Preferences and Income Effects in Monopolistic Competition Models*

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Abstract

This paper develops a novel approach to modeling preferences in monopolistic competition models with a continuum of goods. In contrast to the commonly used CES preferences, which do not capture the effects of consumer income and the intensity of competition on equilibrium prices, the present preferences can capture both effects. The relationship between consumers' incomes and product prices is then analyzed for two cases: with and without income heterogeneity.

Keywords: product prices; intensity of competition; income distribution.

JEL classification: D4

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1 Introduction

Starting with Dixit and Stiglitz (1977), the monopolistic competition framework has been widely used in the economic literature. The most common assumption about preferences in this framework is the constant elasticity of substitution (CES) utility function. This greatly owes to the high analytical tractability of this particular functional form. Despite such a desirable property, the CES utility function has a shortcoming. One of the implications of the CES functional form is that prices set by firms depend only on marginal cost of those firms and the elasticity of substitution. This in turn implies that changes in the intensity of competition (that might follow as a result of opening a country to international trade) or changes in consumer income do not affect the prices that firms set. Meanwhile, the literature on pricing-to-market (see for instance Hummels and Lugovskyy (2008), Simonovska (2011), or Flach (2013)) has demonstrated that prices of the same goods vary with characteristics of the importing markets. Hence, it is desirable to have a tractable monopolistic competition model where prices depend not only on marginal cost, but also on other relevant factors such as the intensity of competition or consumer income.

In this paper, I develop a novel approach to modeling consumer preferences in a monopolistic competition framework with a continuum of goods. I construct a general form of consumer preferences (for instance, CES preferences are a special case of the preferences developed in this paper), which is analytically manageable and at the same time, captures the effects of income and the intensity of competition on equilibrium prices. Specifically, I consider a framework where all potentially available goods are indivisible and consumers purchase at most one unit of each good. Consumers differ in their tastes for a certain good. A taste for a certain good is a realization of a random variable, which is independently drawn for each consumer and each good from a common distribution. The utility function implies that given prices and consumer tastes, goods are arranged so that consumers can be considered as moving down some list in choosing what to purchase. That is, consumers first purchase a good they like best, then move to the second best, and keep on until their income is exhausted. This list of goods is consumer specific and depends on consumer income and tastes. Hence, demand for a certain good is equal to the fraction of consumers who decide to purchase this good multiplied by the total mass of consumers.

This approach to modeling preferences has a number of useful properties. First, it is highly tractable and eminently suitable for monopolistic competition models with a continuum of goods. Second, in the paper I show that the fraction of consumers who purchase a certain good is endogenous and depends not only on the price of the good, but also on the intensity of competition and consumers' incomes. As a result, equilibrium prices depend on the intensity of competition and the distribution of consumer income as well. Finally, this approach appears to be quite general, as by choosing different distributions of consumer tastes, one can generate different demand functions. In particular, a Pareto distribution leads to isoelastic demand (CES preferences) with the possibility of demand satiation, while a uniform distribution results in linear demand (a là Melitz and Ottaviano (2008)). It should be also noted that the constructed preferences are nonhomothetic in general, which is consistent with the empirical evidence (see for example Deaton and Muellbauer (1980) and Hunter and Markusen (1988)). Demand for a certain product depends not only on the aggregate income in the economy, but also on its components: the number of agents (population size) and the income distribution among them. This, for instance, distinguishes the present framework from a framework with oligopolistic firms and CES preferences where consumer income and the intensity of competition also affect prices charged by firms, but only aggregate income matters.

In the paper, I distinguish between two cases: with and without income heterogeneity among consumers. In the case of no income heterogeneity, a rise in consumer income makes demand for products less elastic, which in turn results in higher prices charged by firms and, thereby, higher firm profits in the short run. In the long run, higher firm profits lead to more entry into the market and tougher competition among active firms. As a result, product prices fall. I find that the latter effect completely cancels out the short-run effect of a rise in consumer income. Thus, in the long run, consumer income does not affect product prices.

I then consider an extension of the model where consumers are different not only in their tastes, but also in their incomes. I find that for an arbitrary distribution of tastes, a proportional rise in consumer incomes does not affect product prices in the long run. Moreover, I show that if the distribution of consumer tastes is multiplicatively separable and demand for each product of each income class is strictly positive, then prices charged by firms depend only on the average income in the short run and do not depend on the income distribution in the long run. These findings are similar to the results derived in the case of one income class in the economy. For other distributions, one can expect a more complex relationship between income distribution and prices where other moments of income distribution may matter.

The utility function considered in this paper is reminiscent of the stochastic utility functions developed in Perloff and Salop (1985) and later in Anderson *et al.* (1992). However, my approach is different in at least two ways. First, in Perloff and Salop (1985) and Anderson *et al.* (1992), consumers are allowed to purchase only one unit of the good they like most, which is a rather simplifying way of describing individual demand. In contrast, in my paper consumers are not limited to buying only one good. Second, in Perloff and Salop (1985) and Anderson *et al.* (1992), there are no income effects. In these works, the marginal utility of income is just a parameter in the model. In my approach, the marginal utility of income is an endogenous variable and depends on the observable characteristics of the economic environment including consumer income.

This paper is not the only one that explores the dependence of prices on the characteristics of the economic environment. To capture the impact of consumer income and the intensity of competition on prices, Behrens and Murata (2007, 2012) consider a monopolistic competition framework with a CARA utility function. Saure (2012) and Simonovska (2011) use the non-homothetic log-utility function that assumes the upper bound on the marginal utility from consumption. The present paper formulates an alternative approach for modeling consumer preferences based on a stochastic utility function, which provides an additional complementary insight on the relationship between income distribution and product prices. For instance, while deriving similar results to those in Behrens and Murata (2012) for the case of a multiplicatively separable distribution of tastes, the paper also suggests that, under other distributions of consumer tastes, a more complex relationship between prices and income distribution is possible (where other moments of income distribution can play a role in determining prices). This property of the preferences can be especially important when looking at the relationship between prices and income distribution in the data.

Hummels and Lugovskyy (2008) consider a generalized version of Lancaster's "ideal variety" model that allows for income effects operating through an intensity of preferences for the ideal variety. However, they limit their analysis to a symmetric equilibrium and, therefore, do not allow for firm heterogeneity and hence differences in prices chosen by firms. Meanwhile, the present model remains highly analytically tractable even in the case of the presence of firm heterogeneity, which is for instance important for applications in the industrial organization and international trade literatures. Murata (2009) develops a general equilibrium model with non-homothetic preferences and perfect competition to explore the impact of consumer income on the number and the composition of products consumed. In particular, he shows that, controlling for total income, a higher per capita income results in a higher number of products in consumption. The present paper, besides other findings, replicates the result in Murata (2009).

The remainder of the paper is organized as follows. Section 2 introduces the basic concepts of the model and formulates equilibrium conditions. In Section 3, I consider comparative statics of the model. Section 4 considers the extension of the model where consumers are different not only in their tastes, but also in their incomes. Section 5 concludes.

2 The Model

I consider a model of monopolistic competition with heterogeneous firms and a continuum of consumers and goods.

2.1 Consumption

I assume that all goods are indivisible and consumers purchase at most one unit of each good. In particular, consumer i chooses $\{x(\omega) \in \{0,1\}\}_{\omega \in \Omega}$ to maximize the following utility function:

$$U_i = \int_{\omega \in \Omega} \varepsilon_i(\omega) x(\omega) d\omega \tag{1}$$

subject to

$$\int_{\omega \in \Omega} p(\omega) x(\omega) d\omega = y, \tag{2}$$

where $x(\omega)$ is the consumption of good ω, $\varepsilon_i(\omega)$ is a consumer-specific taste for ω, $p(\omega)$ is the price, y is consumer income (which is identical for all consumers), and Ω is the set of goods available in the economy. I assume that for any i and ω, $\varepsilon_i(\omega)$ is independently drawn from a common distribution. That is,

$$\Pr(\varepsilon_i(\omega) \leq \varepsilon) = F(\varepsilon),$$

where $F(\varepsilon)$ (common for all consumers and goods) is a differentiable function with the support on $[\varepsilon_L, \varepsilon_H]$. Here, $\varepsilon_L \geq 0$.

The utility maximization problem implies that consumer i purchases good ω if and only if

$$\frac{\varepsilon_i(\omega)}{p(\omega)} \geq Q, \tag{3}$$

where Q is the Lagrange multiplier associated with the maximization problem and represents the endogenous marginal utility of income. Note that the Lagrange multiplier Q depends only on consumer income y and does not depend on consumer-specific tastes. Therefore, since all consumers have identical income, Q is the same for all consumers. As $\varepsilon_i(\omega)$ is independently distributed, the proportion of consumers, who purchase good ω, is equal to $1-F\left(p(\omega)Q\right)$. Notice that if the price of ω is sufficiently low (namely, $p(\omega)Q \leq \varepsilon_L$), then all consumers purchase the good. Similarly, if the price is high enough ($p(\omega)Q > \varepsilon_H$), then nobody purchases the good ω. Hence, the demand for good ω is given by

$$D(p(\omega)) = \begin{cases} L, & \text{if } p(\omega) \leq \frac{\varepsilon_L}{Q}, \\ \left(1 - F\left(p(\omega)Q\right)\right) L, & \text{if } \frac{\varepsilon_H}{Q} \geq p(\omega) > \frac{\varepsilon_L}{Q}, \\ 0, & p(\omega) > \frac{\varepsilon_H}{Q}, \end{cases} \tag{4}$$

where L is the total mass of consumers.

The marginal utility of income Q can be found from the budget constraint (2) in the consumer maximization problem. For consumer i,

$$\int_{\omega \in \Omega} p(\omega) x_i(\omega) d\omega = y.$$

If we take the sum across all consumers, we derive

$$\begin{aligned} \int \left(\int_{\omega \in \Omega} p(\omega) x_i(\omega) d\omega \right) di &= yL \iff \\ \int_{\omega \in \Omega} p(\omega) \left(\int x_i(\omega) di \right) d\omega &= yL. \end{aligned}$$

Recall that $x_i(\omega) = 1$ if and only if $\frac{\varepsilon_i(\omega)}{p(\omega)} \geq Q$. Then, by the law of large numbers,

$$\int x_i(\omega) di = LEx_i(\omega) = L \Pr\left(\varepsilon_i(\omega) \geq p(\omega)Q\right) = L\left(1 - F\left(p(\omega)Q\right)\right),$$

which implies that

$$\int_{\omega \in \Omega} p(\omega) \left(1 - F\left(p(\omega)Q\right)\right) d\omega = y. \tag{5}$$

Hence, we have the preference structure that results, in general, in nonhomothetic demand. Namely, given Ω and the prices $\{p(\omega)\}_{\omega\in\Omega}$, economies with identical total income but different population size L and per capita income y will have different demand for a certain good ω. Moreover, this approach to modeling consumer preferences provides a microeconomic foundation for different demand functions in monopolistic competition models. Namely, by choosing different distributions of consumer tastes, one can generate different demand functions. For instance, a Pareto distribution leads to isoelastic demand with the possibility of demand satiation, while a uniform distribution results in linear demand.

2.1.1 A Special Case: Pareto Distribution

Assume that the distribution of consumer tastes is Pareto. That is,

$$F(\varepsilon) = 1 - \left(\frac{\varepsilon_L}{\varepsilon}\right)^{\sigma},$$

where $\sigma > 1$. In the case of a Pareto distribution, the upper bound of the distribution is infinity meaning that $\varepsilon_H = \infty$. This implies that all firms operate in the market. Then, the demand function can be written as follows:

$$D(p(\omega)) = \begin{cases} L, & \text{if } p(\omega) \leq \frac{\varepsilon_L}{Q}, \\ \left(\frac{\varepsilon_L}{p(\omega)Q}\right)^{\sigma} L, & \text{if } p(\omega) > \frac{\varepsilon_L}{Q}. \end{cases} \tag{6}$$

Remember that the marginal utility of income Q can be found from

$$\begin{aligned} \int_{\omega\in\Omega} p(\omega)\left(1 - F\left(p(\omega)Q\right)\right) d\omega &= y \iff \\ Q &= \varepsilon_L \left(\frac{\int_{\omega\in\Omega} \left(p(\omega)\right)^{1-\sigma} d\omega}{y}\right)^{1/\sigma}. \end{aligned} \tag{7}$$

Using expression (7), the demand function in (6) can be rewritten as follows:

$$D(p(\omega)) = \begin{cases} L, & \text{if } p(\omega) \leq \frac{\varepsilon_L}{Q}, \\ \frac{yL}{P}\left(\frac{p(\omega)}{P}\right)^{-\sigma}, & \text{if } p(\omega) > \frac{\varepsilon_L}{Q}, \end{cases}$$

where P being equal to $\left(\int_{\omega\in\Omega} \left(p(\omega)\right)^{1-\sigma} d\omega\right)^{1/(1-\sigma)}$ is the CES price index.

Hence, a Pareto distribution leads to the CES preferences with the possibility of satiated demand for goods with sufficiently low prices. The role of the elasticity of substitution is played by the shape parameter σ. Higher σ leads to a lower variance of the distribution. As a result, consumer tastes become more similar and the elasticity of substitution increases. In the limit case when $\sigma = \infty$, all consumers have identical tastes and demand for good ω is equal to either the mass of consumers L or 0 (see Foellmi and Zweimueller (2006) or Tarasov (2009)).

2.2 Production

The structure of production is similar to that in Melitz (2003) and Melitz and Ottaviano (2008). The only factor of production is labor. I normalize wage per effective unit of labor to unity and assume that each worker is endowed with y effective units of labor. Thus, the total labor endowment in the economy is equal to yL.

There is free entry into the market. Each good ω is produced by a distinct firm. To enter the market, firms have to pay costs f_e (in terms of effective labor units) that are sunk. If a firm incurs the costs of entry, it obtains a draw φ of its productivity from the common distribution $G(\varphi)$ with the support on $[0, \infty)$. This generates ex-post firm heterogeneity. Depending on the productivity drawn, firms choose whether to exit from the market or to stay. Firms that decide to stay engage in price competition with other firms.

Firms choose prices $p(\omega)$ to maximize their profits. I assume that a firm producing good ω with productivity φ incurs marginal cost of $c(\omega) = 1/\varphi$.[1] That is, it takes $1/\varphi$ effective labor units to produce one unit of good ω. Hence, the profit maximization problem of a firm with productivity φ is as follows:

$$\max_p \left\{ (p - 1/\varphi)\, D(p) \right\}, \tag{8}$$

where $D(p)$ is defined by (4). As can be seen, the price of good ω depends only on φ and Q. Therefore, hereafter I omit the notation of ω and consider prices as a function of firm productivity φ and the marginal utility of consumer income Q. Specifically, I denote the price of a product produced by a firm with productivity φ as $p(\varphi, Q)$. Note that because of a continuum of competitors, firms take Q as given.

Notice that the demand function $D(p)$ has a kink at $p = \frac{\varepsilon_L}{Q}$. This implies that for some products, the maximization problem (8) can result in the corner solution with the optimal price

[1] To simplify the analysis, I assume that there are no fixed costs of production. However, the model can be easily extended to the case when firms incur fixed costs as well.

equal to $\frac{\varepsilon_L}{Q}$. While for the other goods, the solution of (8) is interior and satisfies

$$p - \frac{1}{\varphi} = \frac{1 - F(pQ)}{Qf(pQ)}, \tag{9}$$

where $f(\cdot)$ is a density function associated with $F(\cdot)$. The equation in (9) can be rewritten in the following way:

$$\frac{1}{\varphi p} = 1 - \frac{1 - F(pQ)}{pQf(pQ)}. \tag{10}$$

To guarantee the uniqueness of the solution of (9), I assume that the distribution of tastes satisfies the increasing proportionate failure rate (IPFR) property.[2] Namely, $\frac{\varepsilon f(\varepsilon)}{1-F(\varepsilon)}$ is strictly increasing in ε on $[\varepsilon_L, \varepsilon_H]$. The IPFR property implies that the right-hand side of the equation (10) is strictly increasing in p, while the left-hand side is strictly decreasing. Hence, if the solution of (10) exists, then it is unique. In fact, the proportionate failure rate is the price elasticity of demand. Thus, the IPFR property means that the price elasticity is increasing in price, which is a quite general regularity on a demand function. Notice that this property is weaker than the increasing hazard rate property and holds for many distribution families (see Van den Berg (2007)).

It is straightforward to show (see Appendix A) that the necessary and sufficient condition for existence of the solution is

$$\varphi \in \left[\frac{Q}{\varepsilon_H}, \frac{f(\varepsilon_L)Q}{f(\varepsilon_L)\varepsilon_L - 1} \right].$$

If $\varphi > \frac{f(\varepsilon_L)Q}{f(\varepsilon_L)\varepsilon_L - 1}$, then the firm's maximization problem (8) has a corner solution with $p(\varphi, Q) = \frac{\varepsilon_L}{Q}$. Firms with sufficiently low marginal cost choose such a price that all consumers purchase their goods. This is explained by the fact that demand is inelastic if the price is lower than $\frac{\varepsilon_L}{Q}$. Note that if the firm's productivity φ is low enough ($\varphi < \frac{Q}{\varepsilon_H}$), then production of the good yields negative profits. That is, firms with $\varphi < \varphi^*$ (where $\varphi^* = \frac{Q}{\varepsilon_H}$) do not operate in the market. The following lemma summarizes the findings above.

Lemma 1 *If $F(\varepsilon)$ satisfies the IPFR property, then there exists a unique solution of the firm's maximization problem* (8)*. Furthermore, if $\varphi > \frac{f(\varepsilon_L)Q}{f(\varepsilon_L)\varepsilon_L - 1}$, then*

$$p(\varphi, Q) = \frac{\varepsilon_L}{Q},$$

[2] The IPFR property was first established in Singh and Maddala (1976), who describe the size distribution of incomes. The property means that the hazard rate of the distribution does not decrease too fast.

while if $\varphi \in \left[\frac{Q}{\varepsilon_H}, \frac{f(\varepsilon_L)Q}{f(\varepsilon_L)\varepsilon_L - 1}\right]$, $p(\varphi, Q)$ *solves*

$$p - \frac{1}{\varphi} = \frac{1 - F(pQ)}{Qf(pQ)}.$$

Finally, firms with $\varphi < \varphi^* = \frac{Q}{\varepsilon_H}$ *do not operate in the market.*

In the next section, I formulate the equilibrium in the model.

2.3 Equilibrium

In the equilibrium, two conditions need to be satisfied. First, due to free entry, the expected profits of firms have to be equal to zero. Secondly, the goods market clears. Given the pricing rule established in *Lemma* 1, profits of a firm with productivity φ are given by

$$\pi(\varphi, Q) = \begin{cases} \left(\frac{\varepsilon_L}{Q} - \frac{1}{\varphi}\right) L, & \text{if } \varphi > \frac{f(\varepsilon_L)Q}{f(\varepsilon_L)\varepsilon_L - 1}, \\ \frac{(1 - F(p(\varphi,Q)Q))^2}{Qf(p(\varphi,Q)Q)} L, & \text{if } \varphi \in \left[\frac{Q}{\varepsilon_H}, \frac{f(\varepsilon_L)Q}{f(\varepsilon_L)\varepsilon_L - 1}\right] \\ 0, & \text{if } \varphi < \frac{Q}{\varepsilon_H}, \end{cases} \tag{11}$$

where $p(\varphi, Q)$ is determined in *Lemma* 1. Then, the free entry condition implies that

$$\int_0^\infty \pi(\varphi, Q) dG(\varphi) = f_e. \tag{12}$$

The goods market clearing condition means that

$$\int_{\omega \in \Omega} p(\omega) \left(1 - F\left(p(\omega)Q\right)\right) d\omega = y.$$

Let us denote M_e as the mass of firms entering the market. One can think of M_e in terms of there being $M_e g(\varphi)$ different firms with a certain productivity φ. Then, the goods market clearing condition is equivalent to

$$M_e \int_{\varphi^*}^\infty p(\varphi, Q) \left(1 - F\left(p(\varphi, Q)Q\right)\right) dG(\varphi) = y, \tag{13}$$

where $\varphi^* = \frac{Q}{\varepsilon_H}$ (recall that firms with $\varphi < \varphi^*$ do not operate in the market).

Hence, there are two unknowns, Q and M_e, and two equations, (12) and (13). Specifically, the marginal utility of income Q can be found from from the free entry condition (12), while the mass of entrants M_e is determined by (13).

Next, I show that there exists a unique equilibrium in the model. First, I formulate two technical lemmas that describe the relationship between $p(\varphi, Q)$ and Q determined in *Lemma* 1.

Lemma 2 *If $F(\varepsilon)$ satisfies the IPFR property, then for any φ, $p(\varphi, Q)$ is strictly decreasing in Q.*

Proof. In Appendix *A*. ■

The lemma states that higher marginal utility of income results in lower prices charged by firms. In other words, higher Q implies that consumers become more "fastidious" in choosing which goods to purchase. As a result, firms reduce their prices in order to increase their profits. Furthermore, in the next lemma, I show that higher marginal utility of income reduces not only prices, but also demand for some goods.

Lemma 3 *If $F(\varepsilon)$ satisfies the IPFR property, then for any $\varphi \in \left[\frac{Q}{\varepsilon_H}, \frac{f(\varepsilon_L)Q}{f(\varepsilon_L)\varepsilon_L - 1}\right]$, $p(\varphi, Q)Q$ is increasing in Q.*

Proof. In Appendix *A*. ■

Remember that demand for a good produced with productivity φ is given by $(1 - F\left(p(\varphi, Q)Q\right))L$. Therefore, a direct implication of *Lemma* 3 is that demand for goods with sufficiently high marginal cost decreases with a rise in Q. The lemmas allow us to describe the dependence of the profit function $\pi(\varphi, Q)$ on the marginal utility of income Q. Specifically, the following lemma holds.

Lemma 4 *If $F(\varepsilon)$ satisfies the IPFR property, then for any φ, $\pi(\varphi, Q)$ is decreasing in Q.*

Proof. Note that the profit function $\pi(\varphi, Q)$ can be written as follows:

$$\pi(\varphi, Q) = \left(p(\varphi, Q) - \frac{1}{\varphi}\right)(1 - F(p(\varphi, Q)Q))L.$$

As $p(\varphi, Q)$ and $(1 - F\left(p(\varphi, Q)Q\right))L$ are decreasing in Q (see *Lemma* 2 and *Lemma* 3), it is straightforward to see that $\pi(\varphi, Q)$ is decreasing in Q. ■

Remember that the equilibrium value of Q can be found from the free entry condition, which is given by

$$\int_0^\infty \pi(\varphi, Q)dG(\varphi) = f_e.$$

According to *Lemma* 4, the left-hand side of the equation is decreasing in Q. Moreover, it is straightforward to show that

$$\begin{aligned}\int_0^\infty \pi(\varphi, 0) dG(\varphi) &= \infty \text{ and} \\ \int_0^\infty \pi(\varphi, \infty) dG(\varphi) &= 0.\end{aligned}$$

This immediately implies that, for any $f_e > 0$, there exists a unique solution of equation (12). That is, Q is uniquely determined in the equilibrium. Moreover, given Q, M_e is uniquely determined by the goods market clearing condition (see (13). Thus, the following proposition holds.

Proposition 1 *If $F(\varepsilon)$ satisfies the IPFR property, there exists a unique equilibrium in the model.*

3 Comparative Statics

This section explores how consumer income and population size affect product prices, firm profits, and the mass of entrants in equilibrium.

3.1 Short-run Effects

To examine the short-run and long-run effects of consumer income and population size separately, I consider a short-run variation of the model. In constructing the short-run equilibrium, I follow Melitz and Ottaviano (2008). Specifically, I assume that in the short run, entry into the market is not possible. That is, M_e is fixed at some level $\bar{M}_e$. In this case, there is a fixed distribution of firm productivities on $[0, \infty)$ with $\bar{M}_e g(\varphi)$ being different firms with a certain productivity φ. In this framework, depending on its productivity, firms choose whether to produce or not to operate in the market. In addition, it is assumed that firms not operating in the market can restart production without paying the cost of entry. In other words, we have a fixed distribution of firms that can be potentially active in the market.

In the short run, the equilibrium is characterized only by the goods market clearing condition given by

$$\bar{M}_e \int_{\varphi^*}^\infty p(\varphi, Q) \left(1 - F\left(p(\varphi, Q)Q\right)\right) dG(\varphi) = y, \tag{14}$$

where $\varphi^* = \frac{Q}{\varepsilon_H}$. Here firms with $\varphi < \varphi^*$ decide not to operate in the market. In this case, the number of available products is equal to $\bar{M}_e(1 - G(\varphi^*))$. As $\bar{M}_e$ is an exogenous parameter and φ^* is a function of Q, we solve for the equilibrium value of Q from (14).

The results stated in *Lemma* 2 and *Lemma* 3 imply that for any product, $p(\varphi, Q)\left(1 - F\left(p(\varphi, Q)Q\right)\right)$ is strictly decreasing in Q. That is, higher marginal utility of income reduces consumer spendings on all available goods. In addition, an increase in Q decreases the mass of available products, as φ^* rises. Therefore, the following proposition holds.

Proposition 2 *If $F(\varepsilon)$ satisfies the IPFR property, then higher consumer income leads to higher equilibrium prices in the short run.*

Proof. From the previous consideration, $\int_{\varphi^*}^{\infty} p(\varphi, Q)\left(1 - F\left(p(\varphi, Q)Q\right)\right) dG(\varphi)$ is strictly decreasing in Q. Remember that the equilibrium value of Q is determined from the following equation:

$$\bar{M}_e \int_{\varphi^*}^{\infty} p(\varphi, Q)\left(1 - F\left(p(\varphi, Q)Q\right)\right) dG(\varphi) = y.$$

Since the left-hand side of the equation is strictly decreasing in Q, a rise in y leads to lower equilibrium value of Q. From *Lemma* 2, lower Q results in higher equilibrium prices set by firms.

■

The proposition implies that given other things equal, higher consumer income leads to less elastic demand and, thereby, higher prices in the short run. This in turn increases firm profits and, therefore, leads to more firms operating in the market (φ^* falls). Note that in the short run, Q does not depend on population size L. This implies that changes in L do not affect firm prices.

3.2 Long-run Effects

In the long run, entry is possible and, consequently, the equilibrium is characterized by the following system of equations:

$$\begin{aligned} \int_0^{\infty} \pi(\varphi, Q) dG(\varphi) &= f_e, \\ M_e \int_{\varphi^*}^{\infty} p(\varphi, Q)\left(1 - F\left(p(\varphi, Q)Q\right)\right) dG(\varphi) &= y, \end{aligned}$$

where M_e and Q are endogenous. Note that firm profits, $\pi(\varphi, Q)$, depend only on φ, Q, and L and, thus, do not directly depend on consumer income y (see (11)). Therefore, we have a striking result. In the long run, consumer income does not affect Q (as Q can be found from the free entry condition), which implies that the product prices do not depend on consumer income.

The intuition behind this finding is as follows. In the short run, higher consumer income results in higher product prices and, thereby, leads to higher firm profits. This increases the expected profits from entering the market. This means that more firms enter the market in the long run (M_e rises) inducing tougher competition among firms. Tougher competition in turn results in lower prices charged by firms decreasing profits. Hence, we have two effects of a rise in consumer income. The short-run effect increases firm profits, while the long-run effect leads to more entry decreasing firm profits. As a result, two effects are cancelled out, implying no changes in prices and profits.

Indeed, in the equilibrium,

$$M_e = \frac{y}{\int_{\varphi^*}^{\infty} p(\varphi, Q)\left(1 - F\left(p(\varphi, Q)Q\right)\right) dG(\varphi)}.$$

As Q does not depend on y, a rise in y leads to a rise in the mass of entrants M_e. Notice that in the long run, the cutoff φ^* is not affected by consumer income as well. The next proposition summarizes the findings above.

Proposition 3 *In the long run, consumer income does not affect firm prices and profits and positively affects the mass of entrants.*

In the long run, the impact of population size on equilibrium outcomes is also different from that in the short run. Specifically, a rise in L means higher demand for any product and leads to higher $\pi(\varphi, Q)$ for any φ and Q (see (11)). This increases the expected profits from entering the market and, therefore, results in more entry, which induces tougher competition among the firms. Tougher competition in the market in turn implies lower prices charged by firms (Q rises). Hence, in the long run, a rise in population size leads to more entry into the market and lower product prices.[3] The following proposition holds.

Proposition 4 *If $F(\varepsilon)$ satisfies the IPFR property, then in the long run, a rise in population size increases the mass of entrants into the market and decreases product prices charged by firms.*

[3] This effect is similar to that described in Melitz and Ottaviano (2008).

Proof. See Appendix B. ■

The above findings suggest that there is a substantial difference between short-run and long-run effects of consumer income and population size. In the next section, I consider a version of the model with many income classes.

4 The Model with Many Income Classes

To check the robustness of the results derived above, I assume that consumers differ not only in their tastes for goods, but also in their incomes. In particular, consumer j chooses $\{x(\omega) \in \{0,1\}\}_{\omega\in\Omega}$ to maximize

$$U_j = \int_{\omega\in\Omega} \varepsilon_j(\omega)x(\omega)d\omega$$

subject to

$$\int_{\omega\in\Omega} p(\omega)x(\omega)d\omega = y_j,$$

where y_j is her income. As consumers have different incomes, the marginal utility of income varies across them. Specifically, the utility maximization problem implies that consumer j purchases good ω if and only if

$$\frac{\varepsilon_j(\omega)}{p(\omega)} \geq Q_j.$$

I assume that there are N types of consumers indexed by i.[4] A consumer of type i is endowed with y_i efficiency units of labor (wage per efficiency unit is normalized to unity). If we denote α_i as the fraction of type i consumers in the aggregate mass L, the total labor supply in the economy in efficiency units is $L\sum_{i=1}^{N}\alpha_i y_i$.

In this case, it is straightforward to see that the demand for good ω is given by

$$D(p(\omega)) = L\sum_{i=1}^{N}\alpha_i\left(1 - F(p(\omega)Q_i)\right), \tag{15}$$

where Q_i is the marginal utility of income of type i consumers. Here $Q_i > Q_j$ if and only if $y_i < y_j$. Finally, the budget constraint of type i consumer can be written as follows:

$$\int_{\omega\in\Omega} p(\omega)\left(1 - F\left(p(\omega)Q_i\right)\right)d\omega = y_i.$$

[4] The framework can be easily extended to the case of a continuos distribution of income.

The production structure is the same as in the previous section. Firms choose prices $p(\omega)$ to maximize their profits. The profit maximization problem of a firm with productivity φ is as follows:

$$\max_{p} \left\{ \left(p - 1/\varphi\right) D(p) \right\},$$

where $D(p)$ is defined by (15). The solution of the maximization problem yields a certain pricing rule $p(\varphi, Q)$, where $Q = \{Q_i\}_{i=1..N}$.

4.1 Equilibrium

The long-run equilibrium is characterized by the free entry condition and goods market clearing conditions. In particular, the free entry condition implies that

$$\int_0^\infty \pi(\varphi, Q) dG(\varphi) = f_e, \tag{16}$$

where

$$\pi(\varphi, Q) = \left(p(\varphi, Q) - 1/\varphi\right) D(p(\varphi, Q)).$$

The goods market clearing conditions mean that for any i,

$$M_e \int_0^\infty p(\varphi, Q) \left(1 - F\left(p(\varphi, Q) Q_i\right)\right) dG(\varphi) = y_i. \tag{17}$$

Hence, there are $N + 1$ unknowns, $\{Q_i\}_{i=1..N}$ and M_e, and $N + 1$ equations. This determines the equilibrium. Note that depending on Q_i, $1 - F\left(p(\varphi, Q) Q_i\right)$ can be equal to zero for certain products, implying that the baskets of products consumed by different income classes can be different.

Let us denote by M_i^j the number of products consumed by agent j with income y_i. Specifically,

$$M_i^j = \int_{\omega \in \Omega} Ind\left(\varepsilon_j(\omega) \geq p(\omega) Q_i\right) d\omega,$$

where $Ind(\cdot)$ is an indicator function. Using the law of large numbers, it is straightforward to show that

$$\begin{aligned} M_i^j &= \int_{\omega \in \Omega} ((1 - F\left(p(\omega) Q_i\right))) d\omega \\ &= M_e \int_0^\infty \left(1 - F\left(p(\varphi, Q) Q_i\right)\right) dG(\varphi). \end{aligned}$$

As can be inferred, agents with a higher income consume a higher number of goods in equilibrium. Indeed, a higher income implies a lower marginal utility of income (see (17), which in turn results in a higher number of goods consumed. Notice that because of the stochastic utility function, it is not possible to determine the composition of products consumed by a certain income class.

4.2 Income Effects

The equilibrium equations can be rewritten in the following way:

$$\begin{aligned}\int_0^\infty \pi(\varphi, Q)dG(\varphi) &= f_e, \\ \frac{\int_0^\infty p(\varphi, Q)\left(1 - F\left(p(\varphi, Q)Q_i\right)\right)dG(\varphi)}{\int_0^\infty p(\varphi, Q)\left(1 - F\left(p(\varphi, Q)Q_N\right)\right)dG(\varphi)} &= \frac{y_i}{y_N}, \\ M_e \int_0^\infty p(\varphi, Q)\left(1 - F\left(p(\varphi, Q)Q_N\right)\right)dG(\varphi) &= y_N.\end{aligned}$$

Hence, we can find the equilibrium values of $\{Q_i\}_{i=1..N}$ from the first N equations and the value of M_e from the last equation. In the case of many income classes, it is possible to formulate an analogue of *Proposition* 2. In particular, the following proposition holds.

Proposition 5 *A proportional rise in income of all consumers ($\frac{y_i}{y_N}$ does not change) does not affect firm prices and profits and increases the mass of entrants.*

Proof. Since a proportional increase in consumer incomes does not change $\frac{y_i}{y_N}$ ratios. The equations determining the equilibrium values of $\{Q_i\}_{i=1..N}$ do not change as well. That is, the equilibrium values of $\{Q_i\}_{i=1..N}$ do not change and, thereby, firm prices and profits do not change. Finally, as

$$M_e = \frac{y_N}{\int_0^\infty p(\varphi, Q)\left(1 - F\left(p(\varphi, Q)Q_N\right)\right)dG(\varphi)}$$

and the denominator does not change, the mass of entrants M_e rises. ■

The proposition suggests that in the long run, certain changes in the consumer income distribution have no impact on product prices charged by firms. The intuition is the same as in the previous section. On the one hand, a proportional rise in consumer incomes leads to less elastic demand and, thereby, higher prices. On the other hand, higher consumer incomes imply more entry into the market and tougher competition, which in turn reduces the prices charged by firms. As a result, the effects are cancelled out.

In the model, it is quite complicated to explore an impact of arbitrary changes in income distribution on economic outcomes. However, under certain restrictions on the distribution function of consumer tastes, $F(\cdot)$, it is possible to derive an unambiguous relationship between prices and income distribution. Specifically, I consider the case when $F(\cdot)$ is a multiplicatively separable function: i.e., $F(xy) = F_1(x)F_2(y)$ for some $F_1(\cdot)$ and $F_2(\cdot)$. The following proposition holds.

Proposition 6 *If demand for each product of each income class is strictly positive and the distribution function of consumer tastes is multiplicatively separable, then prices charged by firms depend only on average consumer income in the short run and do not depend on income distribution in the long run.*

Proof. If $F(\cdot)$ is multiplicatively separable and demand for each product of each income class is strictly positive, then the aggregate demand for a certain product can be written as follows:

$$D(p) = L\sum_{i=1}^{N} \alpha_i \left(1 - F(pQ_i)\right) = L\left(1 - F_1(p)E\left(F_2(Q)\right)\right), \tag{18}$$

where $E\left(F_2(Q)\right) = \sum_{i=1}^{N} \alpha_i F_2(Q_i)$. Thus, the profit maximizing price charged by a firm with productivity φ solves

$$p - 1/\varphi = \frac{1 - F_1(p)E\left(F_2(Q)\right)}{f_1(p)E\left(F_2(Q)\right)},$$

where $f_1(p) = F_1'(p)$. As a result, the optimal price, $p(\varphi, Q)$, is an implicit function of φ and $E\left(F_2(Q)\right)$. Recall that the budget constraint of consumer i is given by

$$\begin{aligned} M_e \int_0^\infty p(\varphi, Q)\left(1 - F\left(p(\varphi, Q)Q_i\right)\right) dG(\varphi) &= y_i \iff \\ M_e \int_0^\infty p(\varphi, Q)\left(1 - F_1(p(\varphi, Q))F_2\left(Q_i\right)\right) dG(\varphi) &= y_i. \end{aligned}$$

Aggregating across all agents, one can derive that

$$M_e \int_0^\infty p(\varphi, Q)\left(1 - F_1(p(\varphi, Q))E\left(F_2(Q)\right)\right) dG(\varphi) = Ey,$$

where $Ey = \sum_{i=1}^{n} \alpha_i y_i$. Since $p(\varphi, Q)$ is a function of $E\left(F_2(Q)\right)$, $E\left(F_2(Q)\right)$ is determined by the average income Ey in the short run, implying that the prices depend only on the average income in the economy. It is also straightforward to see that in the long run $EF_2(Q)$ is pinned down by the free entry condition and, as a result, the prices do not depend on the income distribution.

■

The proposition implies that, under certain assumptions about consumer tastes, the results established in the model with no income heterogeneity can be also derived in the model with many income classes. Note however that for other distributions one can expect a more complex relationship between income distribution and prices where other moments of income distribution may play an important role. For instance, consider an exponential distribution of consumer tastes:

$$F(\varepsilon) = 1 - \exp\left(-\gamma\varepsilon\right).$$

As the distribution has an infinite upper bound, demand for each product of each income class is strictly positive. In this case, prices charged by firms solve

$$\max_{p} \left\{ (p - 1/\varphi) \left(\sum_{i=1}^{n} \alpha_i \exp\left(-pQ_i\right) \right) \right\},$$

meaning that $p(\varphi, Q)$ solves

$$p - 1/\varphi = \frac{\sum_{i=1}^{n} \alpha_i \exp\left(-pQ_i\right)}{\sum_{i=1}^{n} \alpha_i Q_i \exp\left(-pQ_i\right)}.$$

As can be seen, the relationship between $p(\varphi, Q)$ and $Q = \{Q_i\}_{i=1..n}$ is much more complex compared to the case with a multiplicatively separable distribution function, implying that the average income itself is not enough to determine the prices (other moments of the distribution matter).

In the next subsection, I explore in detail a special case where the distribution of consumer tastes is uniform (and, therefore, satisfies the discussed property of being multiplicative separable).

4.2.1 A Special Case: Uniform Distribution

In this subsection, I illustrate *Proposition* 6 considering a uniform distribution of consumer tastes. Specifically, I assume that $F(\varepsilon)$ is equal to $\frac{\varepsilon}{\varepsilon_H}$ on $[0, \varepsilon_H]$ interval. In addition, I assume that the distribution of productivities $G(\varphi)$ has the support on $[A, \infty)$ (where A is sufficiently high) and the income distribution is not too dispersed. The purpose of the latter assumptions is to guarantee strictly positive demand for each product of each income class. In other words, the income distribution and the parameter A are assumed to be such that for any i and φ, $1-F\left(p(\varphi,Q)Q_i\right)$ is strictly greater than zero in the equilibrium. This will substantially simplify the analysis and allow us to derive the closed-form solution. Later, I discuss these assumptions in more detail.

Given the assumptions, the profit maximization problem is

$$\max_p \left\{ (p-1/\varphi) \left(\sum_{i=1}^{N} \alpha_i \left(1-\frac{pQ_i}{\varepsilon_H}\right)\right)\right\}.$$

Let us denote $\sum_{i=1}^{N} \alpha_i Q_i$ as EQ (the "average" marginal utility of income), then since $\sum_{i=1}^{N} \alpha_i$ equals to one, the profit maximization problem can be rewritten as follows:

$$\max_p \left\{ (p-1/\varphi) \left(1-\frac{pEQ}{\varepsilon_H}\right)\right\}.$$

Hence, the equilibrium prices are given by

$$p(\varphi,Q)=\frac{1}{2\varphi}+\frac{\varepsilon_H}{2EQ}. \tag{19}$$

Recall that according to our assumptions, $1-F\left(p(\varphi,Q)Q_i\right)$ should be strictly greater than zero for any i and φ. By substituting the expression for the price in (19),

$$1-F\left(p(\varphi,Q)Q_i\right)=1-\frac{Q_i}{2\varphi\varepsilon_H}-\frac{Q_i}{2EQ}.$$

Hence, to guarantee that $1-F\left(p(\varphi,Q)Q_i\right)>0$, we firstly need to assume that

$$EQ>\frac{Q_i}{2},$$

meaning that the equilibrium values of $\{Q_i\}_{i=1..N}$ are not too dispersed or, equivalently, the income distribution is not too dispersed. Secondly, we need to assume that for any φ,

$$\varphi>\frac{Q_i EQ}{\varepsilon_H\left(2EQ-Q_i\right)},$$

which is equivalent to

$$A>\frac{Q_i EQ}{\varepsilon_H\left(2EQ-Q_i\right)}.$$

Thus, if the income distribution is not too dispersed and firms in the economy are sufficiently productive (A is sufficiently high), then in the equilibrium demand for each product of each income class is strictly positive. Note that if the income distribution is dispersed enough (for some i, $Q_i > 2EQ$), some firms find it profitable to charge so high prices for their products that relatively poor consumers decide not to buy them, implying zero demand of certain income classes.

Plugging the prices into the budget constraint (17), we have

$$M_e \int_A^\infty \left(\frac{1}{2\varphi} + \frac{\varepsilon_H}{2EQ}\right)\left(1 - \frac{Q_i}{2\varphi\varepsilon_H} - \frac{Q_i}{2EQ}\right) dG(\varphi) = y_i.$$

Taking the weighted sum across all income classes, we derive

$$\begin{aligned} M_e \sum_{i=1}^N \alpha_i \int_A^\infty \left(\frac{1}{2\varphi} + \frac{\varepsilon_H}{2EQ}\right)\left(1 - \frac{Q_i}{2\varphi\varepsilon_H} - \frac{Q_i}{2EQ}\right) dG(\varphi) &= \sum_{i=1}^N \alpha_i y_i \iff \\ M_e \int_A^\infty \left(\frac{1}{2\varphi} + \frac{\varepsilon_H}{2EQ}\right)\left(\frac{1}{2} - \frac{EQ}{2\varphi\varepsilon_H}\right) dG(\varphi) &= Ey, \end{aligned}$$

where Ey is the average consumer income given by $\sum_{i=1}^N \alpha_i y_i$. In addition, it is straightforward to show that

$$\begin{aligned} \pi(\varphi, Q) &= (p(\varphi, Q) - 1/\varphi)\, D(p(\varphi, Q)) \\ &= \frac{L\varepsilon_H}{EQ}\left(\frac{1}{2} - \frac{EQ}{2\varphi\varepsilon_H}\right)^2. \end{aligned}$$

Hence, the equilibrium values of M_e and EQ can be found from the following equations:

$$\frac{L\varepsilon_H}{EQ}\int_A^\infty \left(\frac{1}{2} - \frac{EQ}{2\varphi\varepsilon_H}\right)^2 dG(\varphi) = f_e, \tag{20}$$

$$M_e \int_A^\infty \left(\frac{1}{2\varphi} + \frac{\varepsilon_H}{2EQ}\right)\left(\frac{1}{2} - \frac{EQ}{2\varphi\varepsilon_H}\right) dG(\varphi) = Ey, \tag{21}$$

while $\{Q_i\}_{i=1..N}$ can be found from (17).

As can be seen, in the long-run equilibrium, any changes in income distribution do not affect firm prices. This is because $p(\varphi, Q)$ depends only on average marginal utility of income, which in turn depends only on population size L and the entry cost f_e (see equation (20)). Moreover, the mass of entrants depends only on average income. That is, any changes in income distribution such that average income remains the same do not affect the mass of entrants.

In the short run, entry is not possible. Therefore, EQ should be found from

$$\bar{M}_e \int_A^\infty \left(\frac{1}{2\varphi} + \frac{\varepsilon_H}{2EQ}\right)\left(\frac{1}{2} - \frac{EQ}{2\varphi\varepsilon_H}\right) dG(\varphi) = Ey,$$

where $\bar{M}_e$ is some constant. As can be inferred, in the short run, firm prices depend only on average income Ey. Specifically, a higher average income decreases EQ making demand less elastic and, thereby, increasing the product prices.

5 Conclusion

This paper develops a new family of consumer preferences in the monopolistic competition framework, which can capture the effects of consumer income and the intensity of competition on equilibrium prices. The constructed preferences have two key features. First, goods are indivisible and consumers purchase at most one unit of each good. Second, consumers are allowed to have different tastes for a particular good.

I show that if there is no income heterogeneity, a rise in consumer income leads to higher product prices in the short run and does not affect prices in the long run. In the case of heterogeneous consumer incomes, the distribution of consumer tastes matters. Specifically, if the distribution of consumer tastes is multiplicatively separable (and demand for each product of each income class is strictly positive), prices charged by firms depend only on the average income in the short run and do not depend on the income distribution in the long run. However, in the case of other distributions of tastes, a more complex relationship between prices and income distribution takes place.

The developed approach to modeling preferences is quite flexible and can be used in many various applications requiring variable firm markups. For instance, in the analysis of international trade, Verhoogen (2008) uses a variation of the multinomial-logit demand function with constant consumers' willingness to pay for quality (the analogue of the marginal utility of income) resulting in constant firm markups. The quality of a product can be incorporated in the present model as well.[5] Furthermore, an exponential distribution of consumer tastes results in the analogue of the multinomial-logit demand function. However, in this case, consumers' willingness to pay for quality and, therefore, markups are endogenous.

[5] It is sufficient to introduce some quality index in the utility function.

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Appendix *A*

In this part of Appendix, I analyze the properties of the price function $p(\varphi, Q)$ determined by the profit maximization problem. Recall that if the solution of the maximization problem in (8) is interior, then the optimal price solves

$$\frac{1}{\varphi p} = 1 - \frac{1 - F(pQ)}{pQf(pQ)}. \tag{22}$$

If $F(\varepsilon)$ satisfies the IPFR property, then $\frac{\varepsilon f(\varepsilon)}{1-F(\varepsilon)}$ is strictly increasing on $[\varepsilon_L, \varepsilon_H]$. This implies that the right-hand side of the above equation is strictly increasing as a function of p. Remember that for the interior solution, p must belong to $[\frac{\varepsilon_L}{Q}, \frac{\varepsilon_H}{Q}]$. Hence, taking into account that the left-hand side is strictly decreasing in p, equation (22) has a unique solution on $[\frac{\varepsilon_L}{Q}, \frac{\varepsilon_H}{Q}]$ if and only if

$$\begin{aligned} \frac{1}{\varphi \frac{\varepsilon_L}{Q}} &\geq 1 - \frac{1 - F(\frac{\varepsilon_L}{Q} Q)}{\frac{\varepsilon_L}{Q} Q f(\frac{\varepsilon_L}{Q} Q)} \text{ and} \\ \frac{1}{\varphi \frac{\varepsilon_H}{Q}} &\geq 1 - \frac{1 - F(\frac{\varepsilon_H}{Q} Q)}{\frac{\varepsilon_H}{Q} Q f(\frac{\varepsilon_H}{Q} Q)}. \end{aligned}$$

The latter is equivalent to

$$\varphi \in \left[\frac{Q}{\varepsilon_H}, \frac{f(\varepsilon_L)Q}{f(\varepsilon_L)\varepsilon_L - 1}\right].$$

Next, I provide the proofs of *Lemmas* 2 and 3.

The Proof of *Lemma 2*

From *Lemma 1*, if $\varphi > \frac{f(\varepsilon_L)Q}{f(\varepsilon_L)\varepsilon_L - 1}$, then $p(\varphi, Q)$ is equal to $\frac{\varepsilon_L}{Q}$ and, therefore, is decreasing in Q. If $\varphi \in \left[\frac{Q}{\varepsilon_H}, \frac{f(\varepsilon_L)Q}{f(\varepsilon_L)\varepsilon_L - 1}\right]$, then $p(\varphi, Q)$ is the solution of

$$\frac{1}{\varphi p} = 1 - \frac{1 - F(pQ)}{pQf(pQ)}.$$

As $F(\varepsilon)$ satisfies the IPFR property, $\frac{1-F(pQ)}{pQf(pQ)}$ is decreasing in Q for any p. This implies that for any p, the right-hand side of the above equation is increasing in Q: higher Q shifts the function $1 - \frac{1-F(pQ)}{pQf(pQ)}$ upward. As a result, the value of $p(\varphi, Q)$ decreases.

The Proof of *Lemma 3*

Consider $\varphi \in \left[\frac{Q}{\varepsilon_H}, \frac{f(\varepsilon_L)Q}{f(\varepsilon_L)\varepsilon_L - 1}\right]$. It is straightforward to see that in this case, $p(\varphi, Q)Q$ solves

$$\frac{Q}{\varphi x} = 1 - \frac{1 - F(x)}{x f(x)}$$

with respect to x. Higher Q shifts the left-hand side of the equation upward. This means that the value of $p(\varphi, Q)Q$ increases, as the right-hand side is increasing in x.

Appendix B

In this Appendix, I explore the effects of a rise in population size L on the equilibrium outcomes in the long run. In the long run, the equilibrium is determined by

$$\begin{aligned} \int_0^\infty \pi(\varphi, Q) dG(\varphi) &= f_e, \\ M_e \int_{\varphi^*}^\infty p(\varphi, Q)\left(1 - F\left(p(\varphi, Q)Q\right)\right) dG(\varphi) &= y, \end{aligned}$$

where $\pi(\varphi, Q)$ is given by (11). As can be seen from the equations, population size affects the equilibrium only through $\pi(\varphi, Q)$. Specifically, a rise in L increases $\pi(\varphi, Q)$ for any φ, Q. In other words, higher L increases the firm's expected profits, implying that the left-hand side of the free entry equation rises. From *Lemmas* 2 and 3, it is straightforward to show that $\pi(\varphi, Q)$ is a decreasing function of Q (for any φ). Thus, in order the free entry condition is satisfied, a rise in L has to be compensated by a rise in Q. A rise in Q in turn results in lower prices charged by firms (as it is shown in *Lemma* 2).

Finally, the mass of entrants can be found from

$$M_e = \frac{y}{\int_{\varphi^*}^\infty p(\varphi, Q)\left(1 - F\left(p(\varphi, Q)Q\right)\right) dG(\varphi)}.$$

As the denominator is decreasing in Q and consumer income y is fixed, a rise in L results in more entry into the market (higher M_e).