Labour institutions and vulnerability of developing economies under capital inflows

Vladimir Matveenko

To cite this article: Vladimir Matveenko (2016) Labour institutions and vulnerability of developing economies under capital inflows, Economic Research-Ekonomska Istraživanja, 29:1, 888-903, DOI: 10.1080/1331677X.2016.1193947

To link to this article: http://dx.doi.org/10.1080/1331677X.2016.1193947

© 2016 The Author(s). Published by Informa UK Limited, trading as Taylor & Francis Group

Published online: 23 Nov 2016.

Submit your article to this journal

View related articles

View Crossmark data
Labour institutions and vulnerability of developing economies under capital inflows

Vladimir Matveenko

Department of Economics, National Research University Higher School of Economics, St. Petersburg, Russia

ABSTRACT
The paper argues that labour institutions, such as efficiency wages or their analogues more typical for the former planned economies, can aggravate a vulnerability of developing economies caused by windfalls, such as additional oil income or foreign aid. The economy is modelled by use of the, so-called, fK endogenous growth model based on microfoundations describing the labour institutions. Despite a rather small formal difference, the model differs much in its properties from traditional growth models. The fK model generalises the well-known AK model but is free of the known shortcomings of the latter. Our analytical results as well as computer simulations show that, despite the presence of temporary acceleration opportunities, the unstable windfalls prohibit long-run sustainable growth. The windfall can be followed by a restructuring leading to a long-run economic decline after ceasing the capital inflow. The effect of the capital inflow depends on the relation between the expected inflow growth rate and the economic growth rate of the benchmark model (without inflow), as well as on the time preferences in the economy. If the capital inflow is expected to grow faster than the proper growth rate of the economy, this can lead to a decline in wages.

1. Introduction

It became a consensus last decade that there exists only a weak, if any, correlation between capital flows and growth, and the effectiveness of capital inflows largely depends on the institutions and policies of recipient countries. However, the institutions are path-dependent: they are defined not only by the soundness of the current economic management but also by the trajectory of previous development of the country. The aim of this paper is to argue that labour institutions, such as efficiency wages or their analogues more typical for former planned economies, contribute to a non-effectiveness of the capital inflows.

We will use an fK model – an endogenous growth model, which generalises the popular AK model but is free of the latter’s deficiencies. The production function in the fK model is based on microfoundations taking labour institutions into account. The role of capital inflows, such as foreign aid or additional oil income, is modelled as injecting an extra income into the economy. The form of production function stipulated by the institutions
leads to a presence of zones of economic growth and economic decline in the phase plane describing dynamics of the economy.

The situation with capital inflow in the form of aid or additional oil income, typical for some development economies, is a special case of a more general situation of an inflow of foreign capital improving the country’s position in short foreign assets but, often, changing for the worse its position in long foreign assets. There is a large empirical literature studying interconnections between capital inflows and other macroeconomic variables, in particular, domestic investment and growth rates (see for example, Aizenman, Jinjarak, & Park, 2013; Alfaro, Kalemli-Ozcan, & Volosovych, 2008). These studies show that, despite there being a positive statistical dependence between capital inflows and growth rates, this dependence may be explained by the fact that the main international capital flows are directed to countries with a good investment climate, which, by the way, is a factor of enlarging the inequality between developing countries.

Generally, as both theoretical (e.g. Benigno & Fornaro, 2014; Cardarelli, Elekdag, & Kose, 2010) and empirical studies (e.g. Al-Sadig, 2013; Desai, Foley, & Hines, 2005; Gourinchas & Jeanne, 2013) show, the influence of capital inflows on domestic investment is not simple, especially if possibilities for capital outflow from the country are open. For example, the inflow of capital to Latin America, as against Eastern Asia, matches an increase in consumption but not in domestic investment. Besides, an increase in domestic investment does not explain econometrically an acceleration of economic growth (Rogers, 2003). All this confirms a necessity to account for institutions in models of growth and development, and in models with capital inflows in particular.

Beside the general framework, our paper also contributes to more specialised current discussions on the institutional roots of the Dutch Disease and on the effectiveness of foreign aid, which stress the role of institutions in the country-acceptor (see, for example, Burnside & Dollar, 2000; Easterley, 2003; Djankov, Montalvo, & Reynal-Querol, 2008; Chervin & Van Wijnbergen, 2010; Van der Ploeg, 2011; Frankel, 2012; Hirano, 2013).

The paper is organised as follows. In Section 2 the benchmark fK model is introduced. Its microfoundations based on labour institutions are demonstrated in Sections 3 and 4. In Section 5 basic properties of the fK model are presented. Section 6 formulates the version of the fK model with capital inflow. Section 7 is devoted to analytical studying of the consequences of the windfalls such as foreign aid or additional oil income, and Section 8 reports results of computer simulations. Section 9 concludes.

2. Benchmark fK model

The fK model is an endogenous growth model taking labour institutions into account. The benchmark fK model is formulated in the following way:

\[ Y_t = K_f(V_t), \quad t = 1, 2, \ldots \]

\[ Y_t = K_{t+1} V_{t+1} + I_{t+1}, \]

\[ K_{t+1} = \nu K_t + I_{t+1}, \]

\[ I_{t+1} \geq 0, \]
Here \( Y_t \) is output in time period \( t \), \( K_t \) is capital, \( V_t \) is wage (consumption) per unit of capital (or in a more general meaning, a circulating capital), \( C_t = K_tV_t \) is consumption (total wage bill), \( I_t \) is investment, \( \nu \in (0, 1) \) is the fraction of capital remaining after depreciation (i.e. \( 1 - \nu \) is the depreciation rate). It is assumed that function \( f \) possesses the standard properties:

\[
f(0) = 0, f'(.) > 0, f''(.) < 0,
\]

\[
f'(0) > \nu, \quad \lim_{V \to +\infty} f'(V) < \nu.
\]

Initial values \( Y_0 \) and \( K_0 \) are given.

The non-negativity condition \( I_{t+1} \geq 0 \) implies that the investment is irreversible, i.e. the accumulated capital may not be used for consumption. This condition is equivalent to the inequality \( Y_t \geq \nu K_t V_{t+1} \) which, in its turn, in the non-trivial case of \( K_t > 0 \), is equivalent to \( f(V_t) \geq \nu V_{t+1} \).

Formally, the only serious difference in the formulation of the \( fK \) model in comparison with standard neoclassical growth models is the use of the production function \( Y = f(V)K \). This production function is a generalisation of the function \( Y = AK(A = \text{const}) \) used in the AK model. A justification for the chosen form of production function on the basis of microeconomic models with labour institutions is given in Sections 3 and 4. A detailed mathematical analysis of the benchmark \( fK \) model is provided in Matveenko (2006).

Notice that the variable of consumption-to-capital ratio, \( V \), also plays an important role in some other endogenous growth models – in the Uzawa-Lucas endogenous growth model in particular (see Korolev & Matveenko, 2006; Lucas, 1988; Xie, 1994).

The AK model, generalised by the \( fK \) model, was firstly discussed in Frankel (1962) but attracted special attention much later (see Aghion & Howitt, 2009; Barro & Sala-i-Martin, 1995), which can be explained by two reasons. First, the AK model is the simplest model of endogenous growth, and the interest to such models increased sharply in the last three decades. Second, the AK model belongs to the class of Harrod-Domar type models. The main conclusion from such a kind of models is the presence of a positive relation between the investment rate and the growth rate. For half a century, such models have been used in the practice of international financial organisations (see Dalgard & Erickson, 2006; Easterley, 2002) and have served as a basis for decisions concerning the size of aid to developing countries. A failure of many attempts to accelerate development by the use of foreign aid called a new wave of interest to theoretical and empirical studies based on endogenous growth models.

Despite a relatively small difference in formulation, the \( fK \) model differs much in its properties from traditional neoclassical models, in which all balanced growth paths (BGP) have a common long-run growth rate, and the maximal production level is achieved under minimal wages. The \( fK \) model differs as well from the AK model, where there is no convergence at all, as each path is balanced, and the investment rate and the growth rate are linked by a positive relation in the same way as in the Harrod-Domar model.

In the \( fK \) model a positive dependence between the investment rate and the growth rate takes place only under relatively small investment rates, while for higher investment rates the relation is negative. In such a way, the \( fK \) model demonstrates an inverted-U relationship
between the investment rate and the growth rate. Thus, the empirical results which throw doubt upon the AK model do not contradict the fK model. In particular, the fK model identifies in which cases one ought to expect a failure of an attempt to accelerate growth by increasing investment.

3. Efficiency wages as a microfoundation of the fK model

Let us demonstrate some microeconomic foundations that take labour institutions into account and lead to the production function of the form \( Y = Kf(V) \). In this section, a model based on efficiency wages and, hence, related mostly to a market economy is considered. In Section 4, a model more related to a former planned economy (such as the Russian economy, for example) is proposed.

We will follow a version of the efficiency wages model described in Blanchard & Fisher (1989). Let production function \( F(K, e(w)N) \) depend on capital \( K \) and the effective labour \( e(w)N \). Here, \( e(w) \) is an effort of the worker depending on her wage rate, \( w \), and \( N \) is the labour (the number of workers or working hours). Function \( F \) possesses constant returns to scale and other standard neoclassical properties. Concerning the efforts function, it is assumed that \( e(0) < 0, e'(.) > 0, e''(.) < 0 \).

The firm finds the wage rate \( \tilde{w} \) maximising the effective labour \( e(w)N \) given wage costs \( wN = kV \). This problem is easily reduced to finding the unconditional maximum of the function \( e(w)/w \). Hence, \( \tilde{w} \) is a solution of the equation \( e'(w)w/e(w) = 1 \), which means that the effort function, \( e(w) \), has unit elasticity in point \( \tilde{w} \). In spite of that, we obtain:

\[
F(K, e(\tilde{w})N) = F(K, e(\tilde{w})\frac{kV}{\tilde{w}}) = KF(1, \frac{e(\tilde{w})}{\tilde{w}}V) = Kf(V). \tag{3}
\]

Function \( f(V) \) possesses the standard properties. We come precisely to the form of production function used in the fK model.

We recall that a similar but simpler situation arises in the well-known Lewis’ two-sector development model (see Lewis, 1954; Ranis, 2006). In that model, at an initial stage of development, a competitive wage in industrial sector, \( \tilde{w} \), is tied to a wage in excess of a very low (‘zero’) marginal product in a relatively large agricultural sector with family cultivation. Hence, the production function in the industrial sector is

\[
F(K, L) = F(K, \frac{kV}{\tilde{w}}) = Kf(V). \tag{4}
\]

4. Microfoundations of the fK model taking into account labour institutions of a former planned economy

In this section a two-sector model of labour market is considered, which seems to be suitable for some of the former planned economies, the Russian economy in particular. The model includes an old sector and a new sector of the economy. The old sector consists of state enterprises as well as those former state firms that were privatised but faced no serious changes in management and labour relations. The new sector, in particular, provides moonlighting possibilities for workers of the old sector.
Let a worker of the old sector possess a time $T$ (for a period, e.g. for month) which she divides between time $L_1$ of factual working in the old sector firm, time $L_2$ of moonlighting, and leisure, $Le$:

$$T = L_1 + L_2 + Le. \quad (5)$$

The output of the old sector firm is described by a production function $AF(K, NL_1)$ which possesses constant returns to scale and other standard neoclassical properties. Here $K$ is capital, $N$ is the number of workers employed in the firm, $A$ is the total factor productivity.

In the sphere of moonlighting the individual can find work for any time with an hourly wage rate $w_2$. The wage may include the whole labour income in all various forms of its payment being practised in transition economies.

The individual’s preferences are described by a utility function $U(C, Le)$ with standard properties. Here, $C$ is the total wage received by the individual during the month, both in the old sector and in the sphere of moonlighting.

The old sector firm proposes the individual a package consisting of a factual time of work $L_f$ (this flexible variable may differ from the working time stated in a formal contract) and a per month wage rate, $\bar{w}_1$. The individual may choose one of the following two alternatives:

1. to accept the proposal of the old sector firm and, if it is profitable for her, to supplement the work in the firm by moonlighting, or
2. to reject the proposal of the old sector firm and to work only in the new sector.

Thus, the individual solves the following problem

$$\max\left\{ \max_{L_2 + Le = T - L_f} U(\bar{w}_1 + w_2 L_2, Le), \max_{L_2 + Le = T} U(w_2 L_2, Le) \right\}. \quad (6)$$

It is easy to see that if $\bar{w}_1 < w_2 L_f$ then the individual will prefer the second alternative, i.e. $L_1 = 0$, but if $\bar{w}_1 > w_2 L_f$ she will choose the first alternative, i.e. $L_1 = L_f$. In the boundary case we assume that the first variant is chosen: the old sector worker does not move.

It follows that if for each wage costs, $kV = \bar{w}_1 N$, the firm maximises the factually gained labour, $NL_f$, then it solves the problem

$$\max NL_f \quad \text{s.t. } w_2 L_f \leq \frac{kV}{N} (= \bar{w}_1) \quad (7)$$

Evidently, in the solution point the following equation is fulfilled: $NL_f = kV/w_2$. Consequently the factual hourly wage in the old sector firm, $w_f = kV/(NL_f)$, is equal to the reservation hourly wage, $w_2$, and the output is equal to

$$F(K, NL_f) = F\left( K, \frac{kV}{w_2} \right) = KF\left( 1, \frac{V}{w_2} \right) = Kf(V) \quad (8)$$

where $w_2$ is given parameter, and function $f$ possesses the standard properties. We come to the same fK type production function as earlier.

Thus, the fK model emphasises the role played in some of the former planned economies by the factual labour supply, as well as the role in the formation of the factual labour supply played by the availability of moonlighting. Empirical research into the influence of
the moonlighting possibilities on the labour supply in Russia during the transition period is reported in Matveenko and Saveliev (2005).

5. Properties of the benchmark fK model

The vector $S_t = (K_t, Y_t)$ will be considered as the state of the model in period $t$, and $V_t$ as a control variable. Let a path $\{S_t\}$ with an initial state $S_0 = (K_0, Y_0)$ have a control sequence $\{V_t\}_{t=1}^\infty$. The variable $q(S_t) = \nu K_t + Y_t$ will be referred to as the wealth of the economy in state $S_t$. We see that

$$q(S_0) = \nu K_0 + Y_0 = K_1(1 + V_1),$$

$$q(S_t) = (\nu + f(V_t))K_{t+1}(1 + V_{t+1}), t = 1, 2, \ldots$$

(9)

The growth factors of the wealth, of the capital and of the output are, correspondingly:

$$\alpha(V_{t+1}) = \frac{q(S_{t+1})}{q(S_t)} = \frac{\nu + f(V_{t+1})}{1 + V_{t+1}},$$

$$\eta_t = \frac{K_{t+1}}{K_t} = \frac{1 + V_t}{1 + V_{t+1}},$$

$$\mu_t = \frac{Y_{t+1}}{Y_t} = \frac{\nu + f(V_t)}{f(V_{t+1})}$$

(10)

In the special case of the AK model, these three growth factors do coincide. In the fK model, they coincide on any BGP, where $V_t = V = \text{const.}$

Notice that each path of pure consumption (i.e. such that $I_t \equiv 0$) converges to a BGP of pure consumption defined by the control $V^\nu$ satisfying the equation

$$f(V^\nu) = \nu V^\nu.$$

(11)

For each control $V$ such that $V \leq V^\nu$ the path $\{S_t\}$ defined by the initial state $S_0 = K_0(1, f(V))$ and constant control $V_t \equiv V$ is a BGP with growth factor $\alpha(V)$. Conversely, each BGP can be described in such a way.

The maximal growth factor on a BGP is achieved under the control $\tilde{V}$ which is a solution of the equation $f'(V^\nu) = \nu V^\nu$.

Suppose that $\alpha(\tilde{V}) > 1$, i.e. economic growth (but not decline) takes place. It is not difficult to see that in this case the equation $\alpha(V) = 1$, which defines steady states, has two positive roots, $V_{11}$ and $V_{12}$, let $V_{11} < V_{12}$ for definiteness. The inequality $V_{12} < V^\nu$ takes place.

For any $V \in (V_{11}, V_{12})$ the inequality $\alpha(V) > 1$ is fulfilled, which means that a growth (not decline) takes place on a BGP with such control $V$. For $V \in (0, V_{11}) \cup (V_{12}, V^\nu)$ the inequality $\alpha(V) < 1$ is checked, which means an economic decline. The economic sense of the indicated conditions is that the economic decline takes place on those BGPs where the wage per unit of capital (wage ‘in a workshop’), $V$, is either too small or too high.

For unbalanced paths the condition of decline in period $t + 1 > 1$ is

$$\frac{f(V_{t+1})}{1 + V_{t+1}} < \frac{f(V_t)}{\nu + f(V_t)}$$

(12)

This condition also means that the wage per unit of capital, $V_{t+1}$, is either too small or too high.
In the special case of the AK model, the condition of decline reduces to
\[ V_{t+1} > \nu - 1 + A \]  
(13)
i.e. the decline can be explained either by a too high a wage per unit of capital, or by a too small productivity \((A < 1 - \nu)\). However, in the AK model, in contrast to the fK model, decline cannot be caused by a too low wage per unit of capital, while the latter is an especially important case for many development countries and former planned economies.

Now, we turn our attention to a relation between the growth rate and the investment rate. Paths with constant investment rates, \(s \in (0, 1)\), will be referred as Solow paths. In the fK model:

\[
s_t = \frac{I_{t+1}}{Y_t} = \frac{f(V_t) - \nu V_{t+1}}{(1 + V_{t+1})f(V_t)}
\]
(14)

Hence, a Solow path with investment rate \(s\) is described by the following difference equation:

\[
V_{t+1} = \frac{1 - s}{s + \frac{\nu}{f(V_t)}}
\]
which implies a monotonic convergence of the control sequence \(\{V_t\}_{t=1}^{\infty}\) to a steady control \(V(s)\). The steady control function \(V(s)\) decreases, as does its inverse function, \(s(V)\). The growth factor of output on a Solow path,

\[
\frac{Y_{t+1}}{Y_t} = (1 - s)\frac{f(V_{t+1})}{V_{t+1}}
\]
(15)

converges monotonically to \(\nu + sf(V(s))\).

It follows that in the fK model there is no permanent positive relation between the investment rate and the growth rate. The asymptotic growth factor of a Solow path (the growth factor of a BGP) increases with respect to \(s\) if \(s \in (0, s(\tilde{V}))\) and decreases if \(s \in (s(\tilde{V}), 1)\).

In the AK model, on a Solow path:

\[
V_{t+1} = \frac{1 - s}{s + \frac{\nu}{A}} = \text{const}
\]
(16)
hence, the growth factor is constant and equal to \(\nu + sA\). Thus, in the AK model there is always a positive relation between the investment rate and the growth factor. An assumption of such a kind of relation, being a basis of the methodology of the international aid organisations, is a subject of criticism (see Dalgaard & Erickson, 2006; Easterley, 2002), and the absence of such permanent relation in the fK model seems to be its important merit.

To check consequences of the AK model empirically, Jones (1995) considers 15 OECD countries and comes to a conclusion that, in spite of the investment rates rising in the postwar period, the growth rates did not increase. In his opinion, this fact is witness to an insolvency of the AK model. This conclusion is disputed by McGrattan (1998) who considers a wider sample of countries for a longer time period. However, Nandi (2011) supports the Jones’ conclusion. Neither of these empirical evidences contradict the fK model.
Another difference in the properties of the models is that there is no convergence in the AK model, and Barro and Sala-i-Martin (1995) mark this as the most important empirical shortcoming of the AK model, while the fK model is free of this shortcoming.

Now let us compare optimal paths of the fK model for several alternative optimality criteria. The optimal paths turn out to be constructed by use of a simple common rule. To each of the optimality criteria under consideration its own fixed generating control, \( V \), corresponds. By use of the generating control, a current optimal control, \( V_{t+1} \), is being constructed in the following way:

\[
V_{t+1} = \min \left\{ V, \frac{Y_t}{vK_t} \right\}
\]  

(17)

In other words, if the current output is high enough, then the generating control itself is used (\( V_{t+1} = V \)); otherwise, no investment is made (\( I_{t+1} = 0 \)), i.e. the pure consumption takes place in period \( t + 1 \).

In particular, we compare the following optimality criteria.

A. Stepwise maximal output. Under this criterion, the economy moves from a current state \( S_t \) to that state \( S_{t+1} \) which provides the maximal output \( Y_{t+1} \). The generating control \( \tilde{V} \) for this criterion is the point of maximum of function \( f(V)/(1 + V) \). What is more, \( \tilde{V} \) satisfies the equation \( f'(V) = f(V)/(1 + V) \).

B. Stepwise maximal profit. Under this criterion, the economy moves from a current state \( S_t \) to the state \( S_{t+1} \) which provides the maximal profit, \( Y_{t+1} - V_{t+1}K_{t+1} \). The corresponding generating control satisfies the equation \( f'(V) = 1 \).

C. Pareto efficiency. A path \( \{ \tilde{S}_t \} \) is said to be Pareto efficient if there is no path \( \{ S_t \} \) with the same initial state \( S_0 = \tilde{S}_0 \), such that \( S_k > \tilde{S}_k \) at some period \( k \). The corresponding generating control is \( \tilde{V} \) which provides the maximal long-run growth factor (see above); this reminds us that it is a solution of equation \( f'(V) = \alpha(V) \).

D. Maximal discounted consumption. Let \( \beta \in (0, 1) \) be a subjective discount factor, and let the function

\[
\max \sum_{t=0}^{\infty} \beta^t C_{t+1}
\]

be maximised given initial state \( S_0 \). The optimal path is called \( \beta \)-optimal. For \( \beta \geq 1/\alpha(\tilde{V}) \) the series (1) diverges and no \( \beta \)-optimal path exists. For \( \beta \in (0, 1/\alpha(\tilde{V})) \) the generating control, \( V(\beta) \), is a solution of the equation \( 1 - \beta(V + f(V)) + \beta V f'(V) = 0 \). For sufficiently small values of \( \beta \), when the society is ‘impatient’, the inequality \( V(\beta) \geq V_{12} \) is fulfilled, and economic decline takes place on \( \beta \)-optimal paths.

Various dynamic patterns are possible under criteria A – D. Basic cases are listed in Table 1.

6. The fK model with windfalls

Now, we turn to the fK model with windfalls, such as additional oil income or foreign aid. The model is formulated in the following way:

\[
Y_t = K_t f(V_t), t = 1, 2, ...
\]

(19)
Here \( Y_t \) is GDP in time period \( t \), \( W_t \) is the (exogenous) real value of a windfall, \( K_t \) is capital, \( V_t \) is wage per unit of capital, \( C_t = K_t V_t \) is consumption (total wage bill), \( I_t \) is domestic investment, \( \nu \in (0, 1) \) is the share of capital that remains after depreciation (i.e. \( 1 - \nu \) is the depreciation rate). As above, it is assumed that function \( f \) possesses the standard neoclassical properties.

First of all, let us indicate steady states in which the basic variables are constant:
\[
W_t = W, \quad Y_t = Y, \quad K_t = K, \quad V_t = V.
\]
In such a case, \( K[(1 + V) - (\nu + f(V))] = W > 0 \), which means that \( \alpha(V) < 1 \), where \( \alpha(V) \) is the growth factor of the benchmark economy (without capital inflow) with the same control \( V \). Hence, if the capital inflow is ceased, the economy finds itself on a path of economic decline. It means that the steady capital inflow promotes not a sustainable growth but a sustainable decline of the economy.

Now, assume that the capital inflow increases with a constant growth factor, \( \varepsilon \). Of special interest are BGPs. Evidently, the three growth factors of the economy (growth factors of output, capital, and total consumption) are all equal to \( \varepsilon \). Let \( \omega \) be the inflow-to-capital ratio: \( \omega = W_t/K_t \). It may be seen that the corresponding control \( V \) satisfies the following equation:
\[
\varepsilon = \frac{\nu + f(V) + \omega}{1 + V}
\]
Here again the capital inflow can induce a habit harmful for sustainable growth of the economy.

For example, suppose that parameters \( \bar{g} \) and \( \bar{\omega} \) of the capital inflow are such that the corresponding control is \( \bar{V} \), which provides the Pareto efficient paths in the benchmark economy (criterion C introduced in Section 5). Under these parameters, the economic growth factor under windfall is equal to:
\[
\bar{\varepsilon} = \frac{\nu + f(\bar{V}) + \bar{\omega}}{1 + \bar{V}}
\]

<table>
<thead>
<tr>
<th>( Y_0 &gt; WVK_0 )</th>
<th>( 0 &lt; V &lt; V_{11} )</th>
<th>( V_{11} &lt; V &lt; V_{12} )</th>
<th>( V_{12} &lt; V &lt; \nu V )</th>
<th>( V &gt; \nu V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y_0 &lt; WVK_0 )</td>
<td>Proportional decline</td>
<td>Proportional growth</td>
<td>Proportional decline</td>
<td>Lack of investment and decline</td>
</tr>
</tbody>
</table>

Table 1. Patterns of optimal paths depending on generating control (\( V \)) and initial state (\( S_0 \)).

Source: The author's calculation.
Now, imagine that the oil prices increase, or the donors decide to accelerate the economic growth and increase the growth factor of the aid. With this bigger growth factor, $\dot{\varepsilon}$, equation (22) has a new solution, $\dot{V}$; however,

$$\frac{v + f(V)}{1 + \dot{V}} < \frac{v + f(V)}{1 + \dot{V}}$$

i.e. the temporary acceleration of the current economic growth later harms the sustainable growth when the oil prices fall or the aid is ceased. It is even possible that $\alpha(\dot{V}) < 1$, in which case the high oil prices (or big foreign aid budget) promote sustainable decline of the economy.

7. Consequences of non-stationary capital inflows

As in the benchmark model of Section 5, vector $S_t = (K_t, Y_t)$ will be considered as a state of the economy in period $t$, and $V_t$ as a control. Let the path $\{S_t\}$ with an initial state $S_0 = (K_0, Y_0)$ have the control sequence $\{V_t\}_{t=1}^\infty$, and let $\gamma_t$ be the ratio of the windfall to the wealth of the economy:

$$\gamma_t = \frac{W_t}{vK_t + Y_t} = \frac{W_t}{(v + f(V_t))K_t}$$

(25)

It follows from equations (20) and (21) that

$$K_t f(V_t) + W_t = K_{t+1} V_{t+1} + K_{t+1} - vK_t$$

(26)

From here we find the capital growth factor:

$$\eta_t = \frac{K_{t+1}}{K_t} = \frac{v + f(V_t)}{1 + V_{t+1}} (1 + \gamma_t)$$

(27)

and the output growth factor:

$$\mu_t = \frac{Y_{t+1}}{Y_t} = \frac{f(V_{t+1})}{f(V_t)}$$

(28)

Let $\varepsilon_t$ be an exogenous growth factor of the capital inflow: $\frac{W_{t+1}}{W_t} = \varepsilon_t$. Then

$$\gamma_t = \frac{1}{v + f(V_t)} \frac{W_0}{K_0} \varepsilon_0 \varepsilon_1 \ldots \varepsilon_{t-1}$$

(29)

Hence, the output growth factor is equal to

$$\mu_t = \frac{f(V_{t+1})}{1 + V_{t+1}} \cdot \frac{v + f(V_t)}{f(V_t)} \cdot \left(1 + \frac{1}{v + f(V_t)} \frac{W_0}{K_0} \varepsilon_0 \varepsilon_1 \ldots \varepsilon_{t-1} \right)$$

(30)

In period $t$, the unknown here is the control $V_{t+1}$, which has to be chosen according to an optimality criterion. We consider two different criteria.
If the optimality criterion is A – the stepwise maximisation of the current output (see Section 5) – then $V_{t+1} = \bar{V}$, where, we recall, $\bar{V}$ is a solution of the equation $f(V) = f(V)/(1 + V)$. Under this optimality criterion, the choice of the control does not depend on the dynamics of the capital inflow. For example, if $f(V) = V^\beta$, $0 < \beta < 1$, then $\bar{V} = \frac{\beta}{1-\beta}$.

For any constant control, $V_t = V$, the growth factor of the economy is equal to

$$
\mu_t = \eta_t = \alpha(V) \left( 1 + \frac{1}{\nu + f(V)} \cdot \frac{W_0}{K_0} \cdot \frac{\varepsilon_0 \varepsilon_1 \ldots \varepsilon_{t-1}}{\eta_0 \eta_1 \ldots \eta_{t-1}} \right)
$$

(31)

where, as earlier, $\alpha(V) = (\nu + f(V))/(1 + V)$ is the corresponding growth factor of the benchmark economy (without capital inflows). Given an initial state and a control $V$ a sequence of the growth factors can be found recurrently from equation (31).

Another optimality criterion, which we now consider, is C – the maximal long-run growth rate (see Section 5). If the growth rate of the oil income or aid injections is expected to be less than the proper growth rate of the benchmark economy, then, as is seen from equation (31), the optimal control is $\bar{V}$. Recall that $\alpha(\bar{V})$ is the maximal growth factor of sustainable growth in the benchmark model (without inflow).

Let us assume, as previously, that $\alpha(\bar{V}) > 1$ and $Y_0 > vK_0 \bar{V}$. If the rates $\varepsilon_t$ on average do not exceed $\alpha(\bar{V})$, then, even with the inflows, the economy is unable to support a growth factor greater than $\alpha(\bar{V})$ in a long run.

The matter stands differently if the inflow is expected to grow with a higher growth rate than the own growth rate of the economy. In such a case, under criterion C, equation (31) implies that the long-run economic growth rate increases when the control $V$ approaches zero. Thus, the expectations of a sustainable growth of the windfall lead to a decrease in wages. The wages level in the economy is defined by the expectations concerning the possibilities of a leading growth of the capital inflow.

The explicit conditions of economic growth and recession found in Section 5 start working if the windfall injections become small in the long run in comparison with the volume of the economy. If the economy grows well, it does not need injections. However, it is also possible to support a badly growing (falling) economy by use of injections temporally.

Now, let us turn our attention to the relation between the investment rate and the growth rate. In the fK model with inflows, the saving rate is equal to:

$$
s_t = \frac{I_{t+1}}{Y_t + W_t} = \frac{k_{t+1}}{k_t} - \nu = \frac{k_{t+1}}{k_t} - \frac{W_t}{f(V_t)}
$$

(32)

It follows from here that

$$
\eta_t = \nu + s_t \left[ f(V_t) + \frac{W_0}{K_0} \frac{\varepsilon_0 \varepsilon_1 \ldots \varepsilon_{t-1}}{\eta_0 \eta_1 \ldots \eta_{t-1}} \right]
$$

(33)

On a Solow path (where $s = \text{const}$):

$$
V_{t+1} = \frac{1 - s}{s + \frac{\nu}{f(V_t) + \gamma_t (\nu + f(V_t))}} = \frac{1 - s}{s + \frac{\nu}{f(V_t) + \gamma_t \frac{\nu \varepsilon_0 \varepsilon_1 \ldots \varepsilon_{t-1}}{\eta_0 \eta_1 \ldots \eta_{t-1}}}}
$$

(34)
If inflow injections grow slower than the economy does, then on the Solow path the control sequence \( \{ V_t \}_{i=1}^{\infty} \) converges to a steady control \( V(s) \), the same as in the benchmark fK model. It follows that the asymptotic growth rate of the Solow path increases in \( s \) if \( s \in (0, \tilde{V}) \) and decreases if \( s \in (\tilde{V}, 1) \). The sequence of the output growth factors converges monotonously to the steady value \( \nu + s(f(V(s))) \).

**8. Computer simulations**

Several computer simulations were conducted. The main attention was devoted to the question: how do the dynamics of the economic growth depend on the dynamics of the capital inflow?

The production function of the fK model was specified as \( f(V) = V^\beta \), \( \beta \in (0; 1) \). The growth factor of the capital inflow was taken as constant: \( \varepsilon_t = \varepsilon \). Paths were simulated under two scenarios corresponding to the optimality criteria A (maximisation of the current output) and C (maximisation of the long-run growth rate).

For the first scenario, variants of dynamics of the economic growth factors, \( \alpha \), under different inflow growth factors, \( \varepsilon \), are shown in Figure 1. It is easy to see that the economic growth factor converges to the growth factor of the inflow.

Recall that for BGP of the benchmark fK model (without aid) there are definite conditions of growth and decline (listed in Table 1). In the model with capital inflow under the first scenario, the situation is principally different: the long-run economic growth factor depends only on the inflow growth factor. In particular, under \( \varepsilon > 1 \) (when the inflow increases) the economy enters a BGP of growth (not decline), while under \( \varepsilon < 1 \) (when the inflow decreases) an economic decline takes place. Thus, under the first scenario the economy is totally ‘tied’ to the capital inflow.

The situation changes under the second scenario assuming the goal of long-run economic growth (Figure 2). The economic growth factor converges asymptotically to the maximum of two values: the growth factor of the inflow and the maximal long-run growth factor of the benchmark economy (with no inflow). Formally, \( \eta_t \xrightarrow{t \to \infty} \max \{ \varepsilon \alpha^* \} \).
By use of a computer optimization procedure (under particular parameters of the model) the maximal growth factor, $\alpha^*$, of the economy was found. In Figure 3, where $\alpha^* = 1.12$, the dynamics of the economy are shown under two different values of $\varepsilon_1 = 1.25$ and $\varepsilon_2 = 0.9$.

These results correspond to economic intuition. If the economy is oriented to a short-run criterion connected with maximisation of a current value (such as output), then an ability of some industries to grow and develop depends on the presence of external sources of financing. The leading role in such a situation will probably be played by industries ensuring a fast capital inflow into the country. On the contrary, if a long-run optimality criterion is preferable, such as maximisation of a long-run growth, then a probability appears that in some industries own hidden reserves will be found and used, which will be conductive for development of these industries on the basis of their own potential (these are the knowledge-intensive industries first of all). Of course, in such a situation the capital inflow will not lose its importance as an additional source of investments, but it will not play such an essential role as earlier.
9. Conclusion

We introduce an endogenous growth model (fK model) taking into account labour institutions and capital inflows, such as additional oil income or foreign aid. The presence of the institutions makes the behaviour of the model very different from the standard growth models usually used in policy analysis. The benchmark fK model (without capital inflows) demonstrates an inverse-U relation between the investment rate and the economic growth rate: a positive relation under low investment rates, but a negative relation under higher investment rates.

We study a version of the fK model with capital inflows. First, we analyse balanced growth paths and show that the windfall can be followed by a restructuring leading to a long-run economic decline after ceasing the capital inflow. Concerning non-balanced paths it can be concluded that the effect of the capital inflow depends on the relation between the expected inflow growth rate and the economic growth rate of the benchmark model (without inflow), as well as on the choice of the optimality criterion. If the optimality criterion is the maximisation of the current output, then the dynamics of the economy is entirely defined by the dynamics of the capital inflow.

Another optimality criterion under consideration is the maximisation of the long-run growth rate. We show that if the growth rate of the inflow is expected to be less than the growth rate of the benchmark economy (with no inflow) then the economy will follow a path of long-run sustainable growth. However, if the inflow is expected to grow with a higher rate than the proper growth rate of the economy, this can lead to a decline in wages.

Computer simulations show that, under a constant growth rate of the capital inflow and the short-run optimality criterion, the growth rate of the economy converges to the growth rate of the inflow. Under the long-run optimality criterion, the economic growth rate converges to the maximum of two values: the growth rate of the inflow and the maximal long-run growth rate of the benchmark economy (with no inflow).

All these results mean, in particular, that the volume of the capital inflow (such as additional oil income or aid) and its growth rate must be very carefully selected taking into account economic institutions in the country-recipient (labour institutions in particular). An insufficiently thought-out choice of parameters of the inflow may not lead to visible results, and, moreover, may even lead to promoting an economic decline after the windfall has ceased.

Acknowledgement

The author is grateful to participants of the Fourth Economic Development International Conference of GREThA/GRES (Bordeaux, 2012) and the China Meeting of the Econometric Society (Beijing, 2013) for discussions, and to Philip Ushchev for assistance in conducting computer simulations. Two anonymous reviewers provided helpful comments on earlier drafts of the manuscript.

Disclosure statement

No potential conflict of interests was reported by the author.
**Funding**

This work was supported by the Russian Foundation for Basic Research [grant number 14-01-00448], [grant number 14-06-00253].

**ORCID**

Vladimir Matveenko http://orcid.org/0000-0002-2465-9067

**References**


