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Estimates of the parameters of nonlinear internal waves in a stratified lake

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Abstract

The characteristics of nonlinear internal waves in a shallow stratified lake are considered on the example of the Sankhar Lake (Vladimir region, Russia). It is shown that natural variations of temperature in the lake affect the kinematic characteristics of internal waves, especially the coefficient of the quadratic nonlinearity. The theoretical model based on the extended Korteweg–de Vries equation – the Gardner equation is used to estimate the characteristics of internal waves. It is shown that the first mode soliton is a wave of negative polarity. Its amplitude is less than 3 m (depth of the lake up to 15 m). Solitons of the second mode may be of any shape and polarity (compression or depression wave).

Keywords: nonlinear internal waves, stratified lake, first mode, second mode, soliton

1. Introduction

The internal waves in the seas and lakes are an integral part of the water dynamics. They impact on the ecological processes in the thermocline, which is rich in biomass, due to its vertical movement and mixing layers (Miropolsky, 1981; Morozov, 1985; Konyaev, Sabinin, 1992; Bulatov, Vladimirov, 2010; Hutter, 2012; Vlasenko et al, 2005). One can highlight the studies investigating these processes in the Russian lakes the Baikal (Ravenc, 2000) and the Shira (Semin et al, 2012). Oceanic internal waves can reach considerable heights of several tens of meters, so it is necessary to use nonlinear theory to describe their dynamics (Miropolsky, 1981; Vlasenko et al, 2005; Pelinovsky et al, 2002; Grimshaw et al, 2004, 2007; Talipova, Pelinovsky, 2013; Nakoulima et al, 2004). As for the internal waves in the lakes, there are few observations of the intense internal waves (Hutter, 2012; Hunkins, Fliegel, 1973; Farmer, 1978; Antenucci, Imberger, 2001; Vlasenko, Hutter, 2002; Boegman et al, 2005; Terletskaya et al, 2010). The development of the nonlinear theory of such waves in relation to the processes in the lakes and inland waters is only beginning (Hutter, 2012).

Institute of Applied Physics of the Russian Academy of Sciences has held a series of hydro-acoustic experiments on the Sankhar Lake in 1997 - 1998 years (Vyaznikovsky District, Vladimir region). Geographical coordinates of the lake are N 56° 23' 38" E 42° 24'. The map is shown in Figure 1. This is a deep lake of glacial origin with clear water and latest traces of karst phenomena. It has a blade shape with a lot of bays and stretches from the north to the south in the form of an irregular drop with the wider end in the north. An average depth of the lake is 10-15 meters, maximum length of the lake is 1.5 km, and its area is 3 km^2. Today, the lake is a nature reserve established for the preservation and restoration of lake ecosystems, rare species of plants and animals.

The results of experiments are described in the study (Bredikhin et al, 1999). The experimental data analysis showed the presence of large-scale processes associated with the displacements of the thermocline in the internal waves. They are manifested in the low-frequency variations of the acoustic signals. This paper is devoted to the assessment of the possible parameters of internal waves in the Sankhar Lake.

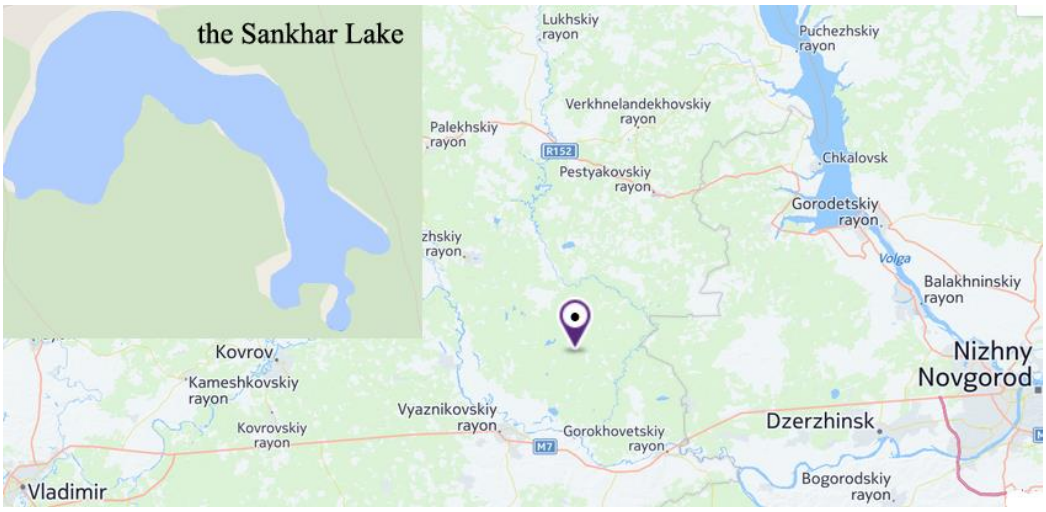

Figure 1. The geographical location of the Sankhar Lake in the Vladimir region (indicated by flag)

2. Hydrology of the Sankhar Lake

The temperature distribution with depth is typical for lakes in summer: upper layer is hot, and the temperature gradual declines to the bottom. The measured vertical profiles of temperature in summer are given in (Bredikhin et al, 1999). They are well approximated by a self-similar solution (Barenblatt, 1996)

$$T(z) = \begin{cases} T_s, & z < h_0, \\ T_s - (T_s - T_b)\left[1 - \left(1 - \dfrac{z - h_0}{h_1 - h_0}\right)^3\right], & h_0 \le z \le h_1, \\ T_b, & z > h_1, \end{cases} \tag{1}$$

here T_s denotes a temperature of the upper layer of mixed fluid of thickness h_0, T_b denotes water temperature at the bottom, and $(h_1 - h_0)$ is thickness of the thermocline. In Ref. (Bredikhin et al, 1999) the following parameters are used: $T_s = 22^0$C, $T_b = 4^0$C, $h_0 = 3$ m and $h_1 = 12$ m. The total depth of the lake at the measuring point is 15 m. The temperature profiles for different days of July 1998 are shown in Figure 2a. Since the lake is freshwater basin, the water density variations are determined only by temperature. To simplify the calculations, one can use a simple formula proposed for the analysis of density stratification in the lagoon Burr (Etang de Berre) near Marseille (Alekseenko, 2013 a,b)

$$\rho(T) = \rho_0\left[1 - \alpha(T - T_0)\right], \tag{2}$$

where $\rho_0 = 1015.27$ kg/m^3, $\alpha = 0.000068$ 1/^{0}C is coefficient of thermal expansion and $T_0 = 10^0$C.

Dependences of the water density on the depth for different days are shown in Figure 2b. Internal waves are determined not by the water density itself but by the buoyancy frequency (Brunt-Väisälä frequency):

$$N(z)=\sqrt{\frac{g}{\rho(z)}\frac{d\rho}{dz}}\cong\sqrt{-g\alpha\frac{dT}{dz}}\ . \tag{3}$$

Buoyancy frequency differs from zero only in the middle layer and is

$$N(z)=\sqrt{\frac{3\alpha g(T_s-T_b)}{h_1-h_0}\left(1-\frac{z-h_0}{h_1-h_0}\right)}, \tag{4}$$

changing linearly with the depth within the thermocline. Plots of buoyancy frequency are shown in Figure 2c. The maximum value of the buoyancy frequency is

$$N_{\max}=\sqrt{\frac{3\alpha g(T_s-T_b)}{h_1-h_0}} \tag{5}$$

and is 0.012 Hz. It determines the upper limit of the internal waves' frequency.

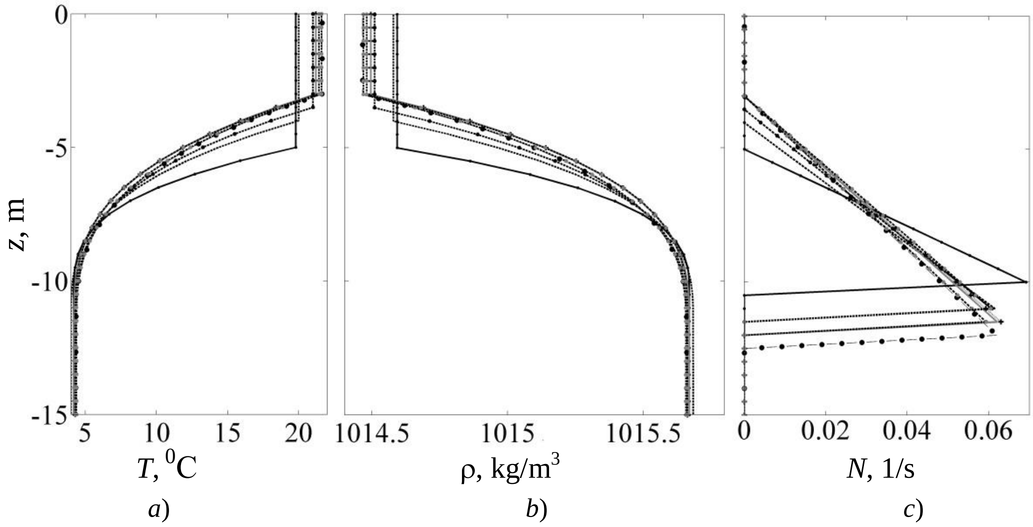

Figure 2. Variation with depth of: *a* – the temperature, *b* – the density and *c* – the buoyancy frequency (– July 2, – July 8, – July 10, – July 11, – July 14, – July 15, – July 16, – July 17)

3. The dispersion characteristics of the internal waves in the lake

First of all, let us consider the dispersion characteristics of linear internal waves in the Sankhar Lake. It is generally known, that the dispersion characteristics are found by solving the boundary value problem of the Sturm-Liouville to determine eigenvalues. As usual in the analysis of the internal waves in lakes, Boussinesq approximation, that is specific for natural reservoirs because of small variation of the density with depth (Miropolsky, 1981; Bulatov, Vladimirov, 2007, 2012), is used:

$$\frac{d^2\Phi}{dz^2}+k^2\frac{N^2(z)-\omega^2}{\omega^2}\Phi=0,\quad \Phi(0)=\Phi(-H)=0\,. \tag{6}$$

Here $\Phi(z, k)$ denotes the vertical structure of the isopycnals displacements field, ω – internal wave frequency, k – its wave number, and H = 15 m is a maximum depth of the lake. Because the lake is relatively small, authors neglect the effects of the Earth's rotation and its curvature.

It is easy to show that the problem (6) has a discrete spectrum with different eigenvalues (Miropolsky, 1981). In the linear theory each internal waves mode propagates independently.

The results of calculations of the dispersion characteristics of the first three modes for two days of observation are shown in Figure 3. Apparently, the dispersion characteristics for different days are similar, although it is clear that the velocity of wave propagation changes (this will be discussed in the next section). It is important to emphasize the great difference in the characteristics of waves of different modes. Thus, velocities of the second and the third modes range between 8 and 4 cm/s. These values are comparable to the velocity of possible background flow and even to the velocity of the first mode waves. Such waves have to be unstable and unlikely propagate as free waves. The vertical structure of the displacement field in the internal waves in the longwave approximation is shown in Figure 4.

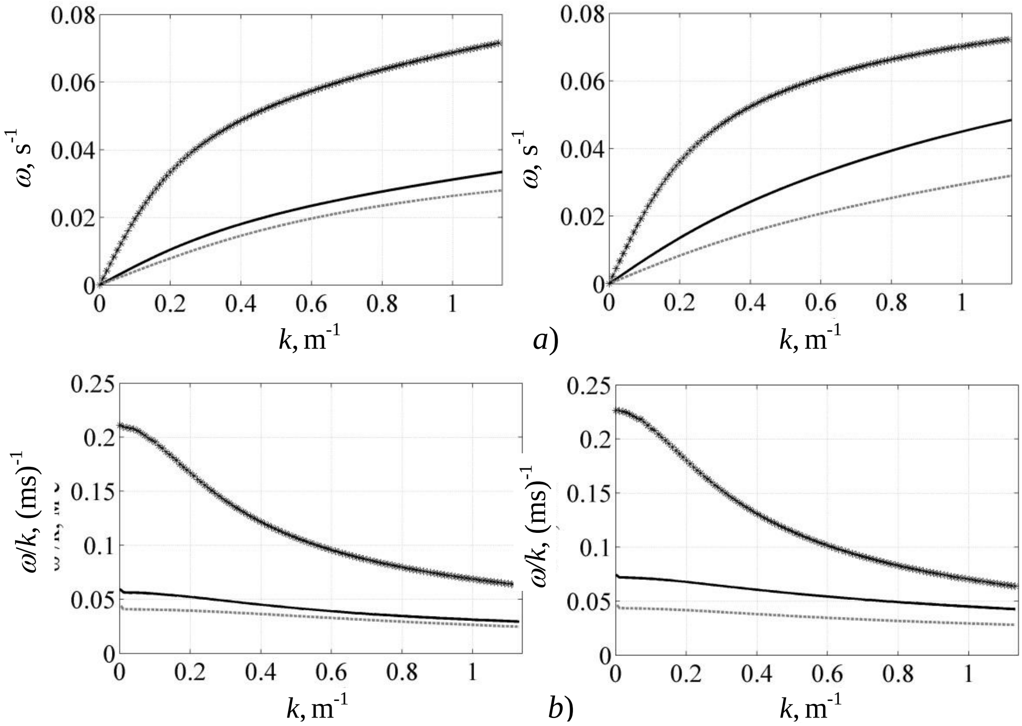

Figure 3. The dispersion relations (a) and propagation velocities (b) for the first three modes (left column – July 8, right column – July 11): —∗— the first mode, ▬▬ the second mode, ▪▪▪▪▪▪▪▪ the third mode

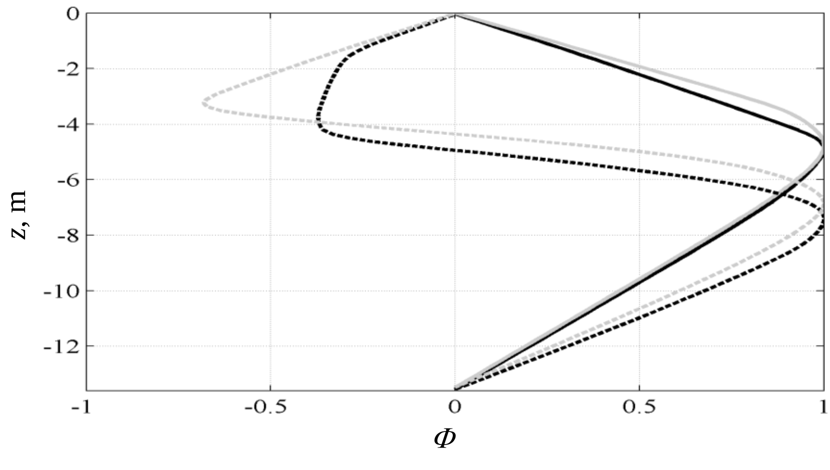

Figure 4. The vertical structure of the displacement field of the first mode (solid) and second mode (dotted line) long internal waves for July 8 (black curves) and 11 July (gray curves)

It should be noted that the solution of the problem does not determine the internal waves' amplitudes. For this purpose it is necessary to investigate their generation of atmospheric processes or baroclinic instability. In this case, authors can use these field observations, summarized in Ref. (Bredikhin et al, 1999). In this study three frequency bands (below 0.10 Hz, 0.01-0.03 Hz and above 0.03 Hz) are identified, in which amplitude spectra ω^{-1}, ω^{-2} и ω^{-1} are observed. Note that such laws are predicted for the nonlinear breaking waves (Ermakov, Pelinovsky, 1975; Ostrovsky, Helfrich, 2011; Kartashova et al, 2013; Pelinovsky D. et al, 2013) and for the so-called soliton turbulence (Costa et al, 2014). The above mentioned facts prove the non-linear character of the internal waves in the lake. The appropriate model is developed in the next section.

4. The nonlinear model of the internal waves

The nonlinear model of internal waves is based on the Korteweg–de Vries equation and its generalizations (Miropolsky, 1981; Vlasenko et al, 2005; Pelinovsky et al, 2002; Grimshaw et al, 2004, 2007; Talipova, Pelinovsky, 2013). In general, the Korteweg–de Vries equation is enough for small-amplitude waves, but it is convenient to include cubic effects into consideration, because they allow us to estimate the internal waves limiting amplitudes in a stratified basin. Proper equation is called the Gardner equation. It has the following form (according (Grimshaw et al, 2004, 2007; Talipova, Pelinovsky, 2013))

$$\frac{\partial\eta}{\partial t}+(\alpha\eta+\alpha_1\eta^2)\frac{\partial\eta}{\partial x}+\beta\frac{\partial^3\eta}{\partial x^3}=0\,, \tag{7}$$

where $\eta(x, t)$ – the wave function, part of the isopycnals vertical displacement (surfaces of equal density) $\zeta(x, y, t)$:

$$\zeta(x,z,t)=\eta(x,t)\Phi(z)\,. \tag{8}$$

Here x – the horizontal and z – vertical coordinates, t – time, $\Phi(z)$ – mode function in the long-wave approximation. It is found from the boundary value problem (6), which in the long-wave approximation is reduced to

$$\frac{d^2\Phi}{dz^2}+\frac{N^2(z)}{c^2}\Phi=0,\quad \Phi(0)=\Phi(-H)=0\,, \tag{9}$$

where c is eigenvalue that determines the propagation velocity of long linear internal waves. Authors will consider each mode of internal waves independently, so in all equations the mode index is omitted. In fact, equation (7) is written in a reference frame moving with velocity c, so that the coordinate x is x – ct. Note that authors use the normalization condition for the function $\Phi(z)$: Φ_{max} = 1, so that the function $\eta(x, t)$ describes the vertical isopycnal displacement at the mode maximum.

Coefficients of equation (7) are called the coefficients of the quadratic and cubic nonlinearity and dispersion, respectively; they are defined by the vertical distribution of the water density

$$\alpha=\left(\frac{3c}{2}\right)\frac{\int_{-H}^{0}(d\Phi/dz)^3dz}{\int_{-H}^{0}((d\Phi/dz)^2dz}\,, \tag{10}$$

$$\alpha_1=\frac{3c}{2}\frac{\int\left[3(dT/dz)-2(d\Phi/dz)^2\right](d\Phi/dz)^2dz}{\int(d\Phi/dz)^2dz}- \\ -\frac{3c}{2}\frac{\int\left\{\alpha^2(d\Phi/dz)^2-\alpha\left[5(d\Phi/dz)^2-4dT/dz\right]d\Phi/dz\right\}dz}{\int(d\Phi/dz)^2dz}, \tag{11}$$

$$\beta = \left(\frac{c}{2}\right)\frac{\int_{-H}^{0} {}^{2}\Phi^2 dz}{\int_{-H}^{0} (d\Phi/dz)^2 dz}. \tag{12}$$

Here, the function T is a nonlinear correction to the mode function Φ, defined by the equation

$$\frac{d^2T}{dz^2} + \frac{N^2(z)}{c^2}T = -\frac{\alpha}{c}\frac{d^2\Phi}{dz^2} + \frac{3}{2}\frac{d}{dz}\left[\left(\frac{d\Phi}{dz}\right)^2\right]. \tag{13}$$

This model is used many times for the calculation of nonlinear internal waves in the ocean (Grimshaw et al, 2004, 2007; Holloway et al, 1997, 1999). It is implemented numerically as the *IGWResearch* package (Tyugin et al, 2011, 2012).

5. The kinematic characteristics of internal waves

It should be noted that the Gardner equation coefficients, determining the kinematic characteristics of internal waves, are set by the rather complicated expressions (10) – (13), which may be sensitive to the hydrology of the basin. Therefore, authors calculated these coefficients for the first mode internal waves for each day of the experiments. These results are summarized in Table 1. Variations of the wave propagation velocity is approximately 10%. It may affect the time propagation of internal waves over long distances. The coefficient of the quadratic nonlinearity varies considerably more than c (about 40%). At the same time, the cubic nonlinearity coefficient varies very little (about 6%). Dispersion parameter varies at about 13%. This sensitivity of the internal waves' kinematic coefficients to the ever-present variability of hydrology should be taken into account in the analysis of experimental data, for which only averaged quantities are typically used.

Table 1. Variation of the coefficients of the Gardner equation for the first mode internal solitary waves

	c, m/s	α, s^{-1}	α_1, $m^{-1}s^{-1}$	β, m^3/s
July 2	0.207	-0.037	-0.014	1.56
July 8	0.21	-0.03	-0.014	1.64
July 10	0.225	-0.041	-0.014	1.72
July 11	0.225	-0.042	-0.014	1.73
July 14	0.227	-0.042	-0.014	1.75
July 15	0.22	-0.036	-0.014	1.71
July 16	0.23	-0.04	-0.015	1.73
July 17	0.23	-0.04	-0.014	1.77

Below authors shall estimate the parameters of a solitary wave. In the framework of the Gardner equation (7) soliton is described by the expression (Grimshaw et al, 2007; Talipova, Pelinovsky, 2013)

$$\eta(x,t) = \frac{A}{1 + B\mathrm{ch}(\gamma(x - Vt))}, \tag{14}$$

$$A = \frac{6\beta\gamma^2}{\alpha},\ B^2 = 1 + \frac{6\alpha_1\beta\gamma^2}{\alpha^2},\ V = \beta\gamma^2, \tag{15}$$

where γ – arbitrary parameter characterizing the inverse width of the soliton. The height of the soliton is

$$a = \frac{A}{1+B}. \tag{16}$$

Since the quadratic nonlinearity coefficient is always negative, the soliton has a negative polarity and has a form of a cavity (convex towards the bottom). Cubic nonlinearity coefficient is always negative, too. This leads to the height restrictions of the soliton by the limit value

$$a_{cr} = \frac{\alpha}{\alpha_1}. \tag{17}$$

Using the values of the coefficients of the quadratic and cubic nonlinearity, authors find that the maximum height of the soliton varies in the range between 2.5 and 3 m. In fact, the height of the wave is small compared with the maximum depth (15 m), so that the importance of a cubic nonlinearity even for the waves of relatively low amplitudes demonstrates in this example.

The shape of the internal solitary wave is shown in Figure 5. Similar estimates are made for the second mode solitons. The calculated coefficients of the Gardner equation for the second mode for the two days are summarized in Table 2.

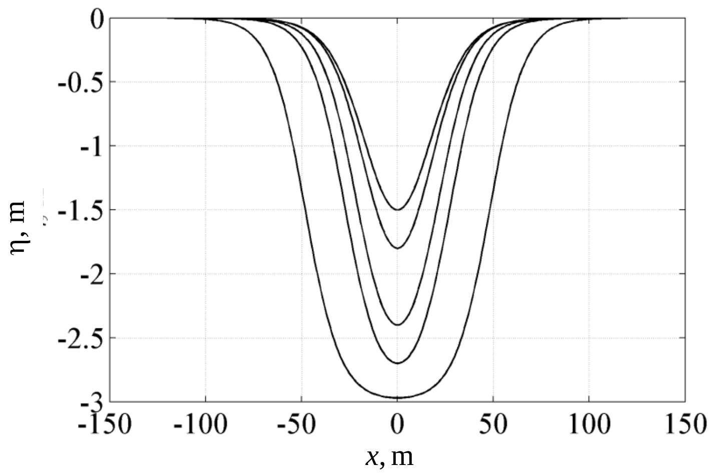

Figure 5. The shape of the internal solitary waves of different amplitudes

Table 2. Variation of the coefficients of the Gardner equation for the second mode internal solitary waves

	c, m/s	α, s^{-1}	α_1, $m^{-1}s^{-1}$	β, m^3/s
July 8	0.056	-0.036	0.178	0.122
July 11	0.072	-0.058	0.306	0.126

Authors have already noted that values of the second mode propagation velocities are small (a few cm/s), so that it is difficult for such waves to be free. If we even imagine that they exist in the lake as free waves, their nonlinear properties, apparently, will differ from those of the first mode. Since the quadratic nonlinearity coefficient is negative, the small-amplitude solitons formally have a negative polarity. However, the vertical structure of the second mode is alternated (Figure 4), so the second mode soliton represents itself as the "depression wave", see Figure 6a. As distinct from the first mode solitons such solitons may be of any amplitude (in the framework of this model), as the cubic nonlinearity coefficient is positive. Furthermore, with the increasing of amplitude here can exist the second mode solitons of positive polarity. Spatially such a soliton represents itself as a "compression wave", see Figure 6b.

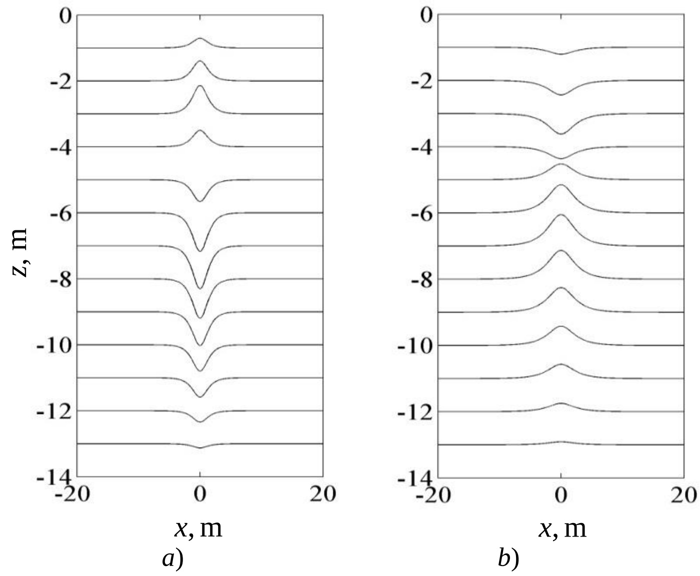

Figure 6. The vertical structure of the second mode internal solitary wave for July 11: *a* – the depression wave; *b* – the compression wave. The maximum amplitude is 0.95 m

These solitons can exist under an amplitude greater than the amplitude of the so-called algebraic soliton (Grimshaw et al, 2007; Talipova, Pelinovsky, 2013)

$$a_{al} = -\frac{2\alpha}{\alpha_1}. \tag{18}$$

This amplitude amounts only 30-40 cm, so that this limitation is not essential. Thus, the existence of the second mode solitary waves is possible in the form of a "wave of compression" and "wave of depression". Finally, authors would like to note that more breathers of the second mode (nonlinear wave packets (Lamb, 2007)) are possible here, so authors do not stop at these findings.

6. Conclusion

On the example of the Sankhar Lake (Vladimir region, Russian Federation) the properties of internal waves in lakes are considered. The effect of the stratification variability on kinematic characteristics of internal waves is studied, the coefficient of quadratic nonlinearity is the most sensitive parameter. The nonlinear theory is used to estimate the internal wave parameters. It is based on the Gardner equation – the extended Korteweg–de Vries equation. It is shown that there may exist the first mode internal solitons with amplitudes up to 3 m in this lake. In the second mode the existence of both two classes of solitons (waves of compression and depression) and breathers is possible.

References

Alekseenko, E. (2013) Numerical modeling of hydrodynamics in the Berre lagoon: application to the recolonization of seagrasses. *Marseille: Ecole Centrale Marseille*, 164

Alekseenko, E., Roux, B., Sukhinov, A., Kotarba, R. & Fougere, D. (2013) Nonlinear hydrodynamics in a Mediterranean lagoon. *Nonlinear Processes Geophysics*, 20, 89-98

Antenucci, J. P. & Imberger, J. (2001) Energetics of long internal gravity waves in large lakes. *Limnology and Oceanography*, 46, 7, 1760-1773

Barenblatt, G. I. (1996) Scaling, Self-Similarity, and Intermediate Asymptotics. *Cambridge: Cambridge Univ. Press*

Boegman, L., Ivey, G. N. & Imberger, J. (2005) The degeneration of internal waves in lakes with sloping topography. *Limnology and Oceanography*, 50, 5, 1620-1637

Bredikhin, V. V., Lebedev, A. V. & Salin, B. M. (1999) The study of fluctuations when the sound is propagating in a closed basin. *Preprint № 490. IAP RAS*, 29 [In Russian]

Bulatov, V. V. & Vladimirov, Yu. V. (2007) Internal gravity waves: theory end applications. *M.: Nauka*, 304

Bulatov, V. V. & Vladimirov, Yu. V. (2010) Dynamics of non-harmonic wave packets in stratified media. *M .: Nauka*, 470 [In Russian]

Bulatov, V. V. & Vladimirov, Yu. V. (2012) Wave dynamics of stratified mediums. *M.: Nauka*, 584

Costa, A., Osborne, A. R., Resio, D. T., Alessio, S., Chrivì, E., Saggese, E., Bellomo, K. & Long C. E. (2014) Soliton turbulence in shallow water ocean surface waves. *Physical Review Letters*, 113, 108501

Ermakov, S. A. & Pelinovsky, E. N. (1975) On the theory of multimode distribution of characteristics of finite amplitude long internal waves. *Proceedings of the Academy of Sciences of the USSR. Physics of the atmosphere and ocean*. 11, 10, 1055-1060 [In Russian]

Farmer, D. M. (1978) Observations of long nonlinear internal waves in a lake. *J. Phys. Oceanography*, 8, 3-73

Grimshaw, R., Pelinovsky, E., Talipova, T. & Kurkin, A. (2004) Simulation of the transformation of internal solitary waves on oceanic shelves. *J. Phys. Oceanography*, 34, 12, 2774-2791

Grimshaw, R., Pelinovsky, E. & Talipova, T. (2007) Modeling internal solitary waves in the coastal ocean. *Survey in Geophysics*. 28, 4, 273-298

Holloway, P., Pelinovsky, E., Talipova, T. & Barnes, B. (1997) A Nonlinear Model of Internal Tide Transformation on the Australian North West Shelf. *J. Physical Oceanography*, 27, 6, 871-896

Holloway, P, Pelinovsky, E. & Talipova, T. (1999) A Generalized Korteweg - de Vries Model of Internal Tide Transformation in the Coastal Zone. *J. Geophys. Res.* 104(C8), 18333-18350

Hunkins, K. & Fliegel, M. (1973) Internal undular surges in Seneca Lake: A natural occurrence of solitons. *J. Geophys. Research*, 78, 539-548

Hutter, K. (2012) Nonlinear internal waves in lakes. *Berlin: Springer*, 294

Kartashova, E., Pelinovsky, E. & Talipova, T. (2013) Fourier spectrum and shape evolution of an internal Riemann wave of moderate amplitude. *Nonlinear Processes in Geophysics*, 20, 571-580

Konyaev, K. V. & Sabinin, K .D. (1992) Waves in the ocean. *L.: Gidrometeoizdat*, 1992. 272 [In Russian]

Lamb, K. G., Polukhina, O., Talipova, T., Pelinovsky, E., Xiao, W. & Kurkin, A. (2007) Breather generation in fully nonlinear models of a stratified fluid. *Physical Review E*, 75, 4, 046306

Miropolsky, Yu. Z. (2001) Dynamics of internal gravity waves in the ocean. *Springer-Verlag, Berlin*

Morozov, E. G. (1985) Ocean internal waves. *M .: Nauka*, 151 [In Russian]

Nakoulima, O., Zahibo, N., Pelinovsky, E., Talipova, T., Slunyaev, A. & Kurkin, A. (2004) Analytical and numerical studies of the variable-coefficient Gardner equation. *Applied Mathematics and Computation*, 152, 2, 449-471

Ostrovsky, L. A. & Helfrich, K. R. (2011) Strongly nonlinear, simple internal waves in continuously-stratified, shallow fluids. *Nonlinear Processes in Geophysics*, 18, 91-102

Pelinovsky, E. N., Poloukhin, N. V. & Talipova, T. G. (2002) Modeling of the characteristics of internal waves in the Arctic Ocean. Surface and internal waves in the Arctic seas (Editors: I.V. Lavrenov and E.G. Morozov). *Saint-Petersburg: Gidrometeoizdat*, 235-279 [In Russian]

Pelinovsky, D., Pelinovsky ,E., Kartashova, E., Talipova, T. & Giniyatullin, A. (2013) Universal power law for the energy spectrum of breaking Riemann waves. *JETP Letters*, 98, 4. 265-269

Ravenc, T. M., Kocsis, O., Wuest, A. & Granin, N. (2000) Small-scale turbulence and vertical mixing in Lake Baikal *Limnology and Oceanography*, 45, 1, 159-173

Semin, S. V., Kurkina, O. E., Kurkin, A. A. & Giniyatullin, A. R. (2012) Numerical simulation of the dynamics of a stratified lake. *Transactions of the Nizhny Novgorod R. E. Alekseev State Technical University*, 2(95), 48-65 [In Russian]

Talipova, T. G. & Pelinovsky, E. N. (2013) Simulation of long propagating internal waves in inhomogeneous ocean: Theory and verification. *Fundamental and Applied hydrophysics*, 6, 2, 46-54 [In Russian]

Terletskaya, E., Malerich, V. & Brovchenko, I. (2010) Strongly nonlinear internal seiche in elongated stratified lakes and the phenomenon of "lake monsters". *Applied Fluid Mechanics*, 12(84), 4, 1-5 [In Russian]

Vlasenko, V. I. & Hutter, K. (2002) Transformation and disintegration of strongly nonlinear internal by topography in stratified lakes. *Annales Geophys*, 20, 1779-1793

Tyugin, D. Yu., Kurkina, O. E. & Kurkin, A. A. (2011) Software package for the numerical simulation of the internal gravity waves in the ocean. *Fundamental and Applied hydrophysics*, 4, 2, 32-44 [In Russian]

Tyugin, D., Kurkin, A., Pelinovsky, E. & Kurkina, O. (2012) Increase of productivity of the program complex for modeling of internal gravity waves IGWresearch with the help of Intel® Parallel Studio XE. *Fundamental and Applied Hydrophysics*, 5, 3, 89-95 [In Russian]

Vlasenko, V., Stashchuk, N. & Hutter, K. (2005) Baroclinic tides: Theoretical modeling and observational evidence. *Cambridge: Cambridge University Press*, 372