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GENERALIZED CALVO APPROACH

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Generalized Calvo Approach

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Abstract

This paper presents a generalized application of the Calvo pricing approach, assuming that in each particular market, both prices and quantities are adjusted according to the Calvo approach. With certain restrictions placed on the parameters, the system of log-linearized equations that I derived using the generalized Calvo approach produces similar dynamics within the economy as are produced by the system that one would obtain assuming Calvo pricing, consumption habit formation, capital adjustment costs, investment adjustment costs, search-and-match at the labor market, and smooth money adjustment. Therefore, the generalized Calvo approach proposed here acts as a unification of many frictions used in conventional dynamic stochastic general equilibrium models.

Key words: DSGE, frictions, Calvo pricing, consumption habit formation, capital adjustment costs, investment adjustment costs, search-and-match.

JEL classification: E1, E3, E5, C62, D59

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A typical modern business cycle model can be viewed as a monetary neoclassical growth model (MNG model), with several market imperfections incorporated. The use of the neoclassical growth framework ensures that the balanced growth path within the frame of the designed model is consistent with the basic facts about long-run growth formulated by Kaldor (1957). However, a pure MNG model is not entirely consistent with short-run data.

A conventional way to build a data-consistent model is to incorporate several frictions into an MNG model. These frictions explain why some variables behave in a different manner than the MNG model otherwise predicts. In fact, almost all variables in a typical dynamic stochastic general equilibrium (DSGE) model require a special "correcting" assumption, which makes it possible to "tune" the trajectory of adjustment of this variable to exogenous disturbances (see Smets and Wouters, 2003, 2007; Christiano et al., 2005, and many others). A typical set of frictions includes Calvo pricing (see Calvo, 1983; Clarida et al., 1999; Bils and Klenow, 2004), consumption habit formation (see Abel, 1990; Campbell and Cochrane, 1999; Fuhrer, 2000), capital or investment adjustment costs, and search-and-match at the labor market (see Pissarides, 2000; Monika and Merz, 1995; Gertler et al., 2008). A model that incorporates all of these frictions provides a reasonable approximation of the data, and, as noted by Smets and Wouters (2007), it produces better macroeconomic forecasts than simple unconstrained vector autoregressive models, at least in the medium- and the long-run.

The need for all these assumptions to make the model accurate casts doubt on the reliability of the theory; usually, researchers are not satisfied with a theory that requires the continuing incorporation of new epicycles to fit the observations made. I show that this critique does not apply to DSGE models because all of the frictions used in a conventional DSGE model can be derived from a single generalized Calvo adjustment assumption rather than from the repeated addition of new adjustments.

The generalized Calvo adjustment assumption is introduced in Section 1 in the following way. Consider a monetary neoclassical growth model as a baseline for the analysis. Assume that in each market, the sellers, who benefit from market power, set prices, and the buyers, who take the prices as given, choose the quantities demanded. Once a seller has chosen her price, she supplies any quantity demanded at this price until the price is revised. In the same manner, once a buyer has chosen the quantity demanded, she purchases this quantity at any price until she changes this decision. Finally, in this model, the decisions are not revised continuously but only at set points in time determined by independent Poisson processes, in the same manner as prices are revised in the Calvo (1983) pricing model.

In Section 2 I compare the model based on the conventional set of frictions with the model derived from the generalized Calvo assumption.

In Subsection 2.2 I compare the habit formation assumption with the generalized Calvo assumption. I show that when the utility is additively separable and there is no trend growth of consumption, these two assumptions produce the very same dynamic equations. The coefficients of the dynamic equations are slightly different under the positive consumption growth rate along the balanced growth path; however, this difference is numerically negligible.

In Subsection 2.3 I compare the investment adjustment costs assumption with the generalized Calvo assumption. In general, these assumptions produce different dynamics of investment. However, when certain restrictions are applied to the parameters and with a zero growth rate of investment along the balanced growth path, the dynamic equations are exactly the same.

The idea of comparing the Calvo adjustment assumption with the adjustment costs assumption is not new in the literature, (see Rotemberg, 1982, 1987; Roberts, 1995; Salvatore and Nistic, 2007; Lombardo and Vestin, 2008). Some evidence in favor of the Calvo pricing assumption and against Rotemberg's price adjustment costs is provided by Ascari et al. (2011). In this paper I compare the Calvo adjustment assumption with an adjustment costs assumption as applied to investment or capital accumulation rather than to prices. As distinguished from the previous literature, in my framework, these approaches are similar only under some restrictions on the parameters.

In Section 2.4 I compare the generalized Calvo assumption with the search-and-match at the labor market assumption. To make these assumptions comparable, I assume that the labor force under the search-and-match assumption is determined exogenously in the model, which is a typical assumption in DSGE literature. If the labor force is exogenous, then the two discrepant assumptions produce dynamic equations that have the same structure but different coefficients.

In Subsection 2.5 I introduce frictions associated with the adjustment of money. Finally, in Section 3 I provide a summary of the discrepancies between the generalized Calvo approach and the conventional set of frictions.

1 DSGE Model

1.1 Sketch of the model

There are four types of agents in the economy: households, firms, the central bank, and the government. Households own capital and labor and rent them to firms. Households use the factor remuneration and the pure profit that they receive from firms to pay for their consumption, lump-sum taxes, and to save physical capital, government debt, and money. Firms use labor and capital to produce final goods. The central bank conducts open market operations by selling or buying government bonds. The government issues government bonds and uses the seigniorage revenue and the lump-sum taxes that it receives from households to pay for government purchases and the interest rate on government bonds.

There are four markets in the economy: the labor market, the capital rent market, the government bonds market, and the final goods market. The generalized Calvo adjustment assumption is applied to the labor market, the capital rent market, and the final goods market. This assumption is not applied to the market for government bonds, where decisions can be revised continuously; otherwise, financial arbitrage would be possible.

There is perfect competition in the market for government bonds and monopolistic competition within the other markets. Households benefit from market power in the labor and capital rent markets, and firms possess market power in the final goods market. Therefore, in the labor, capital, and final goods markets, the sellers possess market power, and the buyers take the prices as given.

Because buyers take prices as given, the adjustment of prices stems from sellers' behavior. The prices are set according to the Calvo adjustment: each seller revises her price only at set points in time determined by an independent Poisson process. Once the price has been set, the seller supplies any quantity demanded at this price until the new Poisson event arrives, and she revises the price that time.

The buyers choose their quantities demanded at given prices. In the labor, capital, and final goods markets, the quantities demanded are also adjusted according to Calvo: each buyer revises her quantity demanded only at set points in time determined by a Poisson process, and once the quantity has been chosen, she buys this quantity demanded at any price until the new Poisson event occurs.

In Section 2, I show that the generalized Calvo assumption can be used instead of all of the frictions used in a conventional DSGE model, including capital adjustment costs. However, a model that assumes Calvo adjustment both for investment and capital would be internally inconsistent. Because capital and investment are related to each other within the capital accumulation equation, I can assume that either capital or investment, but not both, can jump from one value to another. For this reason, in this model remains the standard assumption of capital adjustment costs and applies the generalized Calvo assumption only to investment[1].

1.2 Notation

This paper deploys the following notation. Capital letters denote the absolute values of variables, and almost all lower-case letters denote the relative deviations from the balanced growth path values. The exceptions to this standard are made for interest rates and the inflation rate. for these two variables, lower-case letters denote the absolute, rather than the relative, deviations from the balanced growth path.

Indices i and j denote the individual values of variables associated with household i or firm j, and variables without indices denote the aggregate or average values in the cases where they are appropriable.

Letters without bars or hats denote the actual values of variables. When the Poisson event arrives, and the agent is allowed to revise a particular variable, she resets this variable to some value, which is called the *reset* value of the variable. The reset values are denoted with hats. Finally, to solve the model, I introduce hypothetical *frictionless* values. The frictionless value of a variable is defined as the value of the variable that the agent would chose if she were allowed to revise this value continuously, still setting the values of the other variables according to Calvo. Frictionless values are denoted with bars.

For example, C is the aggregate consumption, C_i is the consumption by household i; $\bar{C}_i$ is the hypothetical frictionless value of consumption by household i; $\hat{C}_i$ is the reset value consumption; and c_i, $\bar{c}_i$, $\hat{c}_i$ are defined as:

$$c_i = \frac{C_i - C_i^*}{C_i^*} \quad \bar{c}_i = \frac{\bar{C}_i - C_i^*}{C_i^*} \quad \hat{c}_i = \frac{\hat{C}_i - C_i^*}{C_i^*}$$

where C_i^* is the i^{th} household's balanced growth path aggregate consumption[2]. The balanced growth path values are not necessarily constant; for example, the expected growth rate of C^* equals the rate of labor-augmenting technological progress.

The number of households and the number of firms are normalized to one, therefore the aggregate consumption is:

$$C = \int_0^1 C_i di$$

[1]The first-order conditions for investment in a model with a Calvo constraint on the investment decisions but without any friction associated capital include the derivative of the Dirac delta function, which complicates the analysis. Therefore, I include some capital adjustment friction into the model, namely capital adjustment costs, to make the analysis relatively simple.

[2]Because households face different realizations of Poisson processes, consumption by different households may converge to different balanced growth paths.

In the same manner, I introduce $\bar{C}$, $\hat{C}$ and C^{*}. Finally, c, $\bar{c}$, and $\hat{c}$ are integrals of c_i, $\bar{c}_i$, and $\hat{c}_i$ over $i \in [0, 1]$.

As mentioned above, the principles of notation are different for interest rates and the inflation rate. For example, Π is actual inflation, Π^{*} is the long-run inflation rate, and π is defined as:

$$\pi = \Pi - \Pi^{*}$$

A complete list of notations is provided in Appendix A.

1.3 Time

This model is simpler in continuous time than in discrete time because when using discrete time, I cannot assume that the probability of simultaneous revision of two or more variables tends to zero. If the model is operated under discrete time, I would need to consider first-order conditions for all possible combinations of variables that potentially may be revised on a given date. Therefore, I consider a continuous-time model.

1.4 Household's problem

Household i maximizes expected utility, which depends on the consumption C_i, the labor L_i, and the real money balances M_i:

$$\max E_0 \int_0^{\infty} e^{-\rho t} U\left(C_i, L_i, M_i\right) dt \tag{1}$$

To ensure the existence of a balanced growth path, I assume the following instantaneous utility function:

$$U\left(C_i, L_i, M_i\right) = \sigma^{-1}\left(C_i^{\theta}\left(1 - L_i - \phi\frac{C_i}{M_i}\right)^{1-\theta}\right)^{\sigma} \tag{2}$$

where $\sigma < 1$, $\theta \in (0, 1)$, and $\phi > 0$ are parameters. The chosen form of the shopping-time function gives Baumol-Tobin's square root formula for money demand.

The household's real wealth A_i consists of physical capital K_i, government bonds B_i and real money holdings M_i. For simplicity, I assume that the household can sell short government debt, which means that B_i may be negative. The household accumulates its wealth according to:

$$\dot{A}_i = \left(R^{b} - \Pi\right) B_i + \left(R_i^{k} - \Pi\right) K_i + D_i + W_i L_i - C_i - \Pi M_i - T_i \tag{3}$$

where R^{b} is the nominal interest rate on the government bonds, Π is the inflation rate, R_i^{k} is the nominal interest rate associated with physical capital, W_i is the wage, D_i is the firms' profits paid to the household.

The individual demand functions for the household's capital and labor stem from the firms' problem, which is defined in the next section (see factor aggregation technologies (9)), and they are given by:

$$K_i = \left(\frac{R^k - \Pi}{R^k_i - \Pi}\right)^{\eta} K \tag{4a}$$

$$L_i = \left(\frac{W}{W_i}\right)^{\xi} L, \tag{4b}$$

where $\eta > 1$ and $\xi > 1$ are the parameters.

The household maximizes its utility with respect to consumption, real wage, capital interest rate, and real money balances. The household is not allowed to revise the values of these variables continuously, but only at set points in time determined by independent Poisson processes[3]. If a particular variable is not revised, it is indexed at the rate of its growth along the equilibrium balanced growth path. Therefore, the process of revision of these variables is formalized as[4]:

$$dC_i = \begin{cases} \hat{C}_i - C_i & \text{prob. } \lambda_c dt \\ \nu C_i dt & \text{prob. } 1 - \lambda_c dt \end{cases} \tag{5a}$$

$$dW_i = \begin{cases} \hat{W}_i - W_i & \text{prob. } \lambda_w dt \\ (\nu - \pi) W_i dt & \text{prob. } 1 - \lambda_w dt \end{cases} \tag{5b}$$

$$dR^k_i = \begin{cases} \hat{R}^k_i - R^k_i & \text{prob. } \lambda_{rk} dt \\ 0 & \text{prob. } 1 - \lambda_{rk} dt \end{cases} \tag{5c}$$

$$dM_i = \begin{cases} \hat{M}_i - M_i & \text{prob. } \lambda_m dt \\ (\nu - \pi) M_i dt & \text{prob. } 1 - \lambda_m dt \end{cases} \tag{5d}$$

where λ_c, λ_w, λ_{rk}, and λ_m are the Poisson arrival rates, and ν is the rate of exogenous labor-augmenting technological progress. I refer to (5) as the *Calvo constraints* on the household's problem.

The household maximizes (1) with respect to $\hat{C}_i$, $\hat{W}_i$, $\hat{R}^k_i$, and $\hat{M}_i$ subject to (3), (4), and (5).

The behavior of the household is described by three systems of equations. The first system it is the set of Calvo constraints (5). This system determines how the actual values of consumption, wages, capital interest rate, and money converge to their reset values.

The second system determines the frictionless values of the variables as functions of the state of the economy. The frictionless value of a variable is defined as the value that the agent would chose if she was exempt from the Calvo constraint associated with this variable, while keeping the Calvo constraints on the other variables. The third system of equations determines the values of the reset values as functionals of the frictionless values.

[3]The assumption of independence of the Poisson processes produces aggregate dynamics which are similar to the dynamics found when we assume a conventional set of frictions. However, the assumption of independence can also be relaxed.

[4]Remember that $\pi = \Pi - \Pi^*$

Taken together, these three system describe the behavior of the households as a set of functions of the state of the economy. If we know the state of the economy, the second system can express the frictionless values as functions of the state, the third system finds the reset values as functionals of the frictionless values, and the Calvo constraints determine the actual dynamics of each variable as a function of its reset value.

The second system of equations, which defines the frictionless values of consumption, wages, capital interest rate, and money, can be written in the following way:

$$U'_C\left(\bar{C}_i, L_i, M_i\right) = \Gamma_i \tag{6a}$$

$$U'_L\left(C_i, L_i(\bar{W}_i), M_i\right) = -\frac{1-\xi}{\xi}\Gamma_i\bar{W}_i \tag{6b}$$

$$\bar{R}^k_i - \Pi^e = \frac{\eta}{\eta-1}\left(R^b - \Pi^e\right) \tag{6c}$$

$$U'_M\left(C_i, L_i, \bar{M}_i\right) = \Gamma_i R^b \tag{6d}$$

$$E\left(\frac{d\Gamma_i}{\Gamma_i}\right) = \left(\rho - R^b + \Pi^e\right)dt \tag{6e}$$

where Γ_i is the shadow price of the household's wealth.

This system of equations (6) looks like a conventional set of first-order conditions for the household's problem within the frame of a monetary neoclassical growth model without any frictions. However, there are two notable features of (6) that distinguish it from the conventional set of first-order conditions.

The first noteworthy feature of (6) is that it distinguishes the actual value of each variable chosen by the household $\left(C_i, W_i, R^k_i, M_i\right)$, and the frictionless values $\left(\bar{C}_i, \bar{W}_i, \bar{R}^k_i, \bar{M}_i\right)$. For example, equation (6a) determines the value of consumption that the household would choose if it was exempt from the Calvo constraint on consumption, while keeping the Calvo constraints on the other variables. This value, $\bar{C}_i$, depends not on the money holding that the household would choose if it was exempt from the Calvo constraint on money, $\bar{M}_i$, but rather on the actual value of money, M_i. This is why on the left-hand side of equation (6a) we see M_i but not $\bar{M}_i$. For the same reason, on the left-hand side of (6a) we see L_i but not $\bar{L}_i$. Roughly speaking, in the first-order condition for a particular variable I mark this variable with a bar, whereas all the other variables are written without bars.

The second notable feature of (6) is that it distinguishes first-order conditions with respect to prices from those with respect to quantities. In a conventional DSGE model, it makes no difference whether household's utility is maximized with respect to the household's labor supply L_i or with respect to the wage W_i because the labor demand function (4b) uniquely determines how much labor the household supplies given the wage or the wage it receives given the value of the labor supply. Under my approach, equation (6b) determines the household's frictionless value of the wage but not the frictionless value of the labor. This distinction is important for the analysis of the adjustment process because it implies that equation (6b) affects the adjustment of the aggregate wage but not of the aggregate labor. Because firms optimize with respect to labor given the aggregate wage, the adjustment process of the aggregate labor stems from firms' behavior.

The final set of the first-order conditions determines the reset-values as functionals of the frictionless values. Explicit formula are derived in Appendix B and log-linearized in Appendix D. For simplicity, they are presented here in log-linearized form, aggregated over all households[5]:

$$\frac{Ed\hat{c}}{dt} = (\lambda_c + \rho + (1-\sigma\theta)\,\nu)\,(\hat{c} - \bar{c}) \tag{7a}$$

$$\frac{Ed\hat{w}}{dt} = (\lambda_w + \rho + (1-\sigma\theta)\,\nu)\,(\hat{w} - \bar{w}) - \pi^e \tag{7b}$$

$$\frac{Ed\hat{r}^k}{dt} = (\lambda_{rk} + \rho)\left(\hat{r}^k - \bar{r}^k\right) \tag{7c}$$

$$\frac{Ed\hat{m}}{dt} = (\lambda_m + \rho + (1-\sigma\theta)\,\nu)\,(\hat{m} - \bar{m}) - \pi^e \tag{7d}$$

where E is the rational expectations operator.

Each equation in (7) has the same structure. To interpret these equations, consider, for example, the integral form of the equation for consumption. Integration forward of (7a) gives:

$$\hat{c}(t) = (\lambda_c + \rho + (1-\sigma\theta)\,\nu)\,E\int_t^{\infty} e^{-(\lambda_c+\rho+(1-\sigma\theta)\nu)(\tau-t)}\bar{c}(\tau)d\tau \tag{8}$$

This integrated equation defines $\hat{c}$ as the weighted average of future values of $\bar{c}$, with the weights determined by the discount rate[6] $(\lambda_c + \rho + (1-\sigma\theta)\,\nu)$. For example, if $\bar{c}$ is constant then $\hat{c} = \bar{c}$. System (7) defines the reset value of each variable as the weighted average of the expected frictionless value of this variable.

1.5 Firm's problem

Labor and capital are differentiated; therefore, each firm uses in production the labor and capital of each household. Let K_{ij} and L_{ij} be the amounts of capital and labor supplied by household i to firm j. Firm j

[5] For simplicity, log-linearized first-order conditions can be derived from a linear-quadratic approximation of the household problem derived using first-order Taylor decomposition for the constraints and second-order Taylor decomposition for the objective around the frictionless values of the variables. Application of this method yields the same solution as the method applied in Appendix B.

[6] Note that because I calculate the expected value of the relative deviation of consumption and not of the absolute deviation, the discount rate is not simply $\lambda_c + \rho$. The term $(1-\sigma\theta)\,\nu$ appears, therefore, as a result of log-linearization.

assembles K_{ij} and L_{ij} into aggregates according to the following technologies:

$$K_j = \left(\int_0^1 K_{ij}^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}} \quad (9a)$$

$$L_j = \left(\int_0^1 L_{ij}^{\frac{\xi-1}{\xi}} di \right)^{\frac{\xi}{\xi-1}} \quad (9b)$$

The firm uses aggregates K_j and L_j to produce intermediate good Y_j using the Cobb-Douglas production function:

$$Y_j = (Z_j K_j)^\alpha \left(L_j e^{\nu t}\right)^{1-\alpha} \quad (10)$$

where Z_j is the rate of capacity utilization, and ν is the rate of labor-augmenting exogenous technological progress.

The final good Y is produced from the intermediate goods Y_j according to:

$$Y = \left(\int_0^1 Y_j^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (11)$$

The capital accumulation by firm j is governed by:

$$\dot{K}_j = H_j - \Delta(Y_j, K_j, L_j, t) - \Omega(Y_j, K_j, L_j, H_j, t) \quad (12)$$

where H_j is the investment by firm j; $\Delta(\cdot)$ is endogenous depreciation function, and $\Omega(\cdot)$ is the capital adjustment costs function.

I assume that the depreciation function can be written in the following form:

$$\Delta(Y_j, K_j, L_j, t) = \Psi(Z_j) K_j^\alpha \left(L_j e^{\nu t}\right)^{1-\alpha} \quad (13)$$

where Z_j is determined by (10), and function $\Psi(\cdot)$ is twice differentiable, increasing, concave and satisfies Inada conditions: $\lim\limits_{Z\to 0_{(+)}} \Psi'(Z) = 0$ and $\lim\limits_{Z\to\infty} \Psi'(Z) = \infty$.

The capital adjustment costs function is given by:

$$\Omega(Y_j, K_j, L_j, H_j, t) = \frac{\zeta}{2} \frac{\left(H_j - \Delta(Y_j, K_j, L_j, t) - \nu K^*\right)^2}{K^*} \quad (14)$$

Let $\Phi(Y_j, K_j, L_j, H_j, t)$ be defined as:

$$\Phi(Y_j, K_j, L_j, H_j, t) = \Delta(Y_j, K_j, L_j, t) + \Omega(Y_j, K_j, L_j, H_j, t) \quad (15)$$

The firm's real profit is given by:

$$D_j = P_j Y_j - (R^k - \Pi) K_j - W L_j - \Phi(Y_j, K_j, L_j, H_j, t) \quad (16)$$

where P_j is the firm's relative price, and the aggregate price level at each date is normalized to 1.

The firm controls its relative price P_j, the value of the labor it uses in production L_j, and its investment H_j. The firm is allowed to revise the values of these variables only at set points in time determined by independent Poisson processes. If a particular variable is not revised, its value is indexed at the rate of its growth along the balanced growth path. Therefore, the process of adjustment of these variables can be formalized by the following Calvo constraints:

$$dP_j = \begin{cases} \hat{P}_j - P_j & \text{prob. } \lambda_p dt \\ -\pi P_j dt & \text{prob. } 1 - \lambda_p dt \end{cases} \tag{17a}$$

$$dH_j = \begin{cases} \hat{H}_j - H_j & \text{prob. } \lambda_h dt \\ \nu H_j dt & \text{prob. } 1 - \lambda_h dt \end{cases} \tag{17b}$$

$$dL_j = \begin{cases} \hat{L}_j - L_j & \text{prob. } \lambda_l dt \\ \nu L_j dt & \text{prob. } 1 - \lambda_l dt \end{cases} \tag{17c}$$

The firm maximizes its market value:

$$\max E_t \int_t^{\infty} e^{-\int_t^{\tau}\left(R^k(s)-\Pi(s)\right)ds} D_j(\tau)d\tau \tag{18}$$

with respect to $\hat{P}_j$, $\hat{H}_j$, and $\hat{L}_j$, subject to (10), (12), (16), (17), and the demand function for its output, which stems from (11):

$$Y_j = P_j^{-\varepsilon} Y \tag{19}$$

The behavior of the firms can be described by three systems of equations. The first system determines the frictionless values $\bar{P}_j$, $\bar{H}_j$, and $\bar{L}_j$ as functions of the state of the economy. The second system determines the reset values of the same variables as functionals of their frictionless values. Finally, the Calvo constraints determine how the actual values of these variables converge to the reset values.

The frictionless values of $\bar{P}_j$, $\bar{H}_j$, and $\bar{L}_j$ are given by:

$$\bar{P}_j = \frac{\varepsilon}{\varepsilon - 1} \cdot Q_j \Phi'_Y \left(Y_j(\bar{P}_j), K_j, L_j, H_j, t\right) \tag{20a}$$

$$Q_j = 1 + Q_j \Phi'_H \left(Y_j, K_j, L_j, \bar{H}_j, t\right) \tag{20b}$$

$$W = -Q_j \Phi'_L \left(Y_j, K_j, \bar{L}_j, H_j, t\right) \tag{20c}$$

where Q is the capital shadow price (the Tobin's q), which evolves according to:

$$\frac{E\left(dQ_j\right)}{dt} = \left(R^k - \Pi + \Phi'_K \left(Y_j, K_j, L_j, H_j\right)\right) Q_j \tag{21}$$

Equation (20a) requires that the firm's frictionless relative price be equal to the product of the markup multiplier $\varepsilon/(\varepsilon-1)$ and the firm's marginal costs. The marginal costs, in turn, are given by the product of $\Phi_Y'(\cdot)$, which determines the firm's marginal costs measured in units of capital, and of the shadow price of capital Q_j. Equation (20b) illustrates Tobin's rule of investment, which implicitly defines the frictionless value of investment as a function of Tobin's q. Equation (20c) has the same structure as (20a) except that there is no markup multiplier in the latter equation. Finally, (21) is Euler's equation for Tobin's q.

The equations that determine the reset values are derived explicitly in Appendix C. For simplicity, they are presented here in log-linearized and aggregated from. The reset-value equations for $\hat{p}$, $\hat{h}$, and $\hat{l}$ are determined by:

$$\frac{Ed\hat{p}}{dt} = \left(\lambda_h + R^{k*} - \Pi^* - \nu\right) \cdot (\hat{p} - \bar{p}) - \pi^e \tag{22a}$$

$$\frac{Ed\hat{h}}{dt} = \left(\lambda_h + R^{k*} - \Pi^* - \nu\right) \cdot \left(\hat{h} - \bar{h}\right) \tag{22b}$$

$$\frac{Ed\hat{l}}{dt} = \left(\lambda_h + R^{k*} - \Pi^* - \nu\right) \cdot \left(\hat{l} - \bar{l}\right) \tag{22c}$$

These equations have the same structure as the reset-value equations in the household's problem (7). Therefore, each reset value is defined as the expected weighted average of the frictionless value.

1.6 The remaining equations

The central bank conducts open market operations to set the government bonds' interest rate according to the Taylor monetary rule:

$$r^b = \upsilon_y \cdot y + \upsilon_\pi \cdot \pi^e \tag{23}$$

where $\upsilon_y > 0$ and $\upsilon_\pi > 1$ are the parameters.

The government collects lump-sum taxes or distributes lump-sum transfers to cover the difference between seigniorage and government spending. Government spending includes the government purchases, G, and the interest payments on the government debt. I assume that the government maintains a fixed ratio of the government debt to the balanced growth path output, therefore $\dot{B} = \nu B$, and the government budget constraint is:

$$\left(R^b - \Pi - \nu\right) B + G = \dot{M} + \Pi M + T \tag{24}$$

where government purchases G are given.

Market clearing requires:

$$Y = C + H + G \tag{25}$$

Taxes and profits are equally distributed among all households:

$$D_i = \int_0^1 D_j dj \tag{26a}$$
$$T_i = T \tag{26b}$$

2 Comparison with conventional DSGE models

This section compares the generalized Calvo approach developed in this paper with conventional assumptions about frictions in DSGE models. This paper substitutes one conventional friction at a time for corresponding friction implied by the generalized Calvo assumptions and analyzes only the equations that have changed. This process will show when the conventional set of frictions and the generalized Calvo adjustment assumption are substitutes for each other.

A detailed comparison is provided in Sections 2.1–2.5, and a summary can be found in Section 3.

2.1 Calvo pricing

I explicitly assume that prices, wages, and capital interest rates are adjusted according to the Calvo pricing approach.

2.2 Generalized Calvo consumption adjustment versus habit formation

Under the generalized Calvo assumption, the adjustment of consumption is governed by the following two equations:

$$\frac{E\left(d\hat{c}\right)}{dt} = \left(\lambda_c + \rho + \left(1 - \theta\sigma\right)\nu\right)\left(\hat{c} - \bar{c}\right) \tag{27a}$$
$$\dot{c} = \lambda_c\left(\hat{c} - c\right) \tag{27b}$$

Equation (27a) repeats (7a), and equation (27b) is the log-linearized and aggregated form of (5a).

This section aims to compare the dynamics of consumption governed by (27) with the dynamics produced by conventional assumption of consumption habit formation. To introduce the assumption of habit formation, I relax the Calvo constraint on consumption, equation (5a), and assume instead that the household's utility depends not on the actual consumption, but on $\tilde{C}_i$ which is defined as:

$$\tilde{C}_i = C_i + \psi_c^{-1}\left(\dot{C}_i - \nu C_i\right) \tag{28}$$

where ψ_c is the habit formation parameter.

Equation (28) formalizes the idea of habit formation. The household draws utility from $\tilde{C}_i$, which includes the actual consumption C_i and the consumption growth $\dot{C}_i$. To make the assumption of habit formation comparable with the generalized Calvo adjustment assumption, I include the "indexation" term $(-\nu C_i)$ on the right-hand side of (28).

Log-linearization of (28) around the balanced growth path and aggregation over $i \in [0, 1]$ give:

$$\dot{c} = \psi_c (\tilde{c} - c) \tag{29}$$

Under assumption $\psi_c = \lambda_c$, equations (27b) and (29) are equivalent to each other if and only if $\tilde{c} = \hat{c}$; therefore, I must compare $\tilde{c}$ and $\hat{c}$.

Let μ_i be the co-state variable associated with consumption in the habit formation version of the model. Instead of first-order condition (7a), which determines $\hat{C}_i$ under the Calvo constraint, I obtain the following first-order conditions for $\tilde{C}$:

$$0 = U'_C\left(\tilde{C}_i, L_i, M_i\right) + \mu_i \psi_c \tag{30a}$$

$$\frac{E(d\mu)}{dt} = \Gamma_i + (\rho + \psi_c - \nu)\,\mu_i \tag{30b}$$

Solving (30b) forward and substituting μ_i from (30a), I obtain:

$$U'_C\left(\tilde{C}_i(t), L_i(t), M_i(t)\right) = E_t \int_t^{\infty} \psi_c e^{-(\rho+\psi_c-\nu)(\tau-t)} \Gamma_i(\tau)\, d\tau \tag{31}$$

Compare (31) with the following equation, which implicitly determines $\hat{C}_i$ (see Appendix B for details), and from which (27a) was derived:

$$0 = E_t \int_t^{\infty} e^{-(\lambda_c+\rho)(\tau-t)} \left(U'_C\left(\hat{C}_i(t)\, e^{\nu(\tau-t)}, L_i(\tau), M_i(\tau)\right) - \Gamma_i(\tau)\right) d\tau \tag{32}$$

There are three discrepancies between (31) and (32). The first discrepancy is that equation (31) determines current marginal utility of consumption, taking into account the current values of consumption, labor, and money, while equation (32) determines the expected future marginal utility of consumption, which depends on the expected future consumption, future labor, and future money.

This discrepancy may be significant when the utility is not additively separable. However, most conventional DSGE models assume an additively separable utility function, under which this discrepancy disappears. For example, if the utility function is:

$$U(C,L,M) = \theta_1 \ln C + \theta_2 \ln(1-L) + \ln M, \tag{33}$$

equations (31) and (32) can be written as:

$$\frac{\theta_1}{\tilde{C}_i(t)} = E_t \int_t^{\infty} \psi_c e^{-(\rho+\psi_c-\nu)(\tau-t)} \Gamma_i(\tau)\, d\tau \tag{34a}$$

$$\frac{\theta_1}{\hat{C}_i(t)} = E_t \int_t^{\infty} (\rho+\lambda_c+\nu)\, e^{-(\rho+\lambda_c)(\tau-t)} \Gamma_i(\tau)\, d\tau \tag{34b}$$

and the first discrepancy disappears.

Before proceeding with the second and third discrepancies, transform (34) into log-linearized equations. Substitution of Γ_i from (6a) into (34), log-linearization and aggregation give:

$$\tilde{c} = E_t \int_0^{\infty} \psi_c e^{-(\psi_c+\rho)(\tau-t)} \bar{c} d\tau \tag{35a}$$

$$\hat{c} = E_t \int_0^{\infty} (\lambda_c+\rho+\nu)\, e^{-(\lambda_c+\rho+\nu)(\tau-t)} \bar{c} d\tau \tag{35b}$$

From (35) we see the remaining discrepancies between the generalized Calvo consumption adjustment assumption and the habit formation assumption. Given that $\psi_c = \lambda_c$ and the rate of technological progress is positive ($\nu > 0$), the discount rate under the habit formation assumption is lower than the discount rate under the Calvo adjustment, this accounting for the second discrepancy between the approaches. Therefore, expected shocks in the distant future are more important under habit formation than in the baseline model. The final discrepancy is that $\tilde{c}$ in absolute value is lower than the expected weighted average of $\bar{c}$, while the value of $\hat{c}$ equals the expected weighted average of $\bar{c}$.

These latter two discrepancies are quantitative, however, and can be ruled out almost entirely by the adjustment of parameters λ_c and ψ_c. Figure 1 illustrates how actual consumption can respond to an expected exogenous temporary increase in $\bar{c}$ under the two assumptions and equal values of the parameters, $\lambda_c = \psi_c = 2$. It is possible to adjust the parameters in such a way that the lines will be closer to each other. Even when the parameters are not adjusted, the discrepancy between the lines does not appear substantial. Plausibly, econometric methods will not be able provide any evidence in favor of one of the approaches over the other.

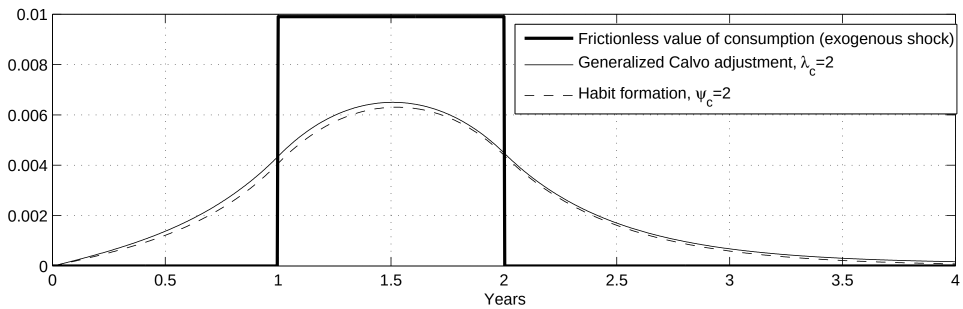

Figure 1: Adjustment of the actual value of consumption to a temporary expected increase in the frictionless value of consumption under the habit formation assumption and under the generalized Calvo adjustment assumption ($\rho = 0.03$, $\nu = 0.018$, and the shock is announced at $t = 0$).

2.3 Generalized Calvo adjustment versus capital or investment adjustment costs

In this section, I compare the dynamics of the investment produced by the generalized Calvo adjustment assumption with the dynamics produced by the investment adjustment costs assumption. A similar analysis can be made to compare the generalized Calvo adjustment assumption with the assumption of capital adjustment costs.

In the baseline model, the investment is governed by:

$$\dot{\hat{h}} = \left(\lambda_h + R^{k*} - \Pi^* - \nu\right)\left(\hat{h} - \bar{h}\right) \tag{36a}$$

$$\dot{h} = \lambda_h \left(\hat{h} - h\right) \tag{36b}$$

where (36a) repeats (22b)), and (36b) is the log-linearized and aggregated equation (22b).

To compare (36) with the conventional investment adjustment costs assumption, relax the Calvo constraint on the investment adjustment, equation (17b), and assume the following capital accumulation and profit equations instead of (12) and (16):

$$\dot{K}_j = H_j - \Phi(Y_j, K_j, L_j, H_j, t) - \frac{\psi_h^{-1}}{2} \frac{\left(\dot{H} - \nu H^*\right)^2}{H^*} \tag{37a}$$

$$D_j = P_j Y_j - \left(R^k - \Pi\right) K_j - W L_j - \Phi(Y_j, K_j, L_j, H_j, t) - \frac{\psi_h^{-1}}{2} \frac{\left(\dot{H} - \nu H^*\right)^2}{H^*} \tag{37b}$$

where the last term in both equations is the investment adjustment costs, and ψ_h is the adjustment costs parameter.

The solution to the model with investment adjustment costs can be written as[7]:

$$\dot{\tilde{h}} = \left(\psi_h + R^{k*} - \Pi^*\right)\left(\tilde{h} - h\right) + \zeta \frac{H^*}{K^*}\left(h - \bar{h}\right) \tag{38a}$$

$$\dot{h} = \psi_h \left(\tilde{h} - h\right) \tag{38b}$$

Note that the frictionless value of investment $\bar{h}$ is not well-defined when there are no capital adjustment costs. Therefore, we can compare the solutions only for $\zeta > 0$.

When ζ tends to zero, the role of the frictionless value of investment in system (38) vanishes. In this case, $\tilde{h}$ is determined solely by transversality conditions to the firm's problem, and the generalized Calvo adjustment assumption becomes incomparable with the investment adjustment costs assumption.

However, when the following condition is satisfied:

$$\psi_h + R^{k*} - \Pi = \zeta \frac{H^*}{K^*} \tag{39}$$

and $\nu = 0$, equations (36) and (38) produce the same dynamics of h. When (39) is satisfied, but $\nu > 0$, the solutions are numerically slightly different, in the same manner as the consumption habit formation assumption produces slightly different dynamics of consumption than does the generalized Calvo assumption (see Figure 1).

Therefore, I conclude that the dynamics of investment are similar under the two discrepant assumptions only for certain parameter values. In the general case, the dynamics are different.

A similar line of reasoning can be applied to compare the capital adjustment costs assumption with the generalized Calvo assumption.

2.4 Calvo-type labor adjustment versus search and match assumption

The adjustment of labor in the baseline model is determined by the following two equations:

$$\frac{E\left(d\hat{l}\right)}{dt} = \left(\lambda_l + R^{b*} - \Pi^*\right)\left(\hat{l} - \bar{l}\right) \tag{40a}$$

$$\dot{l} = \lambda_l \left(\hat{l} - l\right) \tag{40b}$$

To compare the baseline model with a model that assumes search-and-match at the labor market, relax assumption (17c), and assume that a job is created when an unemployed worker meets a free vacancy, which happens with a probability determined by matching function, and job destruction happens with an exogenous

[7] In this equation I use substitution $\tilde{h} \equiv \mu_j^h + h$, where μ_j^h is the co-state variable associated with the investment in the firm's problem.

Poisson probability density ψ_l:

$$\dot{L} = \kappa \left(\breve{L} - L\right)^{1-\beta} \left(\tilde{L} - L\right)^{\beta} - \psi_l L \tag{41}$$

where $\breve{L}$ is the labor force, $\tilde{L}$ is the total number of vacancies and filled workplaces, κ and β are the parameters. The number of unemployed equals $\left(\breve{L} - L\right)$, and the number of vacancies is $\left(\tilde{L} - L\right)$. For simplicity, let us assume that the cost of creating a new vacancy is zero.

The stream of a firm's marginal benefit from a filled vacancy can be found by differentiating (16) with respect to $\tilde{L}$: $\left(-W - \Phi'_L\left(Y_j, K_j, \tilde{L}_j, H_j, t\right)\right)$, where W can be substituted from (20c), $W = -\Phi'_L\left(Y_j, K_j, \bar{L}_j, H_j, t\right)$. Because the cost of the creation of a new vacancy is assumed to be zero, the expected discounted value of this stream until the time when the vacancy will be destroyed must be zero:

$$\begin{aligned} 0 \quad = \quad & E_t \int_t^{\infty} e^{-\int_t^{\tau}\left(R^b(z) - \Pi(z) + \psi_l\right)dz} \left(\Phi'_L\left(Y_j(\tau), K_j(\tau), \tilde{L}_j(t), H_j, t\right) - \right. \\ & \left. \Phi'_L\left(Y_j(\tau), K_j(\tau), \bar{L}_j(\tau), H_j, t\right)\right) d\tau \end{aligned} \tag{42}$$

If $\psi_l = \lambda_d$, equation (42) is the same as the equation for $\hat{L}$ in the baseline model (see Appendix C for details). Therefore, $\tilde{L} = \hat{L}$. It remains to be demonstrated whether under some restrictions matching-destruction function (41) can be approximated by (40b).

In general, (40b) cannot be derived from a matching function because the matching function depends not only on the number of vacancies $\left(\tilde{L} - L\right)$, where $\tilde{L}$ is determined by (42), but also on the number of unemployed $\left(\breve{L} - L\right)$, where the labor force $\breve{L}$ should be derived from the household's problem. Therefore, I had to introduce a new argument into (40b) to make it comparable with the search-and-match assumption.

This is possible to reformulate our model in such a way that $\breve{l}$ would appear in equation (40b): substitute the assumption that wages are set by households and labor by firms with the assumption that there exists some share of households who set wages given the labor demand, and the remaining households set labor supply given the wage level.

The baseline model, however, does not call for this complication because generates empirically plausible labor market dynamics. When frictionless labor is derived from the firm's problem, aggregate demand shocks directly affect labor demand but do not directly affect wages. In this case, the variations of wages caused by shocks in the aggregate demand are much lower than the variations of employment, which is supported by empirical facts.

Moreover, most of the DSGE models use simplifying assumptions to be (40b) wholly valid. For example, if we assume that the labor force does not depend on wages but is determined exogenously, then the matching-destruction function can be approximated by:

$$\dot{l} = \psi_{l1}\left(\tilde{l} - l\right) - \psi_{l2} l \tag{43}$$

where

$$\psi_{l1} = \psi_l (1-\beta)\left(\frac{\breve{L}}{L^*} - 1\right)^{-1} + \psi_{l2}, \qquad \psi_{l2} = \beta\kappa^{\frac{1}{\beta}}\psi_l^{\frac{\beta-1}{\beta}}\left(\frac{\breve{L}}{L^*} - 1\right)^{\frac{\beta-1}{\beta}}$$

and L^* is the balanced growth path value of employment, which can be derived from the parameters of the model.

There are two discrepancies between (43) and (40b). First, according to equation (40b), l converges to $\hat{l}$, while according to (43), it converges to $\frac{\psi_{l1}}{\psi_{l1}+\psi_{l2}}\tilde{l}$. Second, in system (40) the same parameter λ_l appears in both equations, whears in system (42), the corresponding parameters are different.

I conclude, therefore, that when the labor force is determined exogenously, the generalized Calvo approach and the search-and-match approach produce dynamic equations that have the same structure but different coefficient values.

2.5 Money adjustment

The adjustment of money is governed by:

$$\frac{E\left(d\hat{m}\right)}{dt} = \left(\lambda_m + \rho + (1-\theta\sigma)\, g\right)\left(\hat{m} - \bar{m}\right) \tag{44a}$$

$$\dot{m} = \lambda_m\left(\hat{m} - m\right) \tag{44b}$$

Subsystem (44) produces smooth forward-looking dynamics of money. The importance of the process of money adjustment has been recognized in models of hyperinflation, however, I unaware of any DSGE models that use a similar assumption.

3 Conclusion: generalized Calvo approach versus conventional frictions

Table 1 summarizes the relationships between the generalized Calvo approach and the conventional frictions used in DSGE models. When certain restrictions are placed on the parameters, these approaches generate similar dynamics. I provide no evidence favoring of one of the approaches over the other.

Table 1: Generalized Calvo approach versus conventional frictions: comparison of log-linearized adjustment equations

Calvo constraint imposed on:	Conventional friction	Discrepancies
Consumption	Consumption habit formation	1. If the utility is not additively separable, the marginal utility of consumption is calculated using different values of L_i and M_i. 2. Slightly different coefficients when $\nu > 0$.
Investment	Investment adjustment costs	For certain parameter values these assumptions are equivalent; however, in general, under adjustment costs assumption $\hat{h}$ is determined not only by $\bar{h}$, but also by h. When ζ tends to 0, the role of $\bar{h}$ under the adjustment costs assumption vanishes.
Capital*	Capital adjustment costs	The same discrepancy exists as in the investment assumptions.
Prices, wages and interest rates	Calvo pricing	When we impose the Calvo constraints on prices, wages and interest rates, we explicitly assume Calvo pricing.
Money	Smooth money adjustment	There is no baseline model with which to compare.
Labor	Search-and-match at the labor market	1. These assumptions are similar to each other only if the labor force in the search-and-match model is exogenous. 2. If the labor force is exogenous, we obtain adjustment equations with the same structure but different coefficients

* Note that the Calvo constraints cannot be imposed simultaneously on capital and investment.

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Technical appendices

A Notation

Notation in alphabet order:

- A – Household's wealth:

  A_i – real wealth of household i.

- B – Government bonds:

  B_i - the amount of the government's bonds held by household i.

- C – Consumption:

  C_i – consumption by household's i, $C = \int_0^1 C_i di$ – aggregate consumption, C_i^* – balanced growth path consumption of household i, $C^* = \int\limits_0^1 C_i^* di$ – balanced growth path aggregate value of consumption, $\hat{C}_i$ – reset value consumption of household i, $\hat{C} = \int\limits_0^1 \hat{C}_i di$ – aggregate reset value consumption, $\bar{C}_i$ – free-choice consumption of household i, $\bar{C} = \int\limits_0^1 \bar{C}_i di$ – frictionless aggregate consumption, $\tilde{C}_i$ – argument of the utility function in the habit formation version of the model, where $\tilde{C}_i = C_i + \psi_c^{-1}\left(\dot{C} - \nu C_i\right)$, $c_i = \frac{C_i - C_i^*}{C_i^*}$, $c = \frac{C - C^*}{C_*}$, $\hat{c}_i = \frac{\hat{C}_i - C_i^*}{C_i^*}$, $\hat{c} = \frac{\hat{C} - C^*}{C_*}$, $\bar{c}_i = \frac{\bar{C}_i - C_i^*}{C_i^*}$, $\bar{c} = \frac{\bar{C} - C^*}{C_*}$.

- D – Real profit:

  D_j – real profit produced by firm j, D_i – real profit received by household i.

- E – Operator of rational expectations.

- G – Government purchases:

  G – absolute value of government purchases, G^* – balanced growth path value of the government purchases, $g = \frac{G - G^*}{G^*}$.

- H - Investment:

  H_j – investment by firm j, H – aggregate investment, $\hat{H}_j$ - reset value investment by firm j, $\hat{H}$ – aggregate reset value investment, $\bar{H}_j$ – frictionless investment by firm j, $\bar{H}$ – aggregate frictionless investment, H^* – balanced growth path investment (aggregate or individual), $h = \frac{H - H^*}{H^*}$, $h_j = \frac{H_j - H^*}{H^*}$, $\hat{h} = \frac{\hat{H} - H^*}{H^*}$, $\hat{h}_j = \frac{\hat{H}_j - H^*}{H^*}$, $\bar{h} = \frac{\bar{H} - H^*}{H^*}$, $\bar{h}_j = \frac{\bar{H}_j - H^*}{H^*}$,

- i – Counter of the households.

- j – Counter of the firms.

- K – Physical capital:

  K – aggregate value of capital, K_{ij} – the value of capital that household i rents to firm j, K_i – the total amount of capital possessed by household i, K_j – the total amount of capital used by firm j, K^* – the balanced growth path value of capital (aggregate or used by one firm), $k_j = \frac{K_j - K^*}{K^*}$, $k = \frac{K - K^*}{K^*}$.

- L – Labor

  L – aggregate value labor, L_{ij} – the value of labor possessed by household i and employed at firm j, L_i – the total amount of labor of household i, L_j – the total amount of labor used by firm j, L^* – the aggregate balanced growth path value of labor and the balanced growth path value of labor employed by one firm L_i^* – the balanced growth path value of labor of household i, $\hat{L}_j$ – the reset-value of labor of firm j, $\hat{L}$ – the aggregate reset-value of labor, $\bar{L}_j$ – the frictionless value of employment at firm j, $\bar{L}$ – the aggregate frictionless employment at all the firms, $l = \frac{L - L^*}{L^*}$, $l_j = \frac{L_j - L^*}{L^*}$, $\hat{l} = \frac{\hat{L} - L^*}{L^*}$, $\hat{l}_j = \frac{\hat{L}_j - L^*}{L^*}$, $l = \frac{\bar{L} - L^*}{L^*}$, $l_j = \frac{\bar{L}_j - L^*}{L^*}$.

- M – Money: M_i – money holdings by household i, M – aggregate money holdings, $\hat{M}_i$ – reset value of money of household i, $\hat{M}$ – aggregate reset value of money, $\bar{M}_i$ – frictionless value of money of household i, $\bar{M}$ – aggregate frictionless value of money, M_i^* – balanced growth path money holding by household i, M^* – aggregate value of balanced growth path money holdings, $m_i = \frac{M_i - M_i^*}{M_i^*}$, $m = \frac{M - M^*}{M^*}$, $\hat{m}_i = \frac{\hat{M}_i - M_i^*}{M_i^*}$, $\hat{m} = \frac{\hat{M} - M^*}{M^*}$, $\bar{m}_i = \frac{\bar{M}_i - M_i^*}{M_i^*}$, $\bar{m} = \frac{\bar{M} - M^*}{M^*}$.

- P - Prices

  $P = 1$ – aggregate price level, P_j – relative price level of firm j (relative with respect to the aggregate price level), $\bar{P}_j$ – frictionless relative price of firm j, $\bar{P}$ – aggregate frictionless price, $\hat{P}_j$ – reset price by firm j, $\hat{P}$ – aggregate frictionless price, $\bar{p}_j = \bar{P}_j - 1$, $\bar{p} = \bar{P} - 1$, $\hat{p}_j = \hat{P}_j - 1$, $\hat{p} = \hat{P} - 1$.

- Q – Capital shadow price (the Tobin's q):

  Q_j – capital shadow price of firm j, Q – average capital shadow price, $Q^* = 1$ – the balanced-growth-path value of capital shadow price, $q_j = Q_j - 1$, $q = Q - 1$.

- R – Nominal interest rates:

  R^b – nominal interest rate on government bonds, R^{b*} – the balanced growth path value of the nominal interest rate on government bonds, $r^b = R^b - R^{b*}$;

  R_i^k – nominal interest rate on physical capital set by household i, R^k – aggregate value of the nominal interest rate on capital, R^{k*} – balanced growth path value of nominal interest rate on capital (aggregate or set by one household), $\hat{R}_i^k$ – reset-value of nominal interest rate on capital set by household i, $\hat{R}^k$ – aggregate reset-value of nominal interest rate on capital, $\bar{R}_i^k$ – frictionless value value of nominal interest rate on capital set by household i, $\hat{R}^k$ – aggregate frictionless value of nominal interest rate on capital, $r_i^k = R_i^k - R^{k*}$, $r^k = R^k - R^{k*}$, $\bar{r}_i^k = \bar{R}_i^k - R^{k*}$, $\bar{r}^k = \bar{R}^k - R^{k*}$, $\hat{r}_i^k = \hat{R}_i^k - R^{k*}$, $\hat{r}^k = \hat{R}^k - R^{k*}$.

- S – time or aggregate state of the economy:

  s – time (in some integrals), S – vector that describes the aggregate state of the economy (in Bellman functions).

- T – taxes or time:

  T_i – lamp-sum taxes paid by household i, T – aggregate lamp-sum taxes;

  t – time.

- U – Instantaneous utility function.

- W – Wages

- Y – Output: Y_j – output of firm j, Y – aggregate output, Y^* – balanced growth path value of output (aggregated or individual), $y_j = \frac{Y_j - Y^*}{Y^*}$, $y = \frac{Y - Y^*}{Y^*}$.

- Z – factor utilization rate:

  Z_j – factor utilization rate by firm j, Z – average factor utilization rate.

- α – Cobb-Douglas parameter of the production function.

- β – Cobb-Douglas parameter of the search-and-match function.

- Γ, γ – Household's wealth shadow price

- Δ – Endogenous depreciation function.
- ε – Parameter of aggregation of the final output.
- ζ – Parameter of the capital adjustment costs function.
- η – Parameter of aggregation of capital.
- θ – Cobb-Douglas parameter of the utility function.
- κ – Parameter of the search-and-match function.
- λ – Parameters of the Calvo constraints
- μ – Co-state variables in the habit formation and investment adjustment costs versions of the model.
- ν – The rate of exogenous labor-augmenting technological progress.
- ξ – Parameter of aggregation of labor.
- Π, π – The rate of inflation.
- ρ – Household's subjective discount rate.
- σ – Parameter of the utility function.
- τ – Time.
- υ - Parameters of the Taylor rule.
- Φ – Endogenous depreciation plus capital adjustment costs function.
- ϕ – Shopping-time parameter of the utility function.
- Ψ – Subfunction of the endogenous depreciation function.
- ψ – Parameters associated with consumption habit formation, investment adjustment costs and search-and-match at the labor market.
- Ω – Capital adjustment costs function.

B Household's optimization

Let $\mathcal{V}(A_i, C_i, W_i, R_i^k, M_i, S)$ be the value function for the problem, where S is some vector which describes the aggregate state of the economy; vector S includes time t. The Bellman equation for the problem is:

$$\rho\mathcal{V}(A_i, C_i, W_i, R_i^k, M_i, S) =$$

$$\max_{\hat{C}_i, \hat{W}_i, \hat{R}_i^k, \hat{M}_i} \big[U\left(C_i, L\left(W_i, S\right), M_i\right) + \mathcal{V}_A'(A_i, C_i, W_i, R_i^k, M_i, S)\times$$

$$\big[\left(R^b - \Pi\right) \cdot \left(A_i - K\left(R_i^k, S\right) - M_i\right) + \left(R_i^k - \Pi\right) \cdot K\left(R_i^k, S\right) +$$

$$D_i + W_i \cdot L\left(W_i, S\right) - C_i - \Pi M_i - T_i\right] +$$

$$+\mathcal{V}_C'(A_i, C_i, W_i, R_i^k, M_i, S) \cdot \nu \cdot C_i + \mathcal{V}_W'(A_i, C_i, W_i, R_i^k, M_i, S) \cdot (\nu - \pi) \cdot W_i +$$

$$\mathcal{V}_M'(A_i, C_i, W_i, R_i^k, M_i, S) \cdot (nu - \pi) \cdot M_i + \nabla^s \left(\mathcal{V}(A_i, C_i, W_i, R_i^k, M_i, S)\right) \cdot \dot{S} +$$

$$\lambda_c \cdot \Big(\mathcal{V}(A_i, \hat{C}_i, W_i, R_i^k, M_i, S) - \mathcal{V}(A_i, C_i, W_i, R_i^k, M_i, S)\Big) +$$

$$\lambda_w \cdot \Big(\mathcal{V}(A_i, C_i, \hat{W}_i, R_i^k, M_i, S) - \mathcal{V}(A_i, C_i, W_i, R_i^k, M_i, S)\Big) +$$

$$\lambda_{rk} \cdot \Big(\mathcal{V}(A_i, C_i, W_i, \hat{R}_i^k, M_i, S) - \mathcal{V}(A_i, C_i, W_i, R_i^k, M_i, S)\Big) +$$

$$\lambda_m \cdot \Big(\mathcal{V}(A_i, C_i, W_i, R_i^k, \hat{M}_i, S) - \mathcal{V}(A_i, C_i, W_i, R_i^k, M_i, S)\Big)\Big]$$

First-order conditions for the problem are:

$$\mathcal{V}_C'(A_i, \hat{C}_i, W_i, R_i^k, M_i, S) = 0; \quad \mathcal{V}_W'(A_i, C_i, \hat{W}_i, R_i^k, M_i, S) = 0; \tag{45}$$

$$\mathcal{V}_R'(A_i, C_i, W_i, \hat{R}_i^k, M_i, S) = 0; \quad \mathcal{V}_M'(A_i, C_i, W_i, R_i^k, \hat{M}_i, S) = 0$$

Define:

$$\mu_i^c = \mathcal{V}_C'(A_i, C_i, W_i, R_i^k, M_i, S); \quad \mu_i^w = \mathcal{V}_W'(A_i, C_i, W_i, R_i^k, M_i, S);$$

$$\mu_i^{rk} = \mathcal{V}_R'(A_i, C_i, W_i, R_i^k, M_i, S); \quad \mu_i^m = \mathcal{V}_M'(A_i, C_i, W_i, R_i^k, M_i, S);$$

$$\Gamma_i = \mathcal{V}_A'(A_i, C_i, W_i, R_i^k, M_i, S)$$

The envelope theorem gives:

$$\frac{E\left(d\mu_i^c\right)}{dt} = (\rho-\nu)\,\mu_i^c + \left[\Gamma_i - U_C'\left(C_i, L_i, M_i\right)\right] \tag{46a}$$

$$\frac{E\left(d\mu_i^w\right)}{dt} = (\rho-\nu+\pi)\,\mu_i^w + \left[(\xi-1)\,\Gamma_i + \xi\frac{U_L'\left(C_i, L_i, M_i\right)}{W_i}\right] L_i \tag{46b}$$

$$\frac{E\left(d\mu_i^{rk}\right)}{dt} = \rho\mu_i^{rk} + \left[(\eta-1) - \eta\frac{R^b-\Pi}{R_i^k-\Pi}\right] K_i\Gamma_i \tag{46c}$$

$$\frac{E\left(d\mu_i^m\right)}{dt} = (\rho-\nu+\pi)\,\mu_i^m + \left[R^b\Gamma_i - U_M'\left(C_i, L_i, M_i\right)\right] \tag{46d}$$

$$\frac{E\left(d\Gamma_i\right)}{dt} = \Gamma_i\left(\rho - R^b + \Pi\right) \tag{46e}$$

Let's derive the optimal reset value for consumption. Solving forward (46a), we obtain:

$$\mu_i^c(t) = E_t\int_t^\infty e^{-(\rho-\nu)(\tau-t)}\left(U_C'\left(C_i(\tau), L_i(\tau), M_i(\tau)\right) - \Gamma_i(\tau)\right)d\tau \tag{47}$$

Consider a point in time t where consumption is reset to $\hat{C}_i(t)$. By our assumptions, the consumption will evolve according to $C_i(\tau) = \hat{C}_i(t)e^{g(\tau-t)} \equiv \hat{C}_i(t,\tau)$ until the next Poisson event, when the consumption will be reset again. The probability that the consumption will have not been revised until τ is $e^{-\lambda_c(\tau-t)}$. Therefore, (47) can be rewritten as:

$$\begin{aligned}\mu_i^c(t) = {} & E_t\int_t^\infty e^{-(\rho-\nu+\lambda_c)(\tau-t)}\left[U_C'\left(\hat{C}_i(t,\tau), L_i(\tau), M_i(\tau)\right) - \Gamma_i(\tau) + \right. \\ & \left.\lambda_c E_t(\mu_i^c(\tau)|\text{revision at }\tau)\right]d\tau\end{aligned} \tag{48}$$

Since I have assumed that the consumption have been revised at t, from (45) we see that $\mu_i^c(t) = 0$. For the same reason $E_t(\mu_i^c(\tau)|\text{revision at }\tau) = 0$. The first-order condition for the reset value of consumption becomes:

$$0 = E_t\int_t^\infty e^{-(\rho-\nu+\lambda_c)(\tau-t)}\left[U_C'\left(\hat{C}_i(t,\tau), L_i(\tau), M_i(\tau)\right) - \Gamma_i(\tau)\right]d\tau \tag{49}$$

Finally, substitute $\Gamma_i(\tau)$ from (6a):

$$0 = E_t\int_t^\infty e^{-(\rho-\nu+\lambda_c)(\tau-t)}\left[U_C'\left(\hat{C}_i(t,\tau), L_i(\tau), M_i(\tau)\right) - U_C'\left(\bar{C}_i(\tau), L_i(\tau), M_i(\tau)\right)\right]d\tau \tag{50}$$

Using the same approach for the other equations in (46), derive first-order conditions for the remaining household's reset values:

$$0 = E_t \int_t^{\infty} e^{-(\rho-\nu+\lambda_w)(\tau-t)-\int_t^{\tau}\pi(z)dz} \left[\frac{U_L'\left(C_i(\tau), L\left(\bar{W}_i(\tau)\right), M_i(\tau)\right)}{\bar{W}_i(\tau)} - \frac{U_L'\left(C_i(\tau), L\left(\hat{W}_i(t,\tau)\right), M_i(\tau)\right)}{\hat{W}_i(t,\tau)} \right] \left(\frac{W(\tau)}{\hat{W}_i(t,\tau)}\right)^{\xi} L(\tau)d\tau \tag{51a}$$

$$0 = E_t \int_t^{\infty} e^{-(\rho+\lambda_{rk})(\tau-t)} \left(\frac{\hat{R}_i^k(t) - \bar{R}_i^k(\tau)}{\hat{R}_i^k(t) - \Pi(\tau)}\right) \left(\frac{R^k(\tau) - \Pi(\tau)}{\hat{R}_i^k(t) - \Pi(\tau)}\right)^{\eta} K(\tau)d\tau \tag{51b}$$

$$0 = E_t \int_t^{\infty} e^{-(\rho-\nu+\lambda_m)(\tau-t)-\int_t^{\tau}\pi(z)dz} \left[U_M'\left(C_i(\tau), L_i(\tau), \bar{M}_i(\tau)\right) - U_M'\left(C_i(\tau), L_i(\tau), \hat{M}_i(t,t)\right)\right] d\tau \tag{51c}$$

where

$$\hat{C}_i(t,\tau) = \hat{C}_i(t)e^{\nu(\tau-t)} \tag{52a}$$

$$\hat{W}_i(t,\tau) = \hat{W}_i(t)e^{\nu(\tau-t)-\int_t^{\tau}\pi(z)dz} \tag{52b}$$

$$\hat{M}_i(t,\tau) = \hat{M}_i(t)e^{\nu(\tau-t)-\int_t^{\tau}\pi(z)dz} \tag{52c}$$

C Firm's problem

Let $\mathcal{V}(P_j, H_j, L_j, K_j, S)$ be the value function for the problem, where vector S describes the aggregate state of the economy and includes time t. The Bellman equation is:

$$\begin{aligned}
&\left(R^k - \Pi\right)\mathcal{V}(P_j, H_j, L_j, K_j, S) = \\
&\max_{\hat{P}_j, \hat{H}_j, \hat{L}_j} \left[P_j Y_j - \left(R^k - \Pi\right) K_j - W L_j - \Phi(Y_j, K_j, L_j, H_j, t) + \right. \\
&\mathcal{V}_K'(P_j, H_j, L_j, K_j, S) \cdot (H_j - \Phi(Y_j, K_j, L_j, H_j, t)) + \\
&\nabla^s \mathcal{V}(P_j, H_j, L_j, K_j, S)\dot{S} + \mathcal{V}_P'(P_j, H_j, L_j, K_j, S)(-\pi) P_j + \\
&\mathcal{V}_H'(P_j, H_j, L_j, K_j, S)\nu H_j + \lambda_p \left(\mathcal{V}\left(\hat{P}_j, H_j, L_j, K_j, S\right) - \mathcal{V}(P_j, H_j, L_j, K_j, S)\right) + \\
&\lambda_h \left(\mathcal{V}\left(P_j, \hat{H}_j, L_j, K_j, S\right) - \mathcal{V}(P_j, H_j, L_j, K_j, S)\right) + \\
&\left.\lambda_L \left(\mathcal{V}\left(P_j, H_j, \hat{L}_j, K_j, S\right) - \mathcal{V}(P_j, H_j, L_j, K_j, S)\right)\right]
\end{aligned}$$

The first-order conditions are:

$$\mathcal{V}'_P\left(\hat{P}_j, H_j, L_j, K_j, S\right) = 0; \quad \mathcal{V}'_H\left(P_j, \hat{H}_j, L_j, K_j, S\right) = 0; \tag{53}$$

$$\mathcal{V}'_L\left(P_j, H_j, \hat{L}_j, K_j, S\right) = 0.$$

Define:

$$\mu_j^p = \mathcal{V}'_P\left(P_j, H_j, L_j, K_j, S\right); \qquad \mu_j^h = \mathcal{V}'_H\left(P_j, H_j, L_j, K_j, S\right); \tag{54}$$

$$\mu_j^l = \mathcal{V}'_L\left(P_j, H_j, L_j, K_j, S\right); \quad Q_j = 1 + \mathcal{V}'_K\left(P_j, H_j, L_j, K_j, S\right).$$

The envelope theorem gives:

$$\frac{E\left(d\mu_j^p\right)}{dt} = \left(R^k - \Pi^*\right) \cdot \mu_j^p + \frac{(\varepsilon - 1) Y_j}{P_j}\left(P_j - \frac{\varepsilon}{\varepsilon - 1} Q_j \Phi_Y\left(Y_j, K_j, L_j, H_j, t\right)\right) \tag{55a}$$

$$\frac{E\left(d\mu_j^h\right)}{dt} = \left(R^k - \Pi - \nu\right) \cdot \mu_j^h + 1 - Q_j + Q_j \Phi'_H\left(Y_j, K_j, L_j, H_j, t\right) \tag{55b}$$

$$\frac{E\left(d\mu_j^l\right)}{dt} = \left(R^k - \Pi\right) \cdot \mu_j^l + W + Q_j \Phi_L(Y_j, K_j, L_j, H_j, t) \tag{55c}$$

$$\frac{E\left(dQ_j\right)}{dt} = \left(R^k - \Pi + \Phi'_K\left(Y_j, K_j, L_j, H_j, t\right)\right) Q_j \tag{55d}$$

To derive the frictionless value of investment, resolve the problem of the firm relaxing the constraint on investment adjustment, equation (17b), and execute the same operations under $H_j = \hat{H}_j \equiv \bar{H}_j$. Taking into account that in this case $\mu_j^h = 0$, we can see from (55b) that this approach would give:

$$Q_j = 1 + Q_j \Phi'_H\left(Y_j, K_j, L_j, \bar{H}_j, t\right) \tag{56}$$

what implicitly gives $\bar{H}_j$.

Now use the same approach as those in Appendix B: solve forward (55a)-(55c), substitute there (53), cancel out the multipliers which do not depend on τ, apply Bayes' theorem, and substitute definitions (20):

$$0 = E_t \int_t^\infty e^{-\int_t^\tau \left(R^k(z)-\Pi^*+\lambda_p\right)dz} \left(1 - \frac{\bar{P}_j(\tau)}{\hat{P}_j(t,\tau)} \cdot \frac{\Phi'_Y\left(Y(\hat{P}_j(t,\tau),S),K_j,L_j,H_j,\tau\right)}{\Phi'_Y\left(Y(\bar{P}_j(\tau),S),K_j,L_j,H_j\tau\right)}\right) \times \frac{Y(\tau)}{\left(\hat{P}(\tau,t)\right)^\varepsilon} d\tau \tag{57a}$$

$$0 = \int_t^\infty e^{-\int_t^\tau \left(R^k(z)-\Pi(z)+\lambda_h\right)dz} \left(\bar{H}_j(\tau) - \hat{H}_j(t,\tau)\right) d\tau \tag{57b}$$

$$0 = E_t \int_t^\infty e^{-\int_t^\tau \left(R^k(z)-\Pi(z)+\lambda_l\right)dz} Q_j \left(\Phi'_L(Y_j,K_j,\bar{L}_j,H_j,\tau) - \Phi'_L(Y_j,K_j,\hat{L}_j,H_j,\tau)\right) d\tau \tag{57c}$$

where

$$\hat{P}(t,\tau) = \hat{P}(t)e^{-\int_t^\tau \pi(z)dz} \tag{58a}$$

$$\hat{H}(t,\tau) = \hat{H}(t)e^{\nu(\tau-t)} \tag{58b}$$

D Log-linearized adjustment equations

Consider first-order condition (50), which is for convenience rewritten here:

$$0 = E_t \int_t^\infty e^{-(\rho+\lambda^c)(\tau-t)} \left[U'_C\left(\bar{C}_i(\tau),L_i(\tau),M_i(\tau)\right) - U'_C\left(\hat{C}_i(t)e^{\nu(\tau-t)},L_i(\tau),M_i(\tau)\right)\right] d\tau$$

This equation is a functional which implicitly determines $\hat{C}_i$. Since the integrand is an increasing function of $\hat{C}$, if a solution exists it is unique.

Log-linearize the integrand of this equation around the balanced growth path, and taking into account the functional form of (2), consider the following certainty equivalence approximation:

$$\begin{aligned} &E_t\left(U'_C\left(\bar{C}_i(\tau),L_i(\tau),M_i(\tau)\right) - U'_C\left(\hat{C}_i(t)e^{\nu(\tau-t)},L_i(\tau),M_i(\tau)\right)\right) \approx \\ &E_t\left(U''_{CC}\left(C^*_i(\tau),L^*_i(\tau),M^*_i(\tau)\right)\cdot C^*_i(\tau)\cdot\left(\bar{c}_i(\tau) - \frac{C^*_i(t)}{C^*_i(\tau)}\hat{c}_i(t)e^{\nu(\tau-t)}\right)\right) \approx \\ &E_t\left(U''_{CC}\left(C^*_i(t),L^*_i(t),M^*_i(t)\right)\cdot C^*_i(t)\cdot e^{(\theta\sigma-1)\nu(\tau-t)}\cdot\left(\bar{c}_i(\tau)-\hat{c}_i(t)\right)\right) \end{aligned} \tag{59}$$

Substitute (59) into (7a), drop the multipliers which do not depend on τ, and aggregate over i:

$$0 = E_t \int_t^\infty e^{-(\rho+\lambda_c+(1-\theta\sigma)\nu)(\tau-t)} \left(\hat{c}(t) - \bar{c}(\tau)\right) d\tau \tag{60}$$

Integral (60) solves the following differential equation:

$$\frac{E(d\hat{c})}{dt} = (\lambda_c + \rho + (1-\theta\sigma)\nu)(\hat{c} - \bar{c}) \tag{61}$$

The adjustment of consumption is governed by (61) and (5a), which log-linearized version is:

$$\dot{c} = \lambda_c(\hat{c} - c) \tag{62}$$

In the same manner we can derive equations for w, r^k and m from (46b)-(46d) and equations for p, k, and l from the producer's problem. The whole system of equations is given by:

$$\begin{aligned}
\frac{E(d\hat{c})}{dt} &= (\lambda_c + \rho + (1-\theta\sigma)\nu)(\hat{c} - \bar{c}) && \text{(63a)}\\
\frac{E(d\hat{w})}{dt} &= (\lambda_w + \rho + (1-\theta\sigma)\nu)(\hat{w} - \bar{w}) - \pi && \text{(63b)}\\
\frac{E\left(d\hat{r}^k\right)}{dt} &= (\lambda_{rk} + \rho)\left(\hat{r}^k - \bar{r}^k\right) && \text{(63c)}\\
\frac{E(d\hat{m})}{dt} &= (\lambda_m + \rho + (1-\theta\sigma)\nu)(\hat{m} - \bar{m}) - \pi && \text{(63d)}\\
\frac{E\left(d\hat{h}\right)}{dt} &= \left(\lambda_h + R^{k*} - \Pi^* - \nu\right)\left(\hat{h} - \bar{h}\right) && \text{(63e)}\\
\frac{E\left(d\hat{l}\right)}{dt} &= \left(\lambda_l + R^{k*} - \Pi^* - \nu\right)\left(\hat{l} - \bar{l}\right) && \text{(63f)}
\end{aligned}$$

$$\begin{aligned}
\dot{c} &= \lambda_c(\hat{c} - c) && \text{(64a)}\\
\dot{w} &= \lambda_w(\hat{w} - w) - \pi && \text{(64b)}\\
\dot{r}^k &= \lambda_k\left(\hat{r}^k - r^k\right) && \text{(64c)}\\
\dot{m} &= \lambda_m(\hat{m} - m) - \pi && \text{(64d)}\\
\dot{k} &= \lambda_k\left(\hat{k} - k\right) && \text{(64e)}\\
\dot{l} &= \lambda_l\left(\hat{l} - l\right) && \text{(64f)}
\end{aligned}$$

This is possible to write the couple of adjustment equations for prices in the same manner as I have written the adjustment equations for the other Calvo-constrained variables. However, because of normalization $P = 1$, this pair can be simplified, and written as the following single equation:

$$\frac{E(d\pi)}{dt} = \left(R^{k*} - \Pi - \nu\right)\pi - \lambda_p\left(\lambda_p + R^{k*} - \Pi - \nu\right)\bar{p} \tag{65}$$

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