Supplier Selection and Lot-Sizing Optimization Problem under Changing Demand

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The article is devoted to the application of mathematical programming to solve the problem of supplier selection and inventory lot-sizing optimization under the changing demand. In this paper, we propose the model of the stochastic multi item problem and lot-sizing optimization under changing demand, which unlike the current setting of this problem provides an optimal solution to the considerable demand uncertainty.

Key words: Logistics Supply, Inventory Lot-Sizing Problem, Stochastic Programming.

Introduction

A typical problem in logistics is to calculate an optimal batch size of delivery, which turns into a non-trivial task when you need to take into account a large number of restrictions: on contractor proposal and consumer demand, on storage capacity, low cost, etc. Hafiz Ullah and Sultana Parveen [6] provided a review of inventory lot-sizing literature. They also classified inventory control models. The choice of a particular model is influenced by the planning horizon, order quantity, frequency of replenishment, nature of demand (continuous, dynamic or stochastic), number of supply chain levels (single, echelon) etc. In the event that the demand is constant, the classical EOQ model (proposed by Harris in 1913) and its various modifications are used to determine the optimal delivery batch size. In 1958 H.M. Wagner and T.M. Whitin provided the first algorithm [7] of the optimal solution with variable demand. They used dynamic programming for a single level lot-sizing problem.

In recent years, many papers have appeared on the application of dynamic and stochastic programming for production and sales planning. In particular, J. Shapiro considered a multi-period dynamic and stochastic resource allocation problem in his monograph [3]. The monograph by D.B. Yudin and E.G. Holstein [4] considered the problem of acquisition and sale strategy under conditions of changing demand for a single product, i.e. single item lot-sizing problem. In [1], a multi-item inventory lot-sizing scenario was proposed, and in [2], the joint application of the classical EOQ model and linear programming methods to solve the acquisition and sale strategy problem under changing demand was handled.

Mathematical Formulation of the Dynamic Problem of Inventory Lot-Sizing and Supplier Selection

In their work[4], Chirawat Woarawichai, Tarathorn Kullpattaranirun and Vichai Rungreunganun developed a mathematical formulation of lot sizing and supplier selection given the size of storage space and budget constraints. The solution of this problem allows to determine the optimal lot size for each supplier and minimize total procurement costs, which include purchasing, transaction and holding costs. It is assumed that the demand is known throughout the planning period. The problem is formalized as a linear programming problem. We consider its mathematical formulation with the following notation.

Indices:
$i \in \{1,\dots,I\}$ – set of products;
$j \in \{1,\dots,J\}$ – set of suppliers;
$t \in \{1,\dots,T\}$ – set of time periods.
Parameters:
$D_{i,t}$ – demand for product *i* at time *t*;
$P_{i,j}$ – price of *i* product offered by supplier *j*;
H_i – storage costs for *i* product per period;
O_j – transaction cost for supplier *j*;
w_i – storage space for product *i*;
S – total storage space;
B_t – purchasing budget for period *t*.
Decision variables:
$X_{i,j,t}$ – number of products *I* ordered from supplier *j* in
a period of time *t*;
$Y_{j,t}$ – a variable taking value 1 given an order is made from supplier *j* in period *t*, 0 otherwise.
Auxiliary variables:
$R_{i,t}$ – a number of products *i* carried over from period *t* to period *t*+1.

We need to calculate variables $X_{i,j,t}$ and $Y_{j,t}$ turning to a minimum the linear form

$$TC = \sum_i \sum_j \sum_t P_{i,j} X_{i,j,t} + \sum_j \sum_t O_j Y_{j,t} + \sum_i \sum_t H_i \left(\sum_{k=1}^{t} \sum_j X_{i,j,k} - \sum_{k=1}^{t} D_{i,k} \right) \to \min; \tag{1}$$

subject to

$$\begin{cases} R_{i,t} = \sum_{k=1}^{t} \sum_j X_{i,j,k} - \sum_{k=1}^{t} D_{i,k} \geq 0, \forall i,t; & (2) \\ \left(\sum_{k=t}^{T} D_{i,k} \right) Y_{j,t} - X_{i,j,t} \geq 0, \forall i,j,t; & (3) \\ \sum_i w_i \left(\sum_{k=1}^{t} \sum_j X_{i,j,k} - \sum_{k=1}^{t} D_{i,k} \right) \leq S, \forall t; & (4) \\ \sum_i \sum_j P_{i,j} X_{i,j,t} \leq B_t, \forall t; & (5) \\ Y_{j,t} \in \{0,1\}, \forall j,t; & (6) \\ X_{i,j,t} \geq 0, \forall i,j,t. & (7) \end{cases}$$

The objective function is shown in equation (1). It consists of three parts: 1) cost of products, 2) transaction costs for suppliers, and 3) storage cost for remaining products for *t*+1 period.

Constraint (2) indicates that demand restrictions must be made in the period in which they arise: shortage or sending an order back are not acceptable. Constraint (3) implies there are no orders without charging relevant transaction costs. Constraint (4) – is a constraint on storage space. Constraint (5) suggests total cost of purchases for each product can not exceed the budget for the period. Constraint (6) indicates that

$Y_{j,t}$ is a boolean variable with values 0 or 1; constraint (7) denotes that decision variable $X_{i,j,t}$ must take non-negative values.

In general, it's a rather difficult task to find a solution to such models. The interaction between many variables should be taken into account. For example, the stock at the end of the specified time period t is determined by the decisions about purchasing and storaging goods in the period from 1 to T. Therefore, this problem is formalized as a dynamic multi-period linear programming problem and solved by using optimization packages, such as LINGO 12.

Let's consider the example of a numerical solution of this problem.

Example 1. Suppose , a certain company decides to purchase three products A, B and C from three suppliers X, Y and Z over five time periods. It is assumed that the demand for products is known throughout the planning periods. Table 1 shows the demand for three products over five scheduling periods and a budget to buy them for the same period B_t.

Table 1: Demand for three products over five periods $D_{i,t}$ and restrictions on the budget for their purchasing B_t. c.u.

Products	Planning horizon, un.				
	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
A ($i = 1$)	12	15	17	20	13
B ($i = 2$)	20	21	22	23	24
C ($i = 3$)	20	19	18	17	16
Budget, B_t, c.u.	1820	2000	3500	3000	3500

Table 2 represents the price of three products for each of three suppliers X, Y, Z ($P_{i,j}$) and their transaction costs O_j.

Table 2: Price of three types of products for each of three suppliers X, Y, Z ($P_{i,j}$) and their transaction cost (O_j),c.u.

Products	Supplier's price, $P_{i,j}$, , c. u		
	X ($j = 1$)	Y ($j = 2$)	Z ($j = 3$)
A ($i = 1$)	30	33	32
B ($i = 2$)	32	35	30
C ($i = 3$)	45	43	45
Transaction cost, O_j, c. u.	110	80	102

Storage cost for three products A, B, C (H_i) and their storage space w_i are presented in Table 3.

Table 3: Storage cost of three products A, B, C (H_i),c.u. and their storage space (w_i).u.

Data	Products		
	A ($i = 1$)	B ($i = 2$)	C ($i = 3$)
Storage cost, H_i, c. u.	1	2	3
Storage space, w_i, un.	10	40	50

The total storage space S is equal to 200 units. We need to determine the optimal lot size for each vendor and minimize total procurement costs.

The results of solving this problem obtained by optimization package LINGO 12 are shown in Table 4.

Table 4: Order quantity of three products over five periods $X_{i,j,t}$.u.

Products	Order quantity of product i during period t, $X_{i,j,t}$, un.				
	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
A ($i = 1$)	$X_{1,1,1} = 12$	$X_{1,3,2} = 15$	$X_{1,1,3} = 37$	-	$X_{1,3,5} = 13$
B ($i = 2$)	$X_{2,3,1} = 20$	$X_{2,3,2} = 21$	$X_{2,1,3} = 22$	$X_{2,3,4} = 23$	$X_{2,3,5} = 24$
C ($i = 3$)	$X_{3,2,1} = 20$	$X_{3,3,2} = 19$	$X_{3,1,3} = 18$	$X_{3,3,4} = 17$	$X_{3,3,5} = 16$

The total costs for this solution are minimal and amount to TC=10448 c. u.

If we compare the demand data (see Table 1) and order quantity of three products over five periods (see Table 4), we will see that the demand for products B and C is always satisfied in the same period when the demand arises. The demand for the product A is also largely satisfied in the same period when a demand arises, except for the fourth period ($t = 4$). In the third period, the auxiliary variable takes $R_{1,3} = 20$, i.e. 20 units of the product have been purchased in the 3rd period of time to be used in the 4th period.

Thus, the stockpiling is not feasible under constant demand and constant prices for products, and that was shown by a numerical example of the problem.

Technique for Creating and Optimizing a Stochastic Inventory Model for Lot-Sizing and Supplier Selection

The assumption that the demand for goods is known throughout the planning period is, in our view, unrealistic and narrows the scope of usage of this method in the formulation discussed above. We give a stochastic formulation of the linear programming problem.

The development of linear and mixed integer programming (stochastic programming) is a tempting choice for any kind of planning (operational, tactical or strategic), because the manager is able to analyze mistakes and risks in detail. The basic idea is a simultaneous consideration of a number of unknown future scenarios, each with its probability. Each scenario has its own probability. The model simultaneously determines an *optimal random plan* for each scenario and an optimal plan of preemption, which differs from all random plans. Optimization involves maximization (or minimizing) of the expected income (expense), where the term "expected" means multiplying income (expenses) of each scenario by their probabilities.

Consider the technique of creating and optimizing the stochastic linear programming model. With this in view, we transform the discussed above numerical example of choosing a supplier and lot-sizing optimization under constant demand into a problem with changing demand, i.e. a problem of stochastic programming.

Example 2. Suppose, a company is a retailer. The company needs a strategy to acquire goods (products) over two periods. Moreover, the number of products purchased in the first period is known: $X_{1,1,1} = 12, X_{2,3,1} = 20, X_{3,2,1} = 20$.

But the impact of a large advertising campaign on the amount of goods that the company will be able to sell and, accordingly, must purchase in the second period, is unknown. The analysis of previous advertising campaigns and marketing personnel intuitive assessment identified three completely different scenarios shown in Table 5.

Table 5 :Sales forecast in the 2nd period (t = 2)

Products	Demand, $D_{i,t}$, un.	Probability
Low demand (Scenario 1)		
A (i = 1)	$D_{1,2}$ = 13	p_1 = 0,25
B (i = 2)	$D_{2,2}$ = 20	
C (i = 3)	$D_{3,2}$ = 16	
Medium demand (Scenario 2)		
A (i = 1)	$D_{1,2}$ = 17	p_2 = 0,5
B (i = 2)	$D_{2,2}$ = 20	
C (i = 3)	$D_{3,2}$ = 18	
High demand (Scenario 3)		
A (i = 1)	$D_{1,2}$ = 18	p_3 = 0,25
B (i = 2)	$D_{2,2}$ = 20	
C (i = 3)	$D_{3,2}$ = 20	

The data presented in Table 5 shows the need to create a separate sub-model of linear programming for each of three scenarios in the second period, as well as a separate submodel of linear programming in the first period. Obviously, a submodel of linear programming in the first period should be combined with each of three submodels in the second period and then optimized. We consider a mathematical formulation of this problem.

Using the introduced coefficients and variables, we formulate a stochastic linear programming model.

An individual formulation of the stochastic multi-item supplier selection problem and a lot-sizing optimization under the changing demand for two time periods is as follows:

$$TC = TC_0 + 0{,}25 \cdot TC_1 + 0{,}5 \cdot TC_2 + 0{,}25 \cdot TC_3 \rightarrow \min, \quad (8)$$

where TC_0 − total purchasing cost for the 1st period (t = 1); TC_1 − total purchasing cost for the 2nd period (t = 2) under the 1st scenario (s = 1); TC_2 − total purchasing cost for the 2nd period (t = 2) under the 2nd scenario (s = 2); TC_3 − purchasing total cost for the 2nd period (t = 2) under the 3rd scenario (s = 3); p_1 = 0,25, p_2 = 0,5, p_3 = 0,25 – the probability of realization of the 1st 2nd and 3d scenario respectively.

Each summand of the objective function TC_s (8) is a submodel of the following form:

$$TC_s = \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{t} P_{i,j} X_{i,j,t} + \sum_{j=1}^{3}\sum_{t} O_j Y_{j,t} + \sum_{i=1}^{3}\sum_{t} H_i \left(\sum_{k=1}^{t}\sum_{j=1}^{3} X_{i,j,k} - \sum_{k=1}^{t} D_{i,k} \right) + \\ + f \cdot \left(\sum_{j=1}^{3}\sum_{t} \left(\min_j \{P_{i,j}\} \Delta D_{i,k} \right) \right); \tag{9}$$

subject to

$$\begin{cases} R_{i,t} = \sum_{k=1}^{t}\sum_{j=1}^{3} X_{i,j,k} - \sum_{k=1}^{t} D_{i,k} \geq 0, i = 1,2,3; t = 2; & (10) \\ \left(\sum_{k=1}^{T} D_{i,k} \right) Y_{j,t} - X_{i,j,t} \geq 0, i = 1,2,3; j = 1,2,3; t = 2; & (11) \\ \sum_{i=1}^{3} w_i \left(\sum_{k=1}^{t}\sum_{j} X_{i,j,k} - \sum_{k=1}^{t} D_{i,k} \right) \leq S, t = 2; & (12) \\ \sum_{i=1}^{3}\sum_{j=1}^{3} P_{i,j} X_{i,j,t} \leq B_t, t = 2; & (13) \\ Y_{j,t} \in \{0,1\}, j = 1,2,3; t = 2; & (14) \\ X_{1,1,1} = 12, X_{2,3,1} = 20, X_{3,2,1} = 20; & (15) \\ X_{i,j,t} \geq 0, i = 1,2,3; j = 1,2,3; t = 2. & (16) \end{cases}$$

Obviously, we have combined three submodels of the dynamic linear programming (9) - (16) in the objective function (8)). The differences between the model (9) - (16) and the above pattern (1) - (7) are : firstly, the planning horizon covers two time periods; secondly, the objective function (9) takes into account shortage costs in the k -th time period, as

$$F_{j,k} = f \cdot \left(\sum_{j=1}^{3}\sum_{t=k} \left(\min_j \{P_{i,j}\} \Delta D_{i,k} \right) \right),$$

where f – fine for deficit (coefficient taking into account the increasing cost of goods in case of urgent delivery); $\min_j\{P_{i,j}\}$ – cost of goods in case of urgent delivery (if a purchase is carried out at the lowest price); $\Delta D_{i,k} = X_{i,j,k} - D_{i,k}^{\max}$ – shortage of goods in the k-th time period (calculated on the assumption that the demand for goods in the k-th time period is maximum). $D_{i,k}^{\max} = \max_t \{D_{i,t}\}$).

Thirdly, we introduce additional constraints for the variables of the model (15), which means the amount of products acquired in the first week $X_{i,j,1}$ is constant. Thus, we actually find an optimal solution for the second planning period, as the solution for the first period is the initial data of the problem.

The numerical solution of this problem is presented in Table 6.

Table 6: Order quantity of three products for two periods $X_{i,j,t}$, un.

Products	Order quantity of product i during period t, $X_{i,j,t}$, un.			
	$t = 1$	$t = 2$		
		Scenario 1	Scenario 2	Scenario 3
A ($i = 1$)	$X_{1,1,1} = 12$	$X_{1,3,2} = 18$	$X_{1,1,2} = 18$	$X_{1,1,2} = 18$
B ($i = 2$)	$X_{2,3,1} = 20$	$X_{2,3,2} = 20$	$X_{2,1,2} = 20$	$X_{2,3,2} = 20$
C ($i = 3$)	$X_{3,2,1} = 20$	$X_{3,2,2} = 19$	$X_{3,2,2} = 19$	$X_{3,2,2} = 20$

The total cost is TC = 4376,125 c. u.

To analyze the obtained solution, we calculate supplies $I_{i,t}$ in each of two planning periods according to the formula

$$I_{i,t} = R_{i,t} + I_{i,t-1},$$

where $R_{i,t}$ – a number of products I carried over from the period t to the period t+1 (i.e. stockpiling in the current period t); $I_{i,t-1}$– stock of products I stockpiled in the previous period t–1.

Inventory calculations for each of the planning periods are presented in table 7.

Table 7: Amount of Stocks $I_{i,t}$, un., for each of the planning periods

Products	Stock of products i during period t, $I_{i,t}$, un.			
	$t = 1$	$t = 2$		
		Scenario 1	Scenario 2	Scenario 3
A ($i = 1$)	$I_{1,1} = 0$	$I_{1,2} = 5$	$I_{1,2} = 1$	$I_{1,2} = 0$
B ($i = 2$)	$I_{2,1} = 0$	$I_{2,2} = 0$	$I_{2,2} = 0$	$I_{2,2} = 0$
C ($i = 3$)	$I_{3,1} = 0$	$I_{3,2} = 3$	$I_{3,2} = 1$	$I_{3,2} = 0$

The data presented in Table 7, confirm the obvious fact that the higher demand uncertainty for the product, the greater stock of goods. In this example, the demand for product B is unchanged $D_{2,1} = D_{2,2}$ = 20 un., so its order quantity is constant $X_{2,j,1} = X_{2,j,2}$ = 20 un., and the amount of stock is zero. The demand for product A ranges from 13 to 18 units and for product C – from 16 to 20 (see Table 5), therefore you need to stockpile these goods. At the same time, the point is in case of low demand (Scenario 1) the goods should be ordered from suppliers Y and Z; in case of middle demand (scenario 2) – from suppliers X and Y; and in case of high demand (Scenario 3) – from all suppliers X, Y and Z. Thus, the obtained solutions are not reliable for choosing suppliers, i.e. the values of variables $Y_{j,t}$ are not stable. It is not clear, which of them should be given preference to in case of significant fluctuations of demand in the considered planning period. At the same time, the order quantity for three products in the second period $X_{i,j,2}$ is sufficiently stable. So, by increasing demand, the order quantity of product A remains unchanged, and the order quantity of product C varies from 19 to 20, i.e. by one.

Conclusion

The considered example shows the mathematical model of the stochastic multi-item problem and inventory lot-sizing optimization under changing demand is rather complicated: its optimization does not always lead to a stable solution. But a sustainable solution can be obtained with a rather small shift in the demand, therefore this model can be used to determine the optimal batch size for each vendor, which helps minimize the overall cost of procurement. In our opinion, the research in this area should be continued, in particular, the availability to use simulation modeling to solve the problem should be considered.

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