finite proper time of the particle density. The presence of such a solution means that under some initial conditions, backreaction from the particle production can become enormous even in EPP. The last situation should lead to the deformation of the dS metric and probably bring the system to the FRW space-time with a secularly decreasing cosmological constant.

A. Subtleties

To proceed, let us point out some subtleties with field theory in curved space-times, in general, and in dS space, in particular. For illustrative reasons, consider a 4D massless conformally coupled scalar field with the quartic self-interaction $[\square + R/6]\phi = \lambda\phi^3$ on a global dS background. We are interested in the following classical scattering problem. Self-interaction is adiabatically turned on somewhere in the far past and then eventually switched off in the far future. At past infinity, we excite a single free wave and ask the following question: Will the solution of the equation under consideration (at the linear order in λ) contain three free waves in future infinity or will it not?

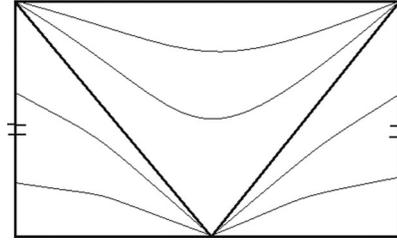


FIG. 1. Penrose diagram of dS space. Left and right sides of the depicted rectangle are glued to each other. Here we show Cauchy surfaces (e.g., constant η slices) for the global dS metric of the form $ds^2 = \frac{1}{\eta^2}[d\eta^2 - d\vec{x}^2]$, where $\eta \in (-\infty, +\infty)$.

In the metric with noncompact spatial slices $ds^2 = \frac{1}{\eta^2}[d\eta^2 - d\vec{x}^2]$, $\eta \in (-\infty, +\infty)$ (see Fig. 1), the problem can be mapped via conformal transformation to the one in Minkowski space. The intensity of the process in question, at the leading λ order, will be proportional to the δ function ensuring energy conservation [7]. The latter will put an obstruction to the process under consideration.

If, however, one addresses the same problem in the metric $ds^2 = dt^2 - \cosh^2(t)d\Omega^2 = \frac{1}{\cos^2(T)}[dT^2 - d\Omega^2]$ (see Fig. 2), then it can be converted to the one in an Einstein static universe with the compact range of time, $T \in [-\frac{\pi}{2}, +\frac{\pi}{2}]$. As a result, instead of the aforementioned δ function one will encounter $\frac{\sin[\frac{\pi\Delta E}{\Delta E}]}{\Delta E}$, where ΔE is the energy deficit in the process in question. Hence, the process will be allowed.

The resolution of the apparent disagreement with the general covariance is that the result of this scattering process strongly depends on the initial/boundary conditions. Approaching the same problem with the use of different ways of spatial slicing of dS space, we implicitly assume different initial conditions. In fact, while in the case of noncompact spatial sections, one has such harmonics as $e^{-i\omega\eta}$; for the situation with the compact sections, the harmonics are $e^{-i\epsilon T}$. The map between these harmonics under the coordinate and conformal transformations should be the main concern for those studying the scattering problem under consideration.

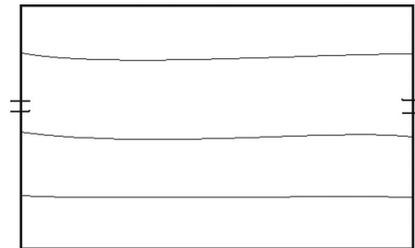


FIG. 2. Here we show Cauchy surfaces (e.g., constant t or T slices) for the global dS metric of the form $ds^2 = dt^2 - \cosh^2(t)d\Omega^2 = \frac{1}{\cos^2(T)}[dT^2 - d\Omega^2]$, where $t \in (-\infty, +\infty)$ and $T \in [-\frac{\pi}{2}, +\frac{\pi}{2}]$.

The observations of the above paragraphs are relevant for us because below we study the IR contributions whose causes are sensitive to the conditions at the boundaries of various patches of the entire space-time. Also, they give one a hint on the explanation of the disagreement between the conclusions of Ref. [8] and those of the present paper. The spatial slicing of Fig. 1 was used in Ref. [8]. Although in the latter paper the entire dS space was considered, the BD state was imposed at the boundary separating contracting CPP and EPP. We do not quite understand the physical meaning of this approach, but we can see that in such a way it is possible to respect dS isometry at each step of the construction of the correlation functions. In fact, as we have already pointed out above, to respect dS isometry in all loops, one has to impose the BD state exactly at the boundary separating EPP from CPP.

At the same time, in the kinetic theory approach, one can use both spatial slicings of Figs. 1 and 2. In this setting there is nothing behind the initial Cauchy surface, and the dS isometry is broken, at least in the loops, unless we start with the exact BD state exactly at the boundary of EPP.

B. The setup of the problem

Our eventual goal is to see whether the backreaction from the particle production in dS space is negligible or not. But in this paper we neglect backreaction and consider self-interacting massive scalar field theory on the fixed dS background. We restrict our attention to the behavior of the solutions of the kinetic equation in the IR limit and try to see if the assumption of the negligible backreaction is self-consistent. Note that in EPP (CPP), the IR limit of the physical momentum corresponds to future (past) infinity.

We study D -dimensional minimally coupled real scalar field theory:

$$L = \sqrt{|g|} \left[\frac{g^{\mu\nu}}{2} \partial_\mu \phi \partial_\nu \phi + \frac{m^2}{2} \phi^2 + \frac{\lambda}{3} \phi^3 + \dots \right]. \quad (1)$$

The dots here stand for the higher power self-interaction terms, which make the theory stable. However, below we will consider formulas only due to the unstable cubic part of the potential. The reason for that is just to simplify the equations. This instability does not affect our conclusions because similar IR contributions do also appear in massive ϕ^4 theory [9] or in the other nonconformal theories in dS space [10–14].

To set the notations, we describe here some of the properties of this space and the free scalar harmonics on it. It is the hyperboloid $X_0^2 - X_i^2 = -1$, $i = 1, \dots, D$ in the $(D + 1)$ -dimensional Minkowski space $ds^2 = dX_0^2 - dX_i^2$. Throughout this paper, we set the curvature of the hyperboloid to 1. The dS isometry group is the $SO(D, 1)$ Lorentz group of the ambient Minkowski space in question. Throughout this paper, we assume that D is even. For odd D , the situation is a bit more subtle, but it is still not much different [3].

The global dS metric is induced through the solution of the equation $X_0^2 - X_i^2 = -1$ via $X_0 = \sinh t$ and $X_i = \omega_i \cosh t$, where $\omega_i^2 = 1$ defines the coordinates on the $(D - 1)$ -dimensional sphere. The induced metric is then $ds^2 = dt^2 - \cosh^2 t d\Omega^2$.

The EPP is the half-space of this hyperboloid. It is defined by the condition $X_0 \geq X_D$. Any other EPP is obtained by a rotation of the latter patch around the X_0 axis, i.e., there is a family of EEPs, which is parametrized by the angle of the rotation around the X_0 axis. Such a patch of dS space respects only some subgroup of the whole isometry group. Its induced metric is $ds^2 = dt^2 - e^{2t} d\vec{x}^2 = \frac{1}{\eta^2} [d\eta^2 - d\vec{x}^2]$, where $\eta = e^{-t}$ is the conformal time, which is flowing in the reverse direction—from infinity (in the past) to zero (in the future).

The CPP also is a half-space of the hyperboloid, which is defined, e.g., by the condition $X_0 \leq X_D$; i.e., it is complementary to EPP within the global dS space. The induced metric on the CPP is $ds^2 = dt^2 - e^{-2t} d\vec{x}^2 = \frac{1}{\eta^2} \times [d\eta^2 - d\vec{x}^2]$, where $\eta = e^t \in [0, +\infty)$.

For large enough D , the theory (1) becomes nonrenormalizable. However, as we explain in the concluding section, if there is a stationary state in this theory, then in the IR limit, we do not care about UV divergences and the renormalizability of the theory in question. We assume that all couplings in all equations take their physical values; i.e., all UV divergent ($\sim \lambda^2 \log \Lambda$) or finite ($\sim \lambda^2$) contributions are absorbed into their renormalization. It is straightforward to do that during the derivation of the kinetic equation from the Dyson-Schwinger one. At this point, one should keep in mind that for the propagators which have proper Hadamard behavior, the UV divergences in dS space are the same as in the flat one. Only BD harmonics provide the proper Hadamard behavior in EPP and CPP. In global dS space, the same is true only for the Euclidean harmonics.

The scalar field harmonics in global dS space satisfy the Klein-Gordon equation

$$\left[\partial_t^2 + (D - 2) \tanh(t) \partial_t + m^2 - \frac{\Delta_{D-1}(\Omega)}{\cosh^2(t)} \right] \phi = 0, \quad (2)$$

where $\Delta_{D-1}(\Omega)$ is the Laplacian on the $(D - 1)$ -dimensional sphere. Any solution of this equation can be expanded in harmonics as follows: $\phi_p(t, \Omega) = g_p(t) Y_{p\bar{m}}(\Omega)$, where $Y_{p\bar{m}}(\Omega)$ are $(D - 1)$ -dimensional spherical functions, and $g_p(t)$ can be expressed through the hypergeometric function [1,2].

For large enough p , we can somewhat simplify Eq. (2) in the limit $t \rightarrow \pm\infty$. In fact, under these conditions and after the change of variables, $\eta_\pm = e^{\mp t}$, the global dS metric, can be approximated by $ds_\pm^2 \approx \frac{1}{\eta_\pm^2} [d\eta_\pm^2 - d\vec{x}^2]$. Here conformal time η_+ flows towards zero ($\rightarrow +0$) in future infinity and thus covers a part of the EPP; similarly, η_- starts from zero ($0+ \rightarrow$) at past infinity and covers a part

of the CPP. Furthermore, the substitution of $\phi(t \rightarrow \pm\infty) \approx \eta_{\pm}^{\frac{D-1}{2}} h_{\pm}(p\eta_{\pm}) e^{\mp i\vec{p}\cdot\vec{x}}$ into the simplified (by the limit $t \rightarrow \pm\infty$) form of (2) reduces it to that of Bessel for $h_{\pm}(p\eta)$. The index of the obtained Bessel equation is $i\mu$, where $\mu = \sqrt{m^2 - (\frac{D-1}{2})^2}$.

Throughout this paper, we will work with the massive fields from the principal series $m > (D-1)/2$. It can be shown Ref. [9], however, that for the fields from the complementary series $m < (D-1)/2$, one-loop IR divergences have the similar logarithmic character as that of massive fields. However, for the fields from the complementary series, it is not clear how to write the kinetic equation of the conventional form, because for them the necessary separation of scales between the time dependence of the harmonics and of the particle density is not obvious.

II. CONTRACTING POINCARÉ PATCH

The reason why we start our discussion with CPP is that the physics of IR divergences there is relevant for the field dynamics in global dS space.

To calculate loops in the nonstationary situation, one has to use the in-in or Schwinger-Keldysh diagrammatic technique. The Feynman rules in EPP can be found in Ref. [15]. In CPP the rules are very similar.

Each particle in the nonstationary situation is described by the matrix of propagators. The constituents of this matrix are the retarded and advanced propagators $D^{R,A}$ that carry information about the particle spectrum of the theory in question, and the Keldysh propagator D^K that carries the statistical information; i.e., it shows which levels are populated.

Because of spatial homogeneity of CPP together with the initial state, we find it convenient to perform the Fourier transform of all quantities along the spatial direction \vec{x} : $D_p^{K,R,A}(\eta_1, \eta_2) \equiv \int d^{D-1}\vec{x} e^{i\vec{p}\cdot\vec{x}} D^{K,R,A}(\eta_1, \vec{x}; \eta_2, 0)$. Then at tree level, the Keldysh propagator is $D_{0p}^K(\eta_1, \eta_2) = (\eta_1 \eta_2)^{\frac{D-1}{2}} \text{Re}[h(p\eta_1)h^*(p\eta_2)]$, the retarded and advanced propagators are $D_{0p}^R(\eta_1, \eta_2) = \theta(\eta_1 - \eta_2) 2(\eta_1 \eta_2)^{\frac{D-1}{2}} \times \text{Im}[h(p\eta_1)h^*(p\eta_2)]$, and $D_{0p}^A(\eta_1, \eta_2) = -\theta(\eta_2 - \eta_1) \times 2(\eta_1 \eta_2)^{\frac{D-1}{2}} \text{Im}[h(p\eta_1)h^*(p\eta_2)]$; $h(p\eta)$ is a solution of the Bessel equation, which we do not specify so far. The choice of the solution defines which harmonic basis we are using to quantize the theory. In other words, it specifies what we have chosen as the background state in our problem.²

²The choice of any other α vacuum except the Bunch-Davies one at past infinity of EPP would affect the UV behavior in the theory. But we are interested in studying the IR properties of the α vacua. At this point, one should keep in mind that Bogoliubov rotation in the low energy effective BCS theory does not affect the UV behavior of quantum electrodynamics (QED).

In the limit $\eta = \sqrt{\eta_1 \eta_2} \rightarrow \infty$, $\eta_1/\eta_2 = \text{const}$ and $\eta_0 \rightarrow 0$, the leading one-loop IR contribution to $D_p^K(\eta_1, \eta_2)$ is contained within the following expression:

$$\begin{aligned} D_{1p}^K(\eta_1, \eta_2) &= (\eta_1 \eta_2)^{\frac{D-1}{2}} [h(p\eta_1)h^*(p\eta_2)n_p(\eta) \\ &\quad + h(p\eta_1)h(p\eta_2)\kappa_p(\eta) + \text{c.c.}], \\ \text{where } n_p(\eta) &\approx \frac{\lambda^2 S_{D-2}}{(2\pi)^{D-1}} \int d^{D-1}q \iint_{\eta_0}^{\eta} d\eta_3 d\eta_4 (\eta_3 \eta_4)^{\frac{D-3}{2}} \\ &\quad \times h^*(p\eta_3)h(p\eta_4)h^*(q\eta_3)h(q\eta_4) \\ &\quad \times h^*(|q-p|\eta_3)h(|p-q|\eta_4), \quad \text{and} \\ \kappa_p(\eta) &\approx -\frac{2\lambda^2 S_{D-2}}{(2\pi)^{D-1}} \int d^{D-1}q \int_{\eta_0}^{\eta} d\eta_3 \\ &\quad \times \int_{\eta_0}^{\eta_3} d\eta_4 (\eta_3 \eta_4)^{\frac{D-3}{2}} \\ &\quad \times h^*(p\eta_3)h^*(p\eta_4)h^*(q\eta_3)h(q\eta_4) \\ &\quad \times h^*(|q-p|\eta_3)h(|q-p|\eta_4). \end{aligned} \quad (3)$$

The details of a similar calculation in EPP can be found in Refs. [4–6]. The largest IR contribution to (3) comes from the region of integration over dq , where $q \gg p$. It is instructive to study the character of such divergences for different choices of the harmonics $h(p\eta)$. If $h(x) \propto \mathcal{H}_{i\mu}^{(2)}(x)$, the IR divergence is present both in n_p and κ_p , and it is proportional to $\log(\eta/\eta_0) = t - t_0$ if $p\eta \ll 1$, which is the proper time elapsed from $\eta_0 = e^{t_0}$ to $\eta = e^t$. If, however, $p\eta \gg 1$, then the divergence is proportional to $\log(1/p\eta_0)$. The explanation for the latter behavior is given at the end of the next subsection.

The in harmonics of the CPP are proportional to the Bessel function $Y_{i\mu}(x)$. In this case, $n_p(\eta)$ diverges as $\log(\eta/\eta_0)$ if $p\eta \ll 1$, and as $\log(1/p\eta_0)$ when $p\eta \gg 1$. At the same time, $\kappa_p(\eta)$ does not have any IR divergence. We interpret this fact as follows: for the in harmonics the backreaction on the background state due to the particle production is small. That is the reason why we choose these harmonics for the proper definition of the quasiparticles in the IR limit [6] (see also the discussion below). For any other type of α harmonics, the IR behavior of n_p and κ_p is similar to that of the $\mathcal{H}_{i\mu}^{(2)}(p\eta)$ harmonics.

A. The physical interpretation of the one-loop IR divergence

Now we will show that n_p given in (3) is the total particle number density per comoving volume $\langle a_p^+ a_p \rangle$ created by the gravitational field during the time passed from η_0 to η . Similarly, κ_p is the measure of the strength of the backreaction $\langle a_p a_{-p} \rangle$ due to the particle production process on the background state during the same time period.

To explain the statement of the last paragraph, for the moment let us forget about the presence of κ_p and consider the situation in flat space with the plane wave harmonics. If the time of the turning on self-interactions is t_0 and the time of the observation is t , then the kinetic equation is (the derivation can be found, e.g., in Ref. [6])

$$\begin{aligned} \frac{dn_p}{dt} \propto & -\lambda^2 \int \frac{d^{D-1}q}{\epsilon(p)\epsilon(q)\epsilon(p-q)} \left\{ \int_{t_0}^t dt' \cos[(-\epsilon(p) + \epsilon(q) + \epsilon(p-q))(t-t')] [(1+n_p)n_q n_{|p-q|} \right. \\ & - n_p(1+n_q)(1+n_{|p-q|})](t) + 2 \int_{t_0}^t dt' \cos[(-\epsilon(q) + \epsilon(q-p) + \epsilon(p))(t-t')] [n_q(1+n_{|q-p|})(1+n_p) \\ & - (1+n_q)n_{|q-p|}n_p](t) + \int_{t_0}^t dt' \cos[(\epsilon(q) + \epsilon(q+p) + \epsilon(p))(t-t')] [(1+n_q)(1+n_{q+p})(1+n_p) \\ & \left. - n_q n_{q+p} n_p](t) \right\}, \end{aligned} \quad (4)$$

where $\epsilon(p)$ is the energy of the particle with the momentum \vec{p} . The rhs of this equation is the collision integral I_{Coll} . In the limit $(t-t_0) \rightarrow \infty$ the dt' integrals transform into the δ functions ensuring energy conservation in some processes, which we will define now.

The expressions multiplying the three dt' integrals in (4) have the following meaning. The first term describes the competition between the following two processes. One is that the particle \vec{p} decays into two excitations: \vec{q} and $\vec{p}-\vec{q}$; it corresponds to the term $n_p(1+n_q)(1+n_{|p-q|})$ and appears with the minus sign in the collision integral because it describes the loss of the excitation with the momentum \vec{p} . The inverse gain process corresponding to the term $(1+n_p)n_q n_{|p-q|}$ with the plus sign is such that two particles \vec{q} and $\vec{p}-\vec{q}$ are merged together to create the excitation \vec{p} .

The second term in (4) also describes two competing processes. The first is that the particle \vec{p} joins with another one, $\vec{q}-\vec{p}$, to create the excitation with the momentum \vec{q} . This is the loss process. The inverse gain process is the one in which the particle \vec{q} decays into two; one of which is with the momentum \vec{p} . The coefficient 2 in front of this term is just the combinatoric factor.

Similarly, the third term describes two processes. The gain process is when three particles, one of which is with the momentum \vec{p} , are created by an external field, if any. The loss process is when three such excitations are annihilated into the vacuum.

All six types of processes are not allowed by the energy-momentum conservation for *massive* fields with ϕ^3 self-interaction in *flat* space-time. Hence, the collision integral is vanishing as $(t-t_0) \rightarrow \infty$.

However, in dS space they are allowed [16–20] because there is no energy conservation. As we show below, the collision integral in dS space can be obtained from (4) as follows. One exchanges dt' for $d\eta'$, multiplies by the proper weight coming from the metric, and then uses under the corresponding integrals the dS harmonics $g_p(\eta) = \eta^{\frac{p-1}{2}} h(p\eta)$ instead of the plane waves $e^{-i\epsilon(p)t}/\sqrt{\epsilon(p)}$. Then, in place of the above-mentioned δ functions, which ensure energy conservation, there are some expressions

whose physical meaning is that they define the differential rates (per given range of \vec{q}) of the processes described in the above three paragraphs.

Now, put n_p to zero under the collision integral (4) with the substitutions described in the previous paragraph. The only term which survives is present in the last line of (4) and describes particle creation by the gravitational field. If one takes the integral of this term over the conformal time from η_0 to η , he obtains exactly the expression for $n_p(\eta)$ represented in (3). Using the collision integral for κ_p derived in Refs. [4,6] (see also the discussion below), one can make a similar observation about the origin of the corresponding expression in (3).

Thus, in the nonstationary situations, when I_{Coll} is not zero, it is natural to expect the linear divergence $\propto (t-t_0)$ in n_p and κ_p . That should be true for the homogeneous in time background fields (such as, e.g., constant electric fields in QED) when particle production rates are constant in time. But, in the time-dependent metric of dS space, one encounters a bit different situation. In particular, in EPP the largest IR contribution has the form of $\lambda^2 \log(p\eta)$, and there is no IR divergence, as one takes the moment of turning on self-interactions to past infinity of the EPP ($\eta_0 \rightarrow +\infty$). That happens because the creation of particles with comoving momentum p effectively starts at $\eta_* \sim 1/p$ rather than at $\eta_0 \rightarrow +\infty$. Similarly, in the case of CPP, the particle creation process goes on until $\eta_* \sim 1/p$ and then stops. We encounter such a situation, because, first of all, the collision integral is not just a constant and depends on time. Second, past (future) infinity in EPP (CPP) corresponds to the UV limit of the physical momentum. In this limit, $g_p(\eta)$ harmonics behave as plane waves $e^{ip\eta}$. As the result, all the rates inside the collision integral can be well approximated by δ functions ensuring energy conservation [6]. The latter fact has a clear physical meaning because the modes whose wavelengths are much smaller than the cosmological horizon size, practically do not feel the curvature of the space-time. Hence, for them the conditions should be similar to those in flat space.

Thus, the collision integral becomes negligible as $p\eta \rightarrow \infty$ both in EPP and CPP. That is the reason why

independently of the type of harmonics, in EPP one can put the moment of turning on self-interactions η_0 to past infinity. Similarly, these observations explain the $\log(1/p\eta_0)$ behavior of the leading IR contribution in the CPP when $\eta > 1/p$.

B. Kinetic equation in the contracting Poincaré patch

In (3) we have found the created particle density n_p and anomalous average κ_p under the implicit assumption that throughout the time period $[\eta_0, \eta]$ both n_p and κ_p have been zero inside the collision integral. In fact, we have been using the tree-level propagators in the calculation of (3). For the proper consideration of the phenomenon, one has to redo the calculation and take into account the change of n_p and κ_p in time. If we care only about IR contributions, this is done with the use of the following ansatz for the IR behavior of the exact Keldysh propagator [4]:

$$D_p^K(\eta_1, \eta_2) = (\eta_1 \eta_2)^{\frac{D-1}{2}} [h(p\eta_1)h^*(p\eta_2)n_p(\eta) + h(p\eta_1)h(p\eta_2)\kappa_p(\eta) + \text{c.c.}],$$

$$\eta = \sqrt{\eta_1 \eta_2}, \quad (5)$$

where one has to assume the usual separation of scales in the kinetic problem; i.e., that n_p and κ_p are slow functions

of η in comparison with the harmonics $h(p\eta)$. Such a separation of scales is quite obvious for the massive fields from the principal series because their harmonics do oscillate in the IR limit. However, for the light fields from the complementary series, the separation of scales is not so transparent because their harmonics do not oscillate in the same limit.

It is not hard to see that after the assumption about the IR behavior of the exact propagator (5), the problem of the calculation of n_p and κ_p reduces to the solution of the Dyson-Schwinger equation in the Schwinger-Keldysh diagrammatic technique [4]. In the IR limit, which is of our interest, the Dyson-Schwinger equation reduces to a variant of the kinetic equation. One can write the kinetic equation both in EPP and CPP [4,6]. With the use of the following matrixes:

$$N_p(\eta_1, \eta_2) = \eta_2^{\frac{D-1}{2}} \begin{pmatrix} n_p(\eta_1)h^*(p\eta_2) & \kappa_p(\eta_1)h(p\eta_2) \\ \kappa_p(\eta_1)h(p\eta_2) & n_p(\eta_1)h^*(p\eta_2) \end{pmatrix},$$

$$P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (6)$$

The system of kinetic equations for n_p and κ_p can be written compactly. The real equation has the form

$$\frac{dn_p(\eta)}{d\eta} = 2\lambda^2 \int \frac{d^{D-1}q}{(2\pi)^{(D-1)}} \int_0^\infty \frac{d\eta'}{(\eta\eta')^D} \text{Re Tr}\{C_{p,q,p-q}(\eta)[(1+N_p^*)N_q N_{p-q} - N_p^*(1+N_q)(1+N_{p-q})](\eta, \eta')$$

$$+ 2C_{q,q-p,p}(\eta)[N_q^*(1+N_{q-p})(1+N_p) - (1+N_q^*)N_{q-p}N_p](\eta, \eta')$$

$$+ D_{p,q,p+q}(\eta)[(1+N_p)(1+N_q)(1+N_{p+q}) - N_p N_q N_{p+q}](\eta, \eta')\} + [N \rightarrow PN]. \quad (7)$$

At the same time the complex equation is as follows:

$$\frac{d\kappa_p(\eta)}{d\eta} = -2\lambda^2 \int \frac{d^{D-1}q}{(2\pi)^{(D-1)}} \int_0^\infty \frac{d\eta'}{(\eta\eta')^D} (\vec{p} \rightarrow -\vec{p}) \text{Tr}\{C_{p,q,p-q}(\eta)[(1+N_p)(1+N_q)(1+N_{p-q}) - N_p N_q N_{p-q}](\eta, \eta')$$

$$+ 2C_{q,q-p,p}^*(\eta)[N_q^*(1+N_{q-p})(1+N_p) - (1+N_q^*)N_{q-p}N_p](\eta, \eta')$$

$$+ D_{p,q,p+q}^*(\eta)[(1+N_p)N_q^* N_{p+q}^* - N_p(1+N_q^*)(1+N_{p+q}^*)](\eta, \eta')\} - [N \rightarrow PN]. \quad (8)$$

The term $[N \rightarrow PN]$ means that we have to add to the explicitly written expressions in (7) and (8) the same quantities with every N substituted by the product PN ; Re Tr means that one has to take the real part and the trace of the expression following after these signs; $(\vec{p} \rightarrow -\vec{p})\text{Tr}$ means that one has to take the trace and add to the expression following after these signs the same term with the exchange $\vec{p} \rightarrow -\vec{p}$. Finally, $C_{k_1 k_2 k_3}(\eta) = \eta^{\frac{3(D-1)}{2}} h^*(k_1 \eta) \times h(k_2 \eta) h(k_3 \eta)$ and $D_{k_1 k_2 k_3}(\eta) = \eta^{\frac{3(D-1)}{2}} h(k_1 \eta) h(k_2 \eta) \times h(k_3 \eta)$.

Actually, the system of Eqs. (7) and (8) represents just a preliminary version of the kinetic equations. It is not yet suitable to sum only the leading IR terms in all loops. It contains the latter contributions, but on top of that it also accounts for some of the subleading expressions.

In particular, one should restrict the dq integrals in (7) and (8) to the region $q \gg p$. It is this region from where the leading IR contribution comes. For more details, see Refs. [4,6] (see also the discussion in the next subsection).

Because of the presence of κ_p in (7), we have extra terms on top of those which are shown in (4). All of them can be obtained from (4) via the simultaneous substitutions of $(1+n_k)$'s and n_k 's by κ_k 's or κ_k^* 's. E.g., we encounter terms of the type

$$[(1+n_p)\kappa_k n_{p-k} - n_p \kappa_k (1+n_{p-k})]. \quad (9)$$

The meaning of (9) is that it describes the following two competing processes: The particle with the momentum \vec{p} is lost (gained) in such a situation, in which instead of the creation (annihilation) of the two particles, \vec{k} and $\vec{p} - \vec{k}$, we

obtain single $\vec{p} - \vec{k}$ excitation, and the missing momentum \vec{k} is gone into (taken from) the background quantum state of the theory.

Similarly, we encounter terms of the type

$$[(1 + n_p)\kappa_k \kappa_{p-k} - n_p \kappa_k \kappa_{p-k}], \quad (10)$$

which describe the processes in which both momenta \vec{k} and $\vec{p} - \vec{k}$ are coming from (going to) the background state. These observations, in particular, justify the interpretation of κ_p as the measure of the strength of the backreaction on the background state of the theory.

C. Solving the system of kinetic equations

Spatially homogeneous kinetic equations in EPP and CPP can be trivially related to each other, because to perform the map from EPP to CPP, one just has to flip the limits of $d\eta'$ integration inside the collision integrals in (7) and (8). On top of that, it is necessary to make the change of $h(x)$ to $h^*(x)$ because the positive energy harmonics are exchanged with the negative ones under the flip of the time direction. As a result, solutions in both patches can also be mapped to each other. That is physically meaningful because the system of kinetic equations (7) and (8) is

$$\begin{aligned} \frac{dn_p(\eta)}{d \log \eta} = & \frac{\lambda^2 S_{D-2}}{(2\pi)^{D-1} \mu} \int_0^\infty dq \eta (q\eta)^{\frac{D-1}{2}} \int_0^\infty d\eta' q (q\eta')^{\frac{D-1}{2}} \times \{ \text{Re}[(q\eta)^{-i\mu} V(q\eta) (q\eta')^{i\mu} V^*(q\eta')] [(1 + n_p)n_q^2 \\ & - n_p(1 + n_q)^2](\eta) + 2 \text{Re}[(q\eta)^{i\mu} W(q\eta) (q\eta')^{-i\mu} W(q\eta')] [n_q(1 + n_q)(1 + n_p) - (1 + n_q)n_q n_p](\eta) \\ & + \text{Re}[(q\eta)^{i\mu} V(q\eta) (q\eta')^{-i\mu} V^*(q\eta')] [(1 + n_q)^2(1 + n_p) - n_q^2 n_p](\eta) \}. \end{aligned} \quad (11)$$

To obtain (11) from (7), we have neglected p with respect to q under the integral and Taylor expanded $h(p\eta)$ and $h(p\eta')$ in the limit $p\eta, p\eta' \rightarrow 0$. The latter approximations allow one to obtain the equation which accounts for only leading IR contributions in all loops [6]. Furthermore, we have subtracted from $h^2(x)$, $(h^*(x))^2$, and $|h(x)|^2$ some polynomials of $1/|x|$ to obtain $V(x) = h^2(x) - \frac{\text{const}}{|x|} - \dots$ and $W(x) = |h(x)|^2 - \frac{\text{const}}{|x|} - \dots$. That makes the collision integral well defined and does not change the IR behavior [6].

Equation (11) does not have the Planckian distribution as its stationary solution because there is no energy conservation. However, one can find a distribution which annihilates the collision integral of (11) in a different manner: while Planckian distribution in ordinary kinetic equation annihilates each term, describing aforementioned couples of competing processes, separately, in the present case we will see the cancellation between different terms. The situation is somewhat similar to the one in which appears the stationary Kolmogorov scaling solution in the turbulent hydrodynamics, as is natural to expect in a system with energy pumping.

For very small densities, the only two terms which survive in the collision integral of (11) are those which

valid for the spatially homogeneous states and is imposed on the quantities which are attributed to the comoving volume.

Before continuing with the solutions of (7) and (8), let us stress that these equations are not yet unambiguously defined because one has to specify harmonics $h(x)$ in their collision integrals. This issue is related to the question of the proper definition of quasiparticles in the IR limit. One should keep in mind, however, that the quantum kinetic (or better to say Dyson-Schwinger) equations and their solutions for different choices of $h(p\eta)$ can be mapped to each other via Bogoliubov transformations.

Intuition from condensed matter should tell one that the proper choice of $h(p\eta)$ has to correspond to the situation in which there is a solution with $\kappa_p = 0$. It was shown in Ref. [6] that in EPP there is such a solution of the IR limit of (7) and (8), if one chooses out-Jost harmonics $h(x) \sim J_{i\mu}(x)$. Moreover, it is stable under small linearized perturbation of κ_p [4]. The systems of kinetic equations for the other harmonics do not allow such a solution.

All these observations can be extended to the CPP. Thus, for the out-Jost (in-Jost) harmonics in EPP (CPP) the system of Eqs. (7) and (8) can be reduced to the single Eq. (7), in which κ_p is set to zero [4,6]:

are responsible for the particle production from the vacuum and for the decay of the produced particles into two [6]. As a result, in the EPP the behavior of the solution in the vicinity of the stationary distribution (i.e., as $p\eta \ll 1$) is as follows [6]: $n_p(\eta) = \frac{\Gamma_2}{\Gamma_1} [C(p\eta)^{\Gamma_1} + 1]$, where C is some real constant which depends on the initial conditions, Γ_2 is the total particle production rate, and Γ_1 is the total particle decay rate. This solution is self-consistent only for heavy enough particles $m \gg 1$ because then the equilibrium distribution $\Gamma_2/\Gamma_1 \approx e^{-2\pi\mu}$ is much less than unity.

Such an EPP's stationary solution is mapped to

$$n_p(\eta) = \frac{\Gamma_2}{\Gamma_1} \left[C \left(\frac{\eta_0}{\eta} \right)^{\Gamma_1} + 1 \right] \quad (12)$$

after the conversion into the CPP. For the given comoving momentum p , it is valid until $\eta_* \sim 1/p$. As $p\eta$ becomes comparable to unity, the approximation in which (11) is derived breaks down. But as we have explained above, right after η_* , the particle production and decay processes stop; i.e., n_p remains constant. One encounters a similar situation in EPP: before $\eta_* \sim 1/p$ we cannot use (11), but

also before this moment of time the particle production is suppressed.

In (12) we see the example of the restoration of dS isometry in CPP after the summation of all leading loop contributions. The question is under what type of initial conditions does the system in question drop into such a stage as (12) in CPP (and/or in EPP)? To obtain the answer, one has to perform the Bogoliubov rotation to the out (in) harmonics in EPP (CPP), and check if the resulting values of n_p and κ_p are small or not, right at the moment when they start to change (which is η_0 in CPP and η_* in EPP). If they are small enough, then it is natural to expect that κ_p flows to zero and n_p flows to the Gibbons-Hawking value in the IR limit.

Note that the number density per physical volume, which scales as $1/\eta^{D-1}$, decays in EPP and grows in CPP. Hence, it may seem that the backreaction from the produced particles eventually will become relevant in CPP. That is indeed true in the case of the presence of any slightly spatially inhomogeneous perturbation. But in the circumstances of the ideal spatial homogeneity, that is not true because the presence of the stationary state means that the dS isometry is restored. As a result, in terms of the invariant expressions such as physical volume and physical momentum $p\eta$, all invariant quantities remain finite. E.g., the total particle density per physical volume is equal to $N = \eta^{D-1} \int d^{D-1} p n_p(\eta)$ and does not explode.

However, we will see now that the kinetic equation in question has yet another peculiar solution both in CPP and EPP. In Ref. [6] it was shown that in CPP there is a solution as follows: $n(\eta) \sim \frac{1}{A - \bar{\Gamma} \log \eta} \sim \frac{1}{\bar{\Gamma} \log \frac{\eta}{\eta_*}}$ for very low momenta $p\eta \rightarrow 0$. It is valid for $\eta < \eta_* = e^{\text{const}/\lambda^2} \gg 1$; A is a constant which depends on the initial state, and $\bar{\Gamma} \propto \frac{\lambda^2}{m^2}$. The interpretation of this solution is as follows: Because of the production of particles at low momenta (both by the gravitational field and by the decays of particles from higher levels) at high enough initial densities, sooner or later we have the explosion of their density per comoving volume.

This solution is mapped to

$$n(\eta) \sim \frac{1}{\bar{\Gamma} \log \frac{\eta}{\eta_*}} \quad (13)$$

in the EPP. It is also valid for very low momenta $p\eta \rightarrow 0$, for $\eta > \eta_*$, where $\eta_* = e^{-\text{const}/\lambda^2} \ll 1$.

It is worth stressing at this moment that the solution (13) is possible because of two specific features of dS space [6]. The first one is the flatness of the IR spectrum in dS space—weak dependence of the harmonics on the momenta in the IR limit. The second is that the background gravitational field makes possible the particle creation and decay processes. In particular, one can show that the scattering processes in ϕ^4 theory, which are allowed even in flat space, do not contribute to the analog of (13); only the particle production and decay processes contribute.

Thus, even in the EPP there is the explosive growth of the particle density if its initial value was big enough. The explosion happens at finite proper time and expansion of EPP will not save the situation. That will cause a modification of the dS metric.

Here, it is also appropriate to ask the following question: Under what initial conditions does the system drop into such an explosive stage both in EPP and CPP? We are not yet in a position to give a conclusive answer to this question.

III. GLOBAL DE SITTER SPACE-TIME

To begin with, following Ref. [5] we would like to show that in global dS space, the moment of turning on self-interactions cannot be taken to past infinity. To find the IR divergences at one loop, we can use the metric of either Fig. 1 or Fig. 2. However, to define the kinetic equation in global dS space, we prefer to work with the slicing depicted in Fig. 2, because as we explained above, for the metric of Fig. 1 the collision integral vanishes for the times $\eta_- > 1/p$ and $\eta_+ > 1/p$, which confuses the whole picture of particle kinetics.

Unlike higher-dimensional dS space with the spatial slicing of Fig. 2, in 2D global dS space the one-loop calculation can be made quite explicit. So let us present it before continuing with the IR divergences in general dimensions. Below we will work with even space-time dimensional dS space. In odd-dimensional dS space the situation is different, but in many respects it is similar.

Because of spatial homogeneity of 2D global dS space, together with the initial state, we find it convenient to perform the Fourier transform of all quantities along the spatial direction $\varphi \in [0, 2\pi]$: $D_p^{K,R,A}(t_1, t_2) \equiv \int_0^{2\pi} d\varphi e^{ip\varphi} \times D^{K,R,A}(t_1, \varphi; t_2, 0)$. Then the one-loop contribution to D_p^K is as follows:

$$\begin{aligned} D_{1p}^K(t_1, t_2) = & -\lambda^2 \sum_{q=-\infty}^{+\infty} \iint_{t_0}^{+\infty} dt_3 dt_4 \cosh(t_3) \cosh(t_4) \left[D_{0p}^R(t_1, t_3) D_{0q}^K(t_3, t_4) D_{0(p-q)}^K(t_3, t_4) D_{0p}^A(t_4, t_2) \right. \\ & + 2D_{0p}^R(t_1, t_3) D_{0q}^R(t_3, t_4) D_{0(p-q)}^K(t_3, t_4) D_{0p}^K(t_4, t_2) + 2D_{0p}^K(t_1, t_3) D_{0q}^K(t_3, t_4) D_{0(p-q)}^A(t_3, t_4) D_{0p}^A(t_4, t_2) \\ & \left. + -\frac{1}{4} D_{0p}^R(t_1, t_3) D_{0q}^R(t_3, t_4) D_{0(p-q)}^R(t_3, t_4) D_{0p}^A(t_4, t_2) - \frac{1}{4} D_{0p}^R(t_1, t_3) D_{0q}^A(t_3, t_4) D_{0(p-q)}^A(t_3, t_4) D_{0p}^A(t_4, t_2) \right]. \end{aligned} \quad (14)$$

Here the Keldysh propagator is $D_{0p}^K(t_1, t_2) = \text{Re}[g_p(t_1)g_p^*(t_2)]$, the retarded and advanced propagators are $D_{0p}^R(t_1, t_2) = \theta(t_1 - t_2)2 \text{Im}[g_p(t_1)g_p^*(t_2)]$, and $D_{0p}^A(t_1, t_2) = -\theta(t_2 - t_1)2 \text{Im}[g_p(t_1)g_p^*(t_2)]$. $g_p(t)$ is a solution of the Klein-Gordon equation, which we do not specify so far; t_0 is the moment when self-interactions are turned on.

The above one-loop expression can be rewritten as

$$\begin{aligned}
D_{1p}^K(t_1, t_2) = & -\frac{\lambda^2}{2} g_p(t_1)g_p^*(t_2) \sum_q \iint_{t_0}^{+\infty} dt_3 dt_4 \cosh(t_3) \cosh(t_4) g_p^*(t_3)g_p(t_4) \times \{-\theta(t_1 - t_3)\theta(t_2 - t_4) \\
& \times [g_q(t_3)g_q^*(t_4)g_{|p-q|}(t_3)g_{|p-q|}^*(t_4) + \text{c.c.}] + [\theta(t_1 - t_3)\theta(t_3 - t_4) + \theta(t_2 - t_4)\theta(t_4 - t_3)] \\
& \times [g_q(t_3)g_q^*(t_4)g_{|p-q|}(t_3)g_{|p-q|}^*(t_4) - \text{c.c.}]\} \\
& -\frac{\lambda^2}{2} g_p(t_1)g_p(t_2) \sum_q \iint_{t_0}^{+\infty} dt_3 dt_4 \cosh(t_3) \cosh(t_4) g_p^*(t_3)g_p^*(t_4) \times \{\theta(t_1 - t_3)\theta(t_2 - t_4) \\
& \times [g_q(t_3)g_q^*(t_4)g_{|p-q|}(t_3)g_{|p-q|}^*(t_4) + \text{c.c.}] + [\theta(t_1 - t_3)\theta(t_3 - t_4) - \theta(t_2 - t_4)\theta(t_4 - t_3)] \\
& \times [g_q(t_3)g_q^*(t_4)g_{|p-q|}(t_3)g_{|p-q|}^*(t_4) - \text{c.c.}]\} + \text{c.c.}
\end{aligned} \tag{15}$$

We need to find the leading divergent contribution to this quantity in the limit $t \equiv \frac{t_1+t_2}{2} \rightarrow +\infty$, $\tau \equiv t_1 - t_2 = \text{const}$ and $t_0 \rightarrow -\infty$. First, to do that we can substitute $t_{1,2}$ by t in the arguments of the θ functions. Second, if $t \rightarrow +\infty$ and $t_0 \rightarrow -\infty$, then the leading IR contributions to D_{1p}^K can appear only from the regions of integration over $t_{3,4}$ in the vicinity of t and t_0 .

As we have explained in the introductory section, for arbitrary dimension in these regions $g_p(t)$ can be approximated by

$$g_p(t) \approx \begin{cases} \eta_+^{\frac{D-1}{2}} h_+(p\eta_+), & \eta_+ = e^{-t}, \quad t \rightarrow +\infty \\ \eta_-^{\frac{D-1}{2}} h_-(p\eta_-), & \eta_- = e^t, \quad t \rightarrow -\infty \end{cases} \tag{16}$$

for the large enough p . Then as $p\eta \rightarrow 0$ and $p\eta_0 \rightarrow 0$, the leading IR contribution to D_{1p}^K can be estimated in any dimension. It is contained in the expressions as follows [5]:

$$\begin{aligned}
D_{1p}^K(t_1, t_2) = & g_p(t_1)g_p^*(t_2)n_p(t) + g_p(t_1)g_p(t_2)\kappa_p(t) + \text{c.c.}, \\
\text{where } n_p(t) \approx & \frac{\lambda^2 S_{D-2}}{(2\pi)^{D-1}} \int d^{D-1}q \iint_{\infty}^{\eta} d\eta_3 d\eta_4 (\eta_3 \eta_4)^{\frac{D-3}{2}} \times h_+(p\eta_3)h_+(p\eta_4)h_+(q\eta_3)h_+(q\eta_4)h_+(|q-p|\eta_3) \\
& \times h_+(|p-q|\eta_4) + \frac{\lambda^2 S_{D-2}}{(2\pi)^{D-1}} \int d^{D-1}q \iint_{\eta_0}^{\infty} d\eta_3 \eta_4 (\eta_3 \eta_4)^{\frac{D-3}{2}} \times h_-(p\eta_3)h_-(p\eta_4)h_-(q\eta_3)h_-(q\eta_4) \\
& \times h_-(|q-p|\eta_3)h_-(|q-p|\eta_4), \quad \text{and} \\
\kappa_p(t) \approx & -\frac{2\lambda^2 S_{D-2}}{(2\pi)^{D-1}} \int d^{D-1}q \int_{\infty}^{\eta} d\eta_3 \int_{\infty}^{\eta_3} d\eta_4 (\eta_3 \eta_4)^{\frac{D-3}{2}} \times h_+(p\eta_3)h_+(p\eta_4)h_+(q\eta_3)h_+(q\eta_4)h_+(|q-p|\eta_3) \\
& \times h_+(|q-p|\eta_4) - \frac{2\lambda^2 S_{D-2}}{(2\pi)^{D-1}} \int d^{D-1}q \int_{\eta_0}^{\infty} d\eta_3 \int_{\eta_0}^{\eta_3} d\eta_4 (\eta_3 \eta_4)^{\frac{D-3}{2}} \times h_-(p\eta_3)h_-(p\eta_4)h_-(q\eta_3) \\
& \times h_-(q\eta_4)h_-(|q-p|\eta_3)h_-(|q-p|\eta_4).
\end{aligned} \tag{17}$$

Here $\eta = e^{-t}$ and $\eta_0 = e^{t_0}$. Thus, the IR divergence in global dS space is just the sum of the contributions from the contracting (past infinity) and expanding (future infinity) regions (Poincaré patches) of the whole space.

Obviously, one can do the same calculation also with the metric of Fig. 1 and obtain the same result as (17) for the IR divergences when $t \rightarrow +\infty$ and $t_0 \rightarrow -\infty$.

A. IR divergence for different harmonics

In this subsection we describe the character of the IR divergences for different $h(p\eta)$ —the solutions of the Bessel equation. That could help to specify the convenient choice of the harmonics for the proper definition of quasiparticles in the IR limit.

We start with the Euclidean harmonics [1,2] which are CPT invariant. I.e., in this case, $h_{\pm}^*(x) = h_{\pm}(x) \propto \mathcal{H}_{i\mu}^{(1)}(x)$, where $\mathcal{H}_{i\mu}^{(1)}$ is the Hankel function of the first kind. Using that $h_{\pm}(x) \approx A_{\pm}x^{i\mu} + A_{\mp}x^{-i\mu}$ as $x \rightarrow 0$ (with some known complex constants A_{\pm}), we obtain the IR divergence as follows:

$$\begin{aligned} n_p(t) &\approx \frac{\lambda^2 S_{D-2}}{(2\pi)^{D-1}} \log\left(\frac{1}{p^2 \eta \eta_0}\right) \iint_0^{\infty} dx_3 dx_4 (x_3 x_4)^{\frac{D-3}{2}} \\ &\quad \times \left[|A_+|^2 \left(\frac{x_4}{x_3}\right)^{i\mu} + |A_-|^2 \left(\frac{x_3}{x_4}\right)^{i\mu} \right] \\ &\quad \times h^2(x_3) [h^*(x_4)]^2, \\ \kappa_p(t) &\approx -\frac{2\lambda^2 S_{D-2}}{(2\pi)^{D-1}} \log\left(\frac{1}{p^2 \eta \eta_0}\right) A_+ A_- \int_{\infty}^0 dx_3 \\ &\quad \times \int_{\infty}^{x_3} dx_4 (x_3 x_4)^{\frac{D-3}{2}} \times \left[\left(\frac{x_4}{x_3}\right)^{i\mu} + \left(\frac{x_3}{x_4}\right)^{i\mu} \right] \\ &\quad \times h^2(x_4) [h^*(x_3)]^2. \end{aligned} \quad (18)$$

It also follows from the region of integration over dq in (17) in which $q \gg p$. Note that to calculate the expression for κ_p , we have used the identity $A_+^* A_-^* = A_+ A_-$. Thus, for the Euclidean vacuum the leading IR divergence is proportional to $\log(1/\eta \eta_0) = (t - t_0)$ and is present both in n_p and κ_p .

For the in harmonics, $h_{-}(x)$ is proportional to the Bessel function $Y_{i\mu}(x)$. Hence, $h_{-}(x) \approx Ax^{-i\mu}$, as $x \rightarrow 0$ (past infinity). At the same time, for this case $h_{+}(x) \approx C_+ x^{i\mu} + C_- x^{-i\mu}$, as $x \rightarrow 0$ (future infinity). Here A and C_{\pm} are some complex constants related to A_+ and A_- via the corresponding Bogoliubov rotation.

Then the leading IR contribution in this case looks as

$$\begin{aligned} n_p(t) &\approx \frac{\lambda^2 S_{D-2}}{(2\pi)^{D-1}} \log\left(\frac{1}{p\eta}\right) \iint_0^{\infty} dx_3 dx_4 (x_3 x_4)^{\frac{D-3}{2}} \\ &\quad \times \left[|C_+|^2 \left(\frac{x_4}{x_3}\right)^{i\mu} + |C_-|^2 \left(\frac{x_3}{x_4}\right)^{i\mu} \right] V_+(x_3) V_+^*(x_4) \\ &\quad + \frac{\lambda^2 S_{D-2}}{(2\pi)^{D-1}} \log\left(\frac{1}{p\eta_0}\right) \iint_0^{\infty} dx_3 dx_4 (x_3 x_4)^{\frac{D-3}{2}} \\ &\quad \times |A|^2 \left(\frac{x_4}{x_3}\right)^{i\mu} V_-^*(x_3) V_-(x_4), \\ \kappa_p(t) &\approx -\frac{2\lambda^2 S_{D-2}}{(2\pi)^{D-1}} \log\left(\frac{1}{p\eta}\right) C_+^* C_-^* \int_{\infty}^0 dx_3 \int_{\infty}^{x_3} dx_4 \\ &\quad \times (x_3 x_4)^{\frac{D-3}{2}} \left[\left(\frac{x_4}{x_3}\right)^{i\mu} + \left(\frac{x_3}{x_4}\right)^{i\mu} \right] V_+(x_3) V_+^*(x_4), \end{aligned} \quad (19)$$

where $V_{\pm}(x) = h_{\pm}^2(x) - \frac{\text{const}}{|x|} - \dots$ [6]. Note that for the in harmonics, $n_p(t)$ depends separately on t and t_0 rather than on their difference ($t - t_0$), and $\kappa_p(t)$ does not diverge as $t_0 \rightarrow -\infty$. A similar situation also appears for the out harmonics with one crucial difference: Now κ_p does diverge as $t_0 \rightarrow -\infty$ but is finite as $t \rightarrow +\infty$. One encounters an analogous situation for the one-loop contribution in the scalar QED on the electric pulse background [21].

For the other α harmonics, the leading IR contributions to n_p and κ_p also unlike the case of the Euclidean harmonics depend on t and t_0 separately, and they both diverge when $t \rightarrow +\infty$ and/or when $t_0 \rightarrow -\infty$.

B. Kinetic equation in global de Sitter space

Having in mind the discussion of CPP in the previous section, it is not hard to derive the system of kinetic equations in global dS space:

$$\begin{aligned} \frac{dn_j}{dt} &= [N \rightarrow PN] + 2\lambda^2 \sum_{j_1, \vec{m}_1; j_2, \vec{m}_2} \text{Re Tr} \left\{ J_{j, \vec{m}; j_1, \vec{m}_1; j_2, \vec{m}_2} \int_{-\infty}^{+\infty} dt' C_{j, j_1, j_2}^*(t) \times [(1 + N_j^*) N_{j_1} N_{j_2} \right. \\ &\quad - N_j^* (1 + N_{j_1})(1 + N_{j_2})](t, t') + 2J_{j_1, \vec{m}_1; j_2, \vec{m}_2; j, \vec{m}} \int_{-\infty}^{+\infty} dt' C_{j_1, j_2, j}^*(t) [N_{j_1}^* (1 + N_{j_2})(1 + N_j) \\ &\quad - (1 + N_{j_1}^*) N_{j_2} N_j](t, t') + I_{j, \vec{m}; j_1, \vec{m}_1; j_2, \vec{m}_2} \int_{-\infty}^{+\infty} dt' D_{j, j_1, j_2}^*(t) [(1 + N_j)(1 + N_{j_1})(1 + N_{j_2}) - N_j N_{j_1} N_{j_2}](t, t') \left. \right\} \end{aligned} \quad (20)$$

and

$$\begin{aligned} \frac{d\kappa_j}{dt} &= -[N \rightarrow PN] - 2\lambda^2 \sum_{j_1, \vec{m}_1; j_2, \vec{m}_2} (\vec{m} \rightarrow -\vec{m}) \text{Tr} \left\{ J_{j, \vec{m}; j_1, \vec{m}_1; j_2, \vec{m}_2} \int_{-\infty}^t dt' C_{j, j_1, j_2}^*(t) \times [(1 + N_j)(1 + N_{j_1})(1 + N_{j_2}) \right. \\ &\quad - N_j N_{j_1} N_{j_2}](t, t') + 2J_{j_1, \vec{m}_1; j_2, \vec{m}_2; j, \vec{m}} \int_{-\infty}^t dt' C_{j_1, j_2, j}^*(t) [N_{j_1}^* (1 + N_{j_2})(1 + N_j) - (1 + N_{j_1}^*) N_{j_2} N_j](t, t') \\ &\quad \left. + I_{j, \vec{m}; j_1, \vec{m}_1; j_2, \vec{m}_2} \int_{-\infty}^t dt' D_{j, j_1, j_2}^*(t) [(1 + N_j) N_{j_1}^* N_{j_2}^* - N_j (1 + N_{j_1}^*)(1 + N_{j_2}^*)](t, t') \right\}. \end{aligned} \quad (21)$$

The terms $[N \rightarrow PN]$ and $(\vec{m} \rightarrow -\vec{m})$ have been defined in Sec. II B. In (20) and (21), instead of the usual flat space δ functions, which ensure energy-momentum conservation, we have

$$\begin{aligned}
J_{j_1\bar{m}_1;j_2\bar{m}_2;j_3\bar{m}_3} &= \int d\Omega Y_{j_1\bar{m}_1}^*(\Omega) Y_{j_2\bar{m}_2}(\Omega) Y_{j_3\bar{m}_3}(\Omega), \\
C_{j_1,j_2,j_3} &= \cosh^{D-1}(t) g_{j_1}^* g_{j_2} g_{j_3}, \\
I_{j_1\bar{m}_1;j_2\bar{m}_2;j_3\bar{m}_3} &= \int d\Omega Y_{j_1\bar{m}_1}(\Omega) Y_{j_2\bar{m}_2}(\Omega) Y_{j_3\bar{m}_3}(\Omega), \\
D_{j_1,j_2,j_3} &= \cosh^{D-1}(t) g_{j_1} g_{j_2} g_{j_3}. \quad (22)
\end{aligned}$$

Unlike the cases of EPP and CPP, we do not yet understand what is the appropriate choice of the harmonics for this system of kinetic equations. Hence, we are not yet in a position to solve it analytically.

IV. CONCLUSIONS

We derive kinetic equations in CPP, EPP, and global dS space for the massive scalar fields from the principal series. Our goal is to see whether the kinetic equation in global dS space has any stationary solutions. In EPP and CPP we find stationary solutions. The possibility of their existence strongly depends on the ratio between the particle production Γ_2 and particle decay Γ_1 probabilities. This ratio defines the equilibrium particle density and is proportional to the Gibbons-Hawking rate $e^{-2\pi\mu}$, $\mu = \sqrt{m^2 - (\frac{D-1}{2})^2}$. It is suppressed when the particle mass is very big. However, as the mass of the particle m approaches the $(D-1)/2$ bound, the solution is not valid anymore. Furthermore, when such a ratio is greater than 1 then the solution in question is unstable under the linearized perturbations [4]. Thus, e.g., for the scalar fields from the complementary series, the picture of the particle kinetics can be quite different from what we observe for the principal series.

Furthermore, in global dS space, particle production and decay rates are different from those in CPP and EPP. As a result, the stationary solutions of EPP and CPP found above are not applicable in global dS space. The question is if (20) together with (21) have any other stationary

solutions. In this respect, it is worth recalling that the solutions of these equations for the different choices of the harmonics are related to each other via Bogoliubov transformations. Hence, if the stationary solution does exist, then it means that there is a choice of $g_j(t)$ harmonics for which the system of equations under consideration has a solution with $\kappa_p = 0$. However, we do not see such a choice so far.

Last, but not least, we would like to mention the UV/IR mixing. In Ref. [5] it was noticed that UV and IR renormalizations are mixed in the coordinate space correlation functions in global dS space. In CPP, the situation is similar and, as we discussed above, it is quite different from EPP (see also Ref. [20] for the related discussion in EPP). In fact, the sufficient condition for this mixing is the presence of IR divergences of the type discussed above. However, the necessary condition for such a mixing is the presence of the nonvanishing collision integral. E.g., one can see that in CPP for the case of the stationary solution, the mixing disappears and the theory becomes renormalizable.

We conclude that in the circumstances when the stationary situation cannot be reached over the fixed dS background, the resolution of the problem comes only when one takes into account the backreaction on the dS metric because then the stationary stage should be achieved in the deformed space.

ACKNOWLEDGMENTS

I would like to acknowledge discussions with A. Sadofyev, M. Johnson, I. Morrison, and A. Polyakov. I would like to thank MPI, AEI, Golm, Germany for hospitality during the final stage of the work on this project. This work was partially supported by the ‘‘Leading Scientific Schools’’ Grants No. NSH-6260.2010.2 and No. RFBR-11-02-01227-a, and by a grant from the Ministry of Education and Science of the Russian Federation, Contract No. 14.740.11.0081.

-
- [1] E. Mottola, *Phys. Rev. D* **31**, 754 (1985).
 - [2] B. Allen, *Phys. Rev. D* **32**, 3136 (1985).
 - [3] A. M. Polyakov, [arXiv:1209.4135](https://arxiv.org/abs/1209.4135); A. Polyakov, ‘‘The Dark and the Red,’’ <http://hep.caltech.edu/ym35/>; ‘‘Quantum Instability of the de Sitter Space,’’ (unpublished).
 - [4] E. T. Akhmedov and P. Burda, *Phys. Rev. D* **86**, 044031 (2012).
 - [5] D. Krotov and A. M. Polyakov, *Nucl. Phys.* **B849**, 410 (2011).
 - [6] E. T. Akhmedov, *J. High Energy Phys.* **01** (2012) 066.
 - [7] E. T. Akhmedov and P. V. Buividovich, *Phys. Rev. D* **78**, 104005 (2008); E. T. Akhmedov, P. V. Buividovich, and D. A. Singleton, *Phys. At. Nucl.* **75**, 525 (2012).
 - [8] D. Marolf, I. A. Morrison, and M. Srednicki, [arXiv:1209.6039](https://arxiv.org/abs/1209.6039); D. Marolf and I. A. Morrison, *Phys. Rev. D* **82**, 105032 (2010); *Gen. Relativ. Gravit.* **43**, 3497 (2011); A. Higuchi, D. Marolf, and I. A. Morrison, *Phys. Rev. D* **83**, 084029 (2011); D. Marolf and I. A. Morrison, *Phys. Rev. D* **84**, 044040 (2011).
 - [9] E. T. Akhmedov, F. Popov, and V. Slepukhin (unpublished).
 - [10] T. Prokopec, N. C. Tsamis, and R. P. Woodard, *Ann. Phys. (Amsterdam)* **323**, 1324 (2008); R. P. Woodard, *J. Phys. Conf. Ser.* **68**, 012032 (2007); T. Prokopec, N. C. Tsamis, and R. P. Woodard, *Classical Quantum Gravity* **24**, 201 (2007); S.-P. Miao and R. P. Woodard, *Phys. Rev. D* **74**,

- 044019 (2006); *Classical Quantum Gravity* **23**, 1721 (2006); N.C. Tsamis and R.P. Woodard, *Nucl. Phys. B* **724**, 295 (2005); R.P. Woodard, *Nucl. Phys. B, Proc. Suppl.* **148**, 108 (2005); N.C. Tsamis and R.P. Woodard, *Nucl. Phys.* **B474**, 235 (1996); *Ann. Phys. (N.Y.)* **253**, 1 (1997); *Classical Quantum Gravity* **11**, 2969 (1994).
- [11] A.D. Dolgov, M.B. Einhorn, and V.I. Zakharov, *Phys. Rev. D* **52**, 717 (1995).
- [12] I. Antoniadis, P.O. Mazur, and E. Mottola, *New J. Phys.* **9**, 11 (2007); E. Mottola, *Phys. Rev. D* **33**, 1616 (1986); **33**, 2136 (1986).
- [13] W. Xue, X. Gao, and R. Brandenberger, *J. Cosmol. Astropart. Phys.* **06** (2012) 035; W. Xue, K. Dasgupta, and R. Brandenberger, *Phys. Rev. D* **83**, 083520 (2011).
- [14] S.B. Giddings and M.S. Sloth, *J. Cosmol. Astropart. Phys.* **07** (2010) 015; **01** (2011) 023; A. Riotto and M.S. Sloth, *J. Cosmol. Astropart. Phys.* **04** (2008) 030.
- [15] M. van der Meulen and J. Smit, *J. Cosmol. Astropart. Phys.* **11** (2007) 023.
- [16] N. Myrhvold, *Phys. Rev. D* **28**, 2439 (1983).
- [17] D. Boyanovsky, H.J. de Vega, and N.G. Sanchez, *Phys. Rev. D* **71**, 023509 (2005); D. Boyanovsky and H.J. de Vega, *Phys. Rev. D* **70**, 063508 (2004).
- [18] J. Bros, H. Epstein, M. Gaudin, U. Moschella, and V. Pasquier, *Commun. Math. Phys.* **295**, 261 (2010); J. Bros, H. Epstein, and U. Moschella, *Ann. Inst. Henri Poincaré, Sect. B* **11**, 611 (2010); *J. Cosmol. Astropart. Phys.* **02** (2008) 003.
- [19] G.E. Volovik, [arXiv:0803.3367](https://arxiv.org/abs/0803.3367); *JETP Lett.* **90**, 1 (2009).
- [20] E.O. Kahya, V.K. Onemli, and R.P. Woodard, *Phys. Rev. D* **81**, 023508 (2010).
- [21] E.T. Akhmedov and S. Guz (unpublished).