Abstract: We consider the possibility to replace an electron beam (EB) by a "hot" equivalent capacitance that will make it possible to use "hot" equivalent circuits and long lines for analysis the EB interaction in TWTs. A sheet EB in the field of the in-phase mode of the slow wave excited by coupled impedance electrodes is considered in this paper.

Keywords: sheet electron beam; equivalent line; dispersion equation; coupling coefficient.

Introduction

Equivalent circuits and equivalent lines are frequently used for analysis of different transmission lines including waveguides [1, 2] and slow-wave structures (SWS) [3, 4]. Using so called "truncated" equivalent line with the specific inductance decreased at the value related to the potential delay, made it possible to use the circuit theory for electro-dynamic models. In all cases, equivalent parameters were calculated for the "cold" structures, structures in the absence of an electron beam (EB). In the same time, it was shown that in the case of a SWS, the decreased inductance is defined by components of the H-wave, while the equivalent capacitance is defined by components of the E-wave, the wave which interacts with an EB in TWTs. This makes it possible, as it is shown below, to include parameters of the EB in the equivalent circuit, which becomes a "hot" circuit or long line.

Initial relations

Consider a "frozen" in the direction of the propagation homogeneous sheet EB with width $H$ significantly larger its thickness $2c$ (Fig. 1). We'll designate the average velocity of the EB as $u_0$ and its current density as $i_0$. From the both sides of the EB, at equal distances $a$ ($x = d$) are placed identical impedance electrodes, supporting a slow wave propagating in the direction of coordinate $z$. We'll assume that the wave front is perpendicular to coordinate $z$ and will neglect by the wave's components dependence on coordinate $y$. Due to symmetry of the model, it's possible to consider only its upper half.

To simplify analysis, we'll consider only an E-wave with components $E_z$, $E_x$, $H_y$ excited at the in-phase mode, when RF potentials of both electrodes are identical. Also, we'll consider only zero spatial harmonic. Remembering that all components of the slow wave are proportional to the so called wave coefficient, $\exp(i\omega_0 z - j\beta z)$, where $\omega$ is the angular frequency, $t$ is time, and $\beta$ is the phase constant, we'll omit this coefficient in writing.

Truncated equivalent line

![Figure 2](image-url)  
**Figure 2.** Equivalent lines for impedance electrodes; a) with capacitance related to two adjacent areas, b) wit capacitance $C_2$, included in inductance $L_1$.

In general, when an impedance electrode is adjacent to two areas, it can be replaced by a three-conductor equivalent line with specific capacitances $C_1$ and $C_2$ and specific inductance $L$ (Fig. 2a) [5]. If inductance $L$ is defined by the magnetic flow induced by transverse current, it satisfies the long line equation, in which phase constant $\beta$ is replaced by transverse constant $\gamma$

$$\gamma^2 = \alpha^2 L(C_1 + C_2),$$

where

$$\gamma^2 = \beta^2 - k, \quad k = \omega \sqrt{\varepsilon_0 \mu_0}$$

and $k$ is the wave number in free space.

Transferring capacitance $C_2$ from the transverse to the longitudinal branch (Fig. 2b), one can write instead of (1)

$$\gamma^2 = \alpha^2 L C_1, \quad L_1 = L(C_1 + C_2)/C_1.$$  

Dispersion equation (3) includes capacitance $C_1$, which is the capacitance of the whole area below the impedance electrode including the EB. This capacitance can be calculated through $E$-type admittance $Y_e(d)$ at the surface of the impedance electrode (at $x = d$). As it was shown in [5],

$$C(x) = H \gamma^2 Y_e(x)/j\omega, \quad Y_e(x) = - H_e(x)/E_z(x).$$  

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It follows from (4)

\[ C_i = -H \gamma^2 Y'(d)/j\omega. \] (5)

**Electro-dynamic admittances**

We'll remind that in the area filled by a homogeneous EB the transverse constant, which we'll designate as \( T \), differs from constant \( \gamma \) and is defined by the next expression [6]:

\[ T^2 = \gamma^2 \left[ 1 - \frac{\beta_e^2}{(\beta_e - \beta_p)^2} \right], \] (6)

where \( \beta_e = \omega / u_e \), \( \beta_p \) is the plasma number. In the absence of the EB, \( \beta_p = 0 \) and \( T = \gamma = \gamma_0 \).

It follows from solutions of the wave equation that at the EB boundary (at \( x = c \))

\[ Y'(c) = -\frac{j\omega e c T}{\gamma} \tanh Tc \] (7)

and, the equivalent capacitance of the EB, \( C_e \), can be defined at the EB surface, i.e. at \( x = c \)

\[ C_e = \varepsilon_0 H T \tanh Tc. \] (8)

It was shown in [6] that admittances at boundaries of the same area are unambiguously related one to another that allows to express \( Y'(d) \) through \( Y'(c) \) and to find expression for capacitance \( C_i \), which includes parameters of the EB.

Introducing the new variable, \( \phi \), satisfying equality

\[ \frac{T}{\gamma} \tanh Tc = \tanh(\phi + \gamma d), \] (9)

where in the absence of the EB \( \phi = \gamma d \) one can obtain

\[ C_i = \varepsilon_0 H T \tanh(\phi + \gamma d). \] (10)

**Expanding in series the dispersion equation**

Substituting \( C_i \) in (3) gives the "hot" dispersion equation

\[ \gamma = \omega^2 L_i \varepsilon_0 H \tanh(\phi + \gamma d). \] (11)

If inductance \( L_i \) does not depend on parameters of the EB, one can write in the absence of electrons the "cold" dispersion equation

\[ \gamma_0 = \omega^2 L_i \varepsilon_0 H \tanh \gamma_0 d. \] (12)

Assuming

\[ T = \gamma_0(1 + y), \gamma = \gamma_0(1 + x), \]
\[ \phi = \gamma_0(1 + z), x, y, z \ll 1, \] (13)

and expanding relation (7) into series in terms of \( \gamma_0 \) with the first order accuracy, we'll obtain

\[ y - x = \frac{\gamma_0 c(z - y)}{\sinh \gamma_0 c \cdot \cosh \gamma_0 c}. \] (15)

After expanding with the same accuracy equation (11) and exclusion from obtained relations \( z \cdot y \), we have the next relation between \( x \) and \( y \), characterizing the EB influence on the transverse constants \( T \) and \( \gamma \)

\[ y - x = x \sinh \gamma_0 d \cdot \cosh \gamma_0 d - \gamma_0 dx. \] (16)

The obtained relation makes it possible to find the coupling coefficient, \( K_c \), as well as the coupling impedance \( R_c \) [6, 7]

\[ K_c = \frac{x \gamma_0^2 / \beta_e^2}{y - x}, \quad R_c = \\sqrt{\frac{\mu_0}{\varepsilon_0 k \beta_e S_e}}, \] (17)

where \( S_e \) is the EB cross section.

**Conclusion**

The possibility to replace an electron beam by the equivalent capacitance was demonstrated on the example of the sheet EB. The obtained results will be useful for evaluation parameters of the TWTs with the sheet electron beams.

**References**
