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The Application of Conflict Measure to Estimating Incoherence of Analyst's Forecasts about the Cost of Shares of Russian Companies

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Abstract

This paper is devoted to modern approaches to the estimation of external conflict in the theory of evidence based on axioms. The conflict measure is defined on the set of beliefs obtained from several sources of information. It is shown that the conflict measure should be a monotone set function with respect to sets of beliefs. Some robust procedures for evaluation of conflict measure that are stable to small changes in evidences are introduced and discussed. The analysis of conflict among forecasts about the value of shares of Russian companies of investment banks is presented. In this analysis the conflict measure estimates inconsistency of recommendations of investment banks, while the Shapley values of this measure on the set of evidences characterize the contribution of each investment bank to the overall conflict. The relationship between conflict and precision of forecasts is also investigated.

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1. Introduction

The quality of combining beliefs which is estimated in the framework of belief functions [1-2] depends on aggregation rules [3] as well as on prior and posterior characteristics that may include reliability, inconsistency and conflict of given information. The analysis of conflict in information given by experts helps us to evaluate inconsistence of information received from different sources and the level of belief to such information. In this case the conflict measure can be used as an indicator of information reliability [4]. Historically, the conflict measure is associated with the Dempster rule [1] where the conflict measure was firstly introduced. Sometimes this measure is called the Dempster conflict [5], but we will call it the canonical conflict measure. This measure

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is equal to the mass function at empty set after applying the non-normalized Dempster rule [1]. There are several approaches to evaluating conflict among beliefs. The metric approach is used in many popular methods [6-7] but there are other ones. At the same time the conflict measure as a functional defined on a pair of belief functions has to satisfy certain properties or axioms [8-9]. Axiomatics and the main existing approaches to the estimation of conflict among evidences are analyzed in Section 3. A new approach to the evaluation of conflict is developed in Section 4. This approach was proposed in [10] and it generalizes the canonical conflict measure. The conflict measure can be considered on the subsets of expert evidences and in this way it defines a monotone set function. It allows us in Section 5 to apply the theory of monotone measures [11] for analyzing conflict. Some conflict measures (for example, the canonical conflict measure) can easily be generalized to arbitrary tuples of evidences. The generalization of others conflict measures (in particular based on the metric approach) requires using specific aggregation functions [12]. These issues are also discussed in Section 5. Another important question concerns the conflict measure robustness, i.e. its stability with respect to small changes in evidences. The robust conflict estimation, based on specialization and generalization transformations of belief functions [13], is discussed in Section 6. Finally, the application of conflict measure to estimating consistence of investment banks forecasts about the value of shares of Russian companies is discussed in Section 7.

2. The belief functions theory and conflict measures

Let X be a finite set and 2^X be a powerset of X. The mass function is the fundamental notion in the theory of evidence. This function $m: 2^X \to [0,1]$ should satisfy the following conditions

$$m(\varnothing)=0,\ \sum_{A\subseteq X} m(A)=1. \tag{1}$$

The value $m(A)$ represents the relative part of evidence that a true alternative from X belongs to the set $A\in 2^X$. A subset $A\in 2^X$ is called a focal element if $m(A)>0$. Let $\mathcal{A}=\{A\}$ be a set of all focal elements. The pair $F=(\mathcal{A},m)$ is called a body of evidence. Let $\mathcal{F}(X)$ be a set of all possible bodies of evidence on X. Evidence $F_A=(A,1)$, $A\in 2^X$ is called categorical. In particular, the evidence $F_X=(X,1)$ is called vacuous because it is totally uninformative.

If we know the body of evidence $F=(\mathcal{A},m)$, then we can estimate the degree of belief that a true alternative from X belongs to set B using the belief function [2] $Bel: 2^X \to [0,1]$, $Bel(B)=\sum_{A\in\mathcal{A}:\, A\subseteq B} m(A)$. Similarly, we can estimate the degree of plausibility that a true alternative from X belongs to B using the dual to Bel the plausibility function $Pl: 2^X\to[0,1]$, $Pl(B)=1-Bel(\overline{B})= \sum_{A\in\mathcal{A}:B\cap A\neq\varnothing} m(A)$.

Let we have two bodies of evidence $F_1=(\mathcal{A}_1,m_1)$ and $F_2=(\mathcal{A}_2,m_2)$ received from two information sources. It is of interest how to measure their conflictness. Historically, the conflict measure associated with the Dempster rule [1-2] was the first among conflict measures:

$$K_0=K_0(F_1,F_2)=\sum_{\substack{B\cap C=\varnothing,\\ B\in\mathcal{A}_1, C\in\mathcal{A}_2}} m_1(B)m_2(C). \tag{2}$$

The value $K_0(F_1,F_2)$ characterizes the amount of conflict between two sources of information described by the bodies of evidence F_1 and F_2. The functional (2) is a one possible way for measuring conflict. We will describe other measures of conflict below.

3. Axiomatics of conflict measures and the main approaches to the estimation of conflict

There are different types of conflict depending on its nature (see, e.g. [14]). For example, there exists the internal conflict in evidence when we can find two focal elements with empty intersection {evidence: value of the shares of the company tomorrow will be in the interval [0,10] or in the interval [30,35]}. But we will consider below only the external conflict, i.e. the conflict between different evidences.

The conflict between evidences depends on the relative position of the focal elements of two evidences and the values of the mass functions. In general, a measure of conflict $K(F_1,F_2)$ should satisfy the following conditions (axioms) [8-9]:

A1: $0 \le K(F_1,F_2) \le 1$ for all $F_1,F_2 \in \mathcal{F}(X)$ (non-negativity and normalization);

A2: $K(F_1,F_2) = K(F_2,F_1)$ for all $F_1,F_2 \in \mathcal{F}(X)$ (symmetry);

A3: $K(F,F) = 0$ for all $F \in \mathcal{F}(X)$ (nilpotency);

A4: $K(F',F) \ge K(F'',F)$, if $F' = (\mathcal{A}',m)$, $F'' = (\mathcal{A}'',m)$, where $\mathcal{A}' = \{A'_i\}$, $\mathcal{A}'' = \{A''_i\}$ и $A'_i \subseteq A''_i$ for all i and $F \in \mathcal{F}(X)$ (antimonotonicity with respect to imprecision of evidence);

A5: $K(F_X,F) = 0$ for all $F \in \mathcal{F}(X)$ (ignorance is bliss [9]);

A6: $K(F_A,F_B) = 1$ if $A \cap B = \varnothing$.

The other axioms for conflict measures are also considered (see, e.g., [9]). Note that the canonical conflict measure K_0 does not satisfy the axiom A3.

There are several approaches to the conflict estimation. The metric approach for estimation of conflict is presented in [6-7] and other works. For example, the metric between the evidences was introduced in [6]:

$$d_J(F_1,F_2) = \sqrt{\tfrac{1}{2}(\mathbf{m}_1 - \mathbf{m}_2)^T \underline{\underline{D}}(\mathbf{m}_1 - \mathbf{m}_2)}\,, \tag{3}$$

where $\mathbf{m}_i$ is a $2^{|X|}$-dimensional column vector whose coordinates are the values of mass function $m_i(A)$, $A \in 2^X$, $i = 1,2$; $\underline{\underline{D}} = (d_{A,B})_{A,B\in 2^X}$, $d_{A,B} = |A \cap B|/|A \cup B|$, if $A,B \ne \varnothing$ and $d_{\varnothing,\varnothing} = 0$. The value $d_{A,B}$ is called Jaccard index (similarity coefficient) [15] and it is a measure of similarity between two sets. This metric is considered as a conflict measure. The conflict measure (3) does not satisfy the conditions A4 and A5. Other indices are used in a number of papers instead of Jaccard index in (3).

The measuring of distance between the two reference measures, corresponding bodies of evidence, is another example of a metric approach to the estimation of conflict. For example, the pignistic probability is considered as such a reference measure. The pignistic probability is defined as [16] $P_F(A) = \sum_B \frac{|A\cap B|}{|B|} m(B)$. For example, the conflict measure $K_P(F_1,F_2) = \max\{|P_{F_1}(A) - P_{F_2}(A)| : A \subseteq X\}$ was introduced in [7]. This measure does not satisfy only the conditions A4 and A5. In addition, the pair of measures $(K_0(F_1,F_2), K_P(F_1,F_2))$ was proposed for characterizing conflict in [7]. The large values of these two measures guarantee large conflict between evidences.

The metric $K(F_1,F_2) = 1 - \mathbf{Pl}_1^T \cdot \mathbf{Pl}_2 / (\|\mathbf{Pl}_1\| \|\mathbf{Pl}_2\|)$ is considered in [17] as a conflict measure, where $\mathbf{Pl}_i$ is the $2^{|X|}$-dimensional column vector whose coordinates are the values of plausibility function $Pl_i(A)$, $A \in 2^X$, $i = 1,2$, $\|\cdot\|$ is the Euclidean norm. This measure does not satisfy the conditions A4, A5, A6. The extensive survey of metrics on the set $\mathcal{F}(X) \times \mathcal{F}(X)$ of pairs of evidence can be found in [17].

The use of the inclusion index of sets $Inc(A,B) = 1$ for $A \subseteq B$ and $Inc(A,B) = 0$ for $A \subsetneq B$, $A,B \in 2^X$, is an example of another approach to the evaluation of conflict between evidences. For example, the degree of inclusion of one set of focal elements to another is introduced in [8]:

$$\delta(\mathcal{A}_1,\mathcal{A}_2) = \max\{d(\mathcal{A}_1,\mathcal{A}_2), d(\mathcal{A}_2,\mathcal{A}_1)\},\ d(\mathcal{A}_1,\mathcal{A}_2) = \frac{1}{|\mathcal{A}_1||\mathcal{A}_2|} \sum_{A_1\in\mathcal{A}_1} \sum_{A_2\in\mathcal{A}_2} Inc(A_1,A_2)\,.$$

Then the conflict measure in [8] is defined as $K(F_1,F_2)=(1-\delta(\mathcal{A}_1,\mathcal{A}_2))\,d_J(F_1,F_2)$, where d_J is the distance defined by (3). This measure does not satisfy only the condition A4.

4. The bilinear conflict measure

The value of the canonical conflict measure is larger when there are many disjoint pairs of focal elements in two evidences with large masses. However, the conflict of two sources of information would be also greater, if there are common elements of two evidences, but the cardinality of their intersection is small with respect to cardinality of comparable focal elements. The conflict measure, which reflects the above observation, was axiomatically introduced in [10]. In general, such a measure depends on degree of intersection of two sets formally defined as follows.

The mapping $r:2^X\times 2^X\to[0,1]$ is called a coefficient of sets intersection if it satisfies the following conditions:

1) $r(A,B)=r(B,A)$;
2) $r(A,B)=0$, if $A\cap B=\varnothing$;
3) $r(A,A)=1$, $A\neq\varnothing$.

The Jaccard index [15] is an example of the coefficient of sets intersection.

Definition [10]. A functional $K_r:\mathcal{F}(X)\times\mathcal{F}(X)\to[0,1]$ is called the *measure of conflict* with respect to intersection coefficient r if:

a) $K_r(F_1,F_2)=K_r(F_2,F_1)$ for all $F_1,F_2\in\mathcal{F}(X)$;
b) $K_r(F',F)\geq K_r(F'',F)$ if $F'=F\cup(A',m)$, $F''=F\cup(A'',m)$ and $r(A',B)\leq r(A'',B)$ for all $B\in\mathcal{A}$;
c) $K_r(F_1,F_2)=1$ if $A\cap B=\varnothing$ for all $A\in\mathcal{A}_1$, $B\in\mathcal{A}_2$.

The above conflict measure does not satisfy in general the conditions A3, A4, A5. But the condition A4 is also valid if the intersection coefficient $r(A,B)$ is monotone with respect to inclusion ($r(A',B)\leq r(A'',B)$ if $A'\subseteq A''$).

The algebraically simplest, bilinear conflict measure is defined in the next definition.

A *measure of conflict* K_r on $\mathcal{F}(X)\times\mathcal{F}(X)$ is called *bilinear* if $K_r(\alpha F_1+\beta F_2,F)=\alpha K_r(F_1,F)+\beta K_r(F_2,F)$ for all $\alpha,\beta\in[0,1]$, $\alpha+\beta=1$, $F,F_1,F_2\in\mathcal{F}(X)$.

Proposition 1 [10]. A functional $K_r:\mathcal{F}(X)\times\mathcal{F}(X)\to[0,1]$ is a *bilinear measure of conflict* $\mathcal{F}(X)\times\mathcal{F}(X)$ iff

$$K_r(F_1,F_2)=\sum_{A\in\mathcal{A}_1,B\in\mathcal{A}_2}\gamma(A,B)m_1(A)m_2(B)=K_0(F_1,F_2)+\sum_{A\cap B\neq\varnothing}\gamma(A,B)m_1(A)m_2(B), \tag{4}$$

where coefficients $\gamma(A,B)\in[0,1]$ satisfy the following conditions:

i) $\gamma(A,B)=\gamma(B,A)$;
ii) $\gamma(A',B)\geq\gamma(A'',B)$, if $r(A',B)\leq r(A'',B)$;
iii) $\gamma(A,B)=1$, if $A\cap B=\varnothing$.

The canonical conflict measure $K_0(F_1,F_2)$ is the smallest among all bilinear conflict measures as follows from formula (4): $K_r(F_1,F_2)\geq K_0(F_1,F_2)$.

For example, the coefficients $\gamma(A,B)=K_r(F_A,F_B)=\psi(r(A,B))$, $A,B\neq\varnothing$, satisfy the conditions i) - iii) of Proposition 1 if ψ is a non-increasing function with $\psi(1)=0$, $\psi(0)=1$ and $r(A,B)=|A\cap B|/\min\{|A|,|B|\}$. In this case $K_r(F_1,F_2)=K_0(F_1,F_2)$ if

$$r(A,B)=\begin{cases}1, & A\cap B\neq\varnothing,\\ 0, & A\cap B=\varnothing\end{cases}$$

is a primitive measure of intersection.

Let us notice that the measure (4) also satisfies the condition A5 if

iv) $\gamma(A,X)=0$ for all $A\neq\varnothing$.

5. The conflict on the set of evidences

A conflict measure can be introduced not only on a pair of evidences, but also on an arbitrary finite set of evidences. Suppose that we have a finite set of evidences $M=\{F_1,...,F_l\}$, $F_i\in\mathcal{F}(X)$, $i=1,...,l$. Let 2^M be the powerset of M. Let by definition $K(B)=0$ if $|B|=1$, $B\in 2^M$ and $K(\varnothing)=0$.

The axioms A1-A6 for the conflict measure K on 2^M can be rewritten in the form:

B1: $0\leq K(B)\leq 1$ for all $B\in 2^M$;

B2: $K(B)=K(B_\pi)$ for all $B=\{F_{i_1},...,F_{i_k}\}\in 2^M$, where $B_\pi=\{F_{\pi(i_1)},...,F_{\pi(i_k)}\}$ and $\pi:\{1,...,k\}\to\{1,...,k\}$ is an arbitrary permutation;

B3: $K(B\cup\{F'\})=K(B)$ if there is a $F''\in B$, $F'=F''$ for all $B\in 2^M$ (indifference to adding of duplicate evidence); if $B=\{F''\}$, then B3 implies that $K(F'',F'')=K(F'')=0$, i.e. the A3 axiom (nilpotency) is true;

B4: $K(B\cup\{F'\})\geq K(B\cup\{F''\})$, if $F'=(\mathcal{A}',m)$, $F''=(\mathcal{A}'',m)$, where $\mathcal{A}'=\{A'_i\}$, $\mathcal{A}''=\{A''_i\}$ и $A'_i\subseteq A''_i$ for all i and $B\in 2^M$;

B5: $K(B\cup\{F_X\})=K(B)$ for all $B\in 2^M$ (indifference to adding of the vacuous evidence);

B6: $K(\{F_{A_1},...,F_{A_k}\})=1$, if $A_i\cap A_j=\varnothing$ for all $i\neq j$, $A_s\subseteq X$.

We will require the satisfaction the additional axiom for the conflict measure K on 2^M:

B7: $K(B)\leq K(C)$, if $B\subseteq C$ and $B,C\in 2^M$.

This condition is obviously the monotonicity of K on 2^M, i.e. adding a new evidence to the set of evidences implies that the conflict measure is not decreased. Some of these axioms (for example, B3 or B6) can be strengthened or weakened. But it is a question of a separate study.

The canonical conflict measure K_0 on 2^M has the form (but there are other ways of extension K_0 on 2^M)

$$K_0(\{F_{i_1},...,F_{i_k}\})=\sum_{A_{i_1}\cap...\cap A_{i_k}=\varnothing} m_{i_1}(A_{i_1})..m_{i_k}(A_{i_k}),\ F_{i_s}=(\{A_{i_s}\},m_{i_s}),\ s=1,...,k. \quad (5)$$

Obviously, it satisfies the monotonicity condition.

Assume that the conflict measure K is initially given only on pairs of evidence $F',F''\in M$. For example, the measure K can be computed with the help of metric (3). We denote through $\tilde{K}$ the extension of this measure on the whole set 2^M. Then $\tilde{K}(B)=K(B)$ for all $B\in 2^M$ with $|B|=2$. In general, we can suppose that $\tilde{K}(B)=A(K(F_{i_1},F_{i_2}),...,K(F_{i_{k-1}},F_{i_k}))$, if $B=\{F_{i_1},...,F_{i_k}\}\in 2^M$, where $A:[0,1]^r\to[0,1]$, $r=C_k^2$, is a symmetric associative aggregation function with the following properties:

1) A is a non-decreasing function of each argument;

2) $A(0,...,0)=0$;

3) $A(x_1,...,x_r)\geq x_i$ for all $i=1,...,k$.

The condition 3) follows from the monotonicity property and property 1) for extension $\tilde{K}$. This implies

4) $A(x_1,...,x_r)=1$, if exist $x_i=1$, $i=1,...,k$;

5) $A(x_1,...,x_r)\geq\max\{x_i:1\leq i\leq k\}$.

A function satisfying the above conditions is a triangular conorm (t-conorm) [12] extended to $[0,1]^r$. The next functions are the examples of t-conorm: $A(x_1,...,x_r)=\max\{x_i:1\leq i\leq k\}$ (maximum), $A(x_1,...,x_r)=1-\prod_{i=1}^k(1-x_i)$ (probabilistic sum), $A(x_1,...,x_r)=\min\{1,\sum_{i=1}^k x_i\}$ (Lukasiewicz t-conorm, bounded sum). If we have a monotone conflict measure K given on the all subset of evidences from M, then we can compute the "contribution" v_i of i-th evidence in total conflict $K(M)$ with the help of Shapley value [18]

$$v_i = \sum_{B\subseteq M,\, F_i\in B} \frac{(l-|B|)!(|B|-1)!}{l!}\left(K(B)-K(B\setminus\{F_i\})\right). \tag{6}$$

Note that the following equality $\sum_{i=1}^{l} v_i = K(M)$ is true. Shapley value can be considered as a characteristic of reliability of information sources.

6. The robust procedures for calculating of conflict measure

The boundary conditions 2) and 4) are the most strong conditions on aggregation function for defining the conflict measure. For example, assume that have three evidences $F_A=(A,1)$, $F_B=(B,1)$ и $F_C=(C,1)$ and $A\cap B\neq\varnothing$, $B\cap C\neq\varnothing$, $A\cap C\neq\varnothing$, but $A\cap B\cap C=\varnothing$. Then we have the following results for the canonical conflict measure $K_0(F_A,F_B)=K_0(F_B,F_C)=K_0(F_A,F_C)=0$. If we will aggregate these partial conflicts by t-conorms $A(x_1,x_2,x_3)$, then we get that the overall conflict is equal zero. But $K_0(F_A,F_B,F_C)=1$ based on extension (5). Partially this problem is related to the instability of some methods of calculating of conflict measure (in particular, the canonical conflict measure) to the "small" changes of evidence. Moreover, these evidences can be generated subjectively and this depends on particularities of information sources. For example, assume that one expert give a “pessimistic” forecast about value of the shares of a some company in interval (30, 40), but another expert give an “optimistic” forecast in interval (38, 45). Then we have two evidence: $F_A=(A,1)$ and $F_B=(B,1)$, where $A=(30,40)$ and $B=(38,43)$. The canonical conflict measure in this case is equal zero: $K_0(F_A,F_B)=0$. Actually, "pessimistic" expert in the refinement of its forecast have in mind the evidence $F_1=(\{A_1,A_2\},m_1(A_i))$, where $A_1=(30,38)$ and $m_1(A_1)=0.8$, $A_2=(38,40)$ and $m_1(A_2)=0.2$. "Optimistic" expert have in mind the evidence $F_2=(\{B_1,B_2,B_3\},m_2(B_i))$, where $B_1=(35,38)$ and $m_2(B_1)=0.2$, $B_2=(38,42)$ and $m_2(B_2)=0.7$, $B_3=(42,44)$ and $m_2(B_3)=0.1$. In this case $K_0(F_A,F_B)=0.7$.

The influence of “bad” evidence (or parts of evidence) can be reduced with the help of specialization-generalization procedures [13]. The use of these procedures improves the robustness of the conflict estimation. The specialization procedure is produced by dividing focal elements on "small" subsets with a new distribution of the mass function on these subsets. Let we have an evidence $F_1=(\{A_i\},m_1)$. Then an evidence $F_2=(\{B_{ij}\},m_2)=S(F_1)$ is the specialization of $F_1=(\{A_i\},m_1)$ (denotation: $F_2\sqsubseteq F_1$), if $\cup_j B_{ij}=A_i$ and $\sum_j m_2(B_{ij})=m_1(A_i)$ for all i. And vice-versa, an evidence $F_3=(\{C_{ij}\},m_3)=G(F_1)$ ($F_1\sqsubseteq F_3$) is a generalization of an evidence $F_1=(\{A_i\},m_1)$ ($F_1\sqsubseteq F_3$), if $\cap_j C_{ij}=A_i$ and $\sum_j m_3(C_{ij})=m_1(A_i)$. The canonical conflict measure can not be decreased after specialization of evidence and it can not be increased after generalization of evidence. Therefore, the axiom A4 (or B4) can be replaced by more general axiom:

A4′: $K(F',F)\geq K(F'',F)$, if $F'\sqsubseteq F''$.

We will consider next specializations-generalizations of evidence $F=(\mathcal{A},m)$ which are close to F. The degree of closeness can be estimated with the help of imprecision index $f:\mathcal{F}(X)\to[0,1]$, which was introduced axiomatically in [19]. The generalized Hartley measure [20] $f(F)=\frac{1}{\ln|X|}\sum_{A\in\mathcal{A}} m(A)\ln|A|$ is an example of imprecision index. It is easy to see that $F_1\sqsubseteq F_2$ implies $Pl_1(A)\leq Pl_2(A)$ for all $A\in 2^X$ [21]. Therefore the inequality $f(F_1)\leq f(F_2)$ is true for any imprecision index f in a view of results presented in [19]. Consequently the imprecision index can not be increased after specialization of evidence and it can not be decreased after generalization of evidence.

Let $S_\varepsilon(F)=\{F'\sqsubseteq F: f(F)-f(F')<\varepsilon\}$ and $G_\varepsilon(F)=\{F\sqsubseteq F': f(F')-f(F)<\varepsilon\}$ be sets of all specializations and generalizations of evidence F correspondingly which are located in ε-neighborhood of evidence with respect to imprecision index f, $SG_\varepsilon(F)=S_\varepsilon(F)\cup G_\varepsilon(F)$. Then the value $K_\varepsilon(F_1,F_2)=\mathrm{MEAN}\{K(\tilde{F}_1,\tilde{F}_2):\tilde{F}_i\in SG_\varepsilon(F_i)\}$ can be used as a conflict measure, where MEAN be a some averaging operator. For example, let $SG_\varepsilon(F_1)=\{F_{1,j}\}$, $SG_\varepsilon(F_2)=\{F_{2,k}\}$, where $F_{i,s}=F_{i,s}(\boldsymbol{\theta})$, $i=1,2$, $\boldsymbol{\theta}\in[0,1]^N$ is a vector of parameters (values of mass function). Then $K(F_{1,j},F_{2,k})=\varphi_{j,k}(\boldsymbol{\theta})$ and

$\boldsymbol{\theta} \in D_{j,k} = \{\boldsymbol{\theta} : |f(F_{1,j}(\boldsymbol{\theta})) - f(F_1)| < \varepsilon, |f(F_{2,k}(\boldsymbol{\theta})) - f(F_2)| < \varepsilon\}$. The average of mean integral values can be used as an operator MEAN:

$$K_\varepsilon(F_1, F_2) = \frac{1}{|\{F_{1,j}\}| \cdot |\{F_{2,k}\}|} \sum_{j,k} I_{j,k},$$

where $I_{j,k} = (1/V(D_{j,k})) \int_{D_{j,k}} \varphi_{j,k}(\boldsymbol{\theta}) d\boldsymbol{\theta}$, V is the Lebesgue measure. There are also other ways for generating the set $SG_\varepsilon(F)$ of evidence F specializations-generalizations.

Let us observe that the notion of minimal conflict $K_{\min}(F_1, ..., F_k) = \min\{K_0(\tilde{F}_1, ..., \tilde{F}_k) : \tilde{F}_i \in S(F_i)\}$ is considered in [22], where $S(F) = \{F' \in \mathcal{F}(X) : F' \sqsubseteq F\}$ is a set of all specialization for F. A similar approach was considered in [9, 23].

Example. Let $X = \{x_1, x_2, x_3\}$, $F_1 = (\{x_1, x_2\}, 1)$, $F_2 = (\{x_3\}, 1)$. Then $K_0(F_1, F_2) = 1$. We will consider the specializations-generalizations of evidence F_1 and F_2. The generalized Hartley measure will be used for estimation of impreciseness. Let $c = \ln 2/\ln 3$. Then $f(F_1) = c$, $f(F_2) = 0$. We have the following evidences as specializations $S(F_1) = \{F_{1,0}, F_{1,1}\}$, where $F_{1,0} = F_1$, $F_{1,1} = (\{\{x_1\}, \{x_2\}\}, \{\theta, 1-\theta\})$, $f(F_{1,1}) = 0$, $S(F_2) = \{F_{2,0}\}$, $F_{2,0} = F_2$. We have the following evidences as generalizations $G(F_1) = \{F_{1,0}, F_{1,2}\}$, where $F_{1,2} = (\{\{x_1, x_2\}, \{x_1, x_2, x_3\}\}, \{\theta, 1-\theta\})$ and $f(F_{1,2}) = c\theta + 1 - \theta$; $G(F_2) = \{F_{2,0}, F_{2,1}, ..., F_{2,4}\}$, where $F_{2,1} = (\{\{x_1, x_3\}, \{x_3\}\}, \{\theta, 1-\theta\})$ and $f(F_{2,1}) = c\theta$, $F_{2,2} = (\{\{x_2, x_3\}, \{x_3\}\}, \{\theta, 1-\theta\})$ and $f(F_{2,2}) = c\theta$, $F_{2,3} = (\{\{x_1, x_3\}, \{x_2, x_3\}\}, \{\theta, 1-\theta\})$ and $f(F_{2,3}) = c$, $F_{2,4} = (\{\{x_1, x_2, x_3\}, \{x_3\}\}, \{\theta, 1-\theta\})$ and $f(F_{2,4}) = \theta$. Then $SG(F_1) = \{F_{1,0}, F_{1,1}, F_{1,2}\}$, $SG(F_2) = \{F_{2,0}, F_{2,1}, ..., F_{2,4}\}$.

Let $\varepsilon < 1 - c$. Then $SG_\varepsilon(F_1) = \{F_{1,0}, F_{1,2}\}$ and $SG_\varepsilon(F_2) = \{F_{2,0}, F_{2,1}, F_{2,2}, F_{2,4}\}$. We have $K_0(F_{1,0}, F_{2,0}) = 1$, $K_0(F_{1,0}, F_{2,1}) = K_0(F_{1,0}, F_{2,2}) = 1 - \theta$ for $0 < c\theta < \varepsilon$, $K_0(F_{1,0}, F_{2,4}) = 1 - \theta$ for $0 < \theta < \varepsilon$, $K_0(F_{1,2}, F_{2,0}) = \theta$ for $0 < (1-\theta)(1-c) < \varepsilon$, $K_0(F_{1,2}, F_{2,1}) = K_0(F_{1,2}, F_{2,2}) = \theta_1(1-\theta_2)$ for $0 < (1-\theta_1)(1-c) < \varepsilon$ and $0 < c\theta_2 < \varepsilon$, $K_0(F_{1,2}, F_{2,4}) = \theta_1(1-\theta_2)$ for $0 < (1-\theta_1)(1-c) < \varepsilon$ and $0 < \theta_2 < \varepsilon$. Then

$$K_\varepsilon(F_1, F_2) = \tfrac{1}{2 \cdot 4} \sum\nolimits_{j,k} I_{j,k}, \text{ where } I_{j,k} = \tfrac{1}{V(D_{j,k})} \int_{D_{j,k}} \varphi_{j,k}(\boldsymbol{\theta}) d\boldsymbol{\theta},$$

i.e. $I_{0,0} = 1$, $I_{0,1} = I_{0,2} = 1 - \varepsilon/(2c)$, $I_{0,4} = 1 - \varepsilon/2$, $I_{2,0} = 1 - \varepsilon/(2(1-c))$, $I_{2,1} = I_{2,2} = (1 - \varepsilon/(2(1-c)))(1 - \varepsilon/(2c))$, $I_{2,4} = (1 - \varepsilon/(2(1-c)))(1 - \varepsilon/2)$. Thus

$$K_\varepsilon(F_1, F_2) = 1 - \varepsilon\left(\tfrac{1}{8} + \tfrac{1}{4c \cdot (1-c)}\right) + \varepsilon^2 \tfrac{(2+c)}{4c(1-c)} \; for \; \varepsilon < 1 - c.$$

7. The study of conflictness of investment banks forecasts

We investigate the conflictness of evidences for analysts' forecasts (of investment banks) as an application of the conflict measure. Also, we will find the contribution of investment banks to conflictness and precision of their forecasts. The conflictness characterizes in this case the degree of inconsistency of forecasts for some set of experts. Conflictness with the precision of forecasts is an important characteristic of the quality of forecasting. The high conflictness of forecasts of some banks with high precision can be associated with the presence of some exclusive techniques of forecasts, and the using of some insider information. The high conflictness with low accuracy probably indicates a low professionalism of analysts.

The database is information on 1307 forecasts of financial analysts, representing 23 investment banks about the value of shares of Russian companies in 2013. Bloomberg and RBC are the sources of information for this study. The forecasts are presented by experts the world largest investment banks, including such famous companies as Goldman Sachs, Credit Suisse, UBS, Deutsche Bank, Renaissance Capital and others.

Each investment bank makes recommendations of three types (“to sell”, “to hold”, “to buy”) and the forecast target price of paper. Target prices of forecasts are recalculated into the so-called relative values of target prices. The relative value of target prices is equal to quotient of price predicted by the analyst and the value of the security on the date of giving the forecast.

Boundaries of relative prices between the recommendations of various types were determined by maximizing number of recommendations that fall into the "corresponding" intervals: [0,0.92), [0.92,1.2), [1.2,+∞). Thus, we have nine sets, each of which represents the interval and a label of recommendation type: $A_1^{(t)} = [0, 0.92)$, $A_2^{(t)} = [0.92, 1.2)$, $A_3^{(t)} = [1.2, +\infty)$, $t \in \{s, h, b\}$, s – sell, h –hold, b – buy.

The set of all intervals $A_k^{(t)}$, $t \in \{s, h, b\}$, $k = 1, 2, 3$ is a set of all focal elements of all evidence. Let we fixed the i -th investment bank, $i = 1, ..., l$ (l is a number of investment banks), $c_{ik}^{(t)}$ is a number of belonging of relative price to interval $A_k^{(t)}$, N_i is a general number of forecasts for i -th investment bank. Then $m_i(A_k^{(t)}) = c_{ik}^{(t)} / N_i$ is a frequency of belonging of relative price to interval $A_k^{(t)}$. The mass function m_i satisfies the normalization condition: $\sum_t \sum_k m_i(A_k^{(t)}) = 1$ for all $i = 1, ..., l$. Then $F_i = (A_k^{(t)}, m_i(A_k^{(t)}))_{k,t}$ is a body of evidence of i -th investment bank, $i = 1, ..., l$. We can find a conflict measure of these evidences.

Note that the formula for calculation of canonical conflict measure $K_0(F_1, ..., F_l)$ considerably will become simpler if all evidence have the same set of focal elements and all focal elements $A_k^{(t)}$ are pairwise disjoint.

Proposition 2. If the bodies of evidence $F_i = (\{A_k\}, m_i(A_k))_k$, $i = 1, ..., l$ satisfy the condition $A_s \cap A_k = \varnothing$ for $s \neq k$, then a canonical conflict measure $K_0(F_1, ..., F_l)$ is equal to $K_0(\{F_1, ..., F_l\}) = 1 - \sum_k \prod_i m_i(A_k)$.

We will use the following measure $K_1(\{F_1, ..., F_l\}) = 1 - \sum_k \min_i m_i(A_k)$ instead of K_0 for estimation of conflict. Formally, the set function K_1 is obtained from K_0 by replacing of triangular norm (t-norm) "multiplication" on the other t-norm "min" [12]. The measure K_1 has a larger range of values than K_0 for large l. Further we can compute a "contribution" of the *i*-th investment bank to the total conflict $K_1(M)$ of set of all investment banks M with the help of the Shapley value (6), which are calculated by the monotone conflict measure K_1.

The results of calculating the estimates of Shapley values of contribution of each investment bank in the total conflict $K_1(M)$ of the forecasts about the cost of shares of Russian companies are shown in Fig 1. The total conflict on all investment banks is equal to $K_1(M) = 0,727$.

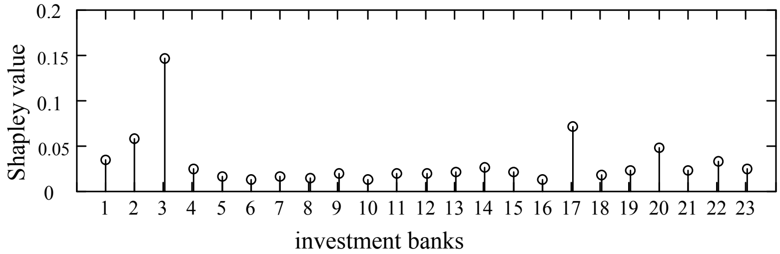

Fig. 1. Estimates of Shapley values, characterizing the contribution of each of the 23 investment banks in general conflict in 2013

Remark. The numbering of investment banks: 1 – Barclays, 2 – Citi group, 3 – Credit suisse, 4 – Deutsche Bank, 5 – HSBC, 6 – Renaissance Capital, 7 – Sberbank CIB, 8 – VTB Capital, 9 – Uralsib Capital, 10 – Goldman Sachs, 11 – Discovery Bank, 12 – Morgan Stanley, 13 – J.P. Morgan, 14 – UBS, 15 – Raiffeisen, 16 – Alfa-Bank, 17 – Aton Bank, 18 – BCS, 19 – Veles Capital, 20 – Gazprombank, 21 – Rye. Man&GorSecurities, 22 – Metropol Bank, 23 – Finam.

Fig 1 shows us that the forecasts of banks 3 (Credit suisse), 17 (Aton Bank), 2 (Citi group), 20 (Gazprombank) are the most conflicting in 2013. Note that $\max v_s / \min v_s \approx 10$, i.e. the set of banks is very heterogeneous with respect to contribution in total conflict.

The share pr_s of successful forecasts can be considered as an estimation of precision of forecasts for s-th investment bank. The distribution of points (v_s, pr_s) for all investment banks are shown in Fig 2.

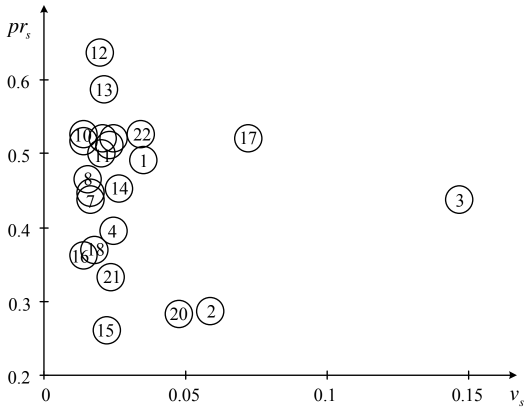

Fig. 2. The distribution of points (v_s, pr_s) for all investment banks (numbers of investment banks are given in the circles)

As can be seen from Fig 2 the investment bank 17 (Aton Bank) has given precise (great precision pr_s) and "unique" (great Shapley value for conflictness v_s) forecasts. The investment bank 2 (Citi group) has given a least precise and "unique" forecast. The investment banks (Credit suisse) and 20 (Gazprombank) have given a "unique" forecast at medium precision. The investment banks 12 (Morgan Stanley) and 13 (J.P. Morgan) have given a more precision forecast at low "uniqueness".

8. Conclusion

The basic axioms for the conflict measure and modern approaches to the estimation of conflict in the framework of the belief functions are studied in this paper. The notion of conflict measure on the set of all subsets of a set of evidences is considered. We postulate axiomatically that a conflict measure has to be a monotone set function. In particular, the conjunctive conflict measure based on the Dempster rule of combination is a monotone set function. We can use the mathematical formalism of monotone measures (e.g. Shapley values) for analysis of the contribution in the general conflict of separate evidences and groups of evidences by monotonicity of conflict measure. The procedures for extending the conflict measure defined on pairs of evidence to the all set of evidences are also considered. Some robust procedures for evaluation of conflict measure that are stable to small changes of evidence are introduced and discussed.

The results of analysis of conflicts forecasts of investment banks about the value of shares of Russian companies are given in the second part of this paper. The frequencies of events {the relative price of shares belong to interval} calculated on the basis of forecasts of investment banks (corresponding to the three types of recommendations: to sell - to hold - to buy) are considered as an evidence. The conflict measure evaluates inconsistency of recommendations of investment banks, while the Shapley values of this measure characterize the contribution of each investment bank to the overall conflict. It is shown that investment banks are extremely heterogeneous in their contribution to the overall conflict. The relationship between conflict and precision of forecasts is also investigated.

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