

Affirmative Action: One Size Does Not Fit All*

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Abstract

This paper identifies a new reason for giving preferences to the disadvantaged using a model of contests. There are two forces at work: the effort effect working against giving preferences and the selection effect working for them. When education is costly and easy to obtain (as in the U.S.), the selection effect dominates. When education is heavily subsidized and limited in supply (as in India), preferences are welfare reducing. The model also shows that unequal treatment of identical agents can be welfare improving, providing insights into when the counterintuitive policy of rationing educational access to some subgroups is welfare improving.

Keywords: contests; educational quotas; private benefits; social welfare.

JEL classification: D61, I23

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1 Introduction

Affirmative action, or more generally, preferences are ubiquitous in much of the world. The idea is often put as “levelling the playing field”, or even tilting it in favor of certain groups so as to atone for past injustices. In this paper we develop a rationale for affirmative action that is different from that offered so far in the literature. We develop a simple model of contests with large numbers of agents (so that strategic effects are removed) and objects. There are two groups of agents: the disadvantaged and the advantaged. Agents are heterogeneous in two dimensions. In one dimension agents from the two groups have the same distribution, in the other dimension, their distributions differ. These dimensions can be interpreted in various ways. Agents could be the same in terms of ability, and differ in terms of polish with advantaged agents drawing from better distributions of polish. Or agents can have different kinds of ability, say native and acquired, with advantaged agents drawing from better distributions of acquired ability. The test, which determines placement, is assumed to value polish (or acquired ability) more highly than does society. For example, the SAT exam has been criticized as being biased against the poor: for example, words like “regatta” are likely to be known to only the rich. In addition, the practice of choosing questions based on how well high performing test takers do on them tends to bias the test against low performing students (see Soares (2012) for more on this). Below we will use the native and acquired ability as the interpretation, though we could use other interpretations that result in the test being biased towards the advantaged.

Agents also choose their effort level. Greater effort does two things. It has intrinsic value in that it raises the agent’s payoffs, independent of its effects on his score. It also raises his score, which weakly improves his admission options. Agents can put in effort for both reasons. Effort is wasteful when expended only to improve the student’s ranking and placement, as such improvements come at the expense of others and so are wasteful for society as a whole. This is explained in more detail later on. In the body of the paper, we present a simplified model with one school and where the agent’s only concern is the effect of effort on his score. In the appendix we show that the results extend to where effort also has intrinsic value.

Our rationale for affirmative action, or giving preferences to the disadvantaged, is based on modelling the trade-off between wasteful effort and selection. On the one hand, we show that conditional on getting in, the disadvantaged tend to put in more wasteful effort, which works against giving preferences to the disadvantaged. On the other hand, conditional on getting in, the disadvantaged have higher native ability than the advantaged. This factor works in favor of having preferences. While there is no one size fits all answer, the results obtained suggest *when* affirmative action is likely to be beneficial. The model would apply to the system in place in India as its rules are very clear cut. It would also apply to the US setting in spirit, though the US admission system is very nuanced. Our results suggest that in countries, like India, where seats are limited, tuition is very low, and preferences are extreme, such

reverse discrimination is very harmful to society. However, in the US, where education is costly, widely available and preferences are marginal, such a policy may well be beneficial.

The examples and simulations provide a number of other interesting results. They suggest that in the presence of rents from admission, even when there are no differences between agents, it may be welfare increasing to *create* them by allowing some agents access to education while denying it to others. This could be done by having a lottery that allocates access, or by allowing access to one group and not another. Such artificial differences may be optimal both when there is no difference between what society values and what the test does, and when there is. The logic of this is simple: when getting in is much better than not, students expend effort to improve their standing and so grab these rents. Unequal treatment relaxes the constraint on seats and reduces this effort at the cost of distorting the allocation of seats. When rents are high, the former outweighs the latter.

We will proceed as follows. In the rest of the introduction, we first provide some background on the prevalence and rationale for preferences in the US and the world. Following this we relate our work to the existing literature. Section 2 lays out the key elements of our model and the *raison d'être* behind them, making it clear how our work differs from the literature. It also works through the properties of basic model. Section 3 defines the “effort effect” and “selection effect” and shows formally that while the former works against giving preferences, the latter works in favor of preferences. Section 4 uses an example and develops some simulations to better understand the case for unequal treatment or preferences. Section 5 discusses extensions and Section 6 concludes.

1.1 Background

As President Lyndon B. Johnson said in a speech at Howard University¹ in 1965:

“Freedom is not enough. You do not take a person who for years has been hobbled by chains .. bring him to the starting point of a race and then say “you’re free to compete” and justly believe that you have been completely fair.”

In 1965 President Johnson issued executive order No. 11246 that required all federal contractors to take affirmative action to promote the hiring of blacks and other minorities. Affirmative action was, ironically, seen as way to ensure that “employees are treated ...without regard to their race, creed, color or national origin.” In Philadelphia for example, specific quotas were set for each of the building and construction trades for blacks, but by the mid seventies, opposition to quotas was growing. In *Bakke vs. the University of California, Davis*, the Supreme court ruled that Allan Bakke, a white medical school applicant, was denied admission on the basis of his race as a quota was set aside for deprived minorities. However, it upheld the use of race as a legitimate criteria in admissions.

¹A leading historically black institution.

Blacks and Hispanics are still given preferences in higher education. Nevertheless, the scope for giving race based preferences has been considerably reduced over the years. While schools used to actively target a given level of minority presence, i.e., meet quotas, such “quotas” are not acceptable today. In 2003, the U.S. Supreme Court approved the use of “points” to promote a diverse student body. However, a number of states, such as Texas, have moved away from using only race based preferences and added other measures. Texas now gives the top 10% of students from public high schools in Texas automatic admission to the state’s flagship public university, UT Austin, in *addition* to using points to help diversify the student body as allowed by the 2003 supreme court ruling. The Supreme Court very recently ruled on Fisher versus the University of Texas which challenged the current policy.² The ruling seems to have further reduced the space in which universities could give preferences. A lucid introduction to the issues, especially in the US context, is Fryer and Loury (2005).

In India, preferences are given in higher education and in public sector jobs to “scheduled castes and tribes” and these preferences are quite extreme. Preferences reserve a fraction of seats, proportional to their population share, for Scheduled Castes and Scheduled Tribes. The former were traditionally relegated to unpleasant tasks as “untouchables”, while the latter were outside the traditional caste system in India.³ India’s allocation system is based on performance in open competitive exams. The difference in cutoff scores for admission between those given preferences and the general category is huge. Moreover, India’s higher educational system is heavily subsidized. Brazil also approved an affirmative action bill in 2012 that reserves half the spots in federal universities for high school graduates of public schools, and distributes the reserved spots among black, mixed race and indigenous students according to the racial makeup of each state.

1.2 Related Literature

Much of the work in economics on the affects of preferences has focused on what is called models of statistical discrimination. If ability is unobserved, but correlated with an observable like race, this line of work argues that race based preferences may be counterproductive or even create inequality where none existed. Another line of work has focused on affirmative action as a way of mitigating differential access to education due to credit constraints. A third line has looked at the issue in the context of contests. We discuss each of these below. While we focus on the theoretical work, we also discuss the relevant empirical work as needed.

1.2.1 Statistical Discrimination

Models of statistical discrimination are the dominant line of research in this area. The key papers are those of Arrow (1973), Phelps (1972) and Coate and Loury (1993). The reason is that preferences create

² Abigail Noel Fisher was denied admission to UT Austin and is white.

³ See Frisancho and Krishna (2012) for more on this.

a “culture of dependence”. Intuitively, less effort is put in by the group given preferences, precisely because preferences make it easy to get jobs even without effort. As a result, people expect the group given preferences to be worse, and these expectations are validated in equilibrium. Even if there are no differences between groups, quotas giving preferences to one group over the other can end up hurting them through this channel. See Fang and Moro (2011) for a comprehensive survey of this literature.

On the empirical side, Ferman and Assuncao (2005) suggest that the effort expended by those offered preferences falls. They exploit a natural experiment which arose when a racial admissions quota was imposed on two of Rio De Janeiro’s top public universities. Using a difference -in-difference approach, they found a 5.5% decrease in standardized test scores among the favored group. This constituted a 25% widening of the achievement gap. Moro (2003) estimates a structural model of statistical discrimination for the US and his work suggests that though wage inequality has declined in the US this is not because of a switch in the equilibrium. A counterfactual exercise suggests that in a color-blind society blacks’ wage would have been on average more than 20% higher. Moro and Norman (2004) incorporate general equilibrium effects in the labor market into a model with statistical discrimination. They find that while affirmative action may increase the minority workers’ incentive to invest in learning while diminishing the non minority’s.

In any case, it is far from clear that affirmative action on the basis of race is optimal. There is concern that admitting students based on preferences creates “mismatch”. In extreme cases, this mismatch may result in their being worse off than if they went to a school or program better suited to their preparation level. See Rothstein and Yoon (2008) for work on Law Schools in the US and Frisncho-Robles and Krishna (2012) for work on quotas in India. On the other hand, Alon and Tienda (2007) suggest that students admitted under the 10% rule in Texas had a higher graduation rate. Arcidiacano et al. (2012) shows that this may be due to their choosing easier courses/majors. In addition, giving preferences on the basis of income rather than race seems to be called for. Cestau, Epple and Sieg (2012) empirically investigate the importance of using race as basis for affirmative action. They develop and estimate a structural model of admission to a gifted and talented program for children entering the first grade kindergarten program in a mid sized urban school district in the US. Their work suggests that once admission to the program is allowed to depend on being part of the free lunch program or not⁴, further conditioning on race gives little benefit to society.

Nor does the literature suggest that test scores alone are a good selection criterion. Selection on the basis of test scores seems to do worse than using more broadly based admission standards, see Bowen and Bok (1998). Part of the reason might well be the importance of non-cognitive skills. Cunha, Heckman and Schennach (2010) and Carniero et al. (2007) show that non-cognitive skills are important for a host of later outcomes, including educational attainment and employment status. This may be why, conditional on performance in tests, the disadvantaged do not seem to do better, at least in college,

⁴This is offered to poor students in the US.

than the advantaged as shown in Rothstein (1994). In fact, he finds that admissions offices could admit better-prepared entering classes by giving explicit admissions preferences to high-SES students and to students from high-SES high schools.

1.2.2 Credit Constraints

The second line of work, distinct from statistical discrimination, looks at preferences in the presence of credit constraints. The basic idea is that markets work well in allocating agents to seats. Monetary bids are just transfers and as long as those with the most to gain from obtaining the seats are also those that society wants to have the seats, i.e., there is no misalignment in social and private benefit, markets will give the first best allocation at the lowest cost. Contests on the other hand involve wasted resources even if they result in the same allocations. Thus, they are dominated by the market. However, if there are credit constraints, then contests may give a better allocation and so be worth the wasted resources they engender, see Fernandez and Gali (1999). Of course, to the extent that credit constraints are income rather than race based, such arguments also favor income based preferences. Also, basic economics (in the form of the principle of targeting which calls for policies to operate on the same margin as the distortion) would suggest that preferences are not the first best solution if the distortion lies in access to credit.⁵ In our model, credit constraints may be what lie behind the differences in acquired ability, as the advantaged may use prep school (that only teaches how to take the test) to a greater extent than the disadvantaged.

1.2.3 Contests

There is considerable recent work on the role of preferences in education couched in the contests setup, both with small numbers of agents so that there is a strategic effect, and with large numbers where there is not. Here the main issue is whether contests encourage effort or not and effort is seen as being good per se.

Fu (2007) and Fain (2009) have a model with two agents, a minority one who is handicapped relative to the non minority agent. They compete to get in to a school. Effort raises performance at some cost, though performance has a random component. Affirmative action (giving more weight to the score of minorities) can increase the effort expended by all agents, and if this is the objective, then affirmative action may be desirable.

More recently, Chade, Lewis, and Smith (2011) model a duopoly setting where colleges set admission standards and the ability of students is only partially observable. Students choose the portfolio of schools to apply to and schools choose admission rules. They show a number of interesting asymmetries arise in the comparative statics and argue that their model helps explain the observed fall in minority applicants to top public universities after affirmative action was removed. Such schools were removed from the

⁵For a nice recent survey on the role of credit constraints in education, see Lochner and Monge-Naranjo (2012).

portfolio of minority students who now had less of a chance of getting in.

These strategic models are less closely related to our work than those based on a competitive setting like that in Fryer and Loury (2013) which looks at the least cost way of achieving a given diversity goal. In their model there are a given number of slots that agents want to acquire. In the first period, agents make investment decisions and these decisions result in a distribution of second period abilities. The minority group is seen as having higher costs of such investments and as a result are on average disadvantaged ex-post in terms of getting these slots. As there are no externalities or distortions, the market allocation of slots is efficient. They ask, when should the intervention occur: Ex ante or ex post? Should intervention should be sighted (based on race say) or blind (based on say being in the top 10%). They show that if the policy can be sighted, then ex post intervention (a subsidy for the disadvantaged group) is all that is needed to meet the target as this subsidy also raises the ex ante incentive to invest by the minority group. In contrast to their work, we explicitly model a possible reason for affirmative action: namely that native ability is more valuable to society than acquired ability and look for when a case for unequal treatment can be made.

Chan and Eyster (2003) look at whether sighted or blind intervention is more efficient. They argue that a ban on affirmative action is inefficient in practice. Universities with a wish for diversity will just obtain it using an alternative criterion which is less efficient at identifying student quality. Fryer, Loury and Yuret (2008) estimate these costs to be four to five times as high as color-conscious affirmative action.⁶

Hickman (2011) compares quotas, admission preferences, and a color blind system and shows that they differ in terms of their effects. He uses an all pay auction setting and then takes the limit of the auction as the number of agents rises. In an auction setting, the distribution of costs (or valuations) of those you are competing with plays an essential role in how you bid. With a quota system, minorities compete for slots only with other minorities who have a worse distribution of costs than non minorities. With a points system, where minorities are given some points and then compete with the general population, competitors have better costs. With a color blind system, that does not give preferences there are no points given to minorities. As a result, with quotas, the best minorities put in less effort than under a system of preferences, while the worst minorities put in more effort. For non minorities, the effects are reversed. Hickman focuses on both the effort elicited and on the gap in attainment and cutoffs. The former is seen as a positive and the latter as a negative. It is worth noting that similar results, though for different reasons, occur in the presence of uncertainty in performance. In this case, the effort profile

⁶There is also the question of whether lower admission standards result in minority students being out of their depth and so performing poorly. There is a literature on mismatch and catchup that relates to this question which we will not say much about here. See Frisanchi-Robles and Krishna (2012) for a literature review and some evidence on this topic using Indian data. Loury and Garman (1993) show that for a given performance, blacks gain more in subsequent earnings from attending selective colleges than whites. However, for many blacks and some whites at such institutions the gain from attending a selective college may be offset by worse performance.

is hump shaped in ability. Agents that are very good need to put in little effort and those that are very bad know that they have little chance of getting in and so choose to put in little effort. Agents who are in between are highly motivated as their chances of getting in rise steeply with effort. Giving preferences to the disadvantaged reduces their cutoff score. This reduces the effort of the more able in the disadvantaged group but raises that of the less able and as the latter are predominant, tends to raise overall effort.

We focus on anonymous contests⁷ and the effect of preferences in such settings. While contests play many roles⁸, we abstract from most of these below and zoom in on their effort costs. Contests encourage effort, and to the extent that such effort is under/over supplied, this directly raises/reduces welfare. Examples of wasteful effort are manifold: managers may undercut their competitors at the cost of the company bottom line and salesmen may steal their rival’s customers. Effort itself tends to be privately beneficial but socially wasteful when there are rents being competed for, or agents incentives are misaligned with those of the principal or society. We focus only on the costs of effort and its social versus private benefits in this paper, abstracting from other dimensions such as those above.

Although we use placement in educational institutions as the example, other applications are possible. Our setting is non strategic with large numbers of applicants who understand they cannot affect the equilibrium cutoffs by their actions. This environment makes it much easier to analyze the case for preferences. A number of the insights in Siegel (2009) hold in our framework, albeit in a more transparent manner. There is also recent related work on contests with head starts, see Siegel (2013). Our setup differs from the work on contests with head starts both in terms of being non strategic, and in allowing intrinsic differences between agents in their abilities. With head starts, all agents typically differ only in terms of how far ahead they are, while we allow for interactions between native and acquired ability and effort. We also explicitly focus on social welfare and how policies affect it.

Our work is also related to the literature on optimal mechanism design. Myerson (1981) shows that in effect the optimal mechanism discriminates in favor of “disadvantaged” buyers as it is optimal to use the “virtual” valuation rather than the actual ones. In effect, this is like having affirmative action.⁹

2 The Basic Model

We first lay out the rationale for the basic assumptions made. Then we setup the model and explain how it works.

⁷Anonymous contests provide a non-strategic setting (analogous to that of monopolistic competition) as this makes more sense in the education application we are interested in.

⁸For example, contests identify latent talent that might not otherwise be unearthed. The international Math Olympics or the Putnam Exam in the US (administered by the Mathematical Association of America) for example allows students to discover their math ability at an early age. See Kenderov (2006) for a discussion of the role of competitions on education in mathematics.

⁹For more on this see Krishna (2002), Chapter 5.2.

2.1 The Setup

Our basic setup makes a few assumptions. First, we posit that performance in the contest is related to total ability which is seen as arising from native ability and acquired ability. This seems reasonable. Both innate ability and training are needed for superior performance, though the extent to which one can substitute for the other is an open question. In this context it is worth mentioning the super 30 program in India which has attracted a lot of attention. Anand Kumar, an India Mathematician from humble beginnings, started a program that tries to level the playing field for scheduled caste/tribe students in the Indian State of Bihar. It trains them to take the JEE (joint entrance exam) for entry into elite engineering schools in India like the IITs (Indian Institutes of Technology).¹⁰ From 2002 onwards, 30 disadvantaged students were chosen on the basis of an aptitude/ability test by the program. What is amazing is that the program has consistently placed over 90%, and often 100% of them these elite institutions.¹¹

The results of the super 30 program suggest that natural ability, once combined with some training (acquired ability), yields large improvements in performance, i.e., the cross partial derivatives of performance with respect to native and acquired ability are positive. We assume that groups may differ in terms of the distribution of acquired ability as differences in background create differences in acquired ability so that even if native ability is similarly distributed between groups as posited, total ability may not be.

Second, we assume that (i) total ability and effort give rise to performance in the exam, (ii) that there is no randomness in outcomes, and (iii) that effort to improve exam performance has no innate value. That higher ability and effort raise performance is uncontroversial. That there is no randomness in exams is less so. After all, everyone has a bad day and this affects performance. Or one may get lucky and get exactly what one studied in the exam and do better than someone who focused on topics that were not tested! Though randomness in performance complicates matters, in many ways our basic results remain unchanged as discussed in Section 5. Finally, the assumption that studying for the exam has no innate value needs some discussion. The education literature distinguishes between assessments based on exams which encourage rote learning, where studying may have no innate value,¹² and those based on “formative” and “authentic” assessments. Formative assessments allow students to both develop their abilities and assess their progress. They tend to integrate teaching and learning with assessment. Often, they do not require formal grading, but a demonstration of the ability to complete some task. Authentic assessments involve demonstration of ability in a real-world context. Studying for such exams may indeed be valuable in itself. We show that even if agents obtain private benefits from studying, what is critical

¹⁰The JEE is fiercely competitive and at a very high level. It is said that it is harder to qualify in this exam than to get into Harvard, MIT or Caltech! Smart upper middle class students with all the advantages of their background and training routinely fail this exam, despite tutoring and maniacal effort. There are preferences for backward castes and tribes that make it much easier for them to get in.

¹¹See http://en.wikipedia.org/wiki/Super_30 for more on this.

¹²There is some evidence that high stakes testing does not improve student learning. See Amerein and Berliner (2002).

to our arguments is that they study more than they would otherwise. This excessive effort is what is wasteful.¹³

Third, we posit that the test values acquired ability more than society. In this way it is biased towards the advantaged. In particular, we assume that the contest allocates seats according to the total ability of the agent. However, society would prefer that seats be allocated to those most likely to contribute to the social good: for example, to agents with high native ability if they are more likely to make the major breakthroughs of use to society than less able, but highly tutored and polished candidates. In other words, society would want Albert Einstein to be educated even if he was not as quick as other people and so fared badly in exams.

2.2 The Specification

There is a continuum of heterogeneous agents, with measure one, who decide whether to take an exam that will be used as the basis for admission. An agent is admitted if his performance exceeds a cutoff performance level denoted by \tilde{P} . This cutoff is determined in equilibrium to fill the available seats. We assume that there is no randomness in outcomes.¹⁴

As there is a continuum of agents, the environment is non strategic. Agents take a summary statistic (the cutoff score) as given, and maximize their objective function. In equilibrium the cutoff score assumed is validated. This is analogous to models of monopolistic competition where firms take the aggregate price index as given and make their choices (on pricing, entry, etc.) to maximize their profits, and where, in equilibrium, the price index that firms take as given is exactly the price index that emerges from the profit maximizing behavior of firms.¹⁵ While the body of the paper looks at admission to a single school, we explain why the results generalize in Section 5 and show this in an Appendix that is available on request.

Agents differ according to their abilities. Specifically, we distinguish between two types of ability: natural and acquired abilities. Natural ability is the ability an agent was born with, while acquired ability is accumulated prior taking the exam (for instance, acquired in high school, or from tutoring). The performance of an agent is denoted by P where:

$$P = f(a_N + a_A, e), \tag{1}$$

where a_N and a_A are natural and acquired abilities respectively, $a = a_N + a_A$ is total ability. For simplicity, we assume that the two types of ability are perfect substitutes. Finally, e is the effort undertaken by the

¹³We ignore any social benefits of studying for the exam in this paper. Acemoglu and Angrist (2000) suggests that such externalities are limited, at least for secondary school.

¹⁴The effects of dropping this assumption are discussed in Section 5.

¹⁵Formally, this is an anonymous game. The game is called anonymous because players' preferences depend only on their own actions and the distribution of all other agents' actions, i.e. on the aggregate behavior of all other agents, not on who plays what.

agent to pass the test. We assume that $f(a, e)$ is increasing in both its arguments and the returns to effort in terms of performance are increasing in ability, i.e., $f_{ae}(\cdot) \geq 0$. Moreover, $f(a, e)$ is concave in e : $f_{ee} \leq 0$. Effort has a private cost denoted by $c(e)$, with $c'(\cdot) > 0$ and $c''(\cdot) \geq 0$ and $c(0) = 0$. As effort is costly, this immediately implies that if an agent decides to take the exam, then the effort expended will be the minimum needed to attain the cutoff denoted by \tilde{P} . If he does not take the exam then his effort level is zero. Hence, the effort level for agents who decide to take the exam is implicitly defined by e in equation (2):

$$f(a, e) = \tilde{P}. \quad (2)$$

Let this effort level be denoted by $e^*(a, \tilde{P})$.

Higher ability agents need to put in less effort to attain a given performance cutoff so that effort levels are decreasing in ability. This follows from totally differentiating equation (2) to get

$$\frac{\partial e^*(a, \tilde{P})}{\partial a} = -\frac{f_a(\cdot)}{f_e(\cdot)} < 0. \quad (3)$$

Moreover, effort rises as the performance cutoff rises,

$$\frac{\partial e^*(a, \tilde{P})}{\partial \tilde{P}} = \frac{1}{f_e(\cdot)} > 0. \quad (4)$$

2.3 Equilibrium

The private returns from education of an agent with ability a are given by $s(a) - T - e^*(a, \tilde{P})$, where T is the tuition level. Without loss of generality, the outside option is normalized to zero. In the text, we assume that effort expended in studying for the exam has no intrinsic value as it does not affect the payoffs, $s(a)$, which depend only on a . In the Appendix, we incorporate an intrinsic value of effort by letting payoffs when educated depend on effort and show that the essence of our results remains. Though effort has two roles in this augmented model; it raises payoffs when educated and it helps you get in, only the former has social value. As a result, when there are rents to be appropriated from getting in, agents exert more effort than socially optimal in order to be admitted which is wasteful.

To make this idea concrete consider a simple example where there are N applicants with identical utility over the schools. They are allocated to N schools on the basis of their exam scores which are ordered in the same way by all applicants with school 1 being the most desirable. It is easy to see that the private gains of the individual moving up in the rankings exceed social gains. If an applicant who is assigned to school 5, puts in more effort and improves his exam results so that he is allocated to school 1, his private benefit is $U(1) - U(5)$. However, other applicants are moved down in the rankings and social gains must include their losses as well. Their losses are:

$$(U(1) - U(2)) + \dots + (U(4) - U(5)) = U(1) - U(5) > 0.$$

In general, in such “rat race” settings, social gains are less than private ones. In this example, the spillovers to other applicants exactly wipe out the gains to the applicant whose position improved.

An individual decides to take the exam if doing so is better than not doing so. That is, an agent with total ability a takes the exam if and only if

$$s(a) - T \geq c(e^*(a, \tilde{P})).$$

We assume that $s(a)$ is increasing in a : more able individuals gain more from education.

Define a^* by:

$$s(a^*) - T = c(e^*(a^*, \tilde{P})).$$

The marginal agent (the agent with ability a^*) is indifferent between getting into university or not. Agents with ability below the cutoff a^* choose not to put in the effort required to pass the exam while students above this cutoff choose to put in the required effort.¹⁶

We assume that the natural and acquired abilities are independently distributed across agents.¹⁷ The distribution functions are given by $H_N(a_N)$ and $H_A(a_A)$ on $[0, a_{\max}^N]$ and $[0, a_{\max}^A]$, respectively. Then, the distribution function for the total ability a is $H(a)$ on $[0, a_{\max}]$, where

$$H(a) = \int_0^{a_{\max}^A} H_N(a - y) dH_A(y) \quad (5)$$

and $a_{\max} = a_{\max}^N + a_{\max}^A$.¹⁸

The equilibrium in the model is determined by two conditions. First, the total mass of agents accepted to the university equals the number of available seats. Second, the agent with ability a^* is indifferent between going to university and not. The former condition means

$$1 - H(a^*) = \alpha, \quad (6)$$

where α is the number of seats (which is exogenously given and strictly less than one). The latter condition means that

$$s(a^*) - T = c(e^*(a^*, \tilde{P})). \quad (7)$$

Hence, we have two equations with two unknowns: a^* and \tilde{P} . In particular, we can solve (6) for a^* and then, given $a^*(\alpha)$, solve (7) for \tilde{P} .¹⁹ Since $s(a)$ is increasing in a and $e^*(a, \tilde{P})$ is decreasing, $s(a) - T -$

¹⁶This corresponds to Siegel (2009) where the reach and power of an agent are critical in defining the equilibrium. In our setup, the “reach” of an agent is the score at which his gain from admission is zero, the cutoff score or “threshold” is the reach of the marginal agent, and the “power” of an agent is his surplus from choosing a score equal to the threshold score. This is also his payoff in equilibrium.

¹⁷This assumption is made in order to simplify the theoretical analysis. The logic of the results in the paper holds if the distribution of natural and acquired ability are assumed to be correlated. One can use numerical simulations to show that.

¹⁸Note that $\Pr(a_N + a_A \leq y) = \sum_{a_A=0}^{a_A=a_{\max}^A} \Pr(a_N < y - a_A) \Pr(a_A)$. Replacing the sum with the integral gives the equation. In other words, the area under the line $a^N + a^A = a$, $H(a)$, is the same as calculating the density of the population below the line $a^N + a^A = a$ at a given a^A , $(H_N(a - a^A)h_A(a^A))$ and then integrating over all a^A .

¹⁹We assume that the number of seats is small enough that an interior equilibrium occurs.

Figure 1: Effort in Equilibrium

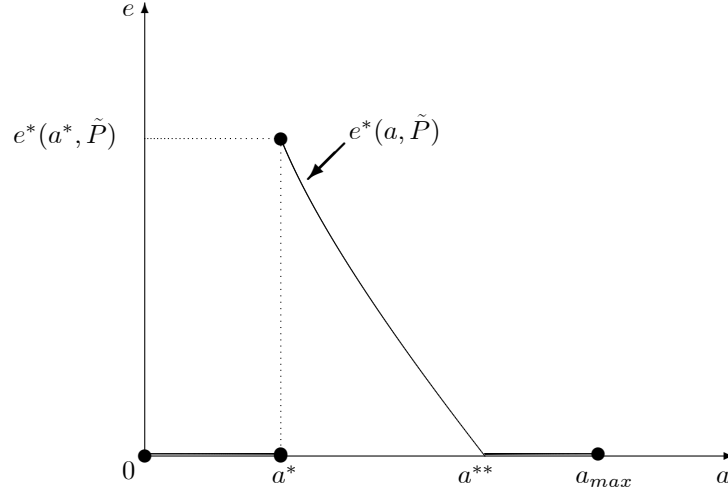
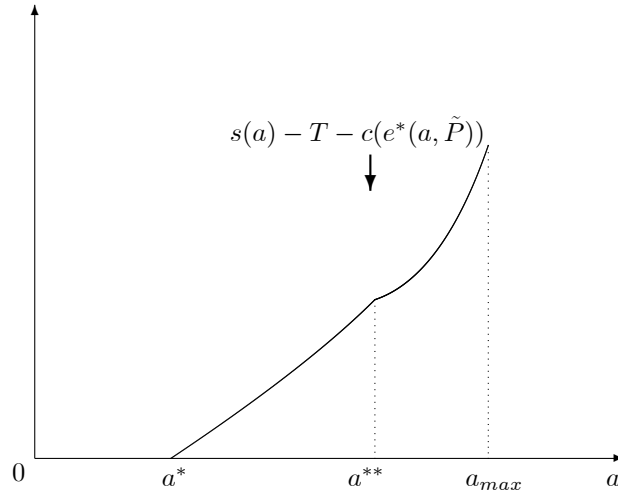


Figure 2: Payoffs from Education



$c(e^*(a, \tilde{P}))$ is increasing in a . Note that the equilibrium value of a^* depends solely on α , while \tilde{P} depends not only on α but also on the tuition level T .

As depicted in Figure 1, in equilibrium, the effort level is zero till ability a^* . At this point it jumps to $e^*(a^*, \tilde{P})$, the effort required to get in. Then, effort falls with ability and at some point may go to zero as depicted if able enough agents can attain the performance cutoff with no effort.²⁰

The next lemma shows what happens if the number of seats is increased or the tuition level rises.

Lemma 1 *A greater number of available seats or a higher tuition level decreases the performance cutoff*

²⁰ As the cost of this effort at a^* is just compensated for by the increase in earnings from going to university, the payoffs depicted in Figure 2 are continuous, though there may be a kink at a^{**} , the ability at which no effort is needed to be admitted. They are also increasing in ability.

\tilde{P} . That is, $\frac{d\tilde{P}}{d\alpha} < 0$ and $\frac{d\tilde{P}}{dT} < 0$. Moreover, a greater number of available seats reduces the ability cutoff though a higher tuition has no effect on it. That is, $\frac{da^*}{d\alpha} < 0$ and $\frac{da^*}{dT} = 0$.

Proof. In the Appendix. ■

A greater number of seats lets agents with lower ability in ($\frac{da^*}{d\alpha} < 0$) and as there are more seats, the cutoff performance falls $\frac{d\tilde{P}}{d\alpha} < 0$. In Figure 1, the effort curve shifts down and to the left with the marginal agent being of lower ability and putting in less effort.

Similarly, a rise in the tuition level decreases the return from being educated of all agents at a given effort level. In order to make the marginal agent indifferent between getting in and not, the effort need to get in, and hence the performance cutoff, must fall as the tuition rises ($\frac{d\tilde{P}}{dT} < 0$). As the performance cutoff falls, all agents need to put in less effort to be admitted. In Figure 1, the effort curve would shift down, with the identity of the marginal agent being unaffected ($\frac{da^*}{dT} = 0$), as the tuition rose.

Due to the lack of randomness in outcomes, agents are able to put in just the effort needed to ensure they attain the cutoff performance, \tilde{P} . However, since effort does not improve ability, these efforts are a social waste, even though they are privately valuable and are necessary to allocate seats.²¹ Effort expended is then the maximum of the effort needed to get in and the effort chosen for other reasons. Effort is excessive when the effort needed to get in exceeds that which would be chosen for other reasons. As the effort needed to get in falls with ability, unless the effort chosen for other reasons falls even faster (which is hard to motivate) lower ability agents are the ones who expend excessive effort and the essence of our results goes through. See the Appendix for details.

2.4 Social Welfare

We postulate that natural ability is more important than acquired ability for society. To capture this we assume that the social gains from education of an agent with abilities a_N and a_A are given by $s(a_N + \beta a_A)$, where $\beta \in [0, 1]$. Here, β represents the relative importance of the acquired ability for the society. If β is equal to zero, then the society cares only about natural ability. If β is equal to one, then natural and acquired abilities are of the same importance for the society. Social welfare is obtained by integrating social surplus over those with total ability over the cutoff level a^* .

$$W = \int_{a_N + a_A \geq a^*} \left(s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P})) - F \right) dH_N(a_N) dH_A(a_A), \quad (8)$$

where F is the social cost of education per student. As tuition is a lump-sum transfer, T does not directly affect welfare. It only affects it via the effort put in by agents. In addition, as there is no uncertainty in our setup, agents below the cutoff ability do not expend any effort.

²¹While assuming efforts have no effect on ability or payoffs might seem a bit extreme, it is not an unreasonable characterization of many entrance exams which involve studying for the test and emphasize memorization rather than deep understanding. In addition, relaxing this assumption does not change the flavor of the results as shown in the Appendix.

Given the number of seats, there are two distortions in the economy. In the model, the effort expended to pass the exam is determined by the tuition level: if the tuition is low, then getting in is very valuable and the performance cutoff is high. To make the cutoff, much effort is needed. This effort is wasteful which results in welfare losses. In (8), this loss is equal to $c(e^*(a_N + a_A, \tilde{P}))$ for an agent with ability $a = a_N + a_A$.

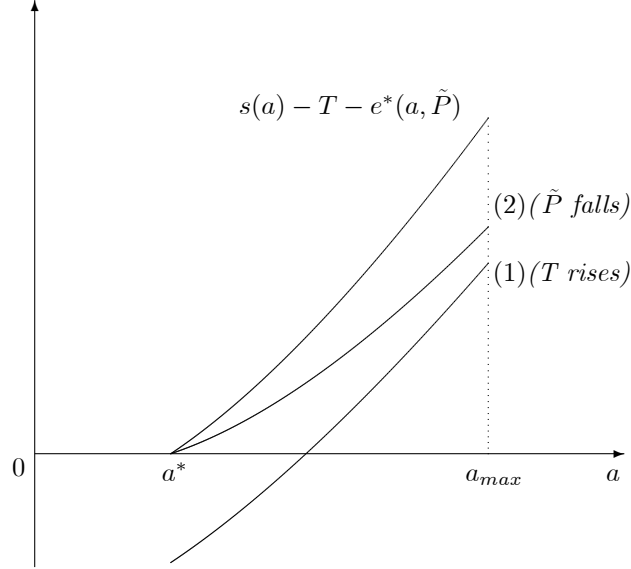
The other distortion, given the number of seats, arises due to social benefit deviating from private benefit. In particular, it is optimal for society that the agents with the highest social benefit get in. However, in practice, those with the highest private gain enter. For example, if admission is on the basis of exam performance, and the latter depends on the sum of natural and acquired ability, but social benefit comes from a function that puts more weight on natural ability than acquired ability, then the wrong people will be admitted. Note that, in the first best case, the number of seats should be set so that the social cost of an additional seat equals the social benefit generated. If the number of seats differs from this level there are welfare costs.

Next, we explore the effects of the tuition level and the number of seats on social welfare. In many countries the best public education is much cheaper and often far better than private education. However, it is rationed by strict performance cutoffs. In India, for example, until recently, all higher education institutions were public, close to free, and seats were allocated by performance in a school leaving exam. Even now, the best colleges remain public. The alternative to a bad domestic placement is to go abroad, where admission to comparable institutions is much easier, and to pay non-resident tuition. As a result, those going abroad to study from India seem to fall into two categories: those admitted with funding who tend to go to the best places abroad, and those without funding who pay their own way, often at less prestigious places as the best places fund whoever cannot pay. Turkey has a similar system. In fact, in most continental European countries, higher education is public and free. In some countries, students even get a government stipend to go to school. It is easy to see that this system encourages agents to put in more effort than is socially optimal when the number of seats is small so that rents from getting in are large. These rents are dissipated by excessive effort which is a real cost.

Another way to reduce these rents is to raise tuition. An increase in T has no effect on the identity of the marginal agent as long as the equilibrium effort level of the marginal agent is positive. In addition, as it is a transfer, it has no direct effect on welfare. Its welfare consequences arise through its effects on agent's actions. As an increase in T reduces the payoffs from admission, it reduces the effort expended by the marginal agent (whose identity is unchanged by the tuition increase) and so reduces the performance cutoff. This reduction in the performance cutoff in turn reduces the effort each agent choosing to become educated needs to incur, which raises welfare. This is the case until T is such that zero effort is expended by the marginal agent. In this way, the optimal tuition removes the distortion caused by wasted efforts.

Tuition increases do nothing but reduce effort until tuition reaches T^{opt} , where the marginal student

Figure 3: Payoffs from Education: a Rise in T



puts in zero effort. This defines T^{opt} to be such that

$$T^{opt} = s(a^*). \quad (9)$$

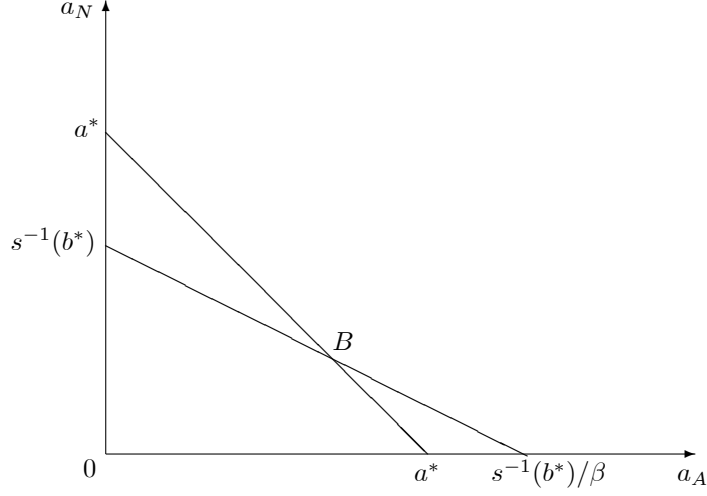
The equilibrium performance cutoff is then determined by

$$e^*(a^*, \tilde{P}) = 0 \iff \tilde{P} = f(a^*, 0).$$

It is worth noting that higher tuition does have a redistributive effect. The least able agents are unaffected as the alternative to going to university is the same and they are indifferent between these two options. The more able lose from an increase in tuition. When tuition rises, there are two effects on the payoffs from education (given by $s(a) - T - c(e^*(a, \tilde{P}))$). First, keeping effort fixed, the increase in tuition shifts the surplus curve down in a parallel fashion. Second, less effort is expended by all agents in order to get in (as \tilde{P} falls) and this shifts the surplus curve up till the surplus of agent a^* is again zero. As the less able put in more effort, this fall in effort is more valuable to them, so that the shift up is greater for less able agents and this flattens the surplus curve. As a result, individual welfare falls with tuition increases, and more so for the more able. Figure 3 illustrates this reasoning. Of course, as wasted effort is reduced, social welfare rises.

As discussed above, the distortion caused by wasted effort can be eliminated by setting a sufficiently high tuition level. However, the distortion caused by selection into education can not be completely removed. Only the agents with the highest social gains from education should fill the available seats. Specifically, given the number of seats to be filled, agents with abilities a_N and a_A , such that $s(a_N + \beta a_A) \geq$

Figure 4: The Selection Distortion



b^* , should be accepted where b^* is determined by

$$\int_{s(a_N + \beta a_A) \geq b^*} dH_N(a_N) dH_A(a_A) = \alpha.$$

However, competition results in the acceptance condition $a_N + a_A \geq a^*$. As a result, some agents, who should not be accepted to the university, are accepted and vice versa, which in turn leads to welfare losses.

Figure 4 illustrates the distortion for the case when $s^{-1}(b^*) < a^* < s^{-1}(b^*)/\beta$. In Figure 4, agents with abilities in the triangle $a^* B s^{-1}(b^*)$ should be accepted to the university on the basis of maximizing social welfare, but they do not apply, as their individual gains from education are less than their outside option. Instead, agents with abilities in the triangle $a^* B s^{-1}(b^*)/\beta$ take the exam and get in, while the social gains from their education are lower than those of the agents in the triangle $a^* B s^{-1}(b^*)$. As the cutoff ability a^* is determined by the number of seats α , the optimal choice of α can limit the welfare losses caused by the selection distortion, but can not completely eliminate them. Without a selection margin, i.e., with $\beta = 1$, setting tuition at the social cost of education and expanding seats so as to prevent excessive effort results in efficiency.²²

3 Reservations and Welfare

In this section, we ask if there is a case for treating groups of agents differently. We assume there are two groups of agents indexed by $i \in \{1, 2\}$, which have identical distributions of natural ability and potentially

²²As pointed out by a referee, when $\beta \neq 1$, whether we put less weight than the test does on acquired ability or put more weight than the test does on native ability, matters. In the former case, the incentive of the agents to obtain education is more than that of society and in the latter it is less.

different distributions of acquired ability. The latter is motivated by the fact that agents with different social backgrounds have had different educational inputs prior to taking the exam, which in turn results in different acquired abilities on their part. In particular, we assume that $H_N^1(a_N) = H_N^2(a_N) \equiv H_N(a_N)$, while $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$ where \succeq_{LR} stands for the likelihood stochastic order. Hence,

$$\frac{h_A^1(a_A)}{h_A^1(x)} > \frac{h_A^2(a_A)}{h_A^2(x)} \text{ for any } a_A, x : a_A > x.$$

This means that group 1 is more favored in terms of acquired ability than group 2. In addition, we assume that the distribution of natural ability has a log-concave density. This assumption is needed to ensure the likelihood stochastic order of the distributions of total ability: i.e., $H^1(a) \succeq_{LR} H^2(a)$.²³

The share of each group in the total mass of agents (which is normalized to unity) is denoted by γ_i , where $\gamma_1 + \gamma_2 = 1$. A share of available seats is reserved for each group of agents. We denote this reservation quota by θ_i , where $\theta_1 + \theta_2 = 1$. If these quotas are binding, then the cutoffs for the two groups will differ.

Rewriting the equilibrium conditions (6) and (7) to reflect this, let

$$H^i(a) = \int_0^{a_{\max,i}^A} H_N(a-y) dH_A^i(y) \quad i = 1, 2. \quad (10)$$

where $a_{\max,i}^A$ is the upper bound of $H_A^i(a_A)$. Then

$$\gamma_i (1 - H^i(a_i^*)) = \theta_i \alpha, \quad (11)$$

$$s(a_i^*) - T = c(e^*(a_i^*, \tilde{P}_i)), \text{ where } i = 1, 2. \quad (12)$$

Here, a_i^* is the total ability of the marginal agent from group i and \tilde{P}_i is the performance cutoff for agents from group i .

Having binding reservations is equivalent to setting different performance cutoffs across groups for acceptance to the university. Social welfare in this case is given by

$$\begin{aligned} W(.) &= \gamma_1 \int_{a_N + a_A \geq a_1^*} \left(s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}_1)) - F \right) dH_N(a_N) dH_A^1(a_A) \\ &\quad + \gamma_2 \int_{a_N + a_A \geq a_2^*} \left(s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}_2)) - F \right) dH_N(a_N) dH_A^2(a_A). \end{aligned}$$

Next, we explore how imposing reservations affects social welfare. To reduce notation, let $\theta_2 = \theta$, so that $\theta_1 = 1 - \theta$. The reservation quota θ has two effects on the social welfare. First, changes in θ have an impact on the performance cutoffs \tilde{P}_1 and \tilde{P}_2 and, therefore, on the effort put in by the agents. For example, a rise in θ (i.e. more seats for the “underprivileged” second group) decreases \tilde{P}_2 and increases \tilde{P}_1 . This in turn implies that agents from group 2 need to put in less effort, while agents from group 1 need to put in more effort to get in. We call this the *effort* effect on the social welfare. Second, there

²³See Theorem 1.C.9 in Shaked and Shanthikumar (2007) for the proof. This assumption is not very restrictive, as a number of commonly used distributions such as the normal, uniform, Gamma, and Beta distributions satisfy it.

is an impact on selection into education, as a_1^* and a_2^* are affected by θ as well. Specifically, a rise in θ decreases a_2^* and increases a_1^* . That is, more agents from group 2 and fewer agents from group 1 are accepted. We call this the *selection* effect. Hence, we have the following:

$$\begin{aligned} \frac{dW(a_1^*, a_2^*, \tilde{P}_1, \tilde{P}_2)}{d\theta} &= \left[\frac{\partial W(a_1^*, a_2^*, \tilde{P}_1, \tilde{P}_2)}{\partial \tilde{P}_1} \frac{\partial \tilde{P}_1}{\partial \theta} + \frac{\partial W(a_1^*, a_2^*, \tilde{P}_1, \tilde{P}_2)}{\partial \tilde{P}_2} \frac{\partial \tilde{P}_2}{\partial \theta} \right] \\ &\quad + \left[\frac{\partial W(a_1^*, a_2^*, \tilde{P}_1, \tilde{P}_2)}{\partial a_1^*} \frac{\partial a_1^*}{\partial \theta} + \frac{\partial W(a_1^*, a_2^*, \tilde{P}_1, \tilde{P}_2)}{\partial a_2^*} \frac{\partial a_2^*}{\partial \theta} \right] \\ &= EE + SE, \end{aligned}$$

where EE and SE stand for the effort (changes in θ affect the performance cutoff and via it effort) and selection effects (changes in θ affect the total ability cutoff and affect selection) on the welfare, respectively. In the Appendix, we derive exact expressions for EE and SE .

3.1 Identical Groups

Before we proceed to the analysis of the general case, we first ask whether there is a case for slight preferences if the groups of agents are the same: i.e., $H_A^1(a_A) \equiv H_A^2(a_A)$, even if social and private benefits from education differ. Not surprisingly, the answer is no. The intuition is that on the margin, the gains of one group are exactly made up for by the losses of the other in this case.

Let us define the non-discrimination quota as the quota which results from market clearing so that $\tilde{P}_1 = \tilde{P}_2$. Note that this immediately implies that $a_1^* = a_2^*$. The following proposition holds.

Proposition 1 *If $H_A^1(a_A) \equiv H_A^2(a_A)$, then whatever be β , welfare is locally unchanged in response to the quota when it is set at the non-discrimination level, i.e., such that $\theta_i = \gamma_i$.*

Proof. In the Appendix. ■

The intuition behind this is straightforward. Given a change in θ , agents in one group have their performance cutoff fall (the cutoff falls by more if the group is small so that asymmetry in the size of the groups does not affect this result), which reduces the effort for all abilities and raises the welfare of this group. The opposite happens for the other group. If the initial equilibrium is non discriminatory and the groups have the same distributions, then the gains to one group exactly cancel the losses to the other so that welfare is unchanged. In other words, the effort effect evaluated at the non-discrimination quota is equal to zero. Similarly, there is no effect on the total welfare through selection. The welfare losses due to the selection effect of one group are completely offset by the gains of the other group.

Even though welfare does not change when a “marginal quota” is imposed when groups are identical in their acquired as well as native abilities, this *does not* mean that quotas are always welfare reducing with identical groups. Strictly binding quotas may well raise welfare. The reason is that under certain conditions, welfare is not concave and the non-discrimination quota gives a minimum, not a maximum of

welfare. The logic behind this result is related to insights from the theory of the second best. When tuition is low, there are rents to be obtained and excess effort is expended to acquire these rents. Discriminating in favor of one or the other of two otherwise identical groups can make this existing distortion less binding and so raise welfare. When this distortion is limited and consequently, there is little excess effort, welfare remains concave and there is no reason to treat ex-ante identical groups differently. We explore this idea using simulations in the next section.

3.2 Non Identical Groups

When groups are not identical, even marginal quotas can affect welfare. That is, starting from a point where the two groups have the same cutoff, but differ in terms of their ability distributions, discriminating in favor of one group can raise the social welfare. To better understand when this can occur we evaluate the effort and selection effects (EE and SE) when the initial equilibrium is non discriminatory. We show that the effort effect is negative, which calls for discrimination against the weaker group, but that the selection effect is positive which calls for discrimination in favor of the weaker group.

The following proposition holds.

Proposition 2 *If $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$, and the distribution of natural ability has a log-concave density, then the effort effect of discriminating in favor of the weaker group (group 2), evaluated at the non-discrimination quota, is negative. That is,*

$$EE_{\theta=\theta^*} < 0.$$

Proof. In the Appendix. ■

Moving seats from group 1 to group 2 increases the performance cutoff for group 1 and decreases it for group 2, implying that agents from group 1 put in more effort to get in, while agents from group 2 put in less effort. As the distribution of total ability in group 1 stochastically dominates that in group 2, in the non-discrimination equilibrium, agents from group 1 put in less effort on average than agents from group 2. As a result, discriminating in favor of group 2 (a rise in θ) raises total wasted effort and decreases the social welfare compared to the non-discrimination equilibrium.

Next we examine the selection effect evaluated at the non-discrimination quota. Specifically, the following proposition holds.

Proposition 3 *If $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$, then the selection effect of discriminating in favor of the weaker group (group 2) evaluated at the non-discrimination quota is positive. That is,*

$$SE_{\theta=\theta^*} > 0.$$

Proof. In the Appendix. ■

The intuition is simple. When group 2 is more disadvantaged in terms of acquired ability, those agents from group 2 who do get must on average have a higher average natural ability.²⁴ In fact, we can show that if $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$, then in the non-discrimination equilibrium, the distribution of natural ability among agents from group 2 accepted to the university stochastically dominates (in the likelihood ratio order) that among accepted agents from group 1.²⁵ Since likelihood ratio dominance ensures first order stochastic dominance, accepted agents from group 2 have a higher natural ability on average than do accepted agents from group 1. As society cares more about the natural ability, than does the exam, a quota in favor of group 2 increases welfare by raising the average natural ability of admitted students.

From the above considerations, we can see that a quota in favor of the disadvantaged group can decrease or increase welfare depending on the magnitudes of the effort and selection effects. Group 2 is the disadvantaged group (because of a worse distribution of acquired ability), and reservations in favor of this group make sense as they allow talented but poorly educated agents to get higher education (the selection effect in the model). However, this comes at a cost, as agents from the disadvantaged group put in on average more effort (the effort effect), which is socially useless. If the selection effect dominates, as is likely when β is low so that acquired ability is worth little to society, then discriminating in favor of the disadvantaged is socially optimal. If β is close to 1, so that acquired ability can easily compensate for native ability in social welfare, then it is actually optimal to discriminate *against* the less advantaged!

Another way of saying this is to note that effort, beyond what would have been undertaken for its own sake, is costly and improves one agent's outcome only by worsening that of others so that there is no gain to all agents as a whole. An increase in such effort must thus always reduce welfare. When college tuition is high and education costly, then being admitted does not require much effort in equilibrium. Analogously, when tuition is low, there is more competition in being admitted to college, and, as a result, the necessary effort to get in is higher in equilibrium. When the affirmative action (e.g. quotas) is implemented, admission becomes more competitive for the advantaged (less so for the disadvantaged) and the necessary effort to get in increases (decreases) in equilibrium. There are two welfare consequences of affirmative action. On the one hand, affirmative action changes the pool of admitted students which can improve welfare when $\beta \neq 1$ (the selection effect); on the other hand, agents from the disadvantaged group exert more effort on average (the effort effect) and this reduces welfare. The effort effect dominates when rents are large, as they would be when education is scarce and priced below its social cost.

The idea that disadvantaged students should be given preferences because, conditional on a performance level in the test, their natural ability is greater than that of advantaged students, and this is something that society values more than the test does, is controversial in practice. The idea intuitively is that society values natural ability more than the test because such students have more potential than

²⁴This is the equivalent of saying in casual conversation that if someone from a bad High School got into Harvard, he/she must be really good!

²⁵The proof is available on request.

similar scoring advantaged ones. This suggests that such students should do better than similar scoring advantaged ones.

This is hard to test for. The disadvantaged may have worse non-cognitive skills and so perform worse than similar scoring advantaged students as discussed earlier. Moreover, the context in which we consider the performance of disadvantaged versus advantaged students with a similar score may also matter. For example, students given preferences in admission to college in the US do not seem to catch up to their peers as one might expect if the affirmative action was slight. With slight preferences these students, though low on polish should be high enough in potential to catch up. However, Frisanco et al. (2013) argue that in college, students have to digest new material so that the goal post is continually moving. In this environment we may not see catch up. They argue that repeated taking of the same exam is a far better place setting in which to look for such evidence. They use administrative data on students choosing to retake the university entrance exam in Turkey. They find that, controlling for selection, less advantaged students have larger learning gains upon retaking.

In the next section we explore when discriminating in favor of the less advantages is welfare improving using a parametric model as an example as well as simulations.

4 Example and Simulations

In this section we look at an example and some simulations to better understand the model and its implications for policy.

4.1 A Special Case

We parametrize the model to derive a closed-form solution and thereby compare the magnitudes of the effort and selection effects. To simplify the analysis, we assume linearity so $s(a) = Sa$, $f(a, e) = a + e$, and $c(e) = Ce$, where S and C are parameters. In this case, the effort put in is given by

$$e^*(a, \tilde{P}) = \max(\tilde{P} - a, 0),$$

which implies that agents with total ability greater than \tilde{P} do not put in any effort. The equilibrium conditions under the non-discrimination quota are given by

$$\gamma_1 (1 - H^1(a^*)) + \gamma_2 (1 - H^2(a^*)) = \alpha, \quad (13)$$

$$Sa^* - T - C(\tilde{P} - a^*) = 0, \quad (14)$$

where the first equation determines the cutoff of total ability, a^* , while the second one determines the performance cutoff:

$$\tilde{P} = \frac{(S + C)a^* - T}{C}.$$

We also assume that both types of ability are uniformly distributed across the agents. That is, $H_N(a_N) = a_N/a_{\max}^N$ on $[0, a_{\max}^N]$ and $H_A^i(a_A) = a_A/a_{\max,i}^A$ on $[0, a_{\max,i}^A]$ where $a_{\max,1}^A \geq a_{\max,2}^A$. Note that under this assumption, $H_A^1(a_A) \not\preceq_{LR} H_A^2(a_A)$ (but $H_A^1(a_A) \succeq_1 H_A^2(a_A)$) so that our assumption about the likelihood stochastic order does not hold anymore. However, as shown below, with uniform distributions of abilities, first-order stochastic dominance is sufficient for all the results formulated in the previous section to hold.

In equilibrium, the marginal ability a^* is pinned down by the number of seats α (see (13)). In our analysis, we consider the case when $a^* \geq a_{\max}^N$ and $a^* \geq a_{\max,i}^A$ for $i = 1, 2$. That is, the number of seats is so low that an agent needs both types of ability to get in. Next, we derive explicit expressions for the effort and selection effects evaluated at the non-discrimination quota.

Proposition 4 *The effort and selection effects evaluated at the non-discrimination quota are given by*

$$\begin{aligned} EE_{\theta=\theta^*} &= -\frac{\alpha(S+C)}{2} \left(a_{\max,1}^A - a_{\max,2}^A + \frac{\left(a_{\max,2} - \min(\tilde{P}, a_{\max,2}) \right)^2}{a_{\max,2} - a^*} - \frac{\left(a_{\max,1} - \min(\tilde{P}, a_{\max,1}) \right)^2}{a_{\max,1} - a^*} \right), \\ SE_{\theta=\theta^*} &= \frac{\alpha(1-\beta)S(a_{\max,1}^A - a_{\max,2}^A)}{2}, \end{aligned}$$

where

$$a_{\max,i} = a_{\max,i}^A + a_{\max}^N.$$

Proof. In the Appendix. ■

As can be seen, the magnitude of the effort effect positively depends on the parameters describing the returns from education and the cost of effort, S and C . Moreover, it is straightforward to see that the magnitude of the effort effect is increasing in the performance cutoff \tilde{P} . Indeed, if $\tilde{P} < a_{\max,i}$ for $i = 1, 2$, so that some of both abilities get in, then

$$EE_{\theta=\theta^*} = -\frac{\alpha(S+C)}{2} \left(\tilde{P} - a^* \right)^2 \frac{a_{\max,1}^A - a_{\max,2}^A}{(a_{\max,2} - a^*)(a_{\max,1} - a^*)},$$

which is negative and decreasing in \tilde{P} . If $a_{\max,2} \leq \tilde{P} \leq a_{\max,1}$, then

$$EE_{\theta=\theta^*} = -\frac{\alpha(S+C)}{2} \left(a_{\max,1}^A - a_{\max,2}^A - \frac{\left(a_{\max,1} - \tilde{P} \right)^2}{a_{\max,1} - a^*} \right),$$

which is also decreasing in \tilde{P} . Finally, if $\tilde{P} > a_{\max,1}$, then the effort effect is given by

$$EE_{\theta=\theta^*} = -\frac{\alpha(S+C)}{2} (a_{\max,1}^A - a_{\max,2}^A),$$

and, therefore, does not depend on \tilde{P} . Thus, all else equal, the effort effect is strictly decreasing in \tilde{P} on $[a^*, a_{\max,1})$ and then is flat with respect to \tilde{P} .

The selection effect depends only on S and the parameter β that describes the difference between the social and private gains from education. Note that if $\tilde{P} > a_{\max,1}$, the overall effect on social welfare is negative:

$$\frac{\partial W}{\partial \theta}_{\theta=\theta^*} = EE_{\theta=\theta^*} + SE_{\theta=\theta^*} = -\frac{\alpha (a_{\max,1}^A - a_{\max,2}^A)}{2} (\beta S + C).$$

In this case, the effort effect dominates over the selection effect. Hence, we can conclude that, for sufficiently high values of the performance cutoff (which represents the level of competition for seats, which in turn depends on tuition and the availability of seats), the effort effect is stronger than the selection effect and, as a result, a quota in favor of disadvantaged results in welfare losses. Whereas, for sufficiently low values of \tilde{P} , the selection effect prevails over the effort effect and a reservation quota in favor of disadvantaged can increase the social welfare. Note that the tuition fee T affects the effort and selection effects only through \tilde{P} . Moreover, a rise in T reduces the performance cutoff \tilde{P} . Thus, we know:

Proposition 5 *There exists a value of the tuition fee, T^{tr} , such that an increase in the quota for the disadvantaged group evaluated at the non-discrimination level, θ^* , raises welfare if and only if $T > T^{tr}$.*

Intuitively, a higher tuition level reduces the magnitude of the effort effect and, as a result, a quota in favor of the disadvantaged is more likely to be welfare improving. For this reason, the model suggests that affirmative action is likely to reduce welfare in a setting where education is subsidized. In India, for example, backward castes and tribes have a share of seats (given by their population share) reserved for them in publicly funded higher education. These reservations result in cutoff entrance exam scores that are much lower for these groups than for the general category.²⁶ As public higher education is not only much cheaper than private, and as the very best institutions are public and seats are scarce, competition to get in is extreme. In such a setting, reservations are likely to be welfare reducing. Higher education is also subsidized in many European countries. However, supply is abundant, and as a result, effort expended to get in is far less than in the Indian context. In the U.S., State Universities tend to be cheaper than private ones of a similar quality. However, the emphasis on need blind admissions and the availability of financial aid significantly reduces the difference in price.

In the next section we turn to some simulations that help us better understand the model. We continue to use the linear setup used in the example in our simulations.

4.2 Simulations

In this subsection, we simulate the model for a number of different values of the parameters. In particular, we assume that the production function $f(a, e)$ and the payoff function $s(a)$ are again linear: $f(a, e) =$

²⁶In the celebrated Indian Institutes of Technology, the entrance exam marks for the general category are in the high nineties and while they are in the low fifties for the reserved category.

$a + e$ and $s(a) = Sa$.²⁷ In this case,

$$e^*(a, \tilde{P}_i) = \max(0, \tilde{P}_i - a).$$

In addition, we assume that the cost of effort is quadratic: $c(e) = Ce^2$. As a result, the equilibrium conditions are given by

$$\begin{aligned} \gamma_i (1 - H^i(a_i^*)) &= \theta_i \alpha, \\ Sa_i^* - T &= C \left(\max(0, \tilde{P}_i - a_i^*) \right)^2 \text{ for } i = 1, 2. \end{aligned}$$

The first condition gives the ability cutoff for each group, while the second equates the benefit from getting in with its full cost for the marginal agent and defines the cutoff performance level for each group. Note that the above implies that a proportional change in S , T , and C does not change either the cutoff ability or the performance cutoff. Therefore, we set C to one and leave S and T to vary in the simulations. We also assume that $F = 1$. In fact, the choice of F does not affect the qualitative implications of the model, as changes in F only shift the welfare function downward or upward and do not affect the equilibrium.

In our simulations, we assume that the distributions of natural and acquired ability take a Gamma form. A Gamma distribution has a density function:

$$f(x, \xi, v) = \frac{1}{v^\xi} \frac{1}{\Gamma(\xi)} x^{\xi-1} e^{-x/v}.$$

It is characterized by the shape parameter ξ and the scale parameter v . It has a mean of ξv and a variance of ξv^2 . There are two advantages of using a Gamma distribution. First, a Gamma distribution has the following property: if X_i has a Gamma distribution with ξ_i and v , then $\sum_i X_i$ has a Gamma distribution with $\sum_i \xi_i$ and v . That is, if the distributions of natural and acquired ability are Gamma with the same scale parameter, then the distribution of total ability is Gamma as well. The second advantage is that if the shape parameter ξ is less than or equal to one, then the density function is decreasing, i.e., the cumulative distribution function (c.d.f.) is concave. For $\xi = 1$, we have the exponential distribution as a special case. Its c.d.f. is also concave. However, for $\xi > 1$, the density at zero is zero, and the density function is first increasing and then decreasing. Thus, the cumulative density is locally convex and then concave. This property is important, as the curvature of the c.d.f. of total ability seems to determine the curvature of the welfare function (see the experiments below). We will assume that $v = 1$ for all distributions. Then, the density functions are given by

$$\begin{aligned} h_N(a_N) &= \frac{1}{\Gamma(\xi_N)} x^{\xi_N-1} e^{-x}, \\ h_A^i(a_A) &= \frac{1}{\Gamma(\xi_A^i)} x^{\xi_A^i-1} e^{-x}, \text{ and} \\ h^i(a) &= \frac{1}{\Gamma(\xi_A^i + \xi_N)} x^{\xi_A^i + \xi_N - 1} e^{-x}, \end{aligned}$$

²⁷If ability and effort are less substitutable than assumed in the simulations, effort is less able to affect outcomes. In this case, the effort effect has a smaller role to play.

Table 1: Parametrization

Variable	Parametrization
Score Production Function:	$f(a, e) = a + e$
Cost of Effort:	$c(e) = Ce^2, C = 1$
Social Cost of Education:	$F = 1$
Scale Parameter of Gamma:	$\nu = 1$
Share of Population in Group i :	$\gamma_1 = \gamma_2 = 0.5$
<i>Shape Parameters of Gamma:</i>	$\xi_A^i, i = 1, 2. \xi_N = .5$
<i>Payoff Function:</i>	$s(a) = Sa$
<i>Tuition:</i>	T
<i>Weight on Acquired Ability in Welfare:</i>	β
<i>Share of Seats in Group i :</i>	$\theta_i, i = 1, 2$
<i>Seats Available:</i>	α

where ξ_N and ξ_A^i take on different values in the simulations. Table 1 summarizes the parametrization. Parameters that are varied in the simulations are in italics.

It is useful to write welfare slightly differently as done below. Given the form we use, welfare is easily broken down into its value when $\beta = 1$ plus an adjustment factor to account for the different weights placed on acquired ability by society and by individual agents. Specifically,

$$\begin{aligned}
W &= \sum_i \gamma_i \int_{a_N + a_A \geq a_i^*} \left(s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}_i)) - F \right) dH_N(a_N) dH_A^i(a_A) \\
&= \sum_i \gamma_i \int_{a_i^*}^{a_{\max, i}^A} \left(Sa - c(e^*(a, \tilde{P}_i)) - T \right) dH^i(a) \\
&\quad - (1 - \beta) S \sum_i \gamma_i \int_0^{a_{\max, i}^A} a_A (1 - H_N(a_i^* - a_A)) dH_A^i(a_A) + \alpha (T - F).
\end{aligned}$$

Note that we use the fact that $s(\cdot)$ is linear in the above. The first part in the expression above stands for private welfare, which is the sum of the private payoffs from being educated. Let us call this PW for private welfare. The second term in the welfare function, which is given by

$$SC = -(1 - \beta) S \sum_i \gamma_i \int_0^{a_{\max, i}^A} a_A (1 - H_N(a_i^* - a_A)) dH_A^i(a_A) + \alpha (T - F),$$

is the difference between social welfare and private welfare. For instance, if the private gains from education are equal to the social gains (i.e., β is equal to one), then the welfare is equal to private welfare if $T = F$. We call this term the "selection" component of welfare and denote it by SC . Thus,

$$W = PW + SC.$$

Next, we construct the welfare as a function of quota θ for different values of $\{\beta, S, \alpha, \xi_A^i\}_{i=1,2}$.

4.2.1 Simulations with only the Effort Effect

In this subsection, we consider the case when β is equal to one. In this case, social welfare is given by

$$W = PW + \alpha(T - F) = \sum_i \gamma_i \int_{a_i^*}^{a_{\max,i}} \left(Sa - c(e^*(a, \tilde{P}_i)) \right) dH^i(a) - \alpha F.$$

We examine two subcases: when the groups have identical distributions of acquired ability (the symmetric case) and when the groups are different in terms of the distributions of acquired ability (the asymmetric case).²⁸

The Symmetric Case When the groups are identical ($\xi_A^1 = \xi_A^2$), our analytical results show that the derivative of welfare with respect to the quota, evaluated at the non-discrimination quota, is equal to zero (see Proposition 1). This implies that if the welfare function is concave (or single-peaked), then the non-discrimination quota is optimal. Recall that the non-discrimination quota when the groups are symmetric is equal to the size of each group (given by γ_i). Simulations verify this result when $\xi_N + \xi_A^i \leq 1$, which ensures that the c.d.f. of total ability is concave which seems to make the welfare function concave as well.

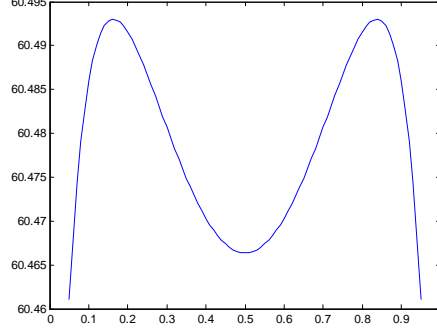
When $\xi_N + \xi_A^i > 1$, interesting things can happen. In the figures in this section, the horizontal axis is the quota in favor of the disadvantaged group. In Figures 5 and 6, we consider the case when $\xi_A^1 = \xi_A^2 = 1$. In this case, the shape parameter of the distribution of total ability is 1.5 and, therefore, the c.d.f. is not concave, nor is welfare. The values of T are 20 and 1 in Figure 5 and 6, respectively. The value of S is 130. The welfare function still has a zero derivative at the non-discrimination quota (as predicted), but this could deliver a local minimum rather than a local maximum.

As can be seen from Figures 5 and 6, there are two global maxima so that it is optimal to discriminate in favor of one group or the other as the two groups are of the same size. In Figure 5 it is optimal not to give all seats to one group. As T falls, we move to Figure 6 and it becomes optimal to exclude one group completely.

Why does this happen? Let us consider what happens, for example, when we move a seat from group 1 to group 2 when the density function is decreasing at the cutoff ability under the non-discrimination quota. As a result of this, the cutoff score for group 2 falls and that for group 1 rises, and the ability of

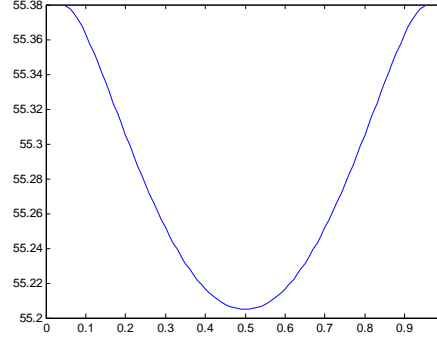
²⁸Note that only the distribution of total ability matters for welfare in this case.

Figure 5: Social Welfare (no selection effect, the symmetric case)



The parametrization: $\beta = 1$, $\alpha = 0.3$, $\xi_A^1 = \xi_A^2 = 1$, $S = 130$, and $T = 20$.

Figure 6: Social Welfare (no selection effect, the symmetric case)



The parametrization: $\beta = 1$, $\alpha = 0.3$, $\xi_A^1 = \xi_A^2 = 1$, $S = 130$, and $T = 1$.

the marginal agent rises in group 1 and falls in group 2. As the density function is decreasing in ability at the cutoff ability, the cutoff for group 2 falls by a smaller amount than the cutoff for group 1 increases even when the two groups are of the same size (as illustrated in Figure 7). This happens because there are fewer agents to the right of the cutoff (as the density is decreasing) than to the left. The more the cutoff falls (rises) the more the effort expended falls (rises). As a result, the increase in effort by the agents in group 1 (which reduces surplus and so welfare) is more than the decrease in effort by agents in group 2 (which raises surplus and welfare). Consequently, welfare falls as this reallocation is made so that non discrimination is optimal.

This is depicted in Figure 8. The surplus

$$s(a) - c(e^*(a, \tilde{P}_i)) - T$$

Figure 7: Decreasing Density: Cutoff Effects

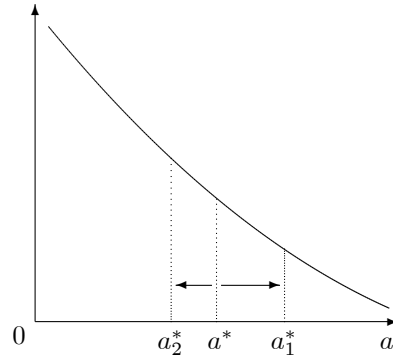


Figure 8: Decreasing Density: Surplus Changes

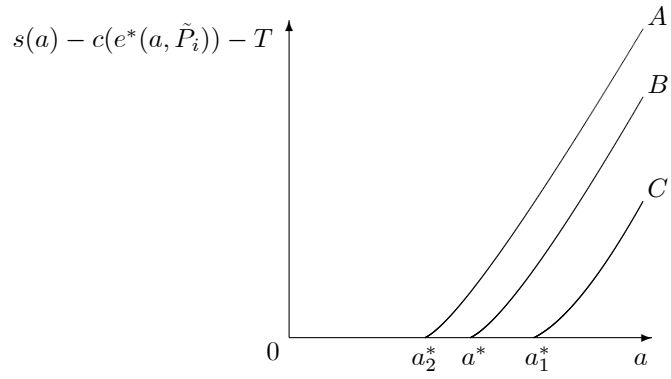
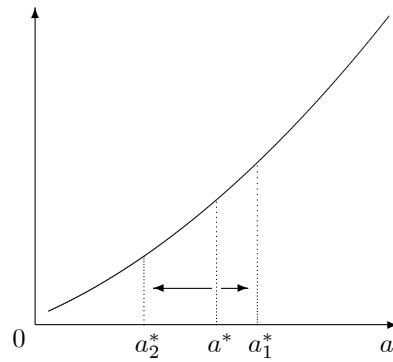


Figure 9: Increasing Density: Cutoff Effects



is zero for the marginal agent and is increasing in ability. With the above reallocation the effort function for group 2 shifts down while that for group 1 shifts up so that their surplus moves in the opposite direction from their effort. As can be seen, the total losses of group 1 (the area $Ca_1^*a^*B$) are greater than the total gains of group 2 (the area $Aa_2^*a^*B$).

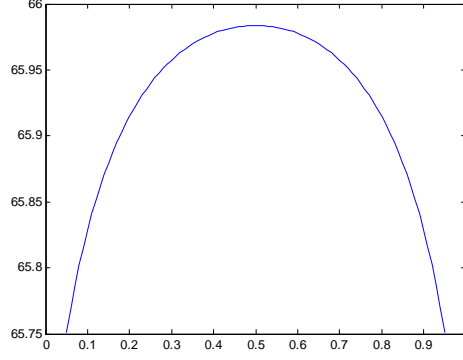
However, if the density function is increasing in ability at the cutoff ability under the non-discrimination quota, then the opposite happens (see Figure 9). The increase in effort by the agents in group 1 (which reduces welfare) is less than the decrease in effort by agents in group 2 (which raises welfare), so that welfare rises as this reallocation is made. Thus, non discrimination is a minimum of welfare not a maximum as depicted in Figures 5 and 6. To summarize, if the density function has a single peak (like the normal distribution), then whether welfare locally rises or falls depends on what side of the peak the non discrimination cutoff ability lies. Of course, this cutoff falls as the number of available seats rises.

In economic terms, this result can be interpreted in terms of the theory of the second best as well. Equal treatment for identical agents is optimal in the absence of other distortions. However, when there are distortions present, as is the case when there are rents that are dissipated by excess effort, equal treatment of identical agents may be far from optimal. Simulations show that *only* when rents are present (for instance, when tuition, T , is low relative to S) does the welfare function become non concave (see Figures 5 and 6 which depicts welfare when T varies). The presence of rents results in excessive effort and if this excessive effort is large enough, it may even be worth preventing some agents from having access to education, since doing so reduces the rents the remaining are competing for and so reduces effort. Of course, those not eligible to compete have no reason to put in effort anyway. This argument also explains why greater discrimination is optimal when T falls (in Figure 6 T is lower than in Figure 5). As T falls, rents increase and the effort expended to get in rises as well. This is increasingly costly due to the convex cost of effort posited. As a result, the saving from reduced effort by discriminating in favor of one group is higher and more discrimination is called for as T falls. In contrast, when T rises, the size of rents falls decreasing the excessive effort and, therefore, making the welfare function "more" concave. In Figure 10 T is equal to 40 (all other parameters take the same values). As can be seen from the figure, the welfare function is concave with the maximum at the non-discrimination quota.

Armed with this understanding let us see what happens when α rises from being 0.3. At $\alpha = 0.3$, we have two peaks as in Figure 5. It may actually become optimal to exclude one group or the other as α rises. The reason is that giving all the seats to one group reduces the effort of that group a lot. It also makes the effort of the other group zero as that group has no seats. This is what is depicted in Figure 11 for $\alpha = 0.4$. If α is even higher, then if all seats are given to one group, there may not be enough agents in this group who wish to avail themselves of the seats (given the tuition level) so that some seats are wasted. In this case, giving fewer seats to this group will raise welfare as depicted in Figure 12 where $\alpha = 0.7$.

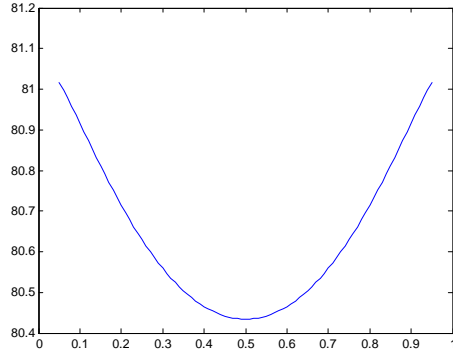
More generally, the simulations suggest that even when there are no differences between agents, it

Figure 10: Social Welfare (no selection effect, the symmetric case)



The parametrization: $\beta = 1$, $\alpha = 0.3$, $\xi_A^1 = \xi_A^2 = 1$, $S = 130$, and $T = 40$.

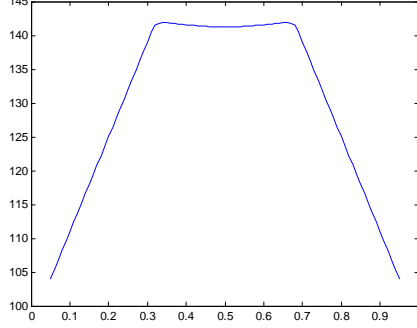
Figure 11: Social Welfare (no selection effect, the symmetric case)



The parametrization: $\beta = 1$, $\alpha = 0.4$, $\xi_A^1 = \xi_A^2 = 1$, $S = 130$, and $T = 20$.

may be welfare increasing to create them by allowing some agents access to education while denying it to others. By granting a smaller share of agents access, surplus per capita in the resulting competition is higher, though of course, the number granted access is lower. What happens to total welfare depends on how fast surplus per capita rises as access is restricted, i.e., its elasticity. This observation is interesting in terms of policy. In practice, differential access to education is the norm and the welfare implications of such differential access is poorly understood. Advantaged groups tend to have better access to education than others. For example, the better off and better connected may have superior information about how and where to apply. These simulations suggest a novel reason why (even when groups are identical ex ante) differential access (perhaps via a lottery) may be welfare improving.

Figure 12: Social Welfare (no selection effect, the symmetric case)



The parametrization: $\beta = 1$, $\alpha = 0.7$, $\xi_A^1 = \xi_A^2 = 1$, $S = 130$, and $T = 20$.

The Asymmetric Case If groups have different distributions of acquired ability and β is equal to one, then, by Propositions 3 and 4, the derivative of the social welfare function at the non-discrimination quota is negative. If the welfare function is concave, this means that the optimal quota should be smaller than the non discrimination quota²⁹, i.e., it should discriminate in favor of the advantaged group. Thus, in this case, θ should be less than non discrimination level.

Under our parametrization, $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$ if $\xi_A^1 > \xi_A^2$. In our experiments, we assume that $\xi_A^1 = 0.5$, $\xi_A^2 = 0.2$, and $\xi_N = 0.5$. Thus, $\xi_A^i + \xi_N < 1$ implying that the density functions of total ability for both groups are decreasing. We also assume that the groups are of the same size: $\gamma_1 = \gamma_2 = 0.5$. Then, the non-discrimination quota solves the following system of equations:

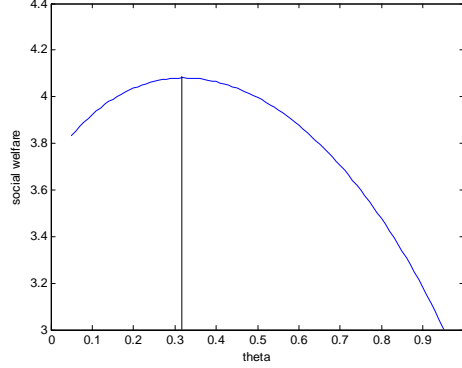
$$\begin{cases} \alpha = 1 - \gamma_1 H^1(a^*) - \gamma_2 H^2(a^*) \\ \theta^* = \gamma_2 (1 - H^2(a^*)) / \alpha. \end{cases}$$

The first equation sets demand for seats under non discrimination equal to the supply to obtain the common cutoff. The second equation sets the fraction of seats that go to the less privileged group at this common cutoff equal, by definition, to the non-discrimination quota. Given the values of the parameters, the non-discrimination quota is equal to 0.3932.

Figures 13 and 14 depict the welfare function for different values of S (we set T to unity in the experiments below). In Figure 13, $S = 10$ and in Figure 14, $S = 100$. As expected, the optimal quota is always less than the non-discrimination quota (as suggested by the theory). Moreover, as can be seen in 13 and 14, the higher is S , the lower the optimal quota. This is explained by the fact that a higher S implies higher performance cutoffs and, thereby, more effort put in by the agents. This in turn increases the welfare losses due to wasted effort. As a result, a larger quota for the advantaged group (which on average put in less effort) is optimal as S rises.

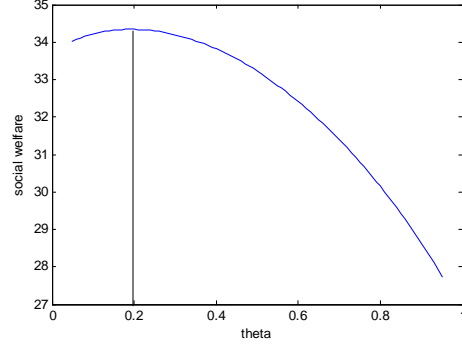
²⁹Recall that θ is the quota in favor of the disadvantaged group.

Figure 13: Social Welfare (no selection effect, the asymmetric case)



The parametrization: $\beta = 1$, $\alpha = 0.3$, $\xi_A^1 = 0.5$, $\xi_A^2 = 0.2$, $S = 10$, and $T = 1$.

Figure 14: Social Welfare (no selection effect, the asymmetric case)



The parametrization: $\beta = 1$, $\alpha = 0.3$, $\xi_A^1 = 0.5$, $\xi_A^2 = 0.2$, $S = 100$, and $T = 1$.

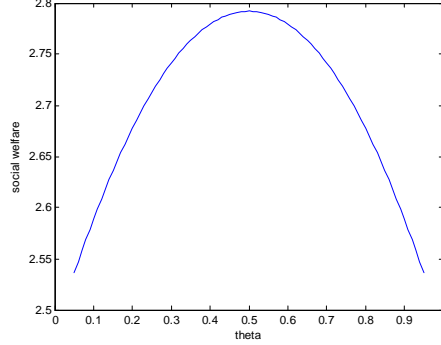
Note that if the distributions of total ability were not concave, then, similar to the symmetric case, the welfare function could be not concave as well. However, in this case, imposing a quota in favor of the advantaged group is still optimal, though the optimal quota is very likely to be equal to zero, i.e., it is optimal to ban the disadvantaged group.

4.2.2 Simulations with the Selection Effect

In this subsection, we assume that β is strictly less than one. In this case, the selection component of the welfare is not zero and, as a result, the optimal quota comes from the interplay of the effort and selection effects. In particular, the selection effect provides a rationale to discriminate in favor of the less advantaged group. We will first consider the symmetric case.

The Symmetric Case When the distribution of acquired ability is the same in both groups, the selection effect evaluated at the non-discrimination quota is equal to zero. This implies that if the total welfare function is concave, then the non-discrimination quota is optimal. Figure 15 illustrates this idea. It depicts the welfare function for the following set of parameters: $\xi_A^1 = \xi_A^2 = 0.4$, $\xi_N = 0.5$, $\gamma_1 = \gamma_2 = 0.5$, $S = 10$, $T = 1$, $\alpha = 0.3$, and $\beta = 0.5$. As can be seen, the non-discrimination quota maximizes the welfare.

Figure 15: Social Welfare (with the selection effect, the symmetric case)



The parametrization: $\beta = 0.5$, $\alpha = 0.3$, $\xi_A^1 = \xi_A^2 = 0.4$, $S = 10$, and $T = 1$.

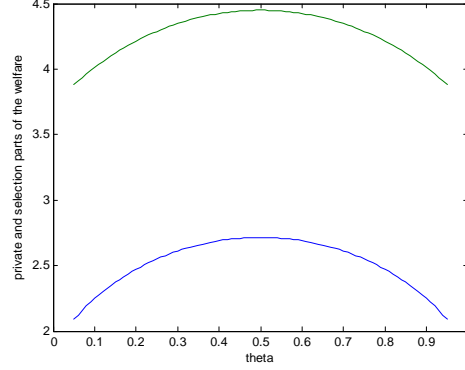
However, in this case it is not enough for the density function to be decreasing for total welfare to be concave. The selection part of the welfare is an additional source of non-concavity of the welfare function (besides the non-concavity of the distributions of total ability). Even though, the distributions of total ability are concave (as in this example), the presence of the selection part of the welfare can make the welfare function non-concave. The idea behind this is as follows. The selection part of welfare is given by

$$SW = -(1 - \beta)S \sum_i \gamma_i \int_0^{a_{\max,i}^A} a_A (1 - H_N(a_i^* - a_A)) dH_A^i(a_A) + \alpha(T - F),$$

which according to our experiments is usually convex in θ . Thus, the welfare function is the sum of a concave (the private component of welfare) and convex (the selection component of welfare) function. As a result, the curvature of the social welfare function is not pinned down when $\beta < 1$ even if private welfare is concave. Moreover, the lower is the value of β , the more likely it is that the convex component dominates, so that the social welfare function is not concave.

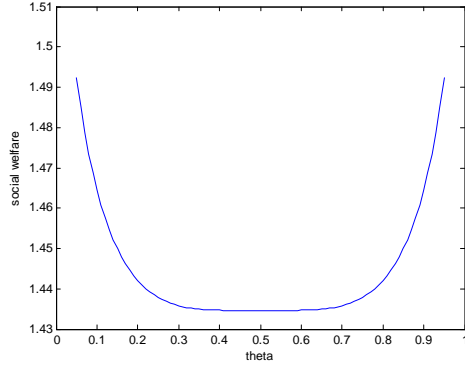
Figure 16 depicts the private part of the welfare (the upper curve) and the negative of the selection components of welfare taken ($-SW$) for $\beta = 0$. As can be inferred, both of these curves are concave with a maximum at the non-discrimination quota. However, the welfare function is the difference between these two curves and, therefore, could be convex or concave. Figure 17 depicts the resulting total welfare function which, as can be seen, is maximized at the corners.

Figure 16: Private and Selection Components of Welfare (the symmetric case)



The parametrization: $\beta = 0$, $\alpha = 0.3$, $\xi_A^1 = \xi_A^2 = 0.4$, $S = 10$, and $T = 1$.

Figure 17: Social Welfare (with the selection effect, the symmetric case)

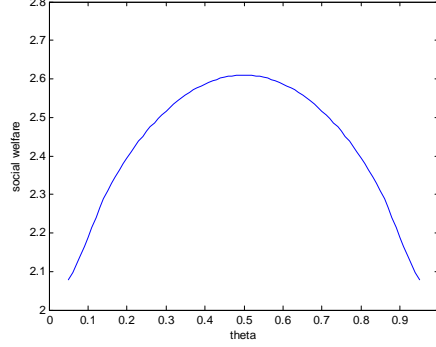


The parametrization: $\beta = 0$, $\alpha = 0.3$, $\xi_A^1 = \xi_A^2 = 0.4$, $S = 10$, and $T = 1$.

Thus, if agents are symmetric, the density of ability is decreasing, and β is close to unity, then (by continuity arguments) no discrimination will be optimal as depicted in Figure 15. If β is very small, so that acquired ability does not matter, then giving all the seats to one group or the other could be optimal.

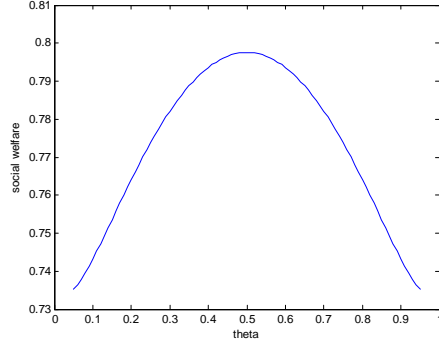
When is such a policy optimal? Intuitively, if there is a lot of wasted effort (wasted effort is large when S is high and/or the number of seats is low), then the private component of welfare is relatively flat. Once it is flat enough, and β is small enough, then the selection component dominates so that giving all seats to one group or the other increases welfare. Figures 18 and 19 illustrate this intuition. In Figure 18 we only increase α from 0.3 to 0.5, in Figure 19 we only decrease S from 10 to 5. As can be seen, in both cases the welfare function is concave and, as a result, the non-discrimination quota is optimal.

Figure 18: Social Welfare (with the selection effect, the symmetric case)



The parametrization: $\beta = 0$, $\alpha = 0.5$, $\xi_A^1 = \xi_A^2 = 0.4$, $S = 10$, and $T = 1$.

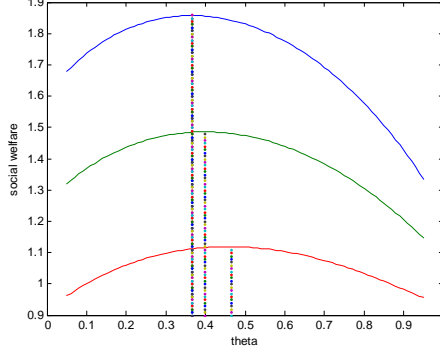
Figure 19: Social Welfare (with the selection effect, the symmetric case)



The parametrization: $\beta = 0$, $\alpha = 0.3$, $\xi_A^1 = \xi_A^2 = 0.4$, $S = 5$, and $T = 1$.

The Asymmetric Case When the groups are different in terms of their acquired ability distribution and $\beta < 1$, then both forces are at play. The presence of the selection effect shifts the optimal quota towards the disadvantaged group. However, the effort effect shifts the quota in favor of the advantaged group. Figure 20 depicts the social welfare function for different values of β . The upper curve is the welfare function when $\beta = 0.8$. As can be seen, in this case the optimal quota is in favor of the advantaged group (optimal θ is lower than the non-discrimination quota given by 0.3932). That is, the effort effect is stronger than the selection effect. When β is equal to 0.5 (the middle curve), the two effects are of the same magnitude and, therefore, the optimal quota is close to the non-discrimination quota. Finally, when $\beta = 0.2$, one can see that the optimal quota is in favor of the disadvantaged group as the selection effect prevails over the effort effect.

Figure 20: Social Welfare (with the selection effect, the asymmetric case)



The parametrization: $\beta = \{0.8, 0.5, 0.2\}$, $\alpha = 0.3$, $\xi_A^1 = 0.5$, $\xi_A^2 = 0.2$, $S = 5$, and $T = 1$.

5 Extensions

In our model, we make several assumptions and it is important to understand the extent to which our results depend on them. First, we assume no uncertainty in terms of the performance in the exam. As a result, agents expend just enough effort to get in, so that more able agents exert less effort, and all agents above an ability threshold get in. Those agents with abilities below this threshold do not get in, nor do they exert any effort. The effort effect follows basically from this: reallocating seats to the advantaged group reduces total effort as this group has higher average total ability and, therefore, tends to expend less (wasteful) effort. This intuition goes through for small enough levels of randomness in performance.

In general, with randomness the effort function becomes hump shaped over ability. The least able put in little or no effort and rely on luck to get in, while the most able need to put in little effort to get in. Those in between need to work to get in. As a result, the disadvantaged could put in more or less effort than the advantaged on average so that the results depend on the position of the hump and the distribution of abilities. If the less advantaged put in more effort, then discriminating against them helps and vice versa.

The selection effect is more general. The exam selects on the basis of total ability which is on average higher in the advantaged group. Society values native ability more than the exam does. Conditional on getting in, the native ability of those from the disadvantaged group is higher and this is true with or without uncertainty. Consequently, in general, the selection effect makes it welfare increasing to discriminate in favor of the less advantaged.

Second, we assume that there is only one school. It is relatively straightforward to show that the intuition remains valid with more schools. When one discriminates in favor of the less advantaged, their cutoffs fall in each school. As a result, they put in less effort. The opposite occurs in the more advantaged

group. As the more advantaged on average put in less effort, welfare falls due to the effort effect. The selection effect of course generalizes (these results are available in the Appendix).

6 Conclusion

We identify two distortions: an effort distortion that arises from wasteful effort, and a selection distortion that arises from society placing a greater weight on native ability than does the placement system. How these two play off against each other and interact results in a one size does not fit all answer. Preferences may be good for one society and bad for another. The disadvantaged group puts in more effort to get in on average than the advantaged one. Given the number of seats, preferences in favor of the disadvantaged group raise the average effort level and this reduces welfare. This effect is large when tuition is low and potential surplus is dissipated via effort. However, if society puts more weight on natural ability than does the placement algorithm, then there is an additional selection effect that operates. The disadvantaged group, *that gets in*, on average has greater natural ability. This makes preferences in their favor desirable. The former effect dominates when tuition is low and the latter when it is high. As a result, our work suggests that while preferences may be a good idea in the US where tuition tends to be high, it may be a very bad idea in India where tuition is very low.

Our model is deliberately kept simple to highlight the trade-off between the selection and effort effects we focus on. It neglects a number of factors that may be very important in the real world. For example, it assumes the distribution of ability is given. However, affirmative action may well affect it. It may reduce the incentive of disadvantaged agents to invest in themselves if they do not need to do so to be competitive. It may increase their prior investment in their ability if affirmative action makes returns from doing so higher by making college possible.³⁰ In addition, to the extent that more effort develops good work habits that are valuable in themselves but are not understood as to be so, there would be too little effort expended, other things constant.

Nor do we deal with on credit constraints. These may act as another selection mechanism. To the extent that credit constraints tend to be more binding on disadvantaged groups, this selection effect will operate in a similar way to that we model. We also neglect non-cognitive skills which have been shown to be important determinants of outcomes along a variety of dimensions, see Carneiro et al. (2007). If the disadvantaged also have worse non-cognitive skills, discrimination in their favor may worsen selection rather than improve it. Ultimately, our point is that the extent of the selection and effort effects drive the desirability or not of affirmative action and those policies in place determine the extent of these effects. Thus, no one answer is possible and the answer to the question of the desirability of affirmative action depends on the setting. It is quite consistent for the answer to be “no” in India and “yes” in the US.

³⁰In other words, preferences may have ex ante effects and operate on the development margin, as well as ex post ones that operate on the assignment margin in the nomenclature of Fryer and Loury (2013).

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