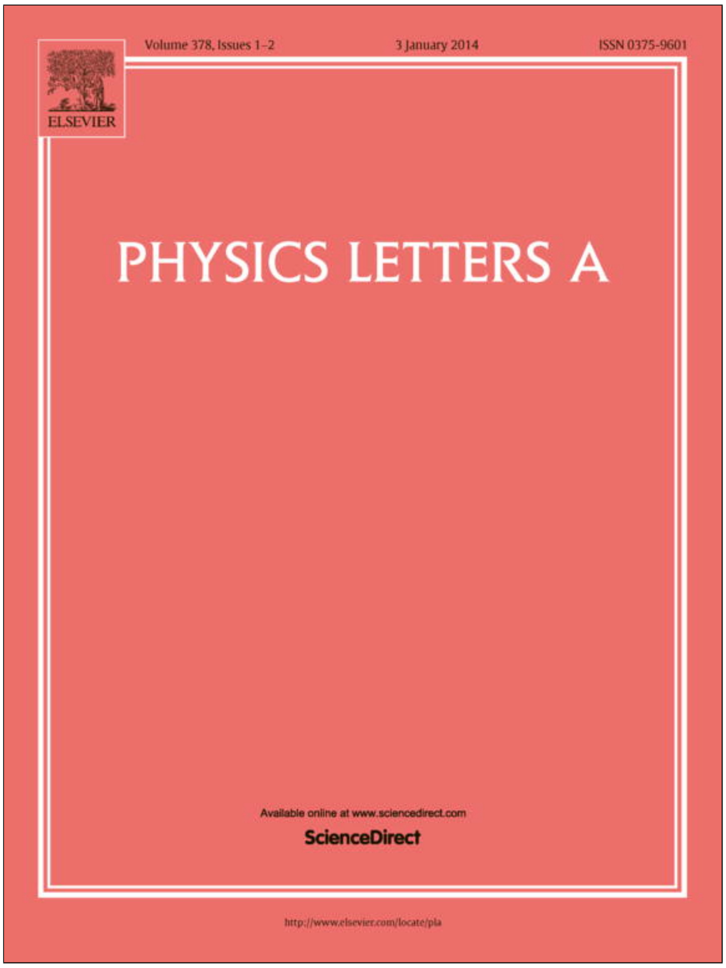
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Edge-wave-driven durable variations in the thickness of the surfactant film and concentration of surface floats

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ABSTRACT

By employing a simple model for small-scale linear edge waves propagating along a homogeneous sloping beach, we demonstrate that certain combinations of linear wave components may lead to durable changes in the thickness of the surfactant film, equivalently, in the concentration of various substances (debris, litter) floating on the water surface. Such changes are caused by high-amplitude transient elevations that resemble rogue waves and occur during dispersive focusing of wave fields with a continuous spectrum. This process can be treated as an intrinsic mechanism of production of patches in the surface layer of an otherwise homogeneous coastal environment impacted by linear edge waves.

1. Introduction

Patches of high concentrations of various adverse impacts (algal blooms, marine litter, oil pollution, etc.) often form at the sea surface or within a certain surface layer owing to natural processes. Patchiness formation in the open ocean and at its surface has been discussed for decades (e.g., Powell and Okubo [18]). The areas with high concentrations of substances and items locked in the surface layer naturally form mostly within large-scale convergence and subduction zones (Pichel et al. [17], Lee and Niiler [14]). Such effects are more probable in semi-sheltered sea domains and coastal regions (e.g., Kononen et al. [10], Granskog et al. [5]), where they are usually associated with current-driven dynamics, which leads to up- or downwelling or to the interaction of jet-like coastal currents and mesoscale vortices. They often reflect long-term correlations of the divergence field and the current-driven advection of floats (Giudici et al. [4], Kalda et al. [8], Samuelsen et al. [20]).

The role of wave motion in the formation of patches of high floating debris or litter concentrations and domains with an increased thickness of the surface film is usually either ignored or underestimated. Although it is widely known that even small-amplitude wind waves provide systematic transport in the surface layer (Starr [21]) and the associated Stokes drift is well understood today (Ardhuin et al. [1]), the related durable variations in the thickness of the surfactant film or the concentrations of surface floats (called surface concentrations below) are negligible even in higher-order Stokes waves. An exception is the nearshore as, for example, debris and litter tend to gather in surf and swash zones where the wave-driven transport ends.

The inability of wind waves to produce durable differences in the surface concentrations of various substances can be associated with the predominantly (at least locally) symmetric nature of the directional spectrum of windseas with respect to the average propagation direction. A probable source of inhomogeneities in these concentrations is intrinsically asymmetric wave systems such as coastally trapped waves or edge waves (Grimshaw et al. [6], Johnson [7]). Edge waves cause small displacements of the sea surface but create notable perturbations of horizontal velocity (Talipova [23]). The resulting velocity field is concentrated at the coast and exponentially decays offshore. Therefore, it is likely that edge waves produce a distortion of the field of water parcels (by transporting faster those parcels that are located closer to the coast) and thus may have quite complicated impact on the dynamics of sediments (Giniyatullin and Kurkin [3]). However, only very few studies are available on the dynamics of surfactant in edge waves.

In this paper we demonstrate that edge waves may generate considerable variations in the field of concentrations of various items on the sea surface. To highlight this effect, we intentionally use the simplest framework of the lowest-order Stokes edge waves propagating along a homogeneous sloping beach.

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2. Concentration field in linear trains of edge waves

Edge waves are interpreted here in a traditional manner, as a subset of relatively high-frequency (for which the rotation of the Earth is negligible) dispersive coastally trapped waves. They propagate along the coast. The maximum water elevation is reached at the shoreline and the range of elevations decays quickly seaward.

We employ the traditional approximation of long edge waves in which the vertical velocity component is a linear function of the location of the water parcel and consider the simplest case of a semi-infinite homogeneous beach where the water depth $h = h(y)$ only varies in the cross-shore direction. The x-axis is directed along the coast and the y-axis across the coast. A particular solution to the equations of motion (LeBlond and Mysak [13])

$$\frac{\partial u}{\partial t} + g\frac{\partial \eta}{\partial x} = 0, \qquad \frac{\partial v}{\partial t} + g\frac{\partial \eta}{\partial y} = 0,$$
$$\frac{\partial \eta}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = 0 \tag{1}$$

is searched using the ansatz of separation of variables (e.g., Rabinovich [19])

$$\eta = \mathrm{Re}\big[F(y)e^{i(\omega t - kx)}\big], \qquad u = \mathrm{Re}\big[U(y)e^{i(\omega t - kx)}\big],$$
$$v = \mathrm{Re}\big[V(y)e^{i(\omega t - kx)}\big], \tag{2}$$

where $u(x, y, t)$ and $v(x, y, t)$ are the horizontal velocity components, $\eta(x, y, t)$ is the surface elevation, ω is the angular frequency (always positive), k is the alongshore component of the wave vector (which can have either sign), the function $F(y)$ reflects the modal structure of the edge wave and satisfies the equation

$$\frac{d^2F}{dy^2} + \frac{1}{h}\frac{dh}{dy}\frac{dF}{dy} + \left(\frac{\omega^2}{gh} - k^2\right)F = 0 \tag{3}$$

and $U = gkF\omega^{-1}$, $V = igF'\omega^{-1}$. The onshore boundary condition for Eq. (3) is the absence of mass flux through the coastline. The function F is assumed to vanish in the offshore at $y \to \infty$. For more details about the structure and properties of edge waves the reader is referred to Mysak [15] and Johnson [7]. For the classical example of the semi-infinite sloping beach along which the water depth is $h(y) = y\tan\beta$ (Fig. 1) there exists an infinite number of discrete edge wave modes and the function $F_n(k, y)$ can be expressed in terms of Laguerre polynomials L_n [15]:

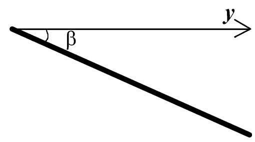

Fig. 1. Scheme of a semi-infinite sloping beach.

$$F_n(k, y) = A_n L_n(2ky)e^{-|k|y}, \tag{4}$$

where A_n and $\omega_n = \sqrt{(2n+1)g|k|\tan\beta}$, $n = 0, 1, 2, \ldots$, are the amplitude at the shoreline and angular frequency of the nth mode and waves can propagate in both the positive and negative x-direction.

The motions in the resulting edge waves are concentrated close to the waterline (Fig. 2). The velocity field causes more intense displacement of water parcels in the immediate vicinity of the nearshore (where the velocities and the amplitude of surface elevation generally are the largest).

The impact of edge wave motion on the surface concentrations can be quantified in terms of modifications to the thickness of the surfactant film $C(x, y, t)$ described by a variation of the standard equation of mass conservation

$$\frac{\partial C}{\partial t} + \frac{\partial (uC)}{\partial x} + \frac{\partial (vC)}{\partial y} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) + \frac{C_0 - C}{\tau}, \tag{5}$$

where D is the horizontal diffusion coefficient, C_0 is the initial (equilibrium) thickness and τ is the characteristic time scale of

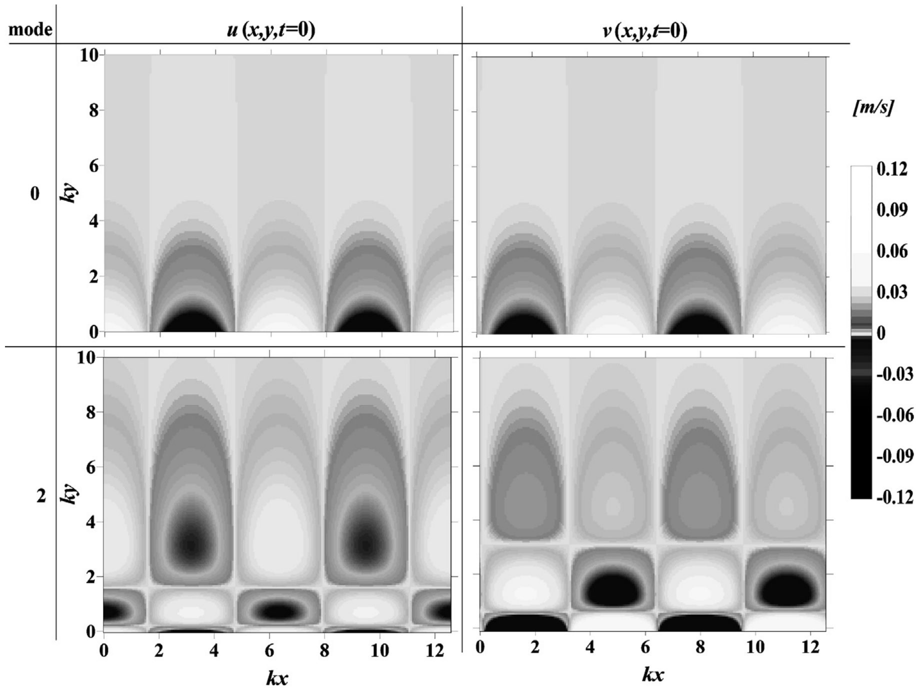

Fig. 2. Example of the velocity field in edge waves of the 0th and 2nd mode for the semi-infinite sloping beach.

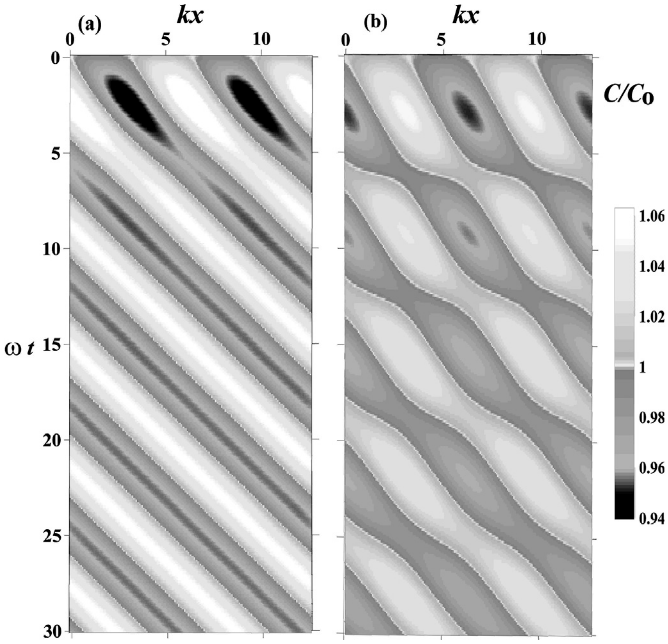

Fig. 3. The relative thickness of the surfactant film $C(x, y, t)$ at the coastline during the propagation of an edge wave in an area of 800 × 800 m for $k \approx 0.0157$, $\omega \approx 0.05$ rad/s: (a) for an infinite sloping beach, $D = 50\ \mathrm{m^2/s}$, $\tau = 180$ s; (b) for a concave exponential beach profile with $a \approx 0.03$, lowest mode and $D = 5\ \mathrm{m^2/s}$, $\tau = 1800$ s.

relaxation of the film (that also characterizes the interaction of the film with the water). The left-hand terms of Eq. (5) describe the advection of the film and the right-hand terms depict the processes of diffusion and restoration of non-equilibrium parts of the film. In the particular case $\tau = \infty$ Eq. (5) also describes the evolution of the total amount (per unit area) of a substance (e.g., debris or litter) that is locked in the surface layer.

Eq. (5) was solved numerically by employing an implicit second-order difference scheme (alternating direction method and three-diagonal matrix algorithm), using the velocities (u, v) in the underlying edge wave system in a rectangular domain, the no-flux boundary condition at the waterline $\partial C/\partial n = 0$ (here n is the normal to the instantaneous boundary), the assumption of a constant value of $C(x, y_{\max}, t) = C_0$ at the seaward border of the computational domain $y = y_{\max}$ and periodic boundary conditions in the wave propagation direction.

The motion of trains of periodic linear edge waves generated a quasi-periodic pattern of alternating symmetric patches of deviations of the thickness of the surfactant film from its equilibrium level (equivalently, the total amount of a substance in the surface layer). Without diffusion and restoration processes this pattern was, as expected, exactly periodic (Fig. 3). It was qualitatively the same for several other versions of bathymetry (not shown in the figure) such as the concave exponential beach profile $h(y) = H_0(1 - e^{-ay})$ and a version of the step-like shelf ($h = h_1 = const$ for $0 \leqslant y \leqslant y_1$ and $h = h_2 = const$ for $y \geqslant y_1$). For an initially homogeneous surface concentration and infinite in the x-direction situation this process generates a weak alongshore current of the surfactant or floats but causes no distortions in their concentration.

The diffusion and restoring processes smoothed the field of deviations to some extent. The smoothing was stronger for large values of the dissipation coefficient and shorter relaxation time scales. An increase in the diffusion led to the lengthening of the patches of the deviation of the film from the equilibrium state. For even stronger diffusion the "phase diagram" of patches (Fig. 3) took a shape of alternating stripes of almost constant thickness. The temporal courses of the maxima and minima of the deviation were symmetrical to each other. The deviations were the largest during the propagation of the first wave crest and tended to reach a nonzero asymptotic value that was higher for stronger diffusion. Not unexpectedly, the overall average of the thickness (equivalently, the total amount of a substance per unit area) did not change along the coast.

Local changes in the thickness as well as its maximum gradients were generally the smallest for the lowest-mode edge wave and, for a given maximum amplitude, increased with the number of the mode. This feature simply reflects the fact that the energy of higher modes with the same amplitude is larger than the energy of lower modes of edge waves. The pattern of deviations of the thickness created by higher modes (not shown in Fig. 3) at the waterline was not exactly symmetric but the overall average thickness again remained the same.

3. Durable changes in the surfactant thickness owing to wave focusing

Let us consider now changes in the surfactant film (concentration of floats) caused by the propagation of a more complicated but still realistic wave field. It is well known that large-amplitude events may be formed in a field of dispersive linear waves when harmonics with larger phase speeds initially travel behind slower waves. If the phases of such waves are properly ordered, their superposition may lead to extremely high-amplitude events [11,16]. For wind waves even such events do not result in a change in the concentration of surface floats (provided the linear theory does not fail) as all the displacements generated by each wave component are simply superposed and none of the components would cause any durable changes in the field of surface floats.

As edge waves are dispersive waves, often produced by a sequence of driving events with possibly different parameters, dispersive focusing eventually happens in the nearshore. Such events are possible for the simplest case of the lowest-mode ($n = 0$) Stokes solution [22] over a small slope $\tan\beta \approx \beta$, for which the corresponding Laguerre polynomial is $L_0(y) = 1$ and the dispersion relation reduces to $\omega_0^2 = g|k|\beta$.

An event of dispersive focusing can be described in terms of a wave system with a continuous amplitude spectrum $A(k)$. The water surface elevation for such a wave system is

$$\eta(x, y, t) = \mathrm{Re}\left[\int_{-\infty}^{\infty} A(k) e^{i(\omega(t-t^*)-kx)-|k|y}\, dk\right]. \quad (5)$$

It is straightforward to show that for the spectrum

$$A(k) = \frac{A_0 L}{2} e^{-L|k|} \quad (6)$$

and at a certain time instant $t = t^*$ (the instant of focusing) the surface elevation obtains the shape of a rogue wave [2,12] (Fig. 4). Here A_0 and L characterize the amplitude and alongshore extension of such an event. Note that all the components (harmonics of the underlying wave field) are linear and, when propagating alone, should cause no durable changes in the thickness of the surfactant film as demonstrated above.

The evolution of the thickness of the surfactant film before an event of focusing (Figs. 5, 6) resembles its reaction to the "usual" (albeit not periodic) edge waves shown in Fig. 3. The patches of deviation of the thickness from the equilibrium one are concentrated in the vicinity of the waterline. At the event of focusing the patches merge together and form a large domain of relatively strong increase in the thickness that is surrounded by equally large domains hosting a decrease in the thickness. The resulting pattern

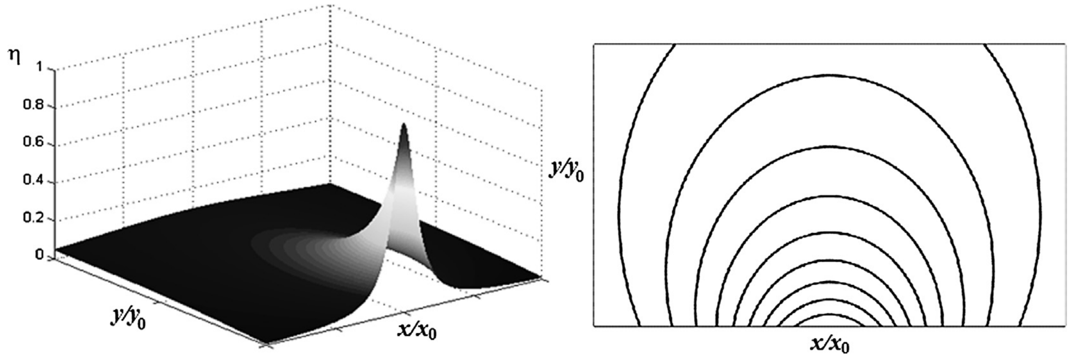

Fig. 4. Nondimensional elevation (left panel) and isolines of the water surface (right panel, distance between isolines is 0.1) at the instant of dispersive focusing of the zeroth-order Stokes edge waves (in normalized variables).

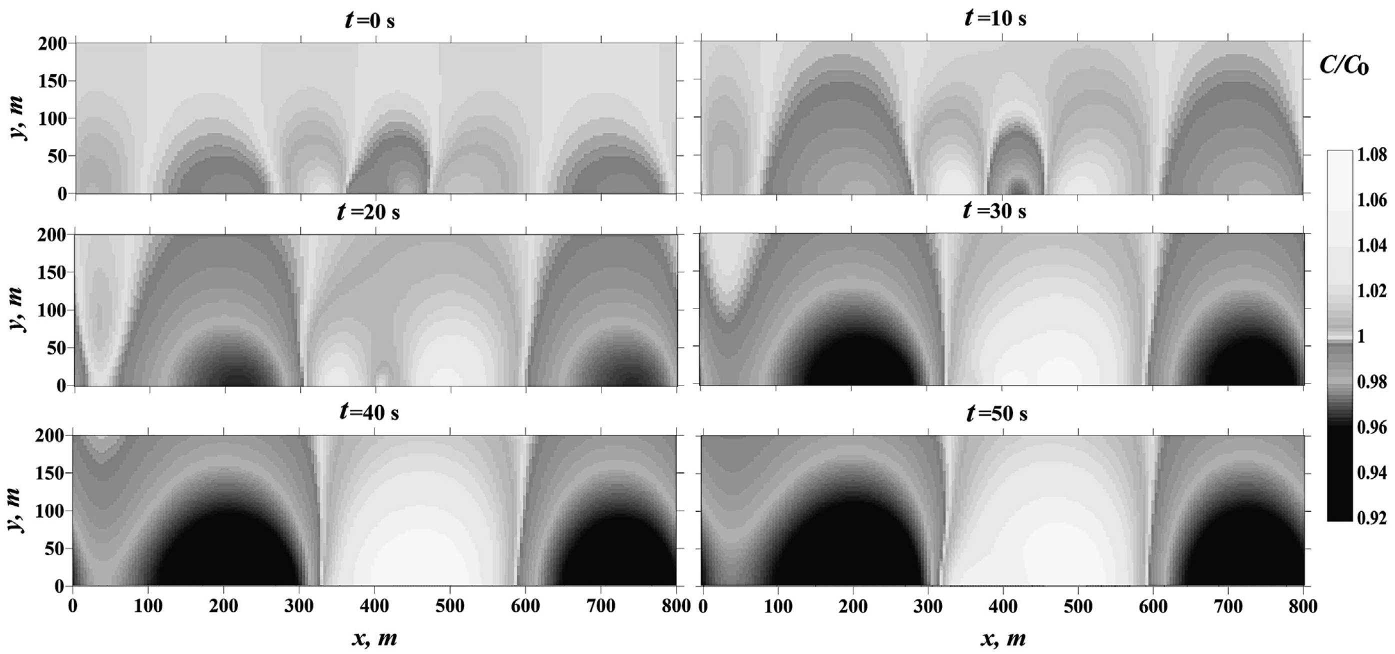

Fig. 5. The relative thickness of the surfactant film during a high-amplitude event of the wave system with a spectrum described by Eq. (6) and $A_0 = 0.01$, $L = 1$. The time instant 30 s corresponds to the instant of focusing.

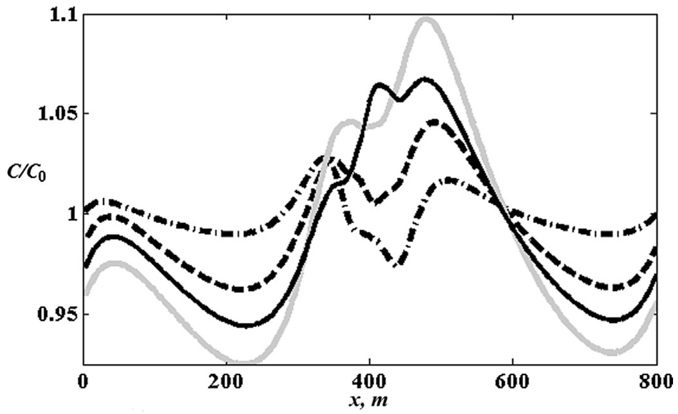

Fig. 6. The relative thickness of the surfactant film during the high-amplitude event of the wave system presented in Fig. 5. The time instant 30 s corresponds to the instant of focusing. The dash-dotted line corresponds to $t = 0$ s, the dashed line to $t = 20$ s, the solid black line to $t = 30$ s and the solid grey line to $t = 50$ s.

is no more symmetric: the maximum increase in the thickness considerably exceeds the thickness decrease in the adjacent domains (cf. Talipova [23]).

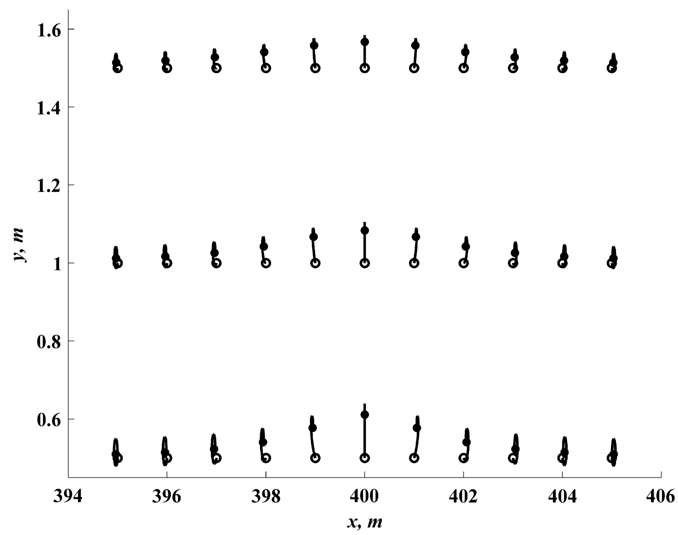

Fig. 7. Original (empty circles) and final (filled circles) locations and trajectories of selected water parcels in the vicinity of the focusing event.

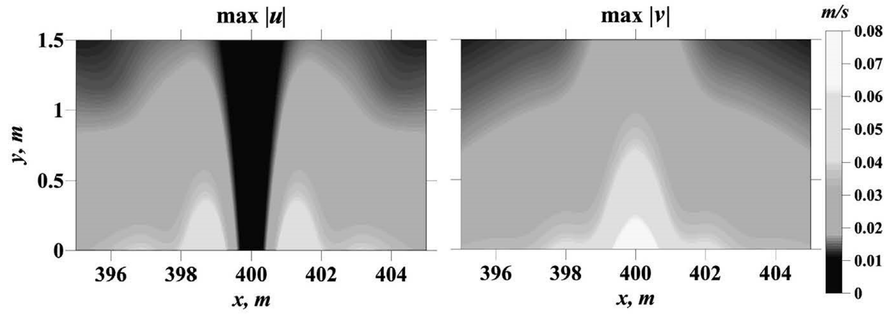

Fig. 8. Spatial distribution of the maxima of alongshore (u) and cross-shore (v) velocity components during a focusing event.

The trajectories of water parcels at the sea surface during focusing (Fig. 7) can be calculated from the horizontal velocity components $u(x, y, t)$ and $v(x, y, t)$ within the linear edge wave theory in a straightforward manner. The maximum displacement of the parcels remains moderate, on the order of 0.1–0.2 m. Considerable displacements only occur in the immediate vicinity of the focusing event, whereas the amplitude of displacements rapidly decreases with the distance from it. Such a pattern of displacements is consistent with the spatial distributions of the maxima of alongshore and cross-shore velocities (Fig. 8). These maxima are also concentrated in the immediate vicinity of the focusing event.

The most interesting outcome of such an event of focusing is that the patches of changes in the thickness of the surfactant do not vanish after the passage of the rogue event. Therefore, the transient pattern of elevation produces here a durable change in the thickness of the surfactant film. In the cased considered in this paper the area of increase in thickness has a single maximum whereas the region of decrease contains two local minima. The effect of diffusion smoothes the pattern noticeably after 40–50 s of further calculation. The maximum of the concentration field occurs much later, about 20 s after the focusing instant.

4. Concluding remarks

By employing a simple model of linear edge waves propagating along a homogeneous beach, we have demonstrated that wave motion causes a certain pattern of changes in the thickness of the surfactant film on the water surfaces. As the model equation also describes the evolution of the concentration of surface floats and of the total amount of various substances (per unit area) that are locked in the surface layer, the results also apply to the evolution of the concentration of, e.g., debris and litter at the sea surface. As expected, the pattern of changes resulting from the propagation of infinite elementary (sine/cosine) wave trains is symmetric with respect to the increase and decrease in the thickness of the film and does not lead to long-term changes in its average thickness (resp. concentration of debris or litter).

The situation radically changes when a system of linear wave components with a continuous spectrum forms a high-amplitude transient event similar to a rogue wave [9]. For ocean wave fields such events normally lead to a sequence of extreme elevations and depressions (not necessarily of equal amplitude). A specific feature of rogue edge waves is that they become evident as large-scale elevations, not surrounded by depressions of comparable amplitude (Pelinovsky et al. [16]).

The resulting changes in the thickness of the film (equivalently, the concentration of debris or litter, or the total amount of dissolved substances in the uppermost layer per unit area) have much larger amplitudes than those caused by typical components in the edge wave field. More importantly, such rogue events of basically linear character lead to durable changes in the discussed surface concentration fields. This process can therefore be treated as a natural mechanism of production of inhomogeneities (or patches) in an otherwise homogeneous nearshore coastal environment impacted by linear waves.

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